QUANTUM FISHER INFORMATION AND TOPOLOGICAL PHASES

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We study the Quantum Fisher Information (QFI) in one-dimensional models as a tool for measuring the Multipartite Entanglement (ME), which can give valuable information about the existence of topological phases. We show that the scaling of the QFI of strictly non-local observables can be used for characterizing the phase diagrams and, in particular, for detecting topological phases, where it scales maximally.

MULTIPARTITE ENTANGLEMENT

Pure state $|\psi\rangle$ of a quantum system with N parts

n–separability

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_n\rangle\,, \qquad n \leq N$$

factorizable in n terms $|\phi_i\rangle$

k–party entanglement

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_m\rangle\,, \quad m \geq N/k$$

each term $|\phi_i\rangle$ involves at most k elements

genuine k-party entanglement	fully separable	maximally entangled
if $\lvert\psi\rangle$ is not producible by $(k-1)$-party entanglement	1-party entangled (N-separable)	N-party entangled (1-separable)

QUANTUM FISHER INFORMATION

PHASE ESTIMATION

$$\rho \to \rho(\theta) = e^{-i\theta\hat{H}}\rho e^{i\theta\hat{H}}$$

Angle θ to be estimated with m measurements and operators $\{\hat{E}_\mu\}$

Quantum Cramér-Rao bound

$$(\Delta\theta)^2 \geq \frac{1}{mF[\rho(\theta),\{\hat{E}_\mu\}]} \geq \frac{1}{mF_Q[\rho,\hat{H}]}$$

limits the precision of the estimation of θ

Fisher information

$$F[\rho(\theta),\{\hat{E}_\mu\}] = \sum_\mu \frac{[\partial_\theta P(\mu|\theta)]^2}{P(\mu|\theta)} \qquad P(\mu|\theta) = \mathrm{Tr}\left[\rho(\theta)\hat{E}_\mu\right]$$

$P(\mu|\theta)$ conditional probabilities of a measure μ given θ

Quantum Fisher information

$$F_Q[\rho,\hat{H}] = \max_{\{\hat{E}_\mu\}} F[\rho(\theta),\{\hat{E}_\mu\}]$$

maximum of F over the set of possible measures $\{\hat{E}_\mu\}$

For pure states: $F_Q[|\psi\rangle,\hat{H}] = 4(\Delta\hat{H})^2 = 4(\langle\hat{H}^2\rangle_\psi - \langle\hat{H}\rangle_\psi^2)$

ENTANGLEMENT CRITERION

Input state with k-party entanglement $\rho_{\text{k-ent}}$

Linear two-mode interferometer:

$$\hat{H}_{\text{lin}} = \frac{1}{2}\sum_j \vec{n}_j \cdot \vec{\sigma}_j = \frac{1}{2}\sum_j \left(\alpha_j\sigma_j^x + \beta_j\sigma_j^y + \gamma_j\sigma_j^z\right)$$

QFI Criterion

$$F_Q[\rho_{\text{k-ent}},\hat{H}_{\text{lin}}] \leq kN$$

QFI density	fully separable	maximally entangled
$f_Q = F_Q/N$	$f_Q[\rho_{\text{sep}},\hat{H}] \leq 1$	$f_Q[\rho_{\text{max}},\hat{H}] \leq N$

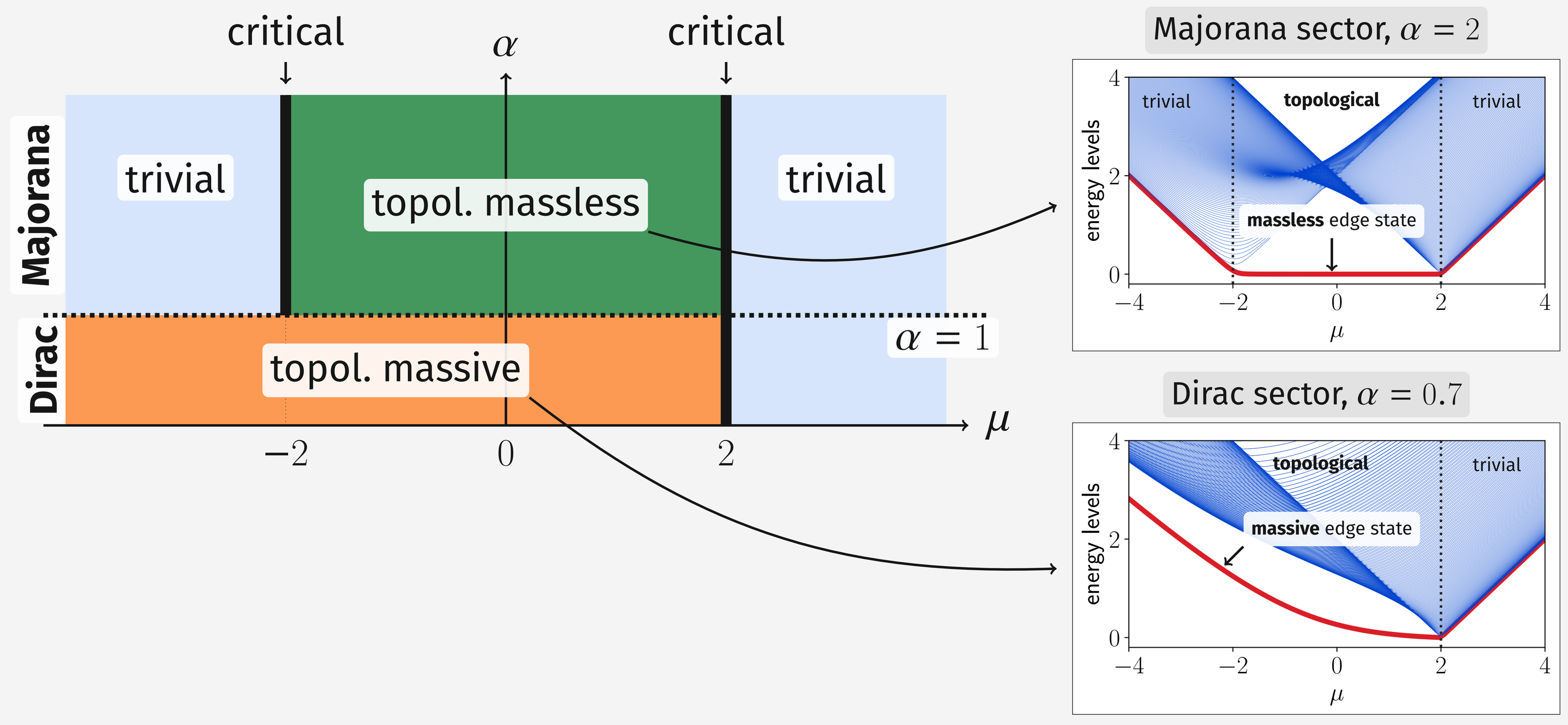
LONG-RANGE KITAEV CHAIN

one-dimensional p-wave superconductor

$$H = \sum_i \left[-tc_i^\dagger c_j - \mu\left(c_j^\dagger c_j - \frac{1}{2}\right) + \frac{\Delta}{2}\sum_r \frac{1}{r^\alpha}c_j^\dagger c_{j+r} + \text{h.c.}\right]$$

superconducting long-range coupling $\sim 1/r^\alpha$

Jordan-Wigner transformation	non-local operators	QFI scaling
$\sigma_j^+ = c_j^\dagger e^{i\pi\sum_{i<j}c_i^\dagger c_i}\quad \sigma_j^- = e^{i\pi\sum_{i<j}c_i^\dagger c_i}c_j$	$\hat{H}_\rho = \sum_j \sigma_j^\rho, \quad \rho = x,y$	$f_Q[\lvert gs\rangle,\hat{H}_\rho] \sim L^b$



BILINEAR–BIQUADRATIC CHAIN

Spin-1 chain

$$H = J\sum_i \left[\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + \beta(\boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1})\right]$$