Data Structures
Sample Midterm Exam

1. (30 pts) Mark by T(=true) or F(=False) each of the following statements. You don't need to prove it.

   判斷下列敘述，標示T表示為真，標示F表示為假。本題無需證明。

   (1) $(6n + 4)(1 + \log n) = \Theta(n \log n)$

   (2) $n^n = O(2^n)$

   (3) $2^n = O(n!)$

   (4) $\sum_{i=1}^{n} i \log i = \Theta(n^2 \log n)$

   (5) If $f(n) = \Theta(g(n))$, then $2^{f(n)} = \Theta(2^{g(n)})$.

   (6) If $d(n)$ is $O(f(n))$ and $e(n)$ is $O(g(n))$, then the difference $d(n) - e(n)$ is $O(f(n) - g(n))$.

2. (5 pts) Consider two functions $f(n) = 3n^2$ and $g(n) = 33n + 36$. Determine n_0 such that f is larger than g for $n \geq n_0$.

   考量$f(n) = 3n^2$ and $g(n) = 33n + 36$，請決定n_0使得對所有$n \geq n_0$，f值大於g。

3. (15 pts) In the following problems, select a Big-O notation among $O(n)$, $O(\log n)$, $O(n \log n)$, $O(\log \log n)$, $O(n^2)$, $O(n^3)$, $O(2^n)$, or $O(n!)$ for the number of times the statement $x = x + 1$ is executed.

   在下列的問題中，請用Big-O符號表式 $x = x + 1$敘述的執行次數。

   (a) (5 pts)

   (b) (5 pts)

   (c) (5 pts)

4. (15 pts) Please consider the following pseudo-code and answer the questions, where A is an $I \times J$ matrix.

   考量下面的虛擬碼並回答問題，A是一個$I \times J$ Array。

   (a) (5 pts) Please state what this pseudo-code does.
   考請説明這虛擬碼的目的。

   (b) (5 pts) Please give the number of operations (steps) for each statement. Then, sum up these numbers to derive the total number of operations.
   請估算這虛擬碼各行敘述的基本運算量以及其總基本運算量。

   (c) (5 pts) What is the asymptotic time of this program if $I = J = n$.
   若$I = J = n$，這虛擬碼的時間複雜度為何?

5. (10 pts) Recall the ***inversion number*** of a permutation in the homework. Please prove by induction that the maximum number of inversions for a permutation P with size n is $\frac{n(n-1)}{2}$.

   在作業中我們曾經介紹過一個排列的反置數量(inversion number)，請用數學歸納法證明一個大小為n的排列P， 其最大反置數量是$\frac{n(n-1)}{2}$。

6. (15 pts) Please answer the following problems simply.
請簡單回答下面的問題：

   (a) (5 pts) Please write the following infix expression in postfix form:
   請將下列中序表示式轉成後序表示式

   (b) (5 pts) Please write the following prefix expression in infix form:
   請將下列前序表示式轉成中序表示式

   (c) (5 pts) Draw the sequence of stack configurations for the evaluation of the following postfix expression. Assume that $a = 2$, $b = 3$, $c = 4$, $d = 5$, and $e = 6$.
   將下列後序表示式計算出相對應的值， 並將計算中堆疊的變化一步步劃出。

   Solution:

   (a)

   (b)

   (c) (suggested way to draw the sequence of stack configurations)

Operation	Contents of Stacks

7. (10 pts) Please answer the following questions shortly and concisely.
請簡潔回答下列問題：

   (a) (5 pts) Suppose each row of an $n \times n$ array A consists of 1's and 0's such that, in any row i of A, all the 1's come before any 0's. Also suppose that the number of 1's in row i is at least the number in row $i + 1$, for $i = 0, 1, \ldots, n - 2$. Assuming A is already in memory, describe a method running in $O(n)$ time (not $O(n^2)$) for counting the number of 1's in A.
   $n \times n$ array A 是由0與1組成，在每一行中所有的1皆在0的前面(左邊)， 且第i行中1的數量不小於第$i + 1$列中1的數量， $i = 0, 1, \ldots, n - 2$，請給一個線性時間的方法來計算出array A中1的數量。

   (b) (5 pts) Describe a recursive algorithm that counts the number of nodes in a singly linked list. Please also provide the pseudo-code and the analysis on time.
   描述一個遞迴演算法算出一個單項鍊結串列的節點數量，並請用虛擬碼表示出演算法及分析此演算法的時間。