Assignment2-solutions

February 5, 2022

[ ]:
```python
import sys
sys.path.append("../")
from dataclasses import dataclass
from typing import Mapping, Dict, Tuple
from rl.distribution import Categorical, FiniteDistribution, Constant
from rl.markov_process import FiniteMarkovProcess, FiniteMarkovRewardProcess,␣
 ↪NonTerminal
import matplotlib.pyplot as plt
import itertools

plt.rcParams['figure.figsize'] = (15, 7)
```

1 Question 1:

The state space is: $S_t = \{1, 2, \cdots, 100\}$ where 100 is the terminal state (we win the game when we reach 100).
The transition probability matrix is:

$$\mathcal{P}_{ij} = \begin{cases} \frac{1}{6}, & \text{if state } i \text{ goes to state } j \\ 0, & \text{otherwise} \end{cases}$$

For example, if I were at state 12, then I have 6 different possibilities for the next state: 13, 14, 15, 16, 17, 18, or 19. If a ladder exists from 16 to 32, then $\mathcal{P}_{12,32} = \frac{1}{6}$ and $\mathcal{P}_{12,16} = 0$ instead. We consider the case of a snake in a similar manner, except that here $i > j$.
Question 2

[ ]:
```python
@dataclass(frozen=True)
class State:
        position : int



class SnakesAndLaddersMP(FiniteMarkovProcess[State]):
        def __init__(self, from_to : Mapping[State, State]):
                '''
        @from_to lists the ladders and the snakes.
        For example, if there was a ladder from state 43 to state 87,
        then this would be represented in the dictionary as:
```

```
        {State(position = 43) : State(position - 87)}, and so on.
        '''
                self.from_to = from_to
                super().__init__(self.get_transition_map())


        def get_transition_map(self) -> Mapping[State,␣
↪FiniteDistribution[State]]:
                d : Dict[State, FiniteDistribution[State]] = {}

                for state in range(1, 100):
                        state_probs_map = {}
                        next_state = 0

                        for j in range(state + 1, min(101, state + 7)):
                                if j in self.from_to.keys():
                                        next_state = self.from_to[j]
                                else:
                                        next_state = j

                                state_probs_map[State(position = next_state)] =␣
↪1. / 6.

            # If I'm at state 97, I need only a 3 to reach 100.
            # If I get more than a 3, I stay in the same position.
                        if state > 94:
                                state_probs_map[State(position = state)] =␣
↪(state - 94.) / 6.

                        d[State(position = state)] =␣
↪Categorical(state_probs_map)

                return d
```

1.0.1 Create an instance of the Snakes and Ladders game

```
[ ]: changes_from = [1, 4, 9, 28, 36, 21, 51, 71, 80, 16, 47, 49, 56, 64, 87, 93,␣
      ↪95, 98]
     changes_to = [38, 14, 31, 84, 44, 42, 67, 91, 100, 6, 26, 11, 53, 60, 24, 73,␣
      ↪75, 78]

     from_to = {fr : to for fr, to in zip(changes_from, changes_to)}
     game = SnakesAndLaddersMP(from_to = from_to)

     # print("Transition Map")
     # print("----------------")
     # print(game)
```

1.0.2 Generate the constant distribution of the starting states and generate traces of the game and count the total number of dice rolls needed per game to finish it

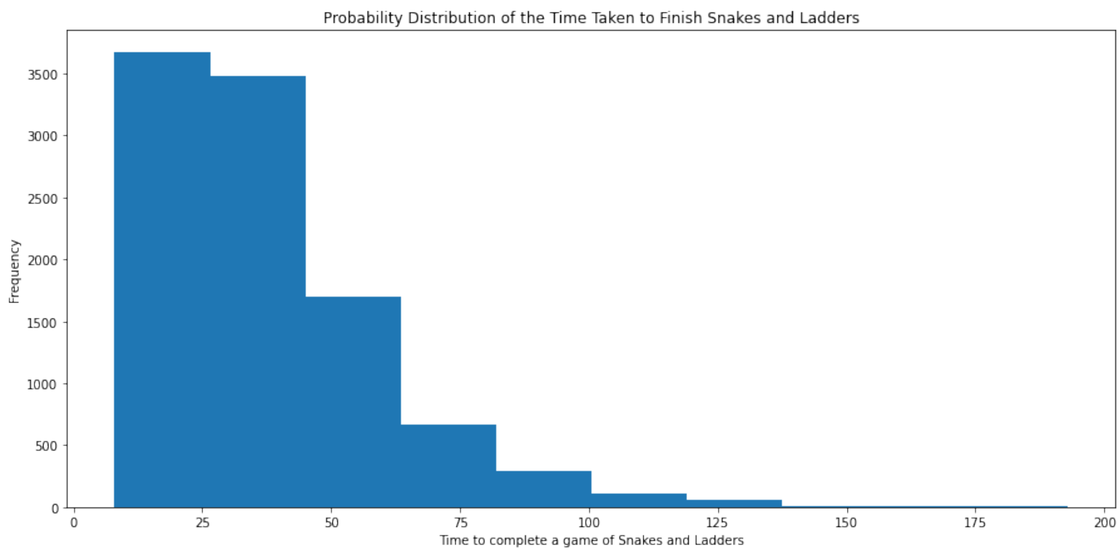
```
[ ]: start_distribution = Constant(value = NonTerminal(State(position = 1)))
     num_traces = 10000

     outcomes = [len([st for st in it]) for it in itertools.islice(game.
      ↪traces(start_distribution), num_traces)]
```

1.0.3 Plot the histogram of the time steps (number of dice rolls)

```
[ ]: plt.hist(outcomes)

     plt.xlabel('Time to complete a game of Snakes and Ladders')
     plt.ylabel('Frequency')
     plt.title("Probability Distribution of the Time Taken to Finish Snakes and␣
      ↪Ladders")
     plt.show()
```



2 Question 3

```
[ ]: @dataclass(frozen=True)
     class State:
             position : int


     class FrogPuzzle(FiniteMarkovProcess[State]):
             length : int = 0 # this is the length of the river between the two banks

             def __init__(self, length : int):
                     self.length = length
                     super().__init__(self.get_transition_map())

             def get_transition_map(self) -> Mapping[State,␣
      ↪FiniteDistribution[State]]:
                     d : Dict[State, FiniteDistribution[State]] = {}

                     for state in range(1, self.length + 2):
                             state_prob_map = {}
                             for next_state in range(state + 1, self.length + 3):
                                     state_prob_map[State(position=next_state)] = 1.␣
      ↪/ (self.length - state + 2)

                             d[State(position = state)] = Categorical(state_prob_map)

                     return d
```

2.0.1 Create an instance of the Markov process

```
[ ]: L = 20

     puzzle = FrogPuzzle(L)
     # print("Transition Map")
     # print("----------------")
     # print(puzzle)
```

2.0.2 Generate traces of it and get the total number of jumps required to get to the other side of the river

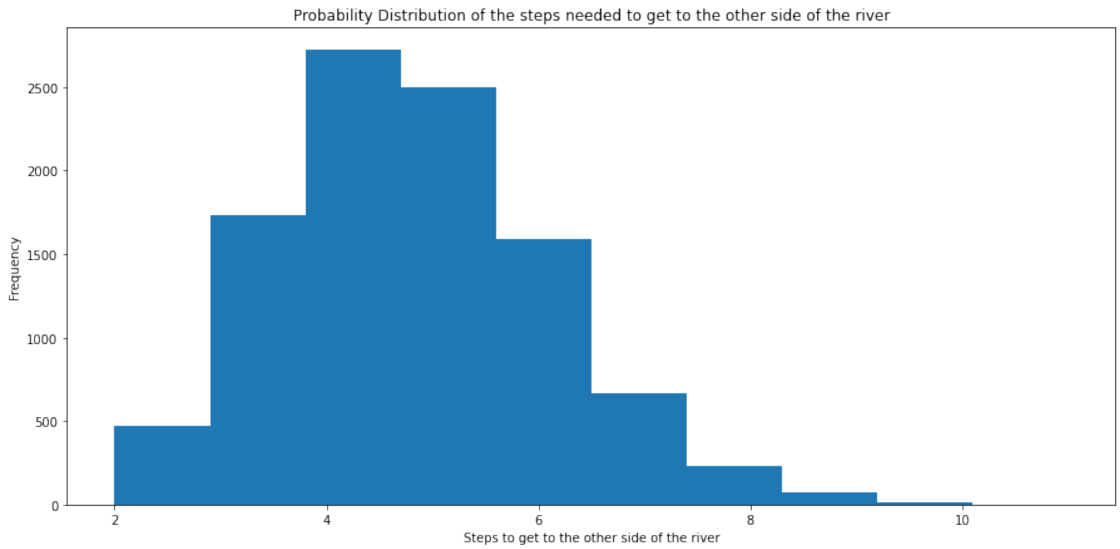
```
[ ]: start_distribution = Constant(value = NonTerminal(State(position = 1)))
     num_traces = 10000
```

```
outcomes = [len([i for i in trace]) for trace in itertools.islice(puzzle.
↪traces(start_distribution), num_traces)]
```

2.0.3 Plot a histogram of the jumps required to get to the other side of the river

[ ]:
```
plt.hist(outcomes)

plt.xlabel('Steps to get to the other side of the river')
plt.ylabel('Frequency')
plt.title("Probability Distribution of the steps needed to get to the other␣
↪side of the river")
plt.show()
```



3 Question 4

Here I recreate the Snakes and Ladders game, but with rewards associated with each state. In order to find the expected number of dice rolls to finish a game, I set the rewards of all states to 1.

[ ]:
```
@dataclass(frozen=True)
class State:
        position : int

class SnakesAndLaddersMRP(FiniteMarkovRewardProcess[State]):
        def __init__(self, from_to : Mapping[State, State]):
                self.from_to = from_to
                super().__init__(self.get_transition_reward_map())
```

```python
    def get_transition_reward_map(self) -> Mapping[State,␣
↪FiniteDistribution[Tuple[State, float]]]:
        d : Dict[State, FiniteDistribution[Tuple[State, float]]] = {}
        reward = 1

        for state in range(1, 100):
            state_probs_map = {}
            next_state = 0

            for j in range(state + 1, min(101, state + 7)):
                if j in self.from_to.keys():
                    next_state = self.from_to[j]
                else:
                    next_state = j

                state_probs_map[(State(position = next_state),␣
↪reward)] = 1. / 6.

            if state > 94:
                state_probs_map[(State(position = state),␣
↪reward)] = (state - 94.) / 6.

            d[State(position = state)] =␣
↪Categorical(state_probs_map)

        return d
```

3.0.1 Create an instance of the game

[ ]:
```python
changes_from = [1, 4, 9, 28, 36, 21, 51, 71, 80, \
                16, 47, 49, 56, 64, 87, 93, 95, 98]
changes_to = [38, 14, 31, 84, 44, 42, 67, 91, 100, \
                6, 26, 11, 53, 60, 24, 73, 75, 78]

from_to = {fr : to for fr, to in zip(changes_from, changes_to)}
game = SnakesAndLaddersMRP(from_to = from_to)
gamma = 1.0 # the discount factor

# print("Transition Map")
# print("--------------")
# print(FiniteMarkovProcess({s.state: Categorical({s1.state: p for s1, p in v.
↪table().items()}) \
#                                      for s, v in game.transition_map.items()}))

# print()
# print("Transition Reward Map")
# print("---------------------")
```

```
# print(game)

# print()
# print("Reward Function")
# print("----------------")
# game.display_reward_function()

# print()
# print("Value Function")
# print("----------------")
# game.display_value_function(gamma = gamma)
```

3.0.2 Use Monte Carlo simulations to find the expected number of dice rolls to win the game and compare this with the value obtained using Markov reward processes:

[ ]:
```
val = game.get_value_function_vec(gamma = gamma)
print(f"The expected number of dice rolls (using the value function) is {val[0]:
 ↪.3f}.")

start_distribution = Constant(value = NonTerminal(State(position = 1)))
num_traces = 10000
outcomes = [len([st for st in it]) for it in itertools.islice(game.
 ↪reward_traces(start_distribution), num_traces)]
print(f"The expected number of dice rolls (using Monte Carlo)        is␣
 ↪{sum(outcomes) / num_traces:.3f}.")
```

```
The expected number of dice rolls (using the value function) is 36.819.
The expected number of dice rolls (using Monte Carlo)        is 36.787.
```