Assginment5

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Contents

1 Question1

1.1 $\rightarrow\; A[t/x] \supset \exists x A$

$$\dfrac{\dfrac{A[t/x] \;\rightarrow\; A[t/x]}{A[t/x] \;\rightarrow\; \exists x A}\,(\exists : right)}{\rightarrow\; A[t/x] \supset \exists x A}\,(\supset : right)$$

1.2 $\forall x A \supset \exists x A$

$$\dfrac{\dfrac{\dfrac{A[t/x] \;\rightarrow\; A[t/x]}{A[t/x] \;\rightarrow\; \exists x A}\,(\exists : left)}{\forall x A \;\rightarrow\; \exists x A}\,(\forall : left)}{\forall x A \subset \exists x A}\,(\supset : right)$$

1.3 $\rightarrow\; (A \supset (B \supset C)) \supset (B \supset (A \supset C))$

$$\dfrac{\dfrac{\dfrac{\dfrac{A, B \;\rightarrow\; A, C \qquad \dfrac{A, B \;\rightarrow\; C, B \qquad C, A, B \;\rightarrow\; C}{A, B, B \supset C \;\rightarrow\; C}\,(\supset : left)}{A, B, A \supset (B \supset C) \;\rightarrow\; C}\,(\supset : left)}{B, A \supset (B \supset C) \;\rightarrow\; A \supset C}\,(\supset : right)}{A \supset (B \supset C) \;\rightarrow\; B \supset (A \supset C)}\,(\supset : right)}{\rightarrow\; (A \supset (B \supset C)) \supset (B \supset (A \supset C))}\,(\supset : right)$$

2 Question2

2.1 $\forall x (x \doteq x)$

$$\dfrac{x \doteq x}{\forall x (x \doteq x)}\,(\forall : right)$$

2.2

$$\dfrac{\dfrac{\dfrac{x \doteq y \;\rightarrow\; y \doteq x}{\rightarrow\; (x \doteq y) \supset (y \doteq x)}\,(\supset : right)}{\rightarrow\; \forall y ((x \doteq y) \supset (y \doteq x))}\,(\forall : right)}{\rightarrow\; \forall x \forall y ((x \doteq y) \supset (y \doteq x))}\,(\forall : right)$$

LK_e system has an axiom as below:
For all L-terms s_1, s_2, t_1, t_2, $s_1 \doteq t_1, s2 \doteq t_2, s_1 \doteq s_2 \;\rightarrow\; t_1 \doteq t_2$.
If we assign y to t_1 and assign x to t_2, s_1, s_2, then $x \doteq y, x \doteq x, x \doteq x \;\rightarrow\; y \doteq x$.

3 Question3

3.1 $\forall x (\forall y (xmuly \doteq y) \supset (x \doteq e))$

$$\dfrac{\dfrac{\dfrac{(xmuly \doteq y) \;\rightarrow\; (x \doteq e)}{\forall y (xmuly \doteq y) \;\rightarrow\; (x \doteq e)}\,(\forall : left)}{\forall y (xmuly \doteq y) \supset (x \doteq e)}\,(\supset : right)}{\forall x (\forall y (xmuly \doteq y) \supset (x \doteq e))}\,(\forall : right)$$

4 Question 4

4.1 +

$+ = \forall x.y \quad l.x + y = if(y = 0, x, S(x + (Iz.l.S(z) = y)))$

4.2 *

$* = \forall x.y \quad l.x * y = if(y = 0, 0, S(x * (Iz.l.S(z) = y)))$

5 Question8

A well order is a total (weak) order such that every non-empty set of elements contains a least element with respect to the ordering. Let w = (L, Γ) be a theory of STT where:

$$L = (\leq, T) where : T(\leq) = l \quad \rightarrow \quad l \quad \rightarrow \quad *$$

Γ contains the following axioms:
1.$\forall x, y : l.x \leq y \wedge y \leq x \quad \rightarrow \quad x = y$
2.$\forall x, y, z : l.x \leq y \wedge y \leq z \quad \rightarrow \quad x \leq z$
3.$\forall x, y : l.x \leq y \vee y \leq x$
4.$\forall p : l * .(\exists x : l.p(x) \quad \rightarrow \quad (\exists y : l.p(y) \wedge (\forall z : l.p(z) \quad \rightarrow \quad y \leq z)))(everynon - emptysetofelementscontainsaleastelement)$.

6 Question 10

Let L = {0, S, push, pop, empty, nil, top, τ}, where τ is defined in the following way:

1. $\tau(nil) = l_2$ (empty stack)
2. $\tau(push) = l_1 \quad \rightarrow \quad l_2 \quad \rightarrow \quad l_2$ (push an element from the stack)
3. $\tau(pop) = l_2 \quad \rightarrow \quad l_2$ (pop an element from the stack)
4. $\tau(empty) = l_2 \quad \rightarrow \quad *$ (is a stack empty?)
5. $\tau(top) = l_2 \quad \rightarrow \quad l_1$ (returns the top element of a stack)

Let Γ be the following set of axioms:

1. empty(nil)
2. $\forall s : l_2.\forall n : l_1.\neg empty(push(n, s))$
3. $\forall s : l_2.\forall n : l_1.pop(push(n, s)) = s$
4. $pop(nil) = \perp_{l2}$
5. $top(nil) = \perp_{l1}$
6. $\forall s : l_2.\forall n : l_1.top(push(n, s)) \neq nil$
7. $\forall s : l_2.\forall n : l_1.top(push(n, s)) = n$
8. $\forall s_1, s_2 : l_2.\forall n_1, n_2 : l_1.push(n_1, n_2) = push(n_2, s_2) \Rightarrow n_1 = n_2 \wedge s_1 = s_2$
9. $\forall p : l_2 \quad \rightarrow \quad *.(p(nil) \wedge (\forall s.l_2.\forall n : l_1.p(s) \Rightarrow p(push(n, s)))) \Rightarrow (\forall s : l_2.p(s))$