from RGB to L*a*b* color space

1. The purpose of the transformation

There are 4 key goals of the transformation from RGB color space to L*a*b* color space.

1. Device-dependent color space → absolute color space

Three primary colors principle is applied in RGB color space, but there is no colormetric definition of red, green and blue, and so we cannot regard the results of mixing primary colors as specified colors, but relative to primary colors (RGB color space, n.d.). It means that the same RGB combination will present slightly differently on different monitors.

L*a*b* color space is a reference color space. To be more specific, colors are device-independent with a given white point which provides information about the light available in the scene in this color space. L*a*b* color space is built on a three-dimensional real number space, where we can define infinite numbers of possible representations of colors, and thus L*a*b* values are absolute in this space with a pre-defined range. The pre-defined range depends on a specified white point. (L*a*b color space, n.d.)

2. Expand the gamut

Gamut is the complete subset of colors in color reproduction. The most common usage refers to the subset of colors which can be accurately represented in a given circumstance. (gamut, n.d.)

Chromaticity is an objective specification of the quality of a color regardless of its luminance. It consists of two independent parameters, often specified as hue (h) and colorfulness (s), where the latter is alternatively called saturation, chroma, intensity, or excitation purity. (chromaticity, n.d.)

RGB color space has a chromaticity gamut that is a color triangle, when the amounts of the three primary colors (red, green, and blue) are constrained to be nonnegative. Although red, green, blue are used widely in computer graphics, RGB color space is not sufficient to reproduce all colors. We can say that RGB color space provides a computationally tractable format to storage images at the cost of displaying a smaller range of colors than human eyes can perceive. (Weller, 2018)

On the contrary, the L*a*b* space is larger than the gamut of computer displays and printers. Its gamut includes both RGB color space and CMYK color model.

3. Fit for computer → fit for human vision

In RGB color space, all three primary colors are in the equal bandwidth to produce any desired color, which can simplify the computation. But human color vision doesn't fall neatly in a perfect cube, thus if our aim is to mimic human color perception, the bandwidth for each channel should be present differently, and additional information about lighting in the environment should be included as well.

Three channels of L*a*b* are L (black to white), a* (green to red), and b* (blue to yellow) are designed to satisfy those requirements mentioned above. This color space is shaped irregularly which is an intuitive way to organize colors for human vision. (Weller, 2018)

4. Perceptually uniform

Another goal of CIELAB color space is to achieve perceptual uniformity, meaning that uniform changes of components in the L*a*b* color space correspond to same amount of changes in perceived color.

2. The Process of the transformation

1. sRGB color space

Images are in sRGB color space by default. sRGB defines the chromaticities of the red, green, and blue primaries, the colors where one of the three channels is nonzero and the other two are zero. The gamut of chromaticities that can be represented in sRGB is the color triangle defined by these primaries. As with any RGB color space, for non-negative values of R, G, and B it is not possible to represent colors outside this triangle, which is well inside the range of colors visible to a human with normal trichromatic vision. (sRGB color space, n.d.)

2. sRGB color space to XYZ color space

sRGB color space is based on the CIE xyY coordinate. The CIE XYZ values must be scaled so that the *Y* of D65 ("white") is 1.0 (*X*, *Y*, *Z* = 0.9505, 1.0000, 1.0890).

The components of sRGB ($R_{srgb}, G_{srgb}, B_{srgb}$) are in the range 0 to 1. The first step is to transform sRGB to RGB which can be viewed as the non-linear-to-linear transformation.

$$C_{linear} = \begin{cases} \dfrac{C_{srgb}}{12.92}, & C_{srgb} \leq 0.0405 \\ \left(\dfrac{C_{srgb} + \alpha}{1 + a}\right)^{2.4}, & C_{srgb} > 0.0405 \end{cases}$$

$$\text{where a} = 0.055 \text{ and where C is R, G, and B}$$

Followed by a matrix multiplication of the linear RGB values to get XYZ:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} R_{linear} \\ G_{linear} \\ B_{linear} \end{bmatrix}$$

Theory of the transformation

The sRGB gamma cannot be represented as a single numerical value, and there is only an approximate median value (2.2). The transformation is based on this decoding gamma, but with the linear portion near zero to avoid having an infinite slope at $K = 0$, which can cause numerical problems. The continuity condition for the curve C_{linear}, which is defined above as a piecewise function of C_{srgb}:

$$\left(\frac{K_0 + \alpha}{1 + \alpha}\right)^{\gamma} = \frac{K_0}{\phi}$$

$\gamma = 2.4$ and $\phi = 12.92$ yields two K values, $K_0 \approx 0.0381548$ and $K_0 \approx 0.0404482$. The IEC 61966-2-1 standard usesrounded value $K_0 = 0.0405$.

3. XYZ color space to L*a*b color space

The last step is to convert XYZ color space to L*a*b* color space. (L*a*b color space, n.d.)

$$L^* = 116\,\mathrm{f}\left(\frac{Y}{Y_n}\right) - 16$$

$$a^* = 500\,(\,\mathrm{f}\left(\frac{X}{X_n}\right) - \mathrm{f}\left(\frac{Y}{Y_n}\right))$$

$$b^* = 200(\,\mathrm{f}\left(\frac{Y}{Y_n}\right) - \mathrm{f}\left(\frac{Z}{Z_n}\right))$$

$$where\ f(t) = \begin{cases} \sqrt[3]{t}, & if\ t > \delta^3 \\ \dfrac{t}{3\delta^2} + \dfrac{4}{29}, & otherwise \end{cases}$$

$$\delta = \frac{6}{29}$$

Here, X_n, Y_n and Z_n are the CIE XYZ tristimulus values of the reference white point (the subscript n suggests "normalized").

Under Illuminant D65 with normalization $Y = 100$, the values are

$$X_n = 95.047$$
$$Y_n = 100.000$$
$$Z_n = 108.883$$

Values for illuminant D50 are

$$X_n = 96.6797$$
$$Y_n = 100.000$$
$$Z_n = 82.5188$$

Theory of the transformation

The division of the domain of the f function into two parts was done to prevent an infinite slope at $t = 0$. The function f was assumed to be linear below some $t = t_0$, and was assumed to match the $t^{1/3}$ part of the function at t_0 in both value and slope. In other words:

$$t_0^{\frac{1}{3}} = mt_0 + c \ (match\ in\ value)$$
$$\frac{1}{3}t_0^{-\frac{2}{3}} = m \ (match\ in\ slope)$$

The intercept $f(0) = c$ was chosen so that L^* would be 0 for $Y = 0$: $c = 16/116 = 4/29$. The above two equations can be solved for m and t_0:

$$m = \frac{1}{3}\delta^{-2} = 7.787037\ldots$$
$$t_0 = \delta^3 = 0.008856\ldots$$

$$\text{where } \delta = \frac{6}{29}$$

Bibliography

chromaticity. (n.d.). Retrieved from wikipedia: https://en.wikipedia.org/wiki/Chromaticity

gamut. (n.d.). Retrieved from wikipedia: https://en.wikipedia.org/wiki/Gamut

*L*a*b color space*. (n.d.). Retrieved from wikipedia: https://en.wikipedia.org/wiki/CIELAB_color_space

RGB color space. (n.d.). Retrieved from wikipedia: https://en.wikipedia.org/wiki/RGB_color_space

sRGB color space. (n.d.). Retrieved from wikipedia: https://en.wikipedia.org/wiki/SRGB

Weller, H. (2018, 6 28). *color spaces*. Retrieved from https://cran.r-project.org/web/packages/colordistance/vignettes/color-spaces.html