

## Assignment 1

### Question 1. Given

$$x = \begin{bmatrix} 2 \\ 2 \\ 6 \end{bmatrix}, X = \begin{bmatrix} -4 & 7 & 5 \\ 1 & -9 & 2 \\ 8 & 3 & 6 \end{bmatrix}, Y = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 0 & 13 \\ -4 & 5 & 5 \end{bmatrix}, Z = \begin{bmatrix} 4 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Calculate the following both by numpy and by hand:(10 pt)

- |                |                         |                |
|----------------|-------------------------|----------------|
| 1) $x^\top XY$ | 3) $2X + 2Y + X^\top Y$ | 5) $Z^\top Zx$ |
| 2) $YX$        | 4) $YZ^\top$            |                |

### Question 2. Write the code that performs the following operations using numpy. Make sure you print the results. (20 pt)

- Create two matrices and two vectors

$$X = \begin{bmatrix} 2 & 3 \\ 9 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, y = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

- Write the code that extracts the 1st column of  $X$ .
- Write the code that extracts the 2nd row of  $Y$ .
- Write the code that extracts the bottom right value of  $X$ .
- Write the code that adds  $X + Y$ .
- Write the code that performs  $X + YX$ .
- Write the code that performs the Hadamard product  $x \odot y$ .
- Write the code that performs the outer product  $x \otimes y$ .
- Write the code that performs the dot product  $XY$ .
- Write the code that performs the inner product  $\langle X, Y \rangle$ .

### Question 3. (30pts) Given

$$x_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad y, w = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad \mathbf{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- Find the derivative for the following functions.
- Verify your derivative using Python.
- Find  $f' \left( \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$ .

- |                                  |                                 |
|----------------------------------|---------------------------------|
| 1) $f(x) = x^\top Ax - x^\top y$ | 3) $f(x) = x^\top \mathbf{1}$   |
| 2) $f(x) = x^\top A \mathbf{1}$  | 4) $f(w) = \sum_i^2 w^\top x_i$ |

**Question 4.** Write out the derivation to get derivatives for all 17 functions in the class. (20pts)

**Question 5.** Use Python to check that all your derivatives are correct. (20pts) (Note: Skip the SVM objective for now.)