QUANTUM FISHER INFORMATION AND TOPOLOGICAL PHASES

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We study the quantum Fisher information in one-dimensional models as a tool for measuring the multipartite entanglement, which can give valuable information about the existence of topological phases. We show that the scaling of quantum fisher information of strictly non-local observables can be used for characterizing the phase diagrams and, in particular, for detecting topological phases.

MULTIPARTITE ENTANGLEMENT

Pure state $|\psi\rangle$ of a quantum system with N parts

n–separability

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_n\rangle\,, \qquad n \leq N$$

factorizable in n terms $|\phi_i\rangle$

k–party entanglement

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_m\rangle\,, \quad m \geq N/k$$

each term $|\phi_i\rangle$ involves at most k elements

genuine k-party entanglement	fully separable	maximally entangled
if $\|\psi\rangle$ is not producible by $(k-1)$-party entanglement	1-party entangled (N-separable)	N-party entangled (1-separable)

QUANTUM FISHER INFORMATION

PHASE ESTIMATION

$$\rho \to \rho(\theta) = e^{-i\theta\hat{H}}\rho e^{i\theta\hat{H}}$$

Angle θ to be estimated with m measurements and operators $\{\hat{E}_\mu\}$

Quantum Cramér-Rao bound

$$(\Delta\theta)^2 \geq \frac{1}{mF[\rho(\theta),\{\hat{E}_\mu\}]} \geq \frac{1}{mF_Q[\rho,\hat{H}]}$$

Fisher information

$$F[\rho(\theta),\{\hat{E}_\mu\}] = \sum_\mu \frac{[\partial_\theta P(\mu|\theta)]^2}{P(\mu|\theta)} \qquad P(\mu|\theta) = \mathrm{Tr}[\rho(\theta)\hat{E}_\mu]$$

$P(\mu|\theta)$ probabilities of a measure with value μ given θ

Quantum Fisher information

$$F_Q[\rho,\hat{H}] = \max_{\{\hat{E}_\mu\}} F[\rho(\theta),\{\hat{E}_\mu\}]$$

For pure states: $F_Q[|\psi\rangle,\hat{H}] = 4(\Delta\hat{H})^2 = 4(\langle\hat{H}^2\rangle_\psi - \langle\hat{H}\rangle_\psi^2)$

ENTANGLEMENT CRITERION

Input state with k-party entanglement $\rho_{\text{k-ent}}$

Linear two-mode interferometer:

$$\hat{H}_{\text{lin}} = \frac{1}{2}\sum_j \vec{n}_j \cdot \vec{\sigma}_j = \frac{1}{2}\sum_j \left(\alpha_j\sigma_j^x + \beta_j\sigma_j^y + \gamma_j\sigma_j^z\right)$$

QFI Criterion

$$F_Q[\rho_{\text{k-ent}},\hat{H}_{\text{lin}}] \leq kN$$

QFI density	fully separable	maximally entangled
$f_Q = F_Q/N$	$f_Q[\rho_{\text{sep}},\hat{H}] \leq 1$	$f_Q[\rho_{\max},\hat{H}] \leq N$

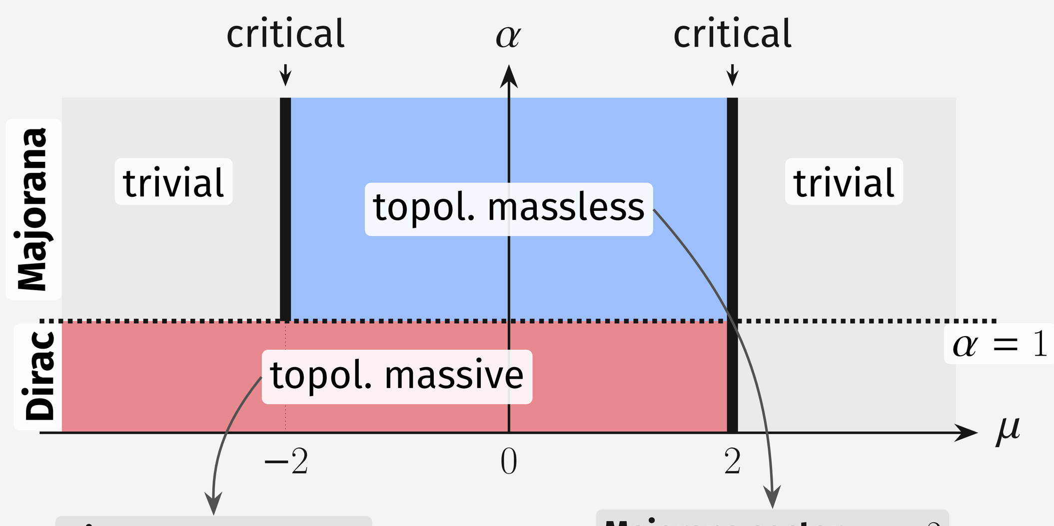
LONG-RANGE KITAEV CHAIN

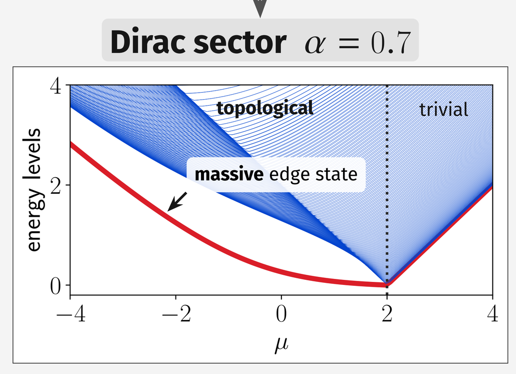
one-dimensional p-wave superconductor

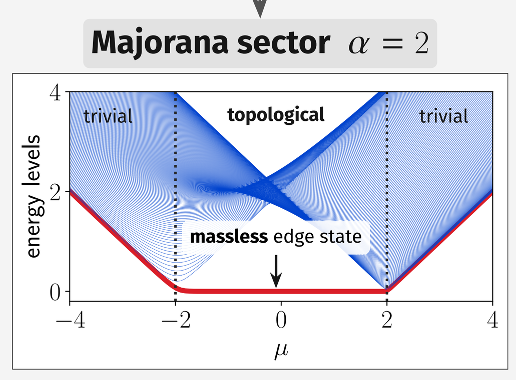
$$H = \sum_i \left[-tc_i^\dagger c_j - \mu\left(c_j^\dagger c_j - \frac{1}{2}\right) + \frac{\Delta}{2}\sum_r \frac{1}{r^\alpha}c_j^\dagger c_{j+r} + \text{h.c.}\right]$$

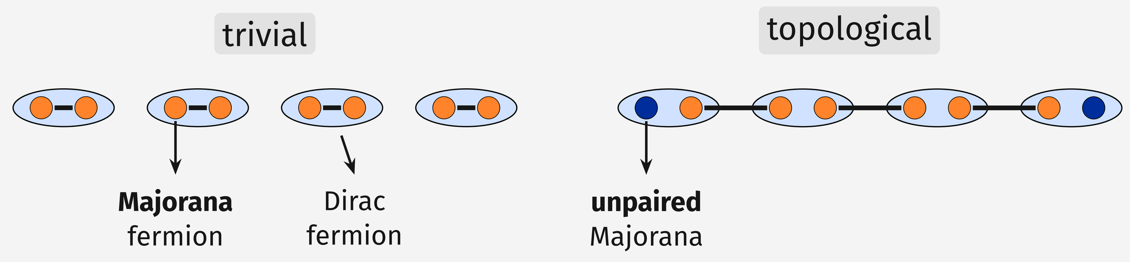
superconducting long-range coupling $\sim 1/r^\alpha$

Jordan-Wigner transformation	non-local operators	QFI from correlators
$\sigma_j^+ = c_j^\dagger e^{i\pi\sum_{i<j}c_i^\dagger c_i}$ $\sigma_j^- = e^{i\pi\sum_{i<j}c_i^\dagger c_i}c_j$	$\hat{H}_\rho = \sum_j \sigma_j^\rho,\quad \rho = x,y$	$f_Q[\|\text{gs}\rangle,\hat{H}_\rho] \sim \sum_{r=1}^L C_\rho(r)$









Correlators from Toeplitz determinants

$$C_\rho(R) = \langle\sigma_1^\rho\sigma_{1+R}^\rho\rangle = \det T_R(\gamma^\rho)$$

$$T_R(\gamma) = \begin{pmatrix} \gamma_0 & \gamma_{-1} & \gamma_{-2} & \cdots & \gamma_{1-N} \\ \gamma_1 & \gamma_0 & \gamma_{-1} & \cdots & \gamma_{2-N} \\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_{3-N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{N-1} & \gamma_{N-2} & \gamma_{N-3} & \cdots & \gamma_{1-N} \end{pmatrix}$$

$$\gamma(\theta) = \sum_n \gamma_n e^{in\theta} \qquad \begin{matrix}\gamma^x = \gamma e^{i\theta} \\ \gamma^y = \gamma e^{-i\theta}\end{matrix}$$

Analytical results

topol. massless	$f_Q \sim L$
topol. massive critical trivial $\alpha < 1$	$f_Q \sim L^{3/4}$
trivial $\alpha > 1$	$f_Q \sim$ const

BILINEAR–BIQUADRATIC CHAIN

Spin-1 chain

$$H = J\sum_i \left[S_i \cdot S_{i+1} + \beta(S_i \cdot S_{i+1})^2\right] = J'\sum_i \left[\cos\theta S_i \cdot S_{i+1} + \sin\theta(S_i \cdot S_{i+1})^2\right]$$

String order parameter	QFI density
$\tilde{S}_j = \left(e^{i\pi\sum_{i<j}S_i^z}\right)S_j,\quad \hat{O} = \sum_j \tilde{S}_j^z$	$f_Q[\|\text{gs}\rangle,\hat{O}] \sim cL^b$