QUANTUM FISHER INFORMATION AS A TOOL FOR DETECTING TOPOLOGICAL PHASES

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Multipartite entanglement

- n-separability

$$|\psi\rangle = \underbrace{|\phi_1\rangle \otimes \cdots \otimes |\phi_n\rangle}_{\text{factorizes in } n \text{ terms}}$$

- ***k*-party entanglement**

$$|\psi\rangle = \bigotimes_i |\phi_i\rangle\,, \qquad |\phi_i\rangle \text{ involves at most } k \text{ parts}$$

Quantum Fisher Information

limit to the achievable precision in a phase estimation protocol $\rho \longrightarrow \rho(\theta)$

$$(\Delta\theta)^2 \geq \frac{1}{mF} \geq \frac{1}{mF_Q}$$

- F: Fisher information
- F_Q: quantum Fisher information
- $f_Q = F_Q/L$: QFI density

Entanglement criterion

$$\boxed{f_Q[\rho_{k\text{-ent}}, \hat{H}_{\text{lin}}] \leq k}$$

$\rho_{k\text{–ent}}$ input state with k–party entanglement

$\hat{H}_{\text{lin}}$ linear interferometer

We look at the multipartite entanglement structure of symmetry protected topological phases using quantum Fisher information of non-local operators one-dimensional p-wave superconductor with **long-range coupling** $\sim 1/r^\alpha$

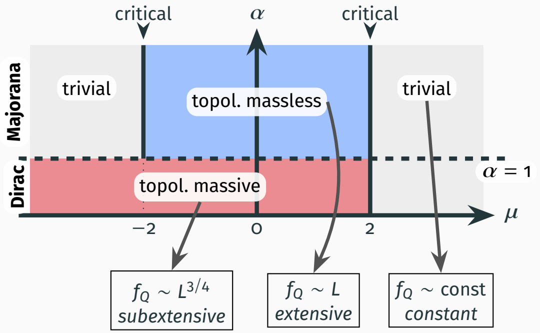
$$H = \sum_j \left[-t c_j^\dagger c_{j+1} - \mu \left(c_j^\dagger c_j - \frac{1}{2} \right) + \frac{\Delta}{2} \sum_r \boxed{\frac{1}{r^\alpha} c_j^\dagger c_{j+r}} + \text{h.c.} \right]$$

QFI of non-local spin degrees of freedom

- $\sigma_j^+ = c_j^\dagger e^{i\pi \sum_{i<j} c_i^\dagger c_i}, \quad \sigma_j^- = e^{i\pi \sum_{i<j} c_i^\dagger c_i} c_j$
- $\hat{H}_{\text{lin}}^\rho = \sum_j \sigma_j^\rho, \ \rho = x, y$

We look at the scaling of $f_Q[|\text{gs}\rangle, \hat{H}_{\text{lin}}^\rho]$ with the system size L

The scaling can be computed analytically using **Toeplitz determinants**



Bilinear-Biquadratic model

most general $SU(2)$–invariant isotropic spin-1 Hamiltonian

$$H = J\sum_i \left[\mathbf{S}_i \cdot \mathbf{S}_{i+1} - \beta(\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2\right] = J' \sum_i \left[\cos\theta\, \mathbf{S}_i \cdot \mathbf{S}_{i+1} - \sin\theta\, (\mathbf{S}_i \cdot \mathbf{S}_{i+1})^2\right]$$

QFI of string operators

- $\widetilde{S}_j^z = \left(e^{i\pi\sum_{i<j} S_i^z}\right) S_j^z$
- $\hat{O} = \sum_j \widetilde{S}_j^z$

We look at the scaling of $f_Q[\,|\mathrm{gs}\rangle\,, \hat{O}]$ with the system size L