Homework Assignment (Problem Set) 1:

Note, Problem Set 1 directly focuses on Modules 1 and 2; Introduction to Decision Analysis and Formulation and Solving Linear Programs.

5 questions

Rubric:

All questions worth 20 points
20 Points: Answer and solution are fully correct and detailed professionally.
16-19 Points: Answer and solution are deficient in some manner but mostly correct.
11-15 Points: Answer and solution are missing a key element or two.
1-10 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.
0 Points: No answer provided.

Question 1:

Clocks in Sox is a small company that manufactures wristwatches in two separate workshops, each with a single watch maker (or horologist, as they are called). Each watchmaker works a different number of hours per month to make the three models sold by Clocks in Sox: Model A, Model B, and Model C. Watchmaker 1 works a maximum of 350 hours per month while Watchmaker 2 works a maximum of 250 hours per month and the time (in hours) and cost of materials for each watch differ by watchmaker due to their experience and equipment (shown below). Each month, Clocks in Sox must produce at least 60 Model A watches, 80 Model B watches, and 50 Model C watches. Clearly formulate a linear program (LP) to minimize the cost of manufacturing the desired amount of watches.

Table 1

Workshop	Model A		Model B		Model C	
	Cost ($)	Time	Cost ($)	Time	Cost ($)	Time
Watchmaker 1	10	2	11	4	12	3
Watchmaker 2	9	9	10	4	13	7

Let:
x1: number of Model A watches made by watchmaker 1.
x2: number of Model B watches made by watchmaker 1.
x3: number of Model C watches made by watchmaker 1.
y1: number of Model A watches made by watchmaker 2.
y2: number of Model B watches made by watchmaker 2.
y3: number of Model C watches made by watchmaker 2.

The objective is to minimize the cost of manufacturing the desired amount of watches, which is given by the cost of material multiplied by the number of watches for each model.

The total cost of manufacturing, the design amount of watches can be calculated as follows:

Cost = 10x1 + 11x2 + 12x3 + 9y1 + 10y2 + 13y3

Subject to the following constraints:

1. Watchmaker 1 can work a maximum of 350 hours per month:
   2x1 + 4x2 + 3x3 <= 350
2. Watchmaker 2 can work a maximum of 250 hours per month:
   9y1 + 4y2 + 7y3 <= 250
3. At least 60 Model A watches, 80 Model B watches, and 50 Model watches must be produced each month:
   x1 + y1 >= 60
   x2 + y2 >= 80
   x3 + y3 >= 50
4. The number of watches cannot be negative:
   x1, x2, x3, y1, y2, y3 >= 0

   Therefore, the complete linear program (LP) formulation to minimize the cost of manufacturing the amount of watches is:

   Minimize Cost = 10x1 + 11x2 + 12x3 + 9y1 + 10y2 + 13y3

   Subject to:
   2x1 + 4x2 + 3x3 <= 350
   9y1 + 4y2 + 7y3 <= 250
   x1 + y1 >= 60
   x2 + y2 >= 80
   x3 + y3 >= 50
   x1, x2, x3, y1, y2, y3 >= 0

Question 2:

Consider the following linear program:

Min $Z = -9x_1 + 18x_2$
Subject To
$-x_1 + 5x_2 \geq 5$
$x_1 + 4x_2 \geq 12$
$x_1 + x_2 \geq 5$
$x_1 \leq 5$
$x_1, x_2 \geq 0$

Part A: Write the LP in standard equality form.

To write the given linear program in standard equality form, we need to introduce slack variable to convert the equities into equations. The standard equity form of the linear program is as follows

Minimize Z = -9x1 + 18x2

Subject to:
-x1 + 5x2 - s1 = 5
x1 + 4x2 - s2 = 12
x1 + x2 - s3 = 5
x1 + s4 = 5

x1, x2, s1, s2, s3, s4 >= 0

Part B: Solve the original LP graphically (to scale). Clearly identify the feasible region and, if one or more exist, the optimal solution(s) (provide exact values for x_1, x_2, and Z).

Optimal solution:
x1 = 5.00
x2 = 2.00
Z = -9.00

Result - Graphical Method

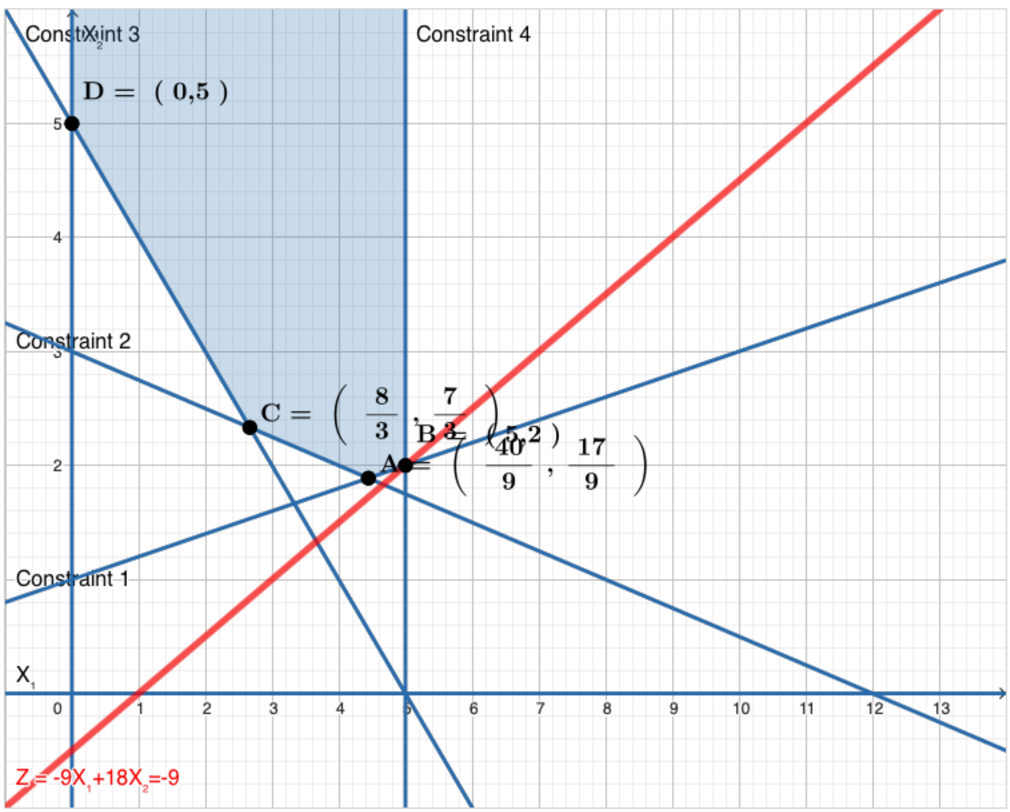

Fig 1: Graph generated on pmcalculators website.

Here are the results of the objective function at each of the points in the feasible region

Point	Coordinates (X_1,X_2)	Objective Function Value $-9X_1+ 18X_2$
A	(40/9,17/9)	-9(40/9)+ 18(17/9) = -6
B	(5,2)	-9(5)+ 18(2) = -9
C	(8/3,7/3)	-9(8/3)+ 18(7/3) = 18
D	(0,5)	-9(0)+ 18(5) = 90

Fig 2: Result of objective function at each point

Verified the solution on atozmath website.

Question 3:

InvestCo currently has \$500 in cash. InvestCo receives revenues at the start of months 1 – 4, after which it pays bills (see Table 2 below). Any money left over should be invested and interest for one month is 0.5%, two months is 2%, three months is 4%, and four months is 8% (total - no compounding). Use linear programming to determine an investment strategy that maximizes cash on hand at the beginning of month 5. Formulate an LP to maximize InvestCo's profit. Do not solve.

Hint: What is coming in and what is going out each month?

Table 2

Month	Revenues (\$)	Bills (\$)
1	600	700
2	900	400
3	300	700
4	500	350

Let's formulate the linear program LP to maximize InvestCo's profit.

Decision Variables:
X11 = amount invested at beginning of Month 1, maturing end of Month 1
X12 = amount invested at beginning of Month 1, maturing end of Month 2
X13 = amount invested at beginning of Month 1, maturing end of Month 3
X14 = amount invested at beginning of Month 1, maturing end of Month 4
X22 = amount invested at beginning of Month 2, maturing end of Month 2
X23 = amount invested at beginning of Month 2, maturing end of Month 3
X24 = amount invested at beginning of Month 2, maturing end of Month 4
X33 = amount invested at beginning of Month 3, maturing end of Month 3
X34 = amount invested at beginning of Month 3, maturing end of Month 4
X44 = amount invested at beginning of Month 4, maturing end of Month 4

Objective Function:
Maximize Cash (Month 5) Z: 1.08x14+ 1.04x24+ 1.02x34+ 1.001x44

Constraints:
Month 1: x11+ x12+ x13+ x14 + 700 <= 1100
Month 2: x22+ x23+ x24+ 400 <= 1.001x11 + 900
Month 3: x33+ x34+ 700 <= 1.001x12+ 1.02x22 + 300
Month 4: x44+ 350 <= 1.001x13+ 1.02x23+ 1.04x33 + 500

Question 4:

Floor is Java sells premium coffee to restaurants. They sell two roasts which they call (cleverly) Roast 1 and Roast 2, each of which is a blend of Columbian and Arabica coffee beans. Columbian beans cost \$20 for a 5 pound box while Arabica beans cost \$15 for a 6 pound box. Roast 1 sells for \$6 per pound and must be at least 75% Columbian beans, while Roast 2 sells for \$5 per pound and must be at least 60% Columbian beans. At most, 40 pounds of Roast 1 and 60 pounds of Roast 2 can be sold each month.

Part A: Formulate an LP to maximize Floor is Java's profit.

Let:
x: represent the number of pounds of Roast 1 sold.
y: represent the number of pounds of Roast 2 sold.

The objective function to maximize profit is:
Maximize $P = 6x + 5y$

Subject to the constraints:
1. Columbian content in Roast 1:
   $0.75x <= 0.5y$
2. Columbian content in Roast 2:
   $0.60x <= 0.5y$
3. Maximum sales of Roast 1:
   $x <= 40$
4. Maximum sales of Roast 2:
   $y <= 60$
5. Non-negativity constraint:
   $x >= 0, y >= 0$

Part B: Solve the LP (provide **exact** values (do not restrict to integer) for all variables and the optimal objective function).

Utilizing linear programming techniques through the PuLP library in Python, we formulated the problem's objective function and constraints. Upon solving, the optimal solution revealed that Floor is Java should produce 40 pounds of Roast 1 and 60 pounds of Roast 2. This production strategy would yield a maximum profit of $540.0. Python program attached in jupyter notebook.

Status: Optimal
Optimal Solution:
Roast1_pounds = 40.0
Roast2_pounds = 60.0

Total Profit (Max Z) = $540.0

Question 5:

Food Beach, a local grocery store, is building a work schedule for its stockers and has specific requirements over each 24 hour period (shown in the table below). Each stocker must work two consecutive shifts.

Shift	Workers
Midnight - 4 am	8
4 am - 8 am	7
8 am - Noon	5
Noon - 4 pm	4
4 pm - 8 pm	4
8 pm - Midnight	7

Part A: Formulate an LP model to minimize the number of workers required to meet requirements.

Let:
x1: number of stockers working the midnight – 4am shift
x2: number of stockers working the 4am – 8am shift
x3: number of stockers working the 8am – noon shift
x4: number of stockers working the noon – 4pm shift
x5: number of stockers working the 4pm – 8pm shift
x6: number of stockers working the 8pm – midnight shift

Objective Function:
The objective is to minimize the total number of workers required.
We can formulate the LP model as follows:

Minimize Z = x1+ x2 + x3 + x4 + x5 + x6

Constraints:

1. Total Number of Workers:
   x1+ x2 + x3 + x4 + x5 + x6 = 35 (Total number of stockers)

2. Each stocker must work two consecutive shifts:
   x1 + x6 >= 8
   x1 + x2 >= 7
   x2 + x3 >= 5
   x3 + x4 >= 4
   x4 + x5 >= 4
   x5 + x6 >= 7

3. The number of stockers cannot be negative:
   x1, x2, x3, x4, x5, x6 >= 0

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).

Output from Python code:
Status: Optimal
Optimal Solution:
Midnight_4am = 6.0
4am_8am = 1.0
8am_Noon = 4.0
Noon_4pm = 0.0
4pm_8pm = 5.0
8pm_Midnight = 2.0
Total Workers Required = 18

The linear programming model was designed to minimize the number of workers required to meet the specified work schedule constraints for Food Beach, a local grocery store. The constraints dictated that each stocker must work two consecutive shifts.
Upon solving the model, the optimal work schedule is as follows:

- **Midnight to 4 am**: 6 workers

- **4 am to 8 am**: 1 worker
- **8 am to Noon**: 4 workers
- **Noon to 4 pm**: 0 workers
- **4 pm to 8 pm**: 5 workers
- **8 pm to Midnight**: 2 workers

This solution indicates that Food Beach needs a total of 18 workers to efficiently cover all shifts, ensuring that each stocker works two consecutive shifts. This optimal staffing solution not only meets the store's operational requirements but also minimizes labor costs by employing the minimum number of workers necessary to maintain productivity.