CAS 701

Logic and Discrete Mathematics

Fall 2017

Exercise Group 1

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You are required to submit your solutions to 10 of the following 15 exercises. **The solutions are due October 3, 2017.** "Gallier" means the textbook *Logic for Computer Science: Foundations of Automatic Theorem Proving, Second Edition* Dover, 2015.

1. [10 pts.] Show that, if A and B are sets, then

   $$(A \cap B) \cup (A \cap \overline{B}) = A.$$

2. [10 pts.] Let $A \setminus B$ denote the difference of A and B and $A \Delta B$ denote the symmetric difference of A and B. Show

   $$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

3. [10 pts.] Exercise 2.1.1(a) on p. 23 of Gallier.

4. [10 pts.] Exercise 2.1.1(b) on p. 23 of Gallier.

5. [10 pts.] Exercise 2.1.1(c) on p. 23 of Gallier.

6. [10 pts.] Exercise 2.1.3 on p. 24 of Gallier.

7. [10 pts.] Exercise 2.1.5 on p. 24 of Gallier.

8. [10 pts.] Let $f : A \to B$ and $g : B \to C$ be total, and let $h = g \circ f : A \to C$ be the composition of g and f. Prove that, if f and g are injective, then h is injective, but the converse is false.

9. [10 pts.] Let $f : A \to B$ and $g : B \to C$ be total, and let $h = g \circ f : A \to C$ be the composition of g and f. Prove that, if f and g are surjective, then h is surjective, but the converse is false.

10. [10 pts.] Determine which of the following functions are bijective from $\mathbb{R}$ to $\mathbb{R}$:

   a. $f(x) = -3x + 4$.
   b. $f(x) = -3x^2 + 7$.
   c. $f(x) = (x + 1)/(x + 2)$.
   d. $f(x) = x^5 + 1$.

11. [10 pts.] Let $R \subseteq (\mathbb{Z}\times\mathbb{Z})\times(\mathbb{Z}\times\mathbb{Z})$ be a relation such that $((a, b), (c, d) \in R$ iff $ad = bc$. Show that R is an equivalence relation. What is the equivalence class of $(1, 2)$? Give an interpretation of the equivalence classes of R.

12. [10 pts.] What is the cardinality of the function space $\mathbb{N} \to \mathbb{N}$?

13. [10 pts.] Let T_n be a full binary tree of height $n \geq 1$. What is the cardinality of the set of nodes in T_n? What is the cardinality of the set of paths in T_n?

14. [10 pts.] Let T_∞ be a full binary tree of infinite height. What is the cardinality of the set of nodes in T_∞? What is the cardinality of the set of paths in T_∞?

15. [10 pts.] Show that the set of real numbers that are solutions of quadratic equations of the form $ax^2 + bx + c = 0$, where a, b, c are integers, is countable.