Assginment3

Lai Wei(Sunny)

ID:400145770

Contents

1 Question1

1.1 Addition

Based on the observation:
add(n,0)=n+0=n
add(n,m+1)=n+(m+1)=(n+m)+1=succ(add(n,m))

we should find two primitive recursive functions
f:$N => N$ and g:$N^3 => N$ such that:
when m=0,add(n,m)=f(n)
when m>0,add(n,m)=g(n,m-1,add(n,m-1)

since add(n,0)=n=>f(n)=n

since add(n,m+1)=succ(add(n,m)),g needs to be a function which maps the ordered triple(n,m,add(n,m))
g:$N^3 => N$ where g(n1,n2,n3)=succ(n3)
because then g(n,m,add(n,m)=succ(add(n,m))
To show g is primitive recursive:
g(n1,n2,n3)=succ(Pr_3^3(n1,n2,n3)),which is achieved by using the projection function
g is obtained by substitution from the basic primitive recursive functions succ and Pr_3^3 and so is primitive recursive

In conclusion, Additon is primitive recursive

1.2 Multiplication

Based on the observation:
mult(n,0)=n*0=0
mult(n,m+1)=n*(m+1)=nm+n=add(n,mult(n,m))

we should find two primitive recursive functions
f:$N => N$ and g:$N^3 => N$ such that:
when m=0,mult(n,m)=f(n)
when m>0,mult(n,m)=g(n,m-1,mult(n,m-1)

since mult(n,0)=0=>f(n)=0,which is the basic primitive recursive function:zero function

since mult(n,m+1)=add(n,mult(n,m)),g needs to be a function which maps the ordered triple(n,m,mult(n,m))
g:$N^3 => N$ where g(n1,n2,n3)=add((Pr_3^1),(Pr_3^3))
g is obtained by substitution from the primitive recursive functions Additon and Pr_3^3, $\mathrm{Pr}_3^1$and so is primitive recursive

In conclusion,Multipilication is primitive recursive

1.3 Exponentiation

Based on the observation:
exp(n,0)=n^0=1
exp(n,m+1)=n*n^m=mult(n,exp(n,m))
we should find two primitive recursive functions
f:$N => N$ and g:$N^3 => N$ such that:
when m=0,exp(n,m)=f(n)
when m>0,exp(n,m)=g(n,m-1,exp(n,m-1)

since exp(n,0)=1=>f(n)=1,which is the basic primitive recursive function:constant function

since exp(n,m+1)=mult(n,exp(n,m)),g needs to be a function which maps the ordered triple(n,m,exp(n,m))
g:$N^3 => N$ where g(n1,n2,n3)=exp((Pr_3^1),(Pr_3^3))
g is obtained by substitution from the primitive recursive functions Multiplication and Pr_3^3, $Pr_3^1$and so is primitive recursive

In conclusion,Multiplication is primitive recursive

2 Question2

We can define the Ackerman function as follows:
A(n,m)=if(m=0,2n,if(n=0,0,if(n=1,2,A(m-1,A(m,n-1)))))
We should show this is well-founded recursive.
For that consider the strict lexicographical order between pairs:
(n1,m1)<(n2,m2)iff(n1<n2) or (n1=n2 and m1<=m2)
Then we have that for base cases the function is well-defined. For the general case we have that:(m -1, A(m, n -1))<(m,n)

3 Question3

Tree is an example.A tree has three nodes. Let A be the root node, B is A's child and C is B's child. This tree is well-founded, because A is the minimum element. The tree is not a partial order,because it is not transitive: A is not C's parent.

4 Question7

4.1 P≡ ¬P

P	¬P	P≡ ¬P
T	F	F
F	T	F

4.2 ((P∧Q) ⟹ (P∨Q))

P	Q	P∧Q	P∨Q	((P∧Q) ⟹ (P∨Q))
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

4.3 ((Q ⟹ ¬P)≡(P≡Q))

Q	¬P	P	∧Q ⟹ ¬P	P≡Q	((Q ⟹ ¬P)≡(P≡Q))
T	T	F	T	F	F
F	T	F	T	T	T
T	F	T	F	T	F
F	F	T	T	F	F

4.4 ((P ⟹ Q)∧(¬P ⟹ Q))

P	Q	¬P	P ⟹ Q	¬P ⟹ Q	((P ⟹ Q)∧(¬P ⟹ Q))
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

4.5 $(((P \implies Q) \implies R) \implies S)$

P	Q	R	R	$P \implies Q$	$(P \implies Q) \implies R$	$((P \implies Q) \implies R) \implies S$
T	T	T	T	T	T	T
F	T	T	T	T	T	T
T	F	T	T	F	T	T
T	T	F	T	T	F	T
T	T	T	F	T	T	F
F	F	T	T	T	T	T
T	F	F	T	F	T	T
T	T	F	F	T	F	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
T	F	T	F	F	T	F
F	F	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	F	T	T	F
T	F	F	F	F	T	F
F	F	F	F	T	F	T

4.6 $(P \wedge Q) \implies (P \implies Q)$

P	Q	$P \wedge Q$	$P \implies Q$	$(P \wedge Q) \implies (P \implies Q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

5 Question8

5.1 Valid

2.2,2.6

5.2 Invalid

2.1,2.3,2.4,2.5

5.3 Satsifiable

2.2,2.3,2.4,2.5

5.4 Unsatsifiable

2.1

6 Question9

Assume B is a knave
=>"A is a knave" is false
=>A is knight
=>"The two of us are both knights" is true, but we B is a knave
=> The assumption is wrong

Assume B is a knight
=>"A is a knave" is true
=>A lies
=>"The two of us are both knights" is false
=> The assumption is right

In conclusion,B is a knight,A is a knave

7 Question10

A	B	
T	T	A:knight,B:knight
F	T	A:knave,B:knight
T	F	A:knight,B:knave
F	F	A:knave,B:knave

8 Question11

Let $\uparrow denotesnandAssume (p \uparrow q) \uparrow r \implies$ p$\uparrow (q \uparrow r)$

Assume (p$\wedge \neg r$) $is \quad true$
$\implies p, \neg r$
$\implies (\neg q) \vee (\neg r) \quad (addition \quad rule)$
$\implies \neg(q \wedge r) = q \uparrow r$
$\implies p \wedge (q \uparrow r) \quad (conjunction \quad rule)$
$\implies \neg\neg(p \wedge (q \uparrow r)) \quad (double \quad negation \quad introduction)$
$\implies \neg(p \uparrow (q \uparrow r)) \quad (nand \quad rule)$

$\neg(p \uparrow q) \vee (\neg r) \quad (addition)$
$\implies \neg((p \uparrow q) \wedge r)$
$\implies (p \uparrow q) \uparrow r \quad (nand \quad rule)$
$\implies \neg\neg((p \uparrow q) \uparrow r) \quad (double \quad negation \quad introduction)$

$(\neg\neg((p \uparrow q) \uparrow r)) \wedge (\neg(p \uparrow (q \uparrow r))) \quad (conjunction \quad rule)$
$\implies \neg((\neg((p \uparrow q) \uparrow r)) \vee (\neg(p \uparrow (q \uparrow r))))$
$\implies \neg((p \uparrow q) \uparrow r) \implies (p \uparrow (q \uparrow r)))$

The assumption is wrong,nand is not associative.

9 Question12

p$\uparrow r \equiv \neg(p \wedge r)$
$\implies p \uparrow r \equiv \neg p \vee \neg r$
$Since\{\neg, \vee\}$ is a complete set,nand is a complete set.

10 Question13

10.1 conjunctive normal form

$\neg(P \wedge Q \wedge R) \wedge \neg(P \wedge Q \wedge \neg R) \wedge \neg(P \wedge \neg Q \wedge R) \wedge (\neg P \wedge Q \wedge \neg R) \wedge \neg(\neg P \wedge \neg Q \wedge R)$

10.2 disjunctive normal form

(P$\wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$