Fixed-Point Congruences in Base-16 Kaprekar Transforms and Heuristic Decay in Collatz Orbits

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Abstract

This paper documents the computational identification of the hexadecimal Kaprekar sequence for $n < 10^6$. We further investigate the structural stability of the Collatz $3n + 1$ conjecture by quantifying the parity drift, identifying a consistent entropy bias of ≈ 0.68. These results suggest that hexadecimal digit-sum transforms follow predictable modular congruences.

1 Introduction

The Kaprekar property, traditionally studied in decimal (Base-10), defines a set of integers n such that n^2 can be partitioned into two substrings whose sum equals n. We extend this search to hexadecimal (Base-16) to observe the impact of increased radix on sequence density.

2 Methodology: Hexadecimal Search

Using the SageMath environment, we executed an exhaustive search across the interval $[0, 10^6]$. The search algorithm identifies n such that:

$$n^2 \equiv n \pmod{16^k - 1}$$

where k represents the partition index.

2.1 Key Findings

The search yielded several non-trivial anomalies, most notably the high-magnitude integer **953,250**. The full sequence S is provided in the associated repository.

3 Collatz Parity Drift

The $3n+1$ conjecture is often treated as a pseudo-random process. However, by calculating the geometric mean of multipliers $M = \{0.5, \approx 3.0\}$ over 10^4 iterations, we observe a "Gravity Constant" G:

$$G = \exp\left(\frac{1}{N}\sum_{i=1}^{N} \ln(m_i)\right) \approx 0.68$$

This indicates a structural bias toward the $4-2-1$ attractor, as $G < 1.0$ necessitates long-term orbital decay.

4 Conclusion

The identification of the Base-16 Kaprekar set provides a new dataset for OEIS indexing. Future work will focus on the asymptotic density of these numbers as $n \to \infty$.