

QUANTUM FISHER INFORMATION AND TOPOLOGICAL PHASES

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We study the Quantum Fisher Information (QFI) in one-dimensional models as a tool for measuring the Multipartite Entanglement (ME), which can give valuable information about the existence of topological phases. We show that the scaling of the QFI of strictly non-local observables can be used for characterizing the phase diagrams and, in particular, for detecting topological phases, where it scales maximally.

MULTIPARTITE ENTANGLEMENT

Pure state $|\psi\rangle$ of a quantum system with N parts

n-separability

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_n\rangle, \quad n \leq N$$

factorizable in n terms $|\phi_i\rangle$

k-party entanglement

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_m\rangle, \quad m \geq N/k$$

each term $|\phi_i\rangle$ involves at most k elements

genuine k-party entanglement

if $|\psi\rangle$ is not producible
by $(k-1)$ -party entanglement

fully separable

1-party entangled
(N -separable)

maximally entangled

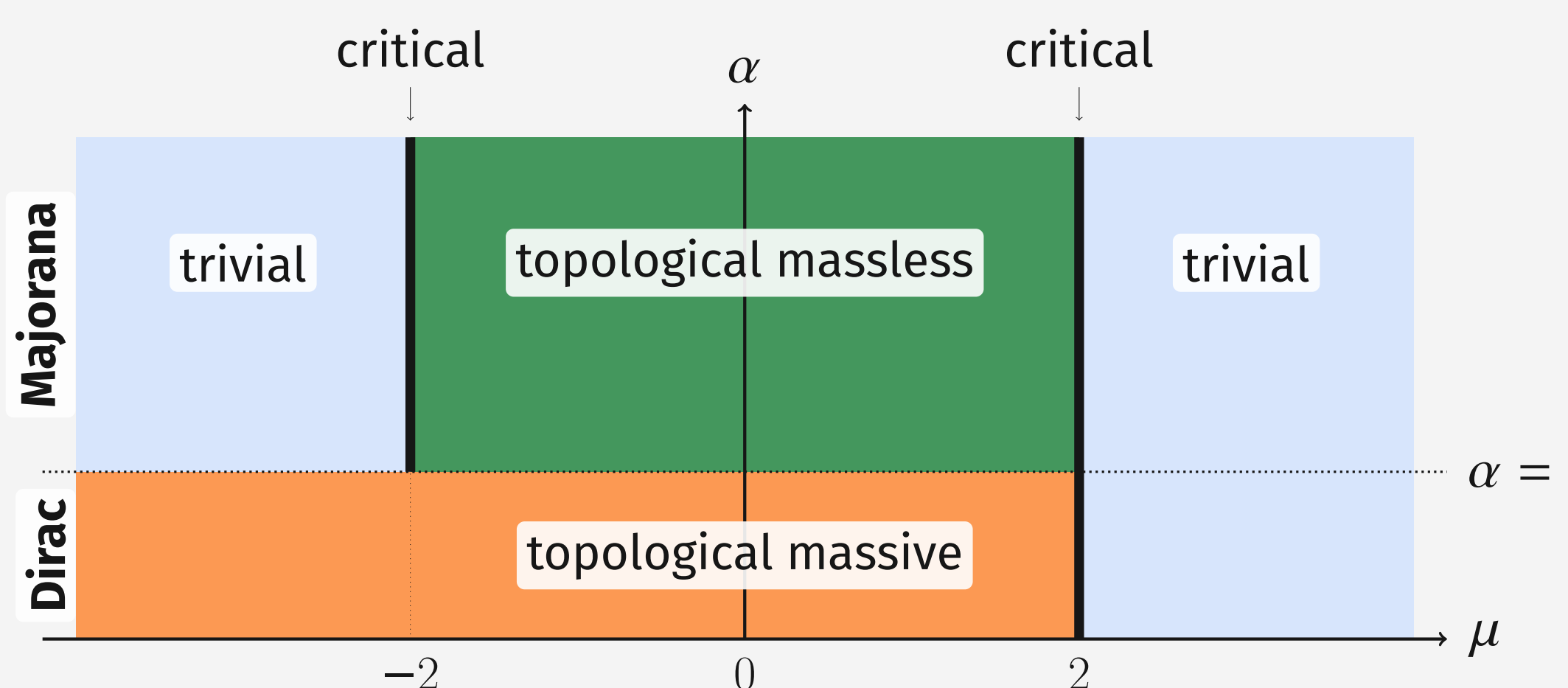
N -party entangled
(1-separable)

LONG-RANGE KITAEV CHAIN

one-dimensional p -wave superconductor

$$H = \sum_i \left[-tc_i^\dagger c_j - \mu \left(c_j^\dagger c_j - \frac{1}{2} \right) + \frac{\Delta}{2} \sum_r \frac{1}{r^\alpha} c_j^\dagger c_{j+r} + \text{h.c.} \right]$$

long-range coupling $\sim 1/r^\alpha$



QUANTUM FISHER INFORMATION

PHASE ESTIMATION

$$\rho \rightarrow \rho(\theta) = e^{-i\theta\hat{H}} \rho e^{i\theta\hat{H}}$$

Angle θ to be estimated with m measurements and operators $\{\hat{E}_\mu\}$

Quantum Cramér-Rao bound

$$(\Delta\theta)^2 \geq \frac{1}{mF[\rho(\theta), \{\hat{E}_\mu\}]} \geq \frac{1}{mF_Q[\rho, \hat{H}]}$$

limits the precision of the estimation of θ

Fisher information

$$F[\rho(\theta), \{\hat{E}_\mu\}] = \sum_\mu \frac{[\partial_\theta P(\mu|\theta)]^2}{P(\mu|\theta)} \quad P(\mu|\theta) = \text{Tr}[\rho(\theta)\hat{E}_\mu]$$

$P(\mu|\theta)$ conditional probabilities of a measure μ given θ

Quantum Fisher information

$$F_Q[\rho, \hat{H}] = \max_{\{\hat{E}_\mu\}} F[\rho(\theta), \{\hat{E}_\mu\}]$$

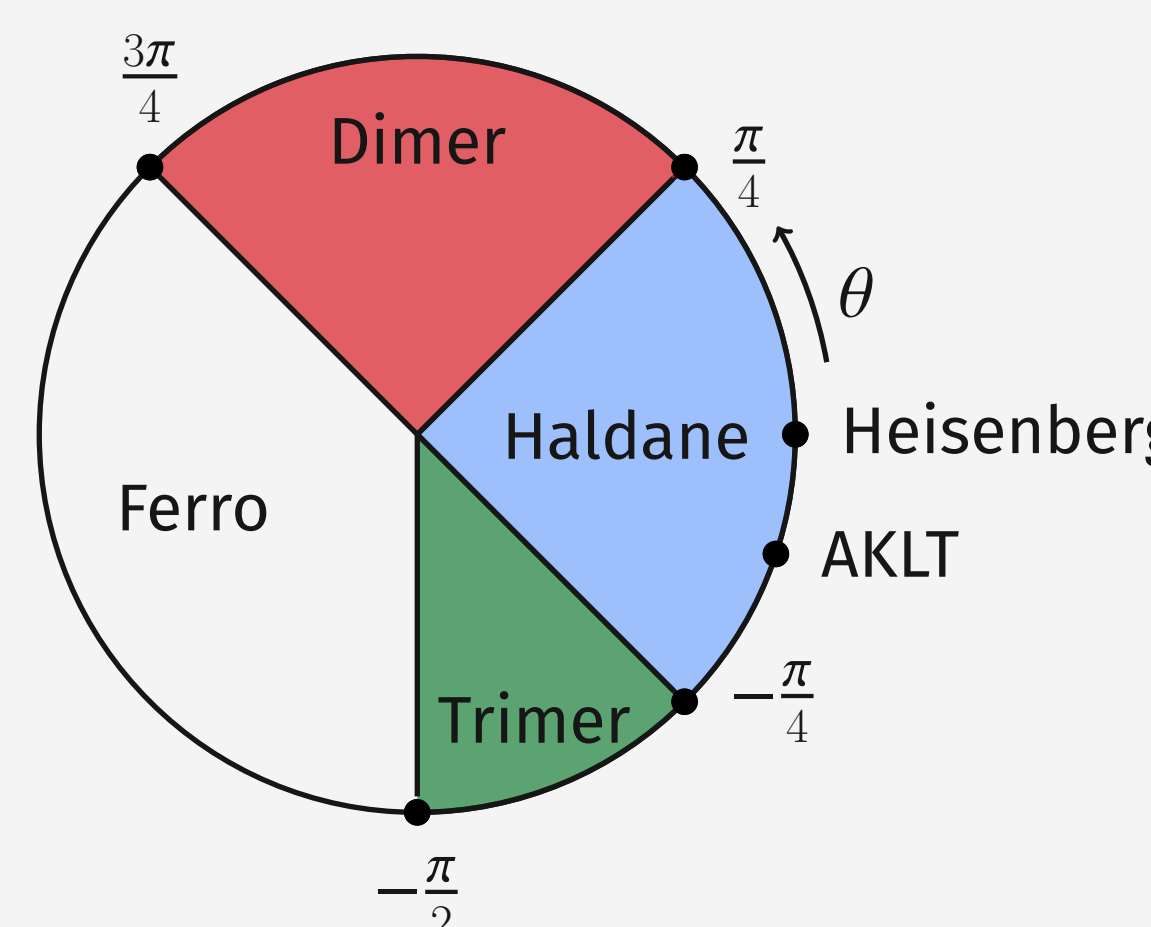
maximum of F over the set of possible measures $\{\hat{E}_\mu\}$

For pure states: $F_Q[|\psi\rangle, \hat{H}] = 4(\Delta\hat{H})^2 = 4(\langle\hat{H}^2\rangle_\psi - \langle\hat{H}\rangle_\psi^2)$

BILINEAR-BIQUADRATIC CHAIN

Spin-1 chain

$$H = J \sum_i [S_i \cdot S_{i+1} + \beta(S_i \cdot S_{i+1})]$$



ENTANGLEMENT CRITERION

Input state with k-party entanglement $\rho_{k\text{-ent}}$

Linear two-mode interferometer:

$$\hat{H}_{\text{lin}} = \frac{1}{2} \sum_j \vec{n}_j \cdot \vec{\sigma}_j = \frac{1}{2} \sum_j \left(\alpha_j \sigma_j^x + \beta_j \sigma_j^y + \gamma_j \sigma_j^z \right)$$

F_Q Criterion

$$F_Q[\rho_{k\text{-ent}}, \hat{H}_{\text{lin}}] \leq kN$$

fully separable

$$F_Q[\rho_{\text{sep}}, \hat{H}] \leq N$$

maximally entangled

$$F_Q[\rho_{\text{max}}, \hat{H}] \leq N^2$$