Thursday, September 3, 2020 8:43 PM

Q1. a) $P(X=1) = \frac{1}{4} + \frac{1}{3} = \boxed{\frac{7}{12}}$

b) $P(X=1|Y=1) = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}} = \boxed{\frac{2}{3}}$

c) $\underline{Var(X) = \sum_x P(X=x)(x-\mu_x)^2}$

$\mu_x = \frac{7}{12}\cdot 1 = \frac{7}{12}$ $= \frac{7}{12}\left(1-\frac{7}{12}\right)^2 + \frac{5}{12}\left(0-\frac{7}{12}\right)^2$

$= \boxed{\frac{35}{144}}$

d) $Var(X|Y=1) = \sum_x P(X=x|Y=1)(x-\mu_{X|Y=1})^2$

$\mu_{X|Y=1} = \frac{2}{3}\cdot 1 + \frac{1}{3}\cdot 0 = \frac{2}{3}$ $= \frac{2}{3}\left(1-\frac{2}{3}\right)^2 + \frac{1}{3}\left(0-\frac{2}{3}\right)^2$

$= \boxed{\frac{2}{9}}$

e) $E[X^3 + X^2 + 3Y^7 | Y=1]$

$= \frac{2}{3}(1+1+3) + \frac{1}{3}(0+0+3)$

$= \boxed{\frac{13}{3}}$