Chapter 1
Number System

Real-Life Connection: We use different number systems because real life needs more than counting: temperature can be negative ($-5°\text{C}$), pizza division gives fractions ($\frac{1}{2}$), and values like $\sqrt{2}$ and π cannot be written as fractions. Complex numbers ($a + bi$) are important in engineering and science.

1.1 Overview

This chapter introduces the fundamental number systems used throughout mathematics and explains how they are related. Starting with the **natural numbers**, we extend step-by-step to **whole numbers, integers, rational numbers, irrational numbers, real numbers, and complex numbers**. These concepts form a strong foundation for algebra, analysis, and higher mathematics.

1.1.1 Ordered List of Number Types

1. **Natural Numbers ($\mathbb{N}$):** counting numbers

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

2. **Whole Numbers ($\mathbb{W}$):** natural numbers together with zero

$$\mathbb{W} = \{0, 1, 2, 3, \dots\}$$

3. **Integers ($\mathbb{Z}$):** whole numbers along with their negative counterparts

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Key Definition: Rational Numbers ($\mathbb{Q}$)
A rational number is any number that can be expressed as

$$\frac{p}{q}, \quad p, q \in \mathbb{Z},\ q \neq 0.$$

Its decimal expansion is either **terminating** or **repeating**.
Examples: $\frac{1}{2},\ -\frac{3}{4},\ 5,\ 0.25,\ 0.\overline{6}$

5. Irrational Numbers ($\mathbb{R} \setminus \mathbb{Q}$): Irrational numbers are real numbers that cannot be expressed as $\frac{p}{q}$. Their decimals are **non-terminating and non-repeating**.

Examples: $\sqrt{2}$, $\sqrt{3}$, π, e

6. Real Numbers ($\mathbb{R}$): The set of all rational and irrational numbers:

$$\mathbb{R} = \mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$$

7. Complex Numbers ($\mathbb{C}$): Numbers of the form

$$a + bi, \quad a, b \in \mathbb{R}, \quad i = \sqrt{-1}.$$

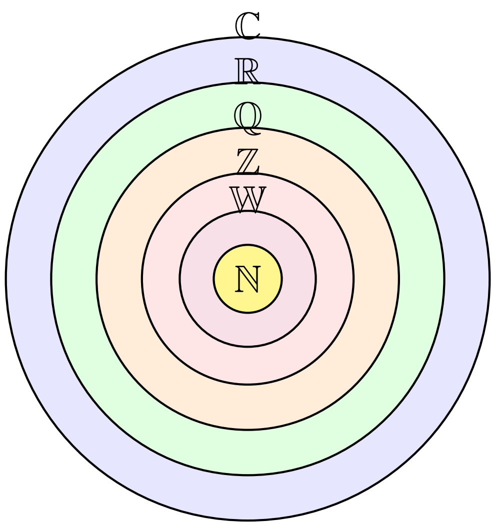
If $b = 0$, the complex number becomes a real number.

Hierarchy (Inclusion Summary)

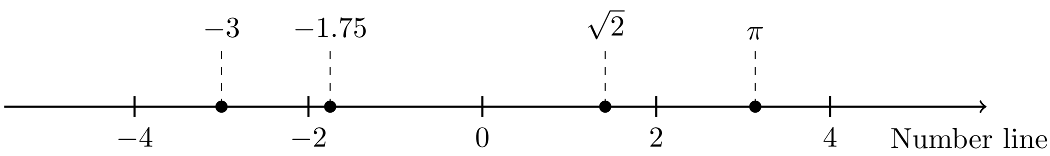
$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

Meaning: Every natural number is a whole number, every whole number is an integer, and so on.

Diagram: Number System Hierarchy



Geometric Representation of Real Numbers (Number Line)



Note: Every point on the number line corresponds to a real number.

Mistake Alert (Very Common Confusions)

- $\sqrt{4} = 2$ is rational, but $\sqrt{2}$ is irrational.
- 0 is a whole number and an integer, but in many textbooks it is not included in $\mathbb{N}$.
- $\frac{p}{q}$ is rational only when $p, q \in \mathbb{Z}$ and $q \neq 0$.
- Every rational number has a decimal expansion that is either terminating or repeating.

1.2 Worked Examples

Example 1: Show that $\sqrt{2}$ is irrational

Solution (Proof by Contradiction): Assume that $\sqrt{2}$ is rational. Then it can be written in lowest terms as

$$\sqrt{2} = \frac{p}{q}, \quad p, q \in \mathbb{Z},\ q \neq 0,\ \gcd(p, q) = 1.$$

Squaring both sides,

$$2 = \frac{p^2}{q^2} \quad \Rightarrow \quad p^2 = 2q^2.$$

So p^2 is even, hence p is even. Let $p = 2k$ for some integer k.
Substitute $p = 2k$:

$$(2k)^2 = 2q^2 \quad \Rightarrow \quad 4k^2 = 2q^2 \quad \Rightarrow \quad q^2 = 2k^2.$$

Thus q^2 is even, so q is even.
Therefore, both p and q are even, which contradicts $\gcd(p, q) = 1$.

Conclusion: The assumption is false, hence

$$\boxed{\sqrt{2} \text{ is irrational.}}$$

Example 2: Show that $0.\overline{81}$ is rational

Solution: Let

$$x = 0.\overline{81} = 0.818181\ldots$$

Multiply by 100:

$$100x = 81.818181\ldots$$

Subtract:

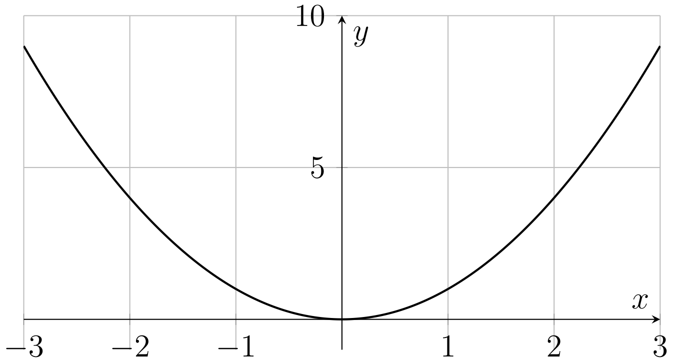
$$100x - x = 81 \quad \Rightarrow \quad 99x = 81 \quad \Rightarrow \quad x = \frac{81}{99} = \frac{9}{11}.$$

Conclusion: Since x is a ratio of integers,

$$\boxed{0.\overline{81} \text{ is rational.}}$$

Graph Example (Visual Learning)

The curve below is an example of a function graph. Such graphs will be studied in detail in later chapters.



This graph represents the quadratic function $y = x^2$.

Practice Questions

1. Write the following in set form: $\mathbb{N}, \mathbb{W}, \mathbb{Z}$.
2. Classify as rational or irrational: (a) 0.25 (b) $0.\overline{3}$ (c) $\sqrt{9}$ (d) $\sqrt{5}$
3. Convert $0.\overline{27}$ into a fraction.
4. Prove that $\sqrt{3}$ is irrational.
5. Find the real and imaginary parts of $z = 5 - 2i$.

Multiple Choice Questions (MCQs)

1. Which of the following is an irrational number?

   (A) $\frac{22}{7}$ (B) $\sqrt{3}$ (C) 0.75 (D) $-\frac{5}{2}$

2. Which set contains the number -7?

   (A) $\mathbb{N}$ (B) $\mathbb{Q}$ (C) $\mathbb{R} \setminus \mathbb{Q}$ (D) None of these

3. The number $0.10100100010000\ldots$ (where the number of zeros increases each time) is:

   (A) rational (B) irrational (C) integer (D) complex but not real

4. Which of the following statements is true?

   (A) Every integer is rational.
   (B) Every rational is integer.

(C) Every rational is real.

(D) Both (A) and (C)

5. Which of the following numbers is transcendental?

(A) $\sqrt{2}$ (B) π (C) $\frac{3}{7}$ (D) $\sqrt{5}$

MCQ Answers: 1(B), 2(B), 3(B), 4(D), 5(B)

Final Quick Summary

$$\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

- Rational numbers: terminating or repeating decimals.
- Irrational numbers: non-terminating and non-repeating decimals.
- Real numbers: all points on the number line.
- Complex numbers: $a + bi$ where $i = \sqrt{-1}$.