Robotics

Lecture 2: Robot Motion

See course website
http://www.doc.ic.ac.uk/~ajd/Robotics/ for up to
date information.

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Robot Motion

- A mobile robot can *move* and *sense*, and must *process information* to link these two. In this lecture we concentrate on robot movement, or locomotion.

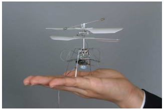
What are the possible goals of a robot locomotion system?

- Speed and/or acceleration of movement.
- Precision of positioning (repeatability).
- Flexibility and robustness in different conditions.
- Efficiency (low power consumption)?

Locomotion

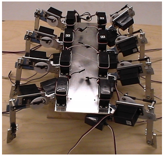
- Robots might want to move in water, in the air, on land, in space...?

AUV



Micro UAV

Zero-G Assistant

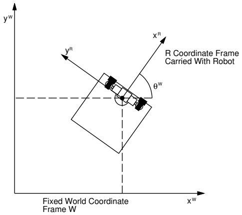


Spider

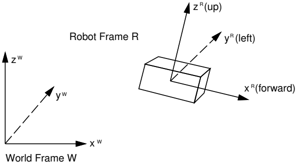
Humanoid

- In this course we will concentrate on wheeled robots which move on fairly flat surfaces.

Motion and Coordinate Frames



2D



3D

- We define two coordinate frames: a *world frame W* anchored in the world, and a *robot frame R* which is carried by and stays fixed relative to the robot at all times.
- Often we are interested in knowing the robot's *location*: i.e. what is the transformation between frames *W* and *R*?

Degrees of Motion Freedom

- A rigid body which translates and rotates along a 1D path has 1 degree of freedom (DOF): translational. Example: a train.
- A rigid body which translates and rotates on a 2D plane has 3 DOF: 2 translational, 1 rotational. Example: a ground robot.
- A rigid body which translates and rotates in a 3D volume has 6 DOF: 3 translational, 3 rotational. Example: a flying robot.

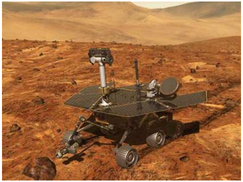
- A *holonomic robot* is one which is able to move instantaneously in any direction in the space of its degrees of freedom.

A Holonomic Ground Robot

- Holonomic robots do exist, but need many motors or unusual designs and are often impractical.
- Ground-based holonomic robots can be made using omnidirectional wheels; e.g. `http://www.youtube.com/watch?v=HkhGr7qfeT0`

Exotic Wheeled Robots

Segway RMP



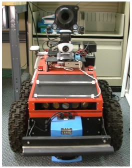
Mars Rover

- Segway platform with dynamic balance gives good height with small footprint and high acceleration. Self-balancing Lego Robots built by DoC students in 2008:
  `http://www.youtube.com/watch?v=fQQctJz7ap4`
- Mars Rover has wheels on stalks to tackle large obstacles.

Standard Wheel Configurations

Rack and Pinion

Differential Drive

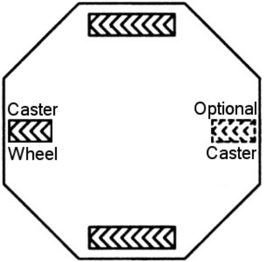


Skid-Steer

Synchro Drive

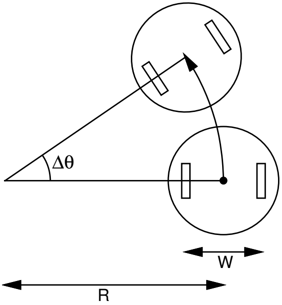
- Simple, reliable, robust mechanisms suitable for robots which essentially move in a plane.
- All of these robots are *non-holonomic* (each uses two motors, but three degrees of movement freedom). For instance, a car-like robot can't instantaneously move sideways.

Differential Drive



- Two motors, one per wheel: steering achieved by setting different speeds.
- Wheels run at equal speeds for straight-line motion.
- Wheels run at equal and opposite speeds to turn on the spot.
- Other combinations of speeds lead to motion in a circular arc.

Circular Path of a Differential Drive Robot



We define the wheel velocities of the left and right wheels respectively to be v_L and v_R (linear velocities of the wheels over the ground: e.g. $v_L = r_L \omega_L$, where r_L is the radius of the wheel and ω_L is its angular velocity). The width between the wheels of the differential drive robot is W.

- Straight line motion if $v_L = v_R$
- Turns on the spot if $v_L = -v_R$
- More general case: moves in a circular arc.

Circular Path of a Differential Drive Robot

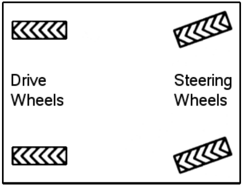
To find radius R of curved path: consider a period of motion Δt where the robot moves along a circular arc through angle $\Delta\theta$.

- Left wheel: distance moved $= v_L \Delta t$; radius of arc $= R - \frac{W}{2}$.
- Right wheel: distance moved $= v_R \Delta t$; radius of arc $= R + \frac{W}{2}$.
- Both wheel arcs subtend the same angle $\Delta\theta$ so:

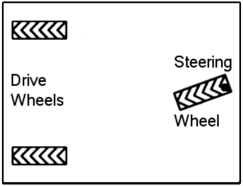
$$\begin{aligned} \Delta\theta &= \frac{v_L \Delta t}{R - \frac{W}{2}} = \frac{v_R \Delta t}{R + \frac{W}{2}} \\ \Rightarrow \frac{W}{2}(v_L + v_R) &= R(v_R - v_L) \end{aligned}$$

$$\boxed{\Rightarrow R = \frac{W(v_R + v_L)}{2(v_R - v_L)} \qquad \Delta\theta = \frac{(v_R - v_L)\Delta t}{W}}$$

Car/Tricycle/Rack and Pinion Drive



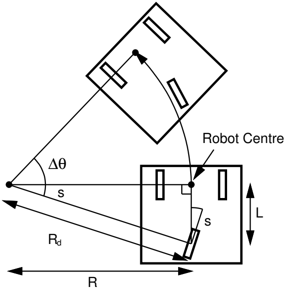
Car



Tricycle

- Two motors: one to drive, one to steer.
- Cannot normally turn on the spot.
- With a fixed speed and steering angle, it will follow a circular path.
- With four wheels, need rear differential and variable ('Ackerman') linkage for steering wheels.

Circular Path of a Car-Like Tricycle Robot



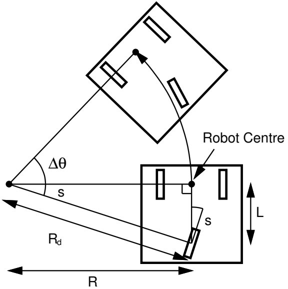
Assuming no sideways wheel slip, we intersect the axes of the front and back wheels to form a right-angle triangle, and obtain:

$$R = \frac{L}{\tan s} \ .$$

The radius of the path that the rear driving wheel moves in is:

$$R_d = \frac{L}{\sin s} \ .$$

Circular Path of a Car-Like Tricycle Robot



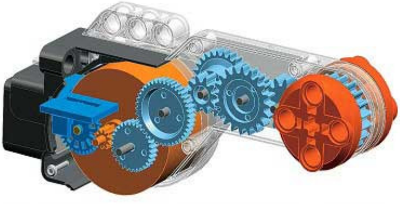
In time Δt the distance along its circular arc moved by the drive wheel is $v\Delta t$, so the angle $\Delta\theta$ through which the robot rotates is:

$$\Delta\theta = \frac{v\Delta t}{R_d} = \frac{v\Delta t \sin s}{L} \; .$$

$$\boxed{R = \frac{L}{\tan s} \qquad \Delta\theta = \frac{v\Delta t \sin s}{L}}$$

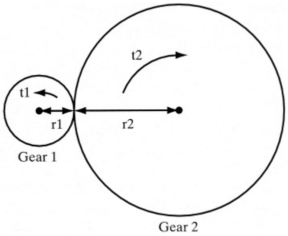
Actuation of Driving Wheels: DC Motors

- Most common motors, available in all sizes and types.
- Simple control with voltage or Pulse Width Modulation (PWM).
- For precision, encoders and feedback can be used for *servo* control (the NXT motors have built-in encoders).



Gearing

- DC motors tend to offer high speed and low torque, so gearing is nearly always required to drive a robot



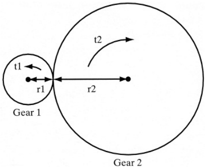
If Gear 1 is driven with torque t_1, it exerts tangential force:

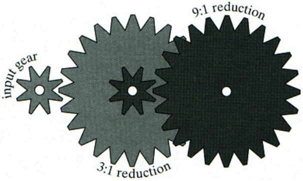
$$F = \frac{t_1}{r_1}$$

on Gear 2. The torque in Gear 2 is therefore:

$$t_2 = r_2 F = \frac{r_2}{r_1} t_1 \ .$$

Gearing



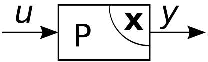


The change in angular velocity between Gear 1 and Gear 2 is calculated by considering velocity at the point where they meet:

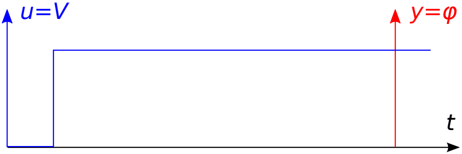
$$\begin{aligned} v &= \omega_1 r_1 = \omega_2 r_2 \\ \Rightarrow \omega_2 &= \frac{r_1}{r_2}\omega_1 \end{aligned}$$

- When a small gear drives a bigger gear, the second gear has higher torque and lower angular velocity in proportion to the ratio of teeth.
- Gears can be chained together to achieve compound effects.

Motor Control — Open Loop



- Let P be a Single-Input-Single-Output (SISO) dynamic system (e.g. Lego Mindstorms geared DC motor). It is described by:
  - an input u,
    here: voltage V or corresponding PWM value,
  - internal states $\mathbf{x}$, whose dynamics follow differential equations,
    here: $\mathbf{x} = [x_1, x_2]^\top = [\omega, \varphi]^\top$, with rotation speed ω and angle φ,
  - an output y as a function of $\mathbf{x}$,
    here: angle φ, i.e. $y = x_2$.
- Qualitative open-loop response on input (voltage) step:

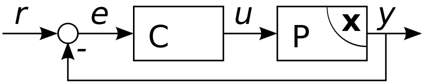


Motor Control — Closed Loop

- How about we want to accurately reach and keep a reference angle?

  $\rightarrow$ **Closed-loop control!**

- Let C be a controller, possibly with internal states:



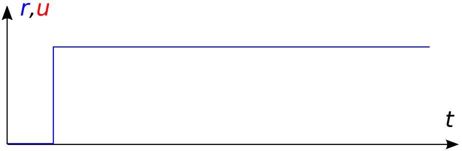
  - r is a reference (desired) output, and
  - e is the error between reference and actual output.

Motor Control — PID

- PID (Proportional-Integral-Differential): a simple controller:

  $$C : u(t) = k_p e(t) + k_i \int_{t_0}^{t} e(\tau) d\tau + k_d \frac{de(t)}{dt},$$

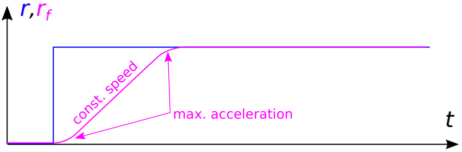
  - with k_p: proportional gain, reduces the error,
  - k_i: integral gain, removes steady-state error,
  - and k_d: differential gain, can reduce settling time.
- Qualitative closed-loop system response to reference step (well-tuned controller):



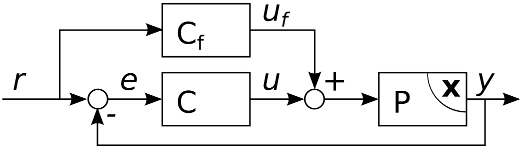
- Simple (heuristic) tuning rule: Ziegler-Nichols:
  - set k_i and k_d to zero. Increase k_p until the system starts oscillating with period P_u (in seconds) — remember this gain as k_u;
  - set $k_p = 0.6k_u$, $k_i = 2k_p/P_u$, and $k_d = k_p P_u/8$.

Motor Control — Additional Tweaks

- Reference filtering: respect physical limits already in the reference:

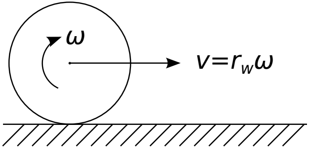


- Anti-Reset-Windup: stops integrating the error for the I-part, when u is at its physical limit.
- Dead-band compensation: add offset to u to compensate friction.
- Feed-forward controller $C_f : u_f(t) = k_f \frac{dr(t)}{dt}$, reduces "work" for the feed-back controller:



Mapping Wheel Rotation Speed to Velocity

- What is the robot speed, when the wheels turn?



- **Note:** the wheel radii r_w need to be **calibrated**, rather than measured (equivalently the wheel baseline W of a diff-drive robot).

Motion and State on a 2D Plane

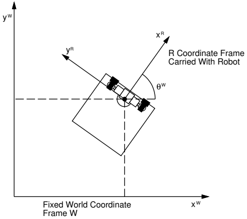
- If we assume that a robot is confined to moving on a plane, its location can be defined with a *state vector* $\mathbf{x}$ consisting of three parameters:

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

- x and y specify the location of the pre-defined 'robot centre' point in the world frame.
- θ specifies the rotation angle between the two coordinate frames (the angle between the x^W and x^R axes).
- The two coordinate frame coincide when the robot is at the origin, and $x = y = \theta = 0$.

Integrating Motion in 2D

- 2D motion on a plane: three degrees of positional freedom, represented by (x, y, θ) with $-\pi < \theta <= \pi$.
- Consider a robot which only drives ahead or turns on the spot:



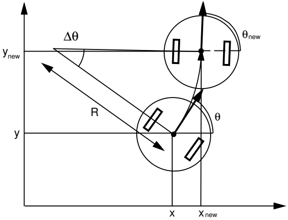
- During a straight-line period of motion of distance D:

$$\begin{pmatrix} x_{new} \\ y_{new} \\ \theta_{new} \end{pmatrix} = \begin{pmatrix} x + D\cos\theta \\ y + D\sin\theta \\ \theta \end{pmatrix}$$

- During a pure rotation of angle angle α:

$$\begin{pmatrix} x_{new} \\ y_{new} \\ \theta_{new} \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta + \alpha \end{pmatrix}$$

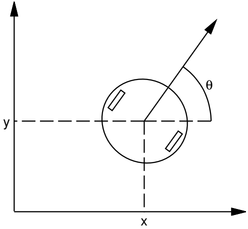
Integrating Circular Motion Estimates in 2D



In the cases of both differential drive and the tricycle robot, we were able to obtain expressions for R and $\Delta\theta$ for periods of constant circular motion. Given these:

$$\begin{pmatrix} x_{new} \\ y_{new} \\ \theta_{new} \end{pmatrix} = \begin{pmatrix} x + R(\sin(\Delta\theta + \theta) - \sin\theta) \\ y - R(\cos(\Delta\theta + \theta) - \cos\theta) \\ \theta + \Delta\theta \end{pmatrix}$$

Position-Based Path Planning



Assuming that a robot has *localisation*, and knows where it is relative to a fixed coordinate frame, then position-based path planning enables it to move in a precise way along a sequence of pre-defined waypoints. Paths of various curved shapes could be planned, aiming to optimise criteria such as overall time or power usage. Here we will consider the specific, simple case where we assume that:

- Our robot's movements are composed by straight-line segments separated by turns on the spot.
- The robot aims to minimise total distance travelled, so it always turns immediately to face the next waypoint and drives straight towards it.

Position-Based Path Planning

In one step of path planning, assume that the robot's current pose is (x, y, θ) and the next waypoint to travel to is at (W_x, W_y).

- It must first rotate to point towards the waypoint. The vector direction it must point in is:

$$\begin{pmatrix} d_x \\ d_y \end{pmatrix} = \begin{pmatrix} W_x - x \\ W_y - y \end{pmatrix}$$

  The absolute angular orientation α the robot must drive in is therefore given by:

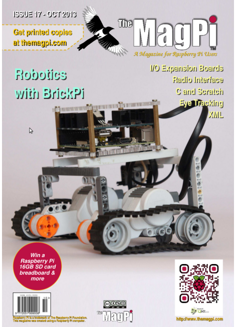
$$\alpha = \tan^{-1} \frac{d_y}{d_x}$$

Care must be taken to make sure that α is in the correct quadrant of $-\pi < \alpha \leq \pi$. A standard $\tan^{-1}$ function will return a value in the range $-\pi/2 < \alpha <= \pi/2$. This can be also achieved directly with an `atan2(dy, dx)` function (available in Python's `math` module).

Position-Based Path Planning

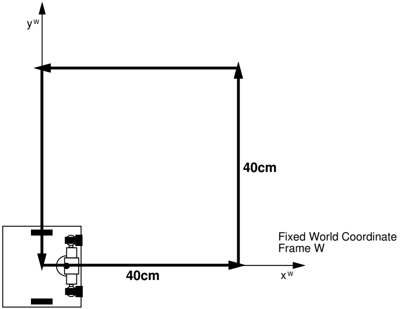
- The angle the robot must rotate through is therefore $\beta = \alpha - \theta$. If the robot is to move as efficiently as possible, care should be taken to shift this angle by adding or subtracting 2π so make sure that $-\pi < \beta \leq \pi$.
- The robot should then drive forward in a straight line through distance $d = \sqrt{d_x^2 + d_y^2}$.

Lego Mindstorms NXT and BrickPi



- Please READ THE WHOLE PRACTICAL SHEET CAREFULLY.
- Lab location: teaching lab 202, one floor down.
- Organise yourselves into groups of 5 and come to us to fill in a form and get a kit.

Week 2 Practical: Locomotion, Calibration and Accurate Motion



- Today's practical is on accurate robot motion. How well is it really possible to estimate robot motion from wheel odometry?
- **Everyone should read the practical sheet fully!**
- This is an **ASSESSED** practical: we will assess your achievement next week at the start of next week's practical. Your whole group should be there to demonstrate and discuss your robot.