#8

$z'' + z' - z = 0$

$r^2 + r - 1 =$ discriminant: $b^2 - 4ac = (1) + 4 = 5 > 0$
real

$$r_1 = \frac{-1 + \sqrt{(-1)^2 - 4(-1)}}{2(-1)} = \frac{1}{2} - \frac{\sqrt{5}}{-2} = -0.618$$

$$r_2 = \frac{-1 - \sqrt{1+4}}{-2} = \frac{1}{2} + \frac{\sqrt{5}}{2} = 1.618$$

$$\cancel{y(t) = C_1 e^{\ldots t} + C_2 e^{\ldots t}}$$

$$y(t) = C_1 e^{-0.618t} + C_2 e^{1.618t}$$

#14 $y'' + y' = 0$; $y(0) = 2$, $y'(0) = 1$

discriminant > 0 ; two reals

$r^2 + r$

$$r_1 = \frac{(-1) + \sqrt{(1)^2 - 4(1)(0)}}{2(1)} = \frac{-1 + \sqrt{1}}{2} = \frac{-1}{2} + \frac{\sqrt{1}}{2} = 0$$

$$r_2 = \frac{-(1) - \sqrt{1}}{2} = \frac{-1 - \sqrt{1}}{2} = \frac{-1}{2} - \frac{\sqrt{1}}{2} = 1$$

$$y(t) = C_1 e^{(0)t} + C_2 e^{(1)t}$$

$$y(0) = C_1 e^{(0)0} + C_2 e^{(1)(0)} = 2$$

$$C_1 + C_2 = 2 \Rightarrow$$

$r_1 C_1 e^{r_1 t} + r_2 C_2 e^{r_2 t}$

$$y'(0) = 0 C_1 e^{(0)(0)} + 1 C_2 e^0 = 1$$

$C_2 = 1$ then $C_1 = 1$

$$y(t) = 1e^{(0)t} + 1e^{1t} = \boxed{y(t) = 1 + e^t}$$

16. $y'' - 4y' + 3y = 0 \ ; \ y(0) = 1, \ y'(0) = 1/3$

$r^2 - 4r + 3$

$A = 1$ $\quad$ discriminant $= (-4)^2 - 4(1)(3)$

$b = -4$ $\quad 16 - 12$

$C = 3$ $\quad = 4 > 0$ two reals

$$r_1 = \frac{-(-4) + \sqrt{(-4)^2 - 4(3)}}{2} = \frac{4 + \sqrt{4}}{2} = 2 + 1 = 3$$

$$r_2 = \frac{-(-4) - \sqrt{4}}{2} = \frac{4 - 2}{2} = \frac{2}{2} = 1$$

$$y(t) = c_1 e^{3t} + c_2 e^{t}$$

$$y(0) = c_1 e^{3(0)} + c_2 e^{0} = 1$$

$$c_1 + c_2 = 1$$

$$y'(0) = 3c_1 e^{3(0)} + 1c_2 e^{0} = 1/3$$

$$\begin{array}{rcrcl} & 3c_1 & + & c_2 & = 1/3 \\ - & c_1 & + & c_2 & = 1 \\ \hline & 2c_1 & & & = -\frac{2}{3} \end{array}$$

$$\frac{2c_1}{2} = \frac{-\frac{2}{3}}{2} = \frac{-1}{3}$$

$$-\frac{1}{3} + \frac{4}{3} = 1$$

$\uparrow \qquad \uparrow$

$c_1 + c_2$ ✓

$$\boxed{y(t) = \left(\frac{-1}{3}\right)e^{3t} + \left(\frac{4}{3}\right)e^{t}}$$

#18 $y'' - 6y' + 9y = 0; \quad y(0) = 2, \quad y'(0) = 28/3$

$a = 1$
$b = -6$
$c = 9$

$-(-6) - 4(1)(9)$

$6 - 36 = -30$

discriminant < 0

$$\mu = \frac{-b}{2a} = \frac{-(-6)}{2} = \frac{6}{2} = 3$$

$$B = \frac{\sqrt{4ac - b^2}}{2a} = \frac{\sqrt{4(9) - 6^2}}{2} = \frac{\sqrt{36 - 36}}{2} = \frac{0}{2} = 0$$

$$y(t) = C_1 e^{\mu t} \cos(Bt) + C_2 e^{\mu t} \sin(Bt)$$

$$y(t) = C_1 e^{3t} \cos(0) + C_2 e^{3t} \sin(0)$$

$$y(0) = C_1 e^{3(0)} \cos(0) + C_2 e^{3(0)} \sin(0) = 2$$

$$C_1(1) \cdot 1 + C_2(1) \cdot 0 =$$

$$C_1 = 2$$

$$y''(t) = 2\left[e^{3t} \cdot -\sin(0) + \cos(0) \cdot 3e^{3t}\right] + C_2\left[e^{3t} \cdot \cos(0) + \sin(0) \cdot 3e^{3t}\right]$$

$$+ C_2\left[e^{3(0)} \cdot (1) + 0 \cdot 3e^{3(0)}\right] = \frac{25}{3}$$

$$y''(0) = 2\left[e^{3(0)} \cdot (0) + (1) \cdot 3e^{3(0)}\right]$$

$$2[0 + 3]$$

$$6 \quad +$$

$$C_2[1 \cdot 1 + 0$$

$$C_2 = \frac{25}{3}$$

$$y''(0) = 6 + C_2 = \frac{25}{3}$$

$$C_2 = \frac{25}{3} - 6$$

$$C_2 = \frac{7}{3}$$

Final:

$$y(t) = 2e^{3t} \cos(0 \cdot t) + \frac{7}{3} e^{3t} \sin(0 \cdot t)$$

$$= y(t) = 2e^{3t} \cos(0) + \frac{7}{3} e^{3t} \sin(0)$$

$$= y(t) = 2e^{3t} \cos(0)$$

(24)

(a) Sub $y = e^{rt}$
find Auxiliary equation for
$$ay' + by = 0$$

(b) Use (a) to find general solution

$$3z' + 11z = 0$$

$$3r(e^{rt}) + 11(e^{rt})$$

$$e^{rt}(3r + 11) = 0$$

$$\frac{e^{rt}(3r+11)}{e^{rt}} = \frac{0}{e^{rt}} = \boxed{3r + 11 = 0} \text{ Auxiliary Eq.}$$

which means r must $= \frac{-11}{3}$

General Solution then is

$$z(t) = 3 \cdot \frac{-11}{3}(e^{rt}) + 11(e^{rt})$$

$$z(t) = -11e^{rt} + 11e^{rt}$$