Question 3

a. We see that:

$$
\begin{aligned}
\log \pi(s,a;\theta) &= \phi(s,a)^T\theta - \log\left(\sum_{b\in\mathcal{S}} e^{\phi(s,b)^T\theta}\right) \\
\therefore\ \nabla_\theta \log \pi(s,a;\theta) &= \phi(s,a) - \frac{\sum_{b\in\mathcal{S}} \phi(s,b)\cdot e^{\phi(s,b)^T\theta}}{\sum_{b\in\mathcal{S}} e^{\phi(s,b)^T\theta}} \\
\therefore\ \nabla_\theta \log \pi(s,a;\theta) &= \phi(s,a) - \sum_{b\in\mathcal{S}} \phi(s,b)\cdot \pi(s,b;\theta) \\
\therefore\ \nabla_\theta \log \pi(s,a;\theta) &= \phi(s,a) - \mathbb{E}_{\pi(s,a;\theta)}\left[\phi(s,a)\right]
\end{aligned}
$$

b. We have an obvious candidate:

$$
\nabla_w Q(s,a;w) = \nabla_\theta \log \pi(s,a;w) \Rightarrow Q(s,a;w) = w^T \nabla_\theta \log \pi(s,a;\theta)
$$

c. By using the above equation for $Q(s,a;w)$, we get:

$$
\begin{aligned}
\mathbb{E}\left(Q(s,a;w)\right) &= \sum_{a\in\mathcal{A}} \pi(s,a;\theta)\cdot Q(s,a;w) \\
&= \sum_{a\in\mathcal{A}} \pi(s,a;\theta)\cdot w^T \nabla_\theta \log \pi(s,a;\theta) \\
&= \sum_{a\in\mathcal{A}} \pi(s,a;\theta)\cdot w^T \left[\frac{1}{\pi(s,a;\theta)}\cdot \nabla_\theta \pi(s,a;\theta)\right] \\
&= \sum_{a\in\mathcal{A}} w^T \nabla_\theta \pi(s,a;\theta) \\
&= w^T \nabla_\theta \left[\sum_{a\in\mathcal{A}} \pi(s,a;\theta)\right] \\
&= w^T \cdot \nabla_\theta (1) \\
\therefore\ \mathbb{E}\left(Q(s,a;w)\right) &= 0 \quad \forall\ s\in\mathcal{N}
\end{aligned}
$$