Development of Laser Rangefinder-based SLAM Algorithm for Mobile Robot Navigation

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Abstract: This paper describes a new implementation of the SLAM algorithm for a mobile robot operating in an outdoor environment such as the IGVC Navigation Challenge, using relative obstacle observation profile from laser rangefinder. The proposed SLAM is possible for the mobile robot to start in an unknown location in an unknown environment and, using relative observations only, incrementally build a perfect map of the world and to compute simultaneously a bounded estimate of mobile robot location by the extended Kalman filter. To confirm the proposed SLAM method, an electric wheelchair based mobile robot is used for implementation and testing.

Keywords: Simultaneous Localization and Map building, extended Kalman filter, mobile robot, laser rangefinder

1. Introduction

Mobile robots need to be able to navigate autonomously in outdoor environments in order to be more useful in today's society. One of the key technologies for achieving this is map building and localization. Although GPS is a useful absolute positioning system and can be used for mobile robot navigation, it is sometimes difficult to estimate self-location during outdoor navigation. Since the availability of spatial signals depends on satellite locations, the accuracy of measuring the mobile robot's location by GPS may vary. To improve the accuracy, we present a new Simultaneous Localization and Mapping (SLAM) algorithm designed for mobile robot navigation. The proposed SLAM algorithm is a landmark-based terrain aided navigation system that features online map building and can simultaneously use the generated global map.

We applied the proposed method to an actual navigation competition named IGVC Navigation Challenge. This competition involves a ground vehicle autonomously navigating from a starting point to a number of target destinations and returning to base using only the coordinates of target waypoints. The waypoint coordinates are given in latitude and longitude. In the competition, we focus on the spatial arrangement between obstacles on the course as landmarks instead of waypoint coordinates. By using the relative range relationship between the mobile robot and the landmarks acquired from the laser rangefinder, we are able to estimate the distance traveled by the mobile robot in the global coordinate system by applying the SLAM algorithm assisted by GPS global waypoints information.

2. Navigation System-based SLAM

Photo 1 shows the mobile robot that we developed and tested.

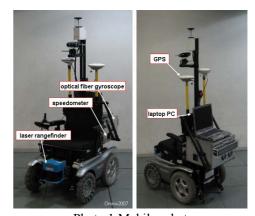


Photo 1 Mobile robot

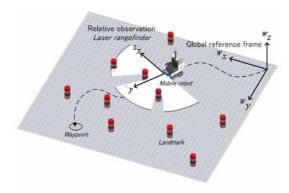


Fig. 1 Schematic diagram of the proposed SLAM system

Fig. 1 shows the environment for the proposed SLAM algorithm. Although various types of SLAM algorithm have been proposed and used, the well-known ones can be classified into two main types: one based on

the extended Kalman filter, and the other based on the particle filter. In general, particle filter-based SLAM algorithms are accurate but slower than extended Kalman filter type algorithms. For real-world competitions, we choose the extended Kalman filter type algorithm since the computational resources of the mobile robot are limited.

As shown in Photo 1, the mobile robot is mounted with a laser rangefinder, and speedometer and optical fiber gyroscope as local sensors. The data from the local sensors is used to estimate the relative location of the mobile robot by dead-reckoning. The data from the laser rangefinder is used to acquire environment distance profile information that can be used to identify obstacles as landmarks.

These data can be combined by applying the extended Kalman filter to build an online map, and simultaneously using the waypoints global map.

2.1 Description of problem in SLAM implementation

The laser rangefinder is used as the primary sensing device for the proposed SLAM algorithm. According to the obstacle locations acquired by the range sensor, we build a global map based on obstacle locations as landmarks. Depending on the sensing range and direction of the range finder, not all of the landmarks can always be observed. We consider a method of updating the observation system according to relation of landmarks, and formulate the state equation and observation equation.

In this study, we examined the following problems:

- (P1) Formulation of state equation and observation equation for SLAM for the mobile robot;
- (P2) How to relate landmarks and observed obstacles from the laser rangefinder;
- (P3) Method of implementing SLAM in real time running in an actual environment;

3. Extended Kalman Filter-based SLAM Implementation

3.1 Formulation of state equation

Fig. 2 shows the coordinate configuration of the extended Kalman filter which is used as the SLAM estimation algorithm. The following constants and variables can be defined.

[Common variables and constants]

k : Discrete time

[Global coordinate system]

- $\begin{bmatrix} W_{x_r}(k) & W_{y_r}(k) \end{bmatrix}$: Positional coordinate of the robot at time k
- $\Delta d(k)$: Traveling distance of the robot in one sampling time
- $\Delta\theta(k)$: Rotation angle of the robot in one sampling time
- $\theta(k)$: Relative angle of traveling direction of the robot
- $\xi(k)$: x -axis component of the robot direction $(\cos \theta(k))$
- $\eta(k)$: y -axis component of the robot direction $(\sin \theta(k))$
- $n_{A\theta}(k)$: Error included in $\Delta\theta(k)$
- $n_{Ad}(k)$: Error included in $\Delta d(k)$
- n: Component number of landmark constructed in global coordinate system
- $\begin{bmatrix} w & x_{ln} & y_{ln} \end{bmatrix}$: Number n positional coordinate of landmark in the global map
- $\begin{bmatrix} w \widetilde{\chi}_{ln} & w \widetilde{\chi}_{ln} \end{bmatrix}$: Number n positional coordinate of landmark profiled by sensors in the global map

[Local coordinate system]

- i: Data points of laser rangefinder according to discrete angle
- r(k,i): Distance of the number i observed at time k
- $[x(k,i) \ y(k,i)]$: Rectangular coordinate system component
- *m*: Component number of the landmark observed in local coordinate system
- $\begin{bmatrix} {}^{L}x_{lm}(k) {}^{L}y_{lm}(k) \end{bmatrix}$: Number m observation value of landmark coordinate
- $[{}^{L}\widetilde{\chi}_{lm}(k) {}^{L}\widetilde{y}_{lm}(k)]$: Number m relative coordinates profiled by sensors

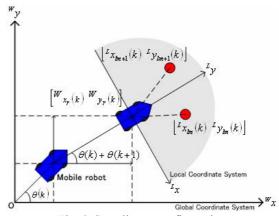


Fig. 2 Coordinate configuration

We assume that the environment is represented by the global coordinate system $[{}^{w}x {}^{w}y]$. The other

system is the local coordinate system of the mobile robot, which is represented by $\begin{bmatrix} L_x & L_y \end{bmatrix}$.

In order to derive the extended Kalman filter, we define the following state space model for the mobile robot and for building the global map.

The state vector in the proposed extended Kalman filter consists of the mobile robot state part (x) and landmarks state part (L). The state vector is given by Eqs. (1) and (2):

$$\begin{bmatrix} \mathbf{x}_{v}(k+1) \\ \mathbf{L}_{1} \\ \vdots \\ \mathbf{L}_{N} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{v}(k+1) & 0 & \cdots & 0 \\ 0 & \mathbf{I}_{n} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \mathbf{I}_{n} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{v}(k) \\ \mathbf{L}_{1} \\ \vdots \\ \mathbf{L}_{N} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_{v}(k) \\ \mathbf{0}_{n} \\ \vdots \\ \mathbf{0}_{n} \end{bmatrix}$$
(1)

$$\mathbf{x}(k+1) = \mathbf{F}(k+1)\mathbf{x}(k) + \mathbf{w}(k)$$
 (2)

State vector $\mathbf{x}_{v}(k)$ for the mobile robot is given by:

$$\mathbf{x}_{v}(k) = \begin{bmatrix} {}^{w}\mathbf{x}_{r}(k) & {}^{w}\mathbf{y}_{r}(k) & \xi(k) & \eta(k) \end{bmatrix}^{T}$$

$$\tag{3}$$

 $\zeta(k) = \cos\theta(k)$ and $\eta(k) = \sin\theta(k)$ are the state variables, so the state transition matrix $\mathbf{F}_{\nu}(k)$ is defined as:

$$\mathbf{F}_{v}(k+1) = \begin{bmatrix} 1 & 0 & \Delta d(k+1) \cdot \cos\Delta\theta(k+1) & -\Delta d(k+1) \cdot \sin\Delta\theta(k+1) \\ 0 & 1 & \Delta d(k+1) \cdot \sin\Delta\theta(k+1) & \Delta d(k+1) \cdot \cos\Delta\theta(k+1) \\ 0 & 0 & \cos\Delta\theta(k+1) & -\sin\Delta\theta(k+1) \\ 0 & 0 & \sin\Delta\theta(k+1) & \cos\Delta\theta(k+1) \end{bmatrix}$$

$$(4)$$

System noise \mathbf{W}_{v} is defined as:

$$\mathbf{w}_{v}(k) = \begin{bmatrix} \xi(k) & -\Delta d(k+1) \cdot \eta(k) \\ \eta(k) & \Delta d(k+1) \cdot \xi(k) \\ 0 & -\eta(k) \\ 0 & \xi(k) \end{bmatrix} \begin{bmatrix} n_{Ad}(k) \\ n_{A\theta}(k) \end{bmatrix}$$

$$= \mathbf{B} (k+1)\mathbf{n} (k)$$
(5)

Landmarks are assumed to be static objects in the global coordinate system. Therefore, if $\begin{bmatrix} w_{X_{ln}}(k)^w y_{ln}(k) \end{bmatrix}^T$ is \mathbf{L}_n , then the state equation of the mobile robot is defined as:

$$\mathbf{L}_{n}(k+1) = \mathbf{L}_{n}(k) = \mathbf{L}_{n} \tag{6}$$

In Eq (1), the process model of the landmark is a unit matrix, so the state vector $\mathbf{x}(k)$ in terms of \mathbf{L}_N and $\mathbf{x}_{\nu}(k)$ is defined as:

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_{v}^{T}(k) & \mathbf{L}_{1}^{T}, \cdots, \mathbf{L}_{n}^{T}, \cdots, \mathbf{L}_{N}^{T} \end{bmatrix}^{T}$$
(7)

3.2 Formulation of observation equation

When the location coordinate $\mathbf{z}_m(k) = \begin{bmatrix} L_{X_{lm}}(k) & L_{Y_{lm}}(k) \end{bmatrix}$ of the local number m landmark which was observed by the laser rangefinder and the number n landmark \mathbf{L}_n constructed in the global map at time k are related, then the observation equation is defined in

terms of state vector $\mathbf{x}(k)$ as follows:

$$\widetilde{\mathbf{z}}_{m}(k) = \mathbf{H}_{n}(k)\mathbf{x}(k) + \mathbf{v} \tag{8}$$

where, \mathbf{v} is a vector with zero mean. Observation matrix $\mathbf{H}_{n}(k)$ is defined by:

$$\mathbf{H}_{n}(k) = \begin{bmatrix} -\eta(k) & \zeta(k) & 0 & 0 & \cdots & \eta(k) & -\zeta(k) & 0 & \cdots & 0 \\ -\zeta(k) & -\eta(k) & 0 & 0 & \cdots & \zeta(k) & \eta(k) & 0 & \cdots & 0 \end{bmatrix}$$
(9)

The observation model $\mathbf{H}_n(k)$ dynamically changes depending on relation of landmark in the global and local coordinates. Depending on detected landmarks, corresponded parameter in $\mathbf{H}_n(k)$ may change instead of $\begin{bmatrix} -\eta(k) & \zeta(k) \\ -\zeta(k) & -\eta(k) \end{bmatrix}$.

3.3 Application of extended Kalman filter

The extended Kalman filter is composed of the state equation and the observation equation. The extended Kalman filter works by a three-step cycle: a prediction step, an observation step and an update step.

(Step 1: Prediction step)

At time k-1, the estimated value of state vector $\mathbf{x}(k-1)$ is defined as $\hat{\mathbf{x}}(k-1|k-1)$, and the covariance matrix is defined as $\mathbf{P}(k-1|k-1)$. The prediction step at time k is defined as:

$$\hat{\mathbf{x}}(k \mid k-1) = \mathbf{F}(k)\hat{\mathbf{x}}(k-1 \mid k-1)$$
(10)

The covariance matrix of estimation error of $\hat{\mathbf{x}}(k \mid k-1)$ is defined:

$$\mathbf{P}(k \mid k-1) = \mathbf{F}(k)\mathbf{P}(k-1 \mid k-1)\mathbf{F}(k)^{T} + \mathbf{B}(k)\mathbf{G}(k)\mathbf{B}(k)^{T}$$
(11)

where, $\mathbf{G}(k)$ is a 2x2 matrix and its diagonal cross-correlations are defined by variances of $n_{\Delta d}(k)$ and $n_{\Lambda \theta}(k)$.

(Step 2: Observation step)

At time k, when number n coordinate value \mathbf{L}_n^T of landmark $[\mathbf{L}_1^T, \dots, \mathbf{L}_n^T, \dots, \mathbf{L}_N^T]^T$ constructed in the global map and number m landmark observed in the local map are related, observation value $\hat{\mathbf{z}}_m(k \mid k-1)$ based on $\hat{\mathbf{x}}(k \mid k-1)$ is defined by the following Eq. (12) with observation matrix (9):

$$\hat{\mathbf{z}}_{m}(k \mid k-1) = \mathbf{H}_{n}(k)\hat{\mathbf{x}}(k \mid k-1) \tag{12}$$

When the covariance matrix of observation errors defines \mathbf{R} , gain $\mathbf{K}_n(k)$ in relation number n landmark is defined as:

$$\mathbf{K}_{n}(k) = \mathbf{P}(k \mid k-1)\mathbf{H}_{n}(k)^{T} (\mathbf{H}_{n}(k)\mathbf{P}(k \mid k-1)\mathbf{H}_{n}(k)^{T} + \mathbf{R})^{-1}$$
(13)

(Step 3: Update step)

When number m landmark positions in local coordinate $\begin{bmatrix} {}^{L}x_{lm}(k) & {}^{L}y_{lm}(k) \end{bmatrix}^{T}$ which was observed by the laser rangefinder defines $\mathbf{z}_{m}(k)$, estimation value $\hat{\mathbf{x}}(k \mid k-1)$ and covariance matrix $\mathbf{P}(k \mid k-1)$ are updated by Eqs. (14) and (15):

$$\hat{\mathbf{x}}(k \mid k) = \hat{\mathbf{x}}(k \mid k-1) + \mathbf{K}_{n}(k)(\mathbf{z}_{m}(k) - \hat{\mathbf{z}}_{m}(k \mid k-1))$$
(14)

$$\mathbf{P}(k \mid k) = \mathbf{P}(k \mid k-1) - \mathbf{K}_{\mathbf{n}}(k)\mathbf{H}_{n}(k)\mathbf{P}(k \mid k-1)$$
(15)

Steps 2 and 3 show the case when the landmark coordinates \mathbf{L}_n in the global map and coordinates $\mathbf{z}_m(k)$ in the local map are related. When number of M landmarks are observed, steps 2 and 3 are repeated M times to determine landmark positions in global map.

3.4 Relation of landmark between coordinates

About P(2), in applying the extended Kalman filter, the observation and update step are calculated based on the relation to observed landmarks. Therefore, the relation and administration of the landmarks \mathbf{L} constructed in global coordinates and the landmark positions in local coordinate $\mathbf{z}_m(k)$ observed in local coordinates are important factors in order to apply the extended Kalman filter normally. We apply the Euclidean distance detection technique as shown in Figs 3 and 4.

The measurement landmark position $\begin{bmatrix} \iota_{x_{lm}}(k) & \iota_{y_{lm}}(k) \end{bmatrix}$ in local coordinate can be calculate to global coordinate $\begin{bmatrix} w_{\widetilde{\chi}_{ln}} & w_{\widetilde{y}_{ln}} \end{bmatrix}$ by following Eq. (16), the calculated global coordinate $\begin{bmatrix} w_{\widetilde{\chi}_{ln}} & w_{\widetilde{y}_{ln}} \end{bmatrix}$ and estimated global coordinate $\begin{bmatrix} w_{\chi_{ln}} & w_{\chi_{ln}} \end{bmatrix}$ by applying the extended Kalman filter should satisfy following Eq. (17):

$$\begin{bmatrix} {}^{W}\widetilde{\chi}_{lN+1} \\ {}^{W}\widetilde{y}_{lN+1} \\ 1 \end{bmatrix} = \begin{bmatrix} \eta(k) & -\xi(k) & \Delta d(k) \cdot \xi(k) + {}^{W}x_{r}(k-1) \\ -\xi(k) & \eta(k) & \Delta d(k) \cdot \eta(k) + {}^{W}y_{r}(k-1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{L}x_{lm}(k) \\ {}^{L}y_{lm}(k) \\ 1 \end{bmatrix}$$
(16)

$$\sqrt{\left({}^{W}x_{\mathrm{ln}} - {}^{W}\widetilde{\chi}_{\mathrm{ln}}\right)^{2} + \left({}^{W}y_{\mathrm{ln}} - {}^{W}\widetilde{y}_{\mathrm{ln}}\right)^{2}} < d_{\mathrm{min}}$$

$$\tag{17}$$

Threshold d_{\min} is set to a larger value than $\Delta d(k)$ which is the distance moved by the mobile robot in one sampling time. If Eq. (17) holds true, the Kalman filter is applied based on the relation to observed landmarks.

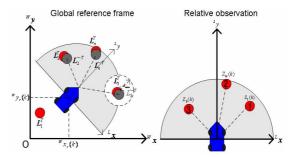


Fig. 3 Relation of landmarks between coordinates

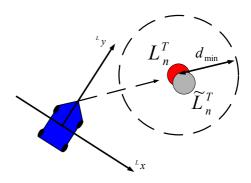


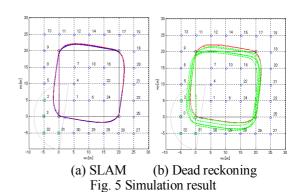
Fig. 4 Discrimination method of relations

4. Experiments

4.1 Simulation experiments

For (P3), we developed simulation software for the mobile robot for SLAM implementation, and examined the waypoints navigation by the proposed SLAM algorithm. The value of each sensor is the input of the system including Gaussian noise of mean zero. Each parameter of the extended Kalman filter requires a value in our experience.

Fig. 5-(a) shows the result estimated by the SLAM algorithm, Fig. 5-(b) shows the estimated self-location by dead reckoning, and the solid line shows the true vehicular swept path of the mobile robot. In Fig. 5-(b), errors accumulate, however in Fig. 5-(a) the errors are canceled and the self-location is estimated exactly.



4.2 Experiments using mobile robot

Photo 2 shows the experiment environment, in which the trees serve as landmarks and are almost equivalent to obstacles in the IGVC navigation challenge.

We measured the coordinates of the waypoints in this environment, and input the given coordinates of waypoints. We examined navigation of the mobile robot by the SLAM algorithm in real time.



Photo 2 Experiment environment outdoors

Fig. 6 shows the locus of travel. The dashed line is the self-location estimated by the SLAM algorithm, and the solid line is the movement estimated by dead reckoning.

In dead reckoning, errors accumulate, whereas with the SLAM algorithm, self-location is estimated exactly and the mobile robot was able to return to the starting point.

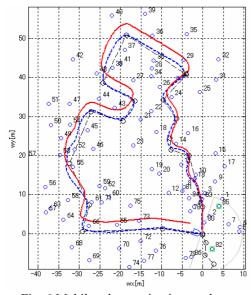


Fig. 6 Mobile robot navigation outdoors

5. Conclusions

In this paper, we described a new SLAM based navigation algorithm which is suitable for the IGVC navigation challenge, and considered obstacles as landmarks to identify self-location.

In order to generate a real-time map, we used the extended Kalman filter based SLAM algorithm instead of the particle filter based algorithm. To confirm the validity of the proposed method, we plan to use this algorithm for the actual IGVC 2007 competition.

Acknowledgments

This research was supported in part by the Ministry of Education, Culture, Sports, Science, and Technology, Grand-in-Aid for Scientific Research (C) 2006, No. 18500442.

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