

Type II Seesaw and Inelastic Dark Matter

from Scalar Triplet

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1 Inelastic Dark Matter

The lepton number conservation in SM leads to the masses of neutrinos being exactly zero at all orders. However, oscillation experiments confirmed that neutrinos mix among themselves. A minimal explanation is incorporating a heavy triplet scalar Δ to the SM of particle physics. The lepton number violating $\Delta L = 2$ interaction of Δ with SM given by the following Lagrangian

$$\mathcal{L} \supset M_{\Delta}^2 \Delta^{\dagger} \Delta + \frac{1}{\sqrt{2}} (\mu_H \Delta^{\dagger} H H + f_{\alpha\beta} \Delta L_{\alpha} L_{\beta} + h.c.)$$

where H and L are the SM Higgs and lepton doublets respectively. After the electroweak symmetry breaking phase transition, Δ acquires a small induced

vev

use \langle \rangle and \langle \rangle, not < and >

$$\langle \Delta \rangle = \mu_H \frac{v^2}{\sqrt{2}M_\Delta^2}$$

where v is the SM Higgs vev 246 GeV. The vev of Δ needs to satisfy condition

that for $x = \frac{\langle \Delta \rangle}{v}$ the ρ parameter

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta} = \frac{1 + 2x^2}{1 + 4x^2} \approx 1$$

hence $\langle \Delta \rangle$ must be smaller than $\mathcal{O}(1)$ GeV. The second term then gives

rise to Majorana mass matrix for the three flavours of light neutrinos you mean the second term of the Lagrangian? be specific.

$$(M_\nu)_{\alpha\beta} = \sqrt{2}f_{\alpha\beta} \langle \Delta \rangle = f_{\alpha\beta} \left(\frac{-\mu_H v^2}{M_\Delta^2} \right)$$

use \left(and \right) to get correct-sized brackets automatically

hence wide range of $f_{\alpha\beta}$ gives rise to neutrino masses. For f on $\mathcal{O}(1)$, the neutrino masses corresponds to M_Δ on the order of 10^{12} GeV. The lepton number violation is therefore in very high scale.

The above Lagrangian is extended by including a Inert Scalar Doublet (SDDM) $\chi = (\chi^+ \chi^0)^T$ and impose a \mathbb{Z}_2 symmetry odd on χ and even on SM fields. you need to state that it's an SU(2) doublet State the \mathbb{Z}_2 charge of the Delta too. The potential part is

$$\begin{aligned} V(\Delta, H, \chi) = & M_\Delta^2 \Delta^\dagger \Delta + \lambda_\Delta (\Delta^\dagger \Delta)^2 \\ & + M_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + M_\chi^2 \chi^\dagger \chi + \lambda_\chi (\chi^\dagger \chi)^2 \\ & + [\mu_H \Delta^\dagger H H + \mu_\chi \Delta^\dagger \chi \chi + h.c.] + \lambda_3 |H|^2 |\chi|^2 + \lambda_4 |H^\dagger \chi|^2 \\ & + \frac{\lambda_5}{2} [(H^\dagger \chi)^2 + h.c.] \end{aligned}$$

I think you probably need to explain explicitly how the contraction over SU(2) indices works for these trilinear couplings. It is probably also worth pointing out earlier that H is the SU(2) Higgs doublet field before EWSB; then you can explain these couplings in terms of the physical Higgs field h and the higgs vev.

where we have neglected the quartic terms involving $\Delta H \chi$ because they are not relevant here due to small vev of Δ . The vacuum stability of the potential requires $\lambda_\Delta, \lambda_H, \lambda_\chi > 0$ and $\lambda_L = \lambda_3 + \lambda_4 - |\lambda_5| > -2\sqrt{\lambda_\chi \lambda_H}$. It was also assumed that $M_\chi^2 > 0$ such that χ does not develop any vev. Then DM is given by the neutral component of the doublet χ .

If replacing χ by vector like fermion doublet $\psi = (\psi_{DM}, \psi_-)$ of hypercharge $Y = -\frac{1}{2}$ and keeping same \mathbb{Z}_2 odd symmetry, the neutral component of ψ is the DM candidate. The Lagrangian includes

$$-\mathcal{L} \supset M_\Delta^2 \Delta^\dagger \Delta + M_D \bar{\psi} \psi + \frac{1}{\sqrt{2}} [\mu_H \Delta^\dagger H H + f_{\alpha\beta} \Delta L_\alpha L_\beta + g \Delta \psi \psi + h.c.]$$

where M_D is on the order $\mathcal{O}(100)$ GeV is the Dirac mass of ψ .

The low-energy spectrum of the SDDM theory constitutes two charged scalars χ^\pm , two real scalars, h , S , and a pseudo scalar A . The masses are given by

$$M_{\chi^\pm} = M_\chi^2 + \lambda_3 \frac{v^2}{2}$$

$$M_h^2 = 2\lambda_H v^2$$

$$M_S = M_\chi^2 + (\lambda_3 + \lambda_4 + \lambda_5) \frac{v^2}{2}$$

$$M_A = M_\chi^2 + (\lambda_3 + \lambda_4 - \lambda_5) \frac{v^2}{2}$$

you should show how these expressions (except the Higgs one) arise

Depending on the sign of λ_5 either S or A constitutes the DM. Assuming λ_5 being negative, then S is the lightest scalar and the next to lightest is A. Their mass difference $\Delta M^2 = M_S^2 - M_A^2 = \lambda_5 v^2$, making the relationship have you defined delta as the mass splitting yet? between δ and parameters in the model is

$$\lambda_5 = \frac{2M_S\delta}{v^2}$$

where λ_5 is one of the coupling strength of four-vertex between χ and Higgs.

Direct detection of dark matter relies on detection of nuclear recoils induced by dark matter scattering elastically or inelastically inside detection region. For inert scalar doublet dark matter (SDDM), our DM candidates you already said this above are the neutral components of SU(2) doublet scalar field

$$\chi = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix} \tag{1}$$

obeying an ad-hoc \mathbb{Z}_2 odd symmetry. As a result, the only non-zero terms relevant for direct detection, at leading order, are the t-channel Z exchange between dark matter candidates and SM particles. Since typical order of magnitude of momentum exchange q is small compared to Z boson mass, the scattering process can be translated into effective vertex of contact interaction between dark matter current and SM vector and axial vector current, denoted

by operator $O_{1,f}^{(6)}$ and $O_{2,f}^{(6)}$ with constant Wilson coefficient at LO, as shown previously,

$$\hat{O}_{1,f}^{(6)} = (\chi^{0*} i \overleftrightarrow{\partial}_\mu \chi^0) (\bar{f} \gamma^\mu f) \quad (2)$$

and

$$\hat{O}_{2,f}^{(6)} = (\chi^{0*} i \overleftrightarrow{\partial}_\mu \chi^0) (\bar{f} \gamma^\mu \gamma_5 f) \quad (3)$$

The mediator of the above interactions is the Z boson. To show that it is possible that elastic DM scattering being suppressed so only inelastic scattering appears, we expand the field χ^0 as its real and imaginary part, two real scalar fields, S and A

$$\chi^0 = \frac{S + iA}{\sqrt{2}} \quad (4)$$

and expand out the covariant derivative $D_\mu = \partial_\mu + i(g_2 T^3 \cos \theta - g_1 Y \mathbb{1} \sin \theta) Z_\mu + \dots$, where other tems inside the covariant derivatives are ^{why?} ignored, T^3 and Y are the third component of electroweak generator and hyper-charge respectively, g_2 is the SU(2) coupling, g_1 is the U(1) coupling, and θ is the Weinberg

angle. Keeping only the relevant terms, we get

$$\begin{aligned}
\mathcal{L}_{UV} &\supset (D_\mu \chi)^\dagger (D^\mu \chi) \\
&\supset (\partial_\mu \chi)^\dagger i Z_\mu (g_2 T^3 \cos \theta - g_1 Y \mathbb{1} \sin \theta) \chi + h.c. \\
&\supset (\partial_\mu \chi^0)^* \left(-\frac{1}{2} g_2 \cos \theta - \frac{1}{2} g_1 \sin \theta \right) \chi^0 + c.c. \\
&= -i Z_\mu \frac{1}{2} \frac{g_2}{\cos \theta} (\partial_\mu \chi^0)^* \chi^0 + c.c. \\
&= -i Z_\mu \frac{1}{2} \frac{g_2}{\cos \theta} \left(\partial_\mu \frac{S - iA}{\sqrt{2}} \right) \frac{S + iA}{\sqrt{2}} + c.c. \\
&= -i Z_\mu \frac{1}{4} \frac{g_2}{\cos \theta} ((\partial_\mu (S - iA)(S + iA)) - (\partial_\mu (S + iA)(S - iA))) \\
&= Z_\mu \frac{1}{2} \frac{g_2}{\cos \theta} ((\partial_\mu S)A - (\partial_\mu A)S)
\end{aligned} \tag{5}$$

where in the third line we used hypercharge value $1/2$ for χ . Explicitly state here that because there are no ZAA or ZSS terms in this expression, there is no elastic scattering.

In scalar triplet model, if coupling λ_5 is non-zero, then the mass splitting between S and A will be $\delta \neq 0$. Then depending on the sign of λ_5 , the lightest scalar particle will be either S or A, you already said this as the dark matter candidate. To find the Wilson coefficients, Eq.(2) and Eq.(3) can be matched to Eq.(5) in the similar procedure as G_F in the 4-Fermi theory,

In a similar fashion, inelastic FDDM in triplet model assumes a hypercharge of $-\frac{1}{2}$ for a vector like fermion doublet

$$\psi = \begin{pmatrix} \psi_{DM} \\ \psi_- \end{pmatrix}$$

obeying an \mathbb{Z}_2 ~~odd~~ symmetry. As a result, for direct detection, the relevant non-zero terms, at leading order, are the t-channel Z exchange between dark matter candidates and SM quarks. In ~~contrast?~~ **contract with SDDM**, the mass splitting m is provided by an induced vev $\langle \Delta^0 \rangle$, of the neutral component of the scalar triplet,

$$\Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \Delta^{++} & -\frac{\Delta^0}{\sqrt{2}} \end{pmatrix}$$

after EW symmetry breaking. This can be seen from that in FDDM, the $\Delta\psi\psi$ coupling term is defined as

$$\begin{aligned} \frac{1}{\sqrt{2}}g\bar{\psi}^c i\tau_2 \Delta\psi = & -\frac{1}{2}g[\sqrt{2}(\bar{\psi}_-^c \psi_- \Delta^{++}) \\ & + (\bar{\psi}_-^c \psi_{DM} + \bar{\psi}_{DM}^c \psi_-)\Delta^+ - \sqrt{2}(\bar{\psi}_{DM}^c \psi_{DM})\Delta^0] \end{aligned}$$

where g is coupling strength and τ_2 is $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. From this we see the induced vev $\langle \Delta^0 \rangle$ gives rise to a small Majorana mass to ψ , $m = \sqrt{2}g \langle \Delta^0 \rangle$.

The Dirac spinor ψ_{DM} can be written as sum of two Majorana spinors $(\psi_{DM})_L$ and $(\psi_{DM})_R$. Suppose a Dirac mass of order 100GeV, this small Majorana mass of order 100keV, the Lagrangian is written as

$$\begin{aligned} \mathcal{L}_{DMmass} = & M_D[(\bar{\psi}_{DM})_L(\psi_{DM})_R + (\bar{\psi}_{DM})_R(\psi_{DM})_L] \\ & + m[(\bar{\psi}_{DM})_L^c(\psi_{DM})_L + (\bar{\psi}_{DM})_R^c(\psi_{DM})_R] \end{aligned} \tag{6}$$

which can be written in terms of matrix form as

$$\mathcal{L}_{DMmass} = \begin{pmatrix} \overline{(\psi_{DM})_R} & \overline{(\psi_{DM})_L} \end{pmatrix} \begin{pmatrix} M_D & m \\ m & M_D \end{pmatrix} \begin{pmatrix} (\psi_{DM})_L \\ (\psi_{DM})_R \end{pmatrix}$$

The mass matrix for ψ_{DM} is then $\begin{pmatrix} M_D & m \\ m & M_D \end{pmatrix}$ in the basis of $\{(\psi_{DM})_L, (\psi_{DM})_R\}$.

Diagonalising it, the mass eigenstates are

$$\psi_1 = \frac{i}{\sqrt{2}}((\psi_{DM})_L - (\psi_{DM})_R)$$

$$\psi_2 = \frac{1}{\sqrt{2}}((\psi_{DM})_L + (\psi_{DM})_R)$$

with Majorana fermion mass eigenvalues $M_{\psi_1} = M_D - m$ and $M_{\psi_2} = M_D + m$.

The mass difference between the two states $\delta = 2m$ is of the order 100 keV. How do we know this? from $\langle \Delta \rangle$. explain.

After expansion of the vector current $\bar{\psi}\gamma_\mu\psi$, it can be shown [01010138

]that in this case the inelastic scattering of DM with nucleons $(\psi_{DM})_1 N \rightarrow$

$(\psi_{DM})_2 N$) via SM Z-exchange dominates the scattering process. why does it dominate in this case?