QUANTUM FISHER INFORMATION AND TOPOLOGICAL PHASES

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We study the quantum Fisher information in one-dimensional models as a tool for measuring the multipartite entanglement, which can give valuable information about the existence of topological phases. We show that the scaling of quantum Fisher information of strictly non-local observables can be used for characterizing the phase diagrams and, in particular, for detecting topological phases.

MULTIPARTITE ENTANGLEMENT

Pure state $|\psi\rangle$ of a quantum system with N parts

n–separability

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_n\rangle\,, \qquad n \leq N$$

factorizable in n terms $|\phi_i\rangle$

k–party entanglement

$$|\psi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \otimes \cdots \otimes |\phi_m\rangle\,, \qquad m \geq N/k$$

each term $|\phi_i\rangle$ involves at most k elements

genuine k-party entanglement

if $|\psi\rangle$ is not producible by $(k-1)$-party entanglement

QUANTUM FISHER INFORMATION

PHASE ESTIMATION

$$\rho \to \rho(\theta) = e^{-i\theta\hat{H}}\rho e^{i\theta\hat{H}}$$

Angle θ to be estimated with m measurements and operators $\{\hat{E}_\mu\}$

Quantum Cramér-Rao bound

$$(\Delta\theta)^2 \geq \frac{1}{mF[\rho(\theta),\{\hat{E}_\mu\}]} \geq \frac{1}{mF_Q[\rho,\hat{H}]}$$

Fisher information

$$F[\rho(\theta),\{\hat{E}_\mu\}] = \sum_\mu \frac{[\partial_\theta P(\mu|\theta)]^2}{P(\mu|\theta)}, \qquad P(\mu|\theta) = \mathrm{Tr}[\rho(\theta)\hat{E}_\mu]$$

$P(\mu|\theta)$ probabilities of a measure with value μ given θ

Quantum Fisher information

$$F_Q[\rho,\hat{H}] = \max_{\{\hat{E}_\mu\}} F[\rho(\theta),\{\hat{E}_\mu\}]$$

For pure states: $F_Q[|\psi\rangle,\hat{H}] = 4(\Delta\hat{H})^2 = 4(\langle\hat{H}^2\rangle_\psi - \langle\hat{H}\rangle_\psi^2)$

ENTANGLEMENT CRITERION

Input state with k-party entanglement $\rho_{\text{k-ent}}$

QFI Criterion

$$f_Q[\rho_{\text{k-ent}},\hat{H}_{\text{lin}}] \leq k \quad \text{where} \quad \hat{H}_{\text{lin}} = \frac{1}{2}\sum_j \vec{n}_j \cdot \vec{\sigma}_j$$

QFI density	fully separable	maximally entangled
$f_Q = F_Q/N$	$f_Q[\rho_{\text{sep}},\hat{H}_{\text{lin}}] \leq 1$	$f_Q[\rho_{\text{max}},\hat{H}_{\text{lin}}] \leq N$

REFERENCES

- Pezzè, Smerzi "*Quantum theory of phase estimation*" arXiv:1411.5164 (2014)
- Vodola et al. "*Kitaev chains with long-range pairing*" PRL 113, 156402 (2014)
- Pezzè et al. "*Multipartite Entanglement in Topological Quantum Phases*" PRL 119, 250401 (2017)
- Kennedy, Tasaki "*Hidden symmetry breaking and the Haldane phase in $S=1$ quantum spin chains*" Commun. Math. Phys 147 431-484 (1992)

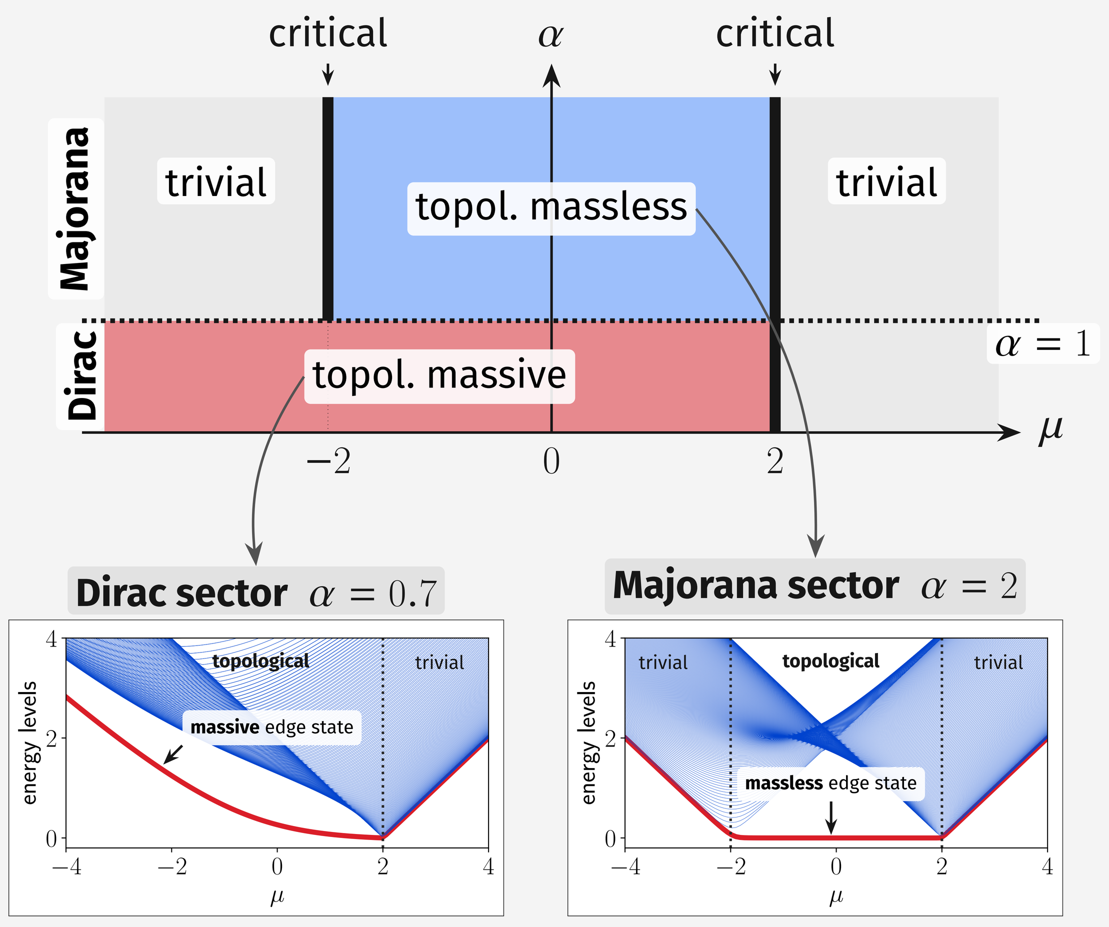
LONG-RANGE KITAEV CHAIN

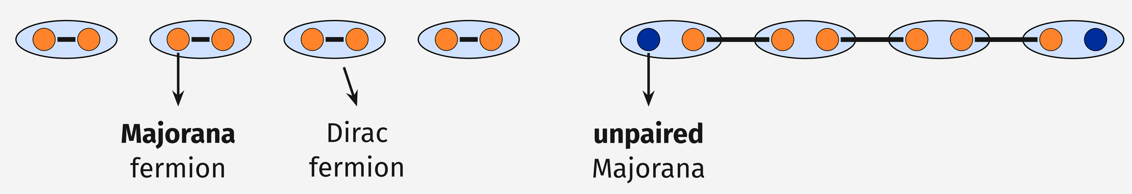
one-dimensional p-wave superconductor

$$H = \sum_i \left[-tc_i^\dagger c_j - \mu\left(c_j^\dagger c_j - \frac{1}{2}\right) + \frac{\Delta}{2}\sum_r \frac{1}{r^\alpha} c_j^\dagger c_{j+r} + \text{h.c.}\right]$$

superconducting long-range coupling $\sim 1/r^\alpha$

Jordan–Wigner transformation	non-local operators	QFI from correlators
$\sigma_j^+ = c_j^\dagger e^{i\pi\sum_{i<j} c_i^\dagger c_i} \quad \sigma_j^- = e^{i\pi\sum_{i<j} c_i^\dagger c_i} c_j$	$\hat{H}_\rho = \sum_j \sigma_j^\rho, \quad \rho = x, y$	$f_Q[\lvert\text{gs}\rangle, \hat{H}_\rho] \sim \sum_{r=1}^L C_\rho(r)$





Correlators from Toeplitz determinants

$$C_\rho(R) = \langle\sigma_1^\rho\sigma_{1+R}^\rho\rangle = \det T_R(\gamma^\rho)$$

$$T_R(\gamma) = \begin{pmatrix} \gamma_0 & \gamma_{-1} & \gamma_{-2} & \cdots & \gamma_{1-N} \\ \gamma_1 & \gamma_0 & \gamma_{-1} & \cdots & \gamma_{2-N} \\ \gamma_2 & \gamma_1 & \gamma_0 & \cdots & \gamma_{3-N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma_{N-1} & \gamma_{N-2} & \gamma_{N-3} & \cdots & \gamma_{1-N} \end{pmatrix}$$

$$\gamma(\theta) = \sum_n \gamma_n e^{in\theta} \qquad \begin{aligned}\gamma^x &= \gamma e^{i\theta} \\ \gamma^y &= \gamma e^{-i\theta}\end{aligned}$$

the γ_ns are computed directly from the couplings in H

Analytical results from Toeplitz determinants

The scaling of f_Q depends only on the topological properties of the function $\gamma(\theta)$.

topological massless	$f_Q \sim L$
topological massive critical trivial $\alpha < 1$	$f_Q \sim L^{3/4}$
trivial $\alpha > 1$	$f_Q \sim \text{const}$

Reproduces the numerical results from [Pezzè 2017]

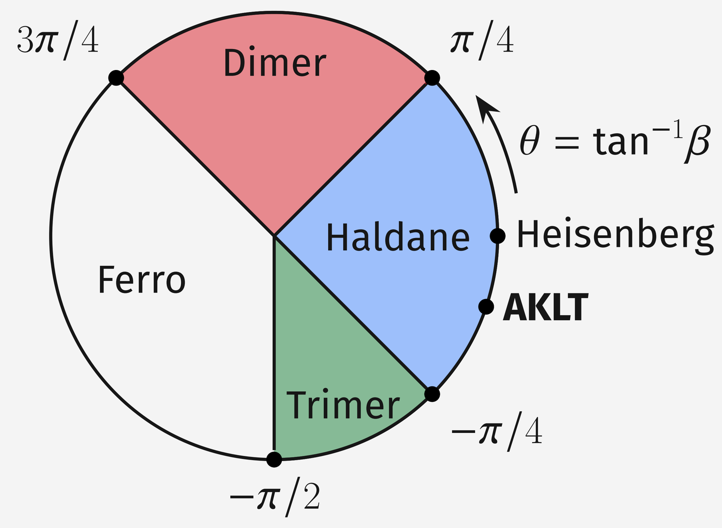
BILINEAR–BIQUADRATIC CHAIN

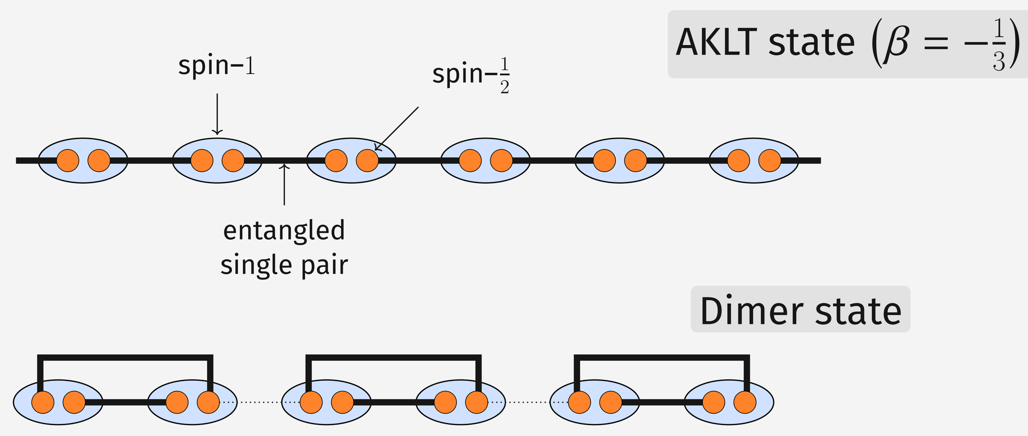
Spin-1 chain

$$H = J\sum_i \left[\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1} - \beta(\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1})^2\right] = J'\sum_i \left[\cos\theta\,\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1} - \sin\theta(\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+1})^2\right]$$

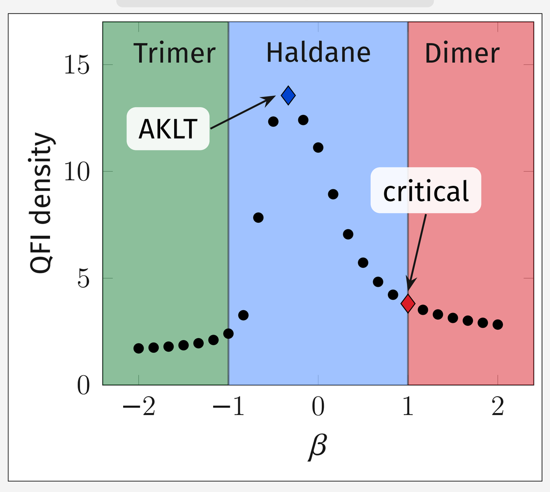
most general $SU(2)$–invariant isotropic spin-1 Hamiltonian

String order parameter	QFI density
$\tilde{S}_j^z = \left(e^{i\pi\sum_{i<j} S_i^z}\right) S_j^z, \quad \hat{O} = \sum_j \tilde{S}_j^z$	$f_Q[\lvert\text{gs}\rangle, \hat{O}] \sim cL^b$



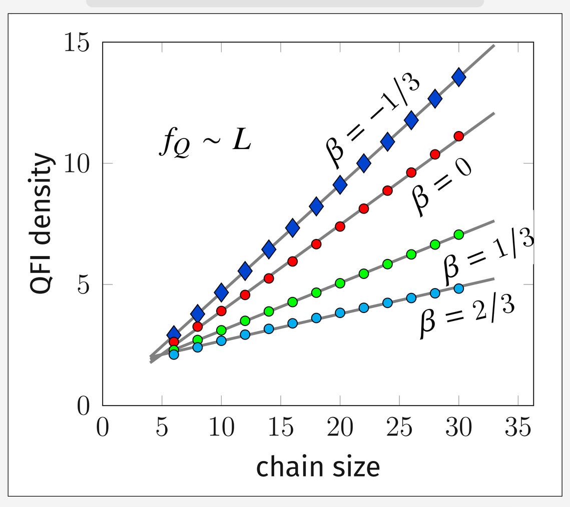


QFI density vs β



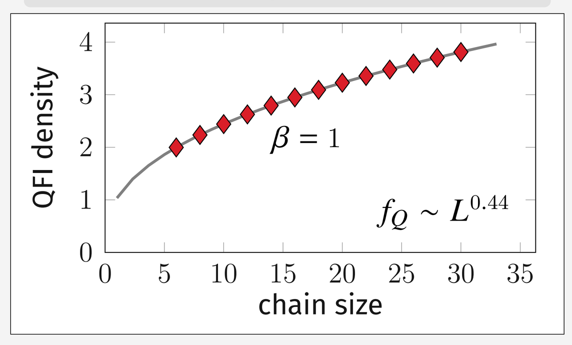
f_Q grows inside the Haldane phase and is maximum on the AKLT point

Inside Haldane phase



f_Q scales *linearly* inside the Haldane *topological* phase

Critical point $\beta = 1$ $(\theta = \pi/4)$



f_Q scales with a *power-law* for a quantum phase transition point