$$p(x_n \mid Z,\pi) = \pi \boxed{N(x_n \mid Z,\tau^2)} + (1-\pi)\boxed{U(x_n \mid Z_{\min}, Z_{\max})}$$

x_n 是观察到深度
Z是真实深度
π 是内点概率
τ_n 一个像素单位对应的深度
$Z_{\min}, Z_{\max}$ 深度范围

- 高斯分布建模内点，均匀分布建模外点

$$
\begin{aligned}
p(Z,\pi \mid x_1,\ldots,x_n) \\
&= \frac{p(x_1,\ldots,x_n \mid Z,\pi)\, p(Z,\pi)}{p(x_1,\ldots,x_n)} \\
&= \frac{p(x_1,\ldots,x_{n-1}, Z,\pi)\, p(x_n \mid Z,\pi)}{p(x_1,\ldots,x_n)} \\
&= \frac{p(Z,\pi \mid x_1,\ldots,x_{n-1})\, p(x_1,\ldots,x_{n-1})\, p(x_n \mid Z,\pi)}{p(x_1,\ldots,x_n)} \\
&\propto p(Z,\pi \mid x_1,\ldots,x_{n-1})\, p(x_n \mid Z,\pi)
\end{aligned}
$$

$$p(Z,\pi \mid \overset{a_n,b_n,\mu_n,\sigma_n}{\cancel{x_1,\dots,x_n}})$$

$$\propto p(Z,\pi \mid \underset{a_{n-1},b_{n-1},\mu_{n-1},\sigma_{n-1}}{\cancel{x_1,\dots,x_{n-1}}})\, p(x_n \mid Z,\pi)$$

$$p(Z,\pi \mid a_n,b_n,\mu_n,\sigma_n) = \mathrm{N}(\mathrm{Z} \mid \mu_n,\sigma_n^2)\,\mathrm{Beta}(\pi \mid a_n,b_n)$$

通过矩比较法进行参数估计

Z的一阶距和二阶距

$$\begin{aligned}
\int Zp(Z,\pi \mid a_n,b_n,\mu_n,\sigma_n)\,\mathrm{dZd}\pi &= \int \mathrm{ZN}(\mathrm{Z} \mid \mu_n,\sigma_n^2)\,\mathrm{Beta}(\pi \mid a_n,b_n)\,\mathrm{dZd}\pi \\
&= \int \mathrm{ZN}(\mathrm{Z} \mid \mu_n,\sigma_n^2)\,\mathrm{dZ} \\
&= \mu_n
\end{aligned}$$

$$\begin{aligned}
\int Z^2 p(Z,\pi \mid a_n,b_n,\mu_n,\sigma_n)\,\mathrm{dZd}\pi &= \int \mathrm{ZN}(\mathrm{Z} \mid \mu_n,\sigma_n^2)\,\mathrm{Beta}(\pi \mid a_n,b_n)\,\mathrm{dZd}\pi \\
&= \int \mathrm{Z}^2\mathrm{N}(\mathrm{Z} \mid \mu_n,\sigma_n^2)\,\mathrm{dZ} \\
&= \mu_n^2 + \sigma_n^2
\end{aligned}$$

π的一阶距和二阶距

$$\int \pi p(Z,\pi \mid a_n,b_n,\mu_n,\sigma_n)\,\mathrm{dZd}\pi = \int \pi \mathrm{N}(Z \mid \mu_n,\sigma_n^2)\,\mathrm{Beta}(\pi \mid a_n,b_n)\,\mathrm{dZd}\pi$$

$$= \int \pi\, \mathrm{Beta}(\pi \mid a_n,b_n)\,\mathrm{d}\pi$$

$$= \frac{a_n}{a_n+b_n}$$

$$\int \pi^2 p(Z,\pi \mid a_n,b_n,\mu_n,\sigma_n)\,\mathrm{dZd}\pi = \int \pi^2\, \mathrm{Beta}(\pi \mid a_n,b_n)\,\mathrm{d}\pi$$

$$= \frac{\Gamma(a_n+b_n)}{\Gamma(a_n)\Gamma(b_n)}\frac{\Gamma(a_n+2)\Gamma(b_n)}{\Gamma(a_n+b_n+2)}$$

$$= \frac{a_n(a_n+1)}{(a_n+b_n)(a_n+b_n+1)}$$

$$p(Z,\pi \mid a_{n-1},b_{n-1},\mu_{n-1},\sigma_{n-1})\,p(x_n \mid Z,\pi)$$
$$=(\pi \mathrm{N}(x_n \mid Z,\tau^2)+(1-\pi)\,\mathrm{U}(x_n \mid Z_{\min},Z_{\max}))\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1})$$
$$=\frac{a_{n-1}}{a_{n-1}+b_{n-1}}\mathrm{N}(x_n \mid Z,\tau^2)\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}\mathrm{U}(x_n \mid Z_{\min},Z_{\max})\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)$$
$$=\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1\mathrm{N}(Z \mid m,s^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)$$

$$C=\int p(Z,\pi \mid a_{n-1},b_{n-1},\mu_{n-1},\sigma_{n-1})\,p(x_n \mid Z,\pi)\,dZd\pi$$
$$=\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2$$

$$C_1=\frac{1}{2\pi(\tau^2+\sigma_{n-1}^2)}\exp\{-\frac{x_n^2}{2\tau^2}-\frac{\mu_{n-1}^2}{2\sigma_{n-1}^2}+\frac{m^2}{2s^2}\}$$

$$C_2=\mathrm{U}(x_n \mid Z_{\min},Z_{\max})$$

Z的一阶距和二阶距

$$\frac{1}{C}\int Z\left\{\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1\mathrm{N}(Z \mid m,s^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)\right\}d\,Zd\pi$$
$$=\frac{1}{C}\{\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1 m+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2\mu_{n-1}\}$$

$$\frac{1}{C}\int Z^2\left\{\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1\mathrm{N}(Z \mid m,s^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)\right\}d\,Zd\pi$$
$$=\frac{1}{C}\{\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1(m^2+s^2)+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2(\mu_{n-1}^2+\sigma_{n-1}^2)\}$$

$$p(Z,\pi \mid a_{n-1},b_{n-1},\mu_{n-1},\sigma_{n-1})p(x_n \mid Z,\pi)$$

$$=(\pi \mathrm{N}(x_n \mid Z,\tau^2)+(1-\pi)\mathrm{U}(x_n \mid Z_{\min},Z_{\max}))\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1})$$

$$=\frac{a_{n-1}}{a_{n-1}+b_{n-1}}\mathrm{N}(x_n \mid Z,\tau^2)\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}\mathrm{U}(x_n \mid Z_{\min},Z_{\max})\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)$$

$$=\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1\mathrm{N}(Z \mid m,s^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)$$

$$C=\int p(Z,\pi \mid a_{n-1},b_{n-1},\mu_{n-1},\sigma_{n-1})p(x_n \mid Z,\pi)dZd\pi$$

$$=\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2$$

$$C_1=\frac{1}{2\pi(\tau^2+\sigma_{n-1}^2)}\exp\{-\frac{x_n^2}{2\tau^2}-\frac{\mu_{n-1}^2}{2\sigma_{n-1}^2}+\frac{m^2}{2s^2}\}$$

$$C_2=\mathrm{U}(x_n \mid Z_{\min},Z_{\max})$$

π的一阶距和二阶距

$$\frac{1}{C}\int \pi\left\{\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1\mathrm{N}(Z \mid m,s^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)\right\}d\,Zd\pi$$

$$=\frac{1}{C}\{\frac{a_{n-1}(a_{n-1}+1)}{(a_{n-1}+b_{n-1})(a_{n-1}+b_{n-1}+1)}C_1m+\frac{b_{n-1}a_{n-1}}{(a_{n-1}+b_{n-1})(a_{n-1}+b_{n-1}+1)}C_2\mu_{n-1}\}$$

$$\frac{1}{C}\int \pi^2\left\{\frac{a_{n-1}}{a_{n-1}+b_{n-1}}C_1\mathrm{N}(Z \mid m,s^2)\,\mathrm{Beta}(\pi \mid a_{n-1}+1,b_{n-1})+\frac{b_{n-1}}{a_{n-1}+b_{n-1}}C_2\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)\,\mathrm{Beta}(\pi \mid a_{n-1},b_{n-1}+1)\right\}d\,Zd\pi$$

$$=\frac{1}{C}\{\frac{a_{n-1}(a_{n-1}+1)(a_{n-1}+2)}{(a_{n-1}+b_{n-1})(a_{n-1}+b_{n-1}+1)(a_{n-1}+b_{n-1}+2)}C_1m+\frac{b_{n-1}a_{n-1}(a_{n-1}+1)}{(a_{n-1}+b_{n-1})(a_{n-1}+b_{n-1}+1)(a_{n-1}+b_{n-1}+2)}C_2\mu_{n-1}\}$$

$$\int \mathrm{N}(x_n \mid Z,\tau^2)\mathrm{N}(Z \mid \mu_{n-1},\sigma_{n-1}^2)dZ$$

$$= \frac{1}{4\pi^2\tau^2\sigma_{n-1}^2}\int \exp\{-\frac{(x_n - Z)^2}{2\tau^2} - \frac{(Z-\mu_{n-1})^2}{2\sigma_{n-1}^2}\}dZ$$

$$= \frac{1}{4\pi^2\tau^2\sigma_{n-1}^2}\int \exp\{-\frac{\sigma_{n-1}^2(x_n - Z)^2 + \tau^2(Z-\mu_{n-1})^2}{2\tau^2\sigma_{n-1}^2}\}dZ$$

$$= \frac{1}{4\pi^2\tau^2\sigma_{n-1}^2}\int \exp\{-\frac{(\sigma_{n-1}^2+\tau^2)Z^2 - 2(\sigma_{n-1}^2 x_n + \tau^2\mu_{n-1})Z + \sigma_{n-1}^2 x_n^2 + \tau^2\mu_{n-1}^2}{2\tau^2\sigma_{n-1}^2}\}dZ$$

$$= \frac{1}{4\pi^2\tau^2\sigma_{n-1}^2}\int \exp\{-\frac{(\sigma_{n-1}^2+\tau^2)Z^2 - 2(\sigma_{n-1}^2 x_n + \tau^2\mu_{n-1})Z + \sigma_{n-1}^2 x_n^2 + \tau^2\mu_{n-1}^2}{2\tau^2\sigma_{n-1}^2}\}dZ$$

$$= \frac{1}{4\pi^2\tau^2\sigma_{n-1}^2}\exp\{-\frac{x_n^2}{2\tau^2} - \frac{\mu_{n-1}^2}{2\sigma_{n-1}^2}\}\int \exp\{-\frac{(\sigma_{n-1}^2+\tau^2)Z^2 - 2(\sigma_{n-1}^2 x_n + \tau^2\mu_{n-1})Z}{2\tau^2\sigma_{n-1}^2}\}dZ$$

$$= \frac{1}{4\pi^2\tau^2\sigma_{n-1}^2}\exp\{-\frac{x_n^2}{2\tau^2} - \frac{\mu_{n-1}^2}{2\sigma_{n-1}^2} + \frac{m^2}{2s^2}\}\int \exp\{-\frac{(Z-m)^2}{2s^2}\}dZ$$

$$= \frac{1}{2\pi(\tau^2+\sigma_{n-1}^2)}\exp\{-\frac{x_n^2}{2\tau^2} - \frac{\mu_{n-1}^2}{2\sigma_{n-1}^2} + \frac{m^2}{2s^2}\}\frac{1}{2\pi s^2}\int \exp\{-\frac{(Z-m)^2}{2s^2}\}dZ$$

$$= C_1\int N(Z \mid \mathrm{m}, s^2)dZ$$

$$\frac{1}{s^2} = \frac{1}{\tau^2} + \frac{1}{\sigma_{n-1}^2} \qquad m = \frac{\sigma_{n-1}^2 x_n + \tau^2\mu_{n-1}}{\sigma_{n-1}^2 + \tau^2}$$

$$C_1 = \frac{1}{2\pi(\tau^2+\sigma_{n-1}^2)}\exp\{-\frac{x_n^2}{2\tau^2} - \frac{\mu_{n-1}^2}{2\sigma_{n-1}^2} + \frac{m^2}{2s^2}\}$$