Precise Orbit Determination of Satellites through 3 Observations (Gauss method)

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March-April 2013

Abstract

Since Jan.1,2009 any computation involving Earth's gravitational field should be performed by means of the new precession-nutation model and this paper presents how to determine the orbital elements of a satellite being known times and topocentric polar coordinates from a given location. The position of Intermediate Pole is taken from IERS and strongly interpolated to have its parameters precise at each time of the three observations while the application for testing the algorithms comes from [1]

1 Preamble

The precession-nutation new model has been approved during the IAU Conference held in Praha in 2006 and 187 routines cleverly codified by SOFA group.
Most of them involve the parameters shown in Fig. 1 called by a devoted Fortran-90 code we have carried out since longtime.

$$(x_p, y_p) \Rightarrow [W]$$
$$(\Delta UT1) \Rightarrow [R]$$
$$(\text{dX,dY}) \Rightarrow [PN]$$
$$\text{LOD} \Rightarrow \vec{\omega}_\oplus$$

Figure 1: Earth Oriented Parameters (EOP): to execute frame transformations

In details, (x_p, y_p) are the instantaneous cooedinates of CIP, acronym of Celestial Intermediate Pole, expressed in arcseconds.

$\Delta UT1$ is the difference in time seconds between Universal Time UT1 and Universal Coordinated Time (UTC), say $\Delta UT1 = UT1 - UTC$.

Celestial Pole Offsets (dX,dY) are corrections to the conventional celestial pole position according to precession-nutation model.

Finally, Length of Day (LOD) is the time difference between the duration of a mean solar day and 86,400 seconds. Aberration neglected due to light-time gap (0.02 s) lower than data tolerances (1 s).

Such quantities are daily issued by IERS [1] (International Earth Rotation and Reference Systems Service) and should be interpolated using this database [2], which is settled into two main groups,

[1] http://www.iers.org/IERS/EN/Science/EarthRotation/EOP.html
[2] http://maia.usno.navy.mil/ser7/finals.all

labelled "I" (definitive values) and "P" (predicted).
EOP data available at the date of paper have been embedded in *finals-iers.txt* file, used as database to be called from the main program. For the sake of reader's information, a glossary of acronyms regarding Earth oriented parameters is downloadable from here[3]

2 Theory of Gauss method

Three vectors *instantaneously* govern the satellite motion in each one of the 3 observed positions and are shown in Fig. 2.
They connect Earth's center O, the topocenter T, where the body is observed and measured from, and the position of satellite P.

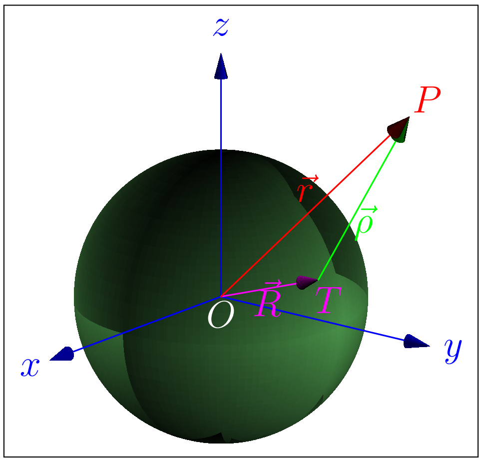

Figure 2: Celestial Triangle (OTP) and its vectorial sum: $\vec{r} = \vec{R} + \vec{\rho}$

The Gauss procedure involves the knowlwdge of data coming from observations carried out from a location (*Site*) and summarized in Table 1 p.447 of [1]

Table 1: Observation times, Right Ascension of Ascending Node, Declination, Site Coordinates

Times		**RAAN**			**Dec**	
11:40:28	00	03	45.48	18	40	03.78
11:48:28	03	00	06.18	35	39	53.07
11:52:28	04	31	32.80	36	59	47.70
Date	**Site Geodetic Coordinates**					
Aug. 20, 2012	$\lambda = -110$		$\phi = 40$		$H = 2000$	

We assume the ellipsoid WGS84 as reference for computing the topocentric coordinates (T_1, T_2, T_3), at observations times (t_1, t_2, t_3).

2.1 Vector $\vec{R}$

The new model of precession-nutation, which deals with the intermediate-frame-of-date matrix given the CIP (X, Y) and the CIO locator (s), should be adopted on determine the precise coordinates of such a vector.
Most of calculations, available through the web and involving satellites orbit determination, opt to use the *same* parameters of Fig. 1 for the three times of observation, or abruptly to neglect them.

[3] http://astrodinamica.altervista.org/PDF/Glossary-IERS.pdf

In one of the forthcoming section of this paper, we show how to perform such parameters by means of a robust interpolation of some tables.

The coordinates transformation from terrestrial reference system (TRS) into celestial reference system (CRS) at the observation epoch t can be written as follows:

$$[\mathrm{CRS}] = \mathrm{Q(t)} * \mathrm{R(t)} * \mathrm{W(t)} * [\mathrm{TRS}]$$

where $Q(t), R(t)$andW(t) are the transformation matrices arising from the motion of celestial pole in the celestial system, from the rotation of the Earth around the axis of the pole and from polar motion respectively.

The SOFA organization [4] cleverly provides public domain routines to perform those matrices and a lot of other facilities, which will be spreadly used in Fortran90 code of this paper; see below Sofa purposes.

```
The International Astronomical Union's SOFA service has the task of
establishing and maintaining an accessible and authoritative set of
algorithms and procedures that implement standard models used in
fundamental astronomy. The service is managed by an international
panel, the SOFA Board, appointed through IAU Division A - Fundamental
Astronomy. SOFA also works closely with the International Earth
Rotation and Reference Systems Service (IERS).
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The Earth centered coordinates of the Site are established from the chosen ellipsoid and comes directly from the routine *iau_GD2GC()*, which transforms geodetic coordinates of location in geocentric ones.

For each observation, the Earth Rotation Angle (ERA) is computed in order to determine the three vectors (R_1, R_2, R_3), by applying some transformations until reaching the celestial-terrestrial matrix (including polar motion). So, the coordinates R_{1x}, R_{1y}, R_{1z} of vector $\vec{R}_1 = \vec{OT}_1$ are determined and, similarly, the ones of $\vec{R}_2$ and $\vec{R}_3$.

They can be grouped in **matrix form**, as follows:

$$R = \begin{pmatrix} R_{1x} & R_{2x} & R_{3x} \\ R_{1y} & R_{2y} & R_{3y} \\ R_{1z} & R_{2z} & R_{3z} \end{pmatrix}$$

[4]http://www.iausofa.org/current_F.html#Downloads

2.2 Vector $\vec{\rho}$

The cosine directions of 3 vectors $\vec{\rho}_{i=1,2,3}$ are determined by means of following:

$$\begin{cases} \ell_1 = & \cos\alpha_1 \cdot \cos\delta_1 \qquad m_1 = \sin\alpha_1 \cdot \cos\delta_1 \qquad n_1 = \sin\delta_1 \\ \ell_2 = & \cos\alpha_2 \cdot \cos\delta_2 \qquad m_2 = \sin\alpha_2 \cdot \cos\delta_2 \qquad n_2 = \sin\delta_2 \\ \ell_3 = & \cos\alpha_3 \cdot \cos\delta_3 \qquad m_3 = \sin\alpha_3 \cdot \cos\delta_3 \qquad n_3 = \sin\delta_3 \end{cases} \tag{1}$$

whose (α, δ) values come from the observations at times (t_1, t_2, t_3).
Written each vector of equations (1) in matrix **L** form, we obtain

$$L = \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$$

So, by naming L^{-1} its **inverse** [5], the new matrix **M** is formed as shown below:

$$M = \begin{pmatrix} \overline{\ell_1} & \overline{\ell_2} & \overline{\ell_3} \\ \overline{m_1} & \overline{m_2} & \overline{m_3} \\ \overline{n_1} & \overline{n_2} & \overline{n_3} \end{pmatrix} \cdot \begin{pmatrix} R_{1x} & R_{2x} & R_{3x} \\ R_{1y} & R_{2y} & R_{3y} \\ R_{1z} & R_{2z} & R_{3z} \end{pmatrix}$$

2.3 Times and Distances

As a matter of principle, for objects orbiting under terrestrial gravitational field, times are taken in seconds and distances in kilometers, and then the correspondent parameter $\mu = G \cdot M_{Earth}$ is expressed in $km^3 s^{-2}$. Its actual value is $398,600.4418$.
Nevertheless, most of astronomers prefer to deal with the so-called **Canonical Units**, in order to have $\mu = 1$ and consequently the given times should be multiplied by a suitable constant, that we call, in analogy of solar system motions, *kgauss*.
Being taken as *unit of length* the equatorial radius of Earth, $RT = 6,378.137\,km$, which defines the circular orbit of an hypotetical satellite, the *unit time* to elapse the arc of 1 radiant is 806.811634148 s and then its inverse becomes $kgauss = 0.001239446678$. So, all observed times should use such a constant to apply canonical units.

2.4 Parameters for middle range magnitude (r_2)

The radar interceptions of Earth satellites are nowadays so sophisticated that observations can be scanned at desired intervals of time, as shown in Table 1, and astronomer's practice decides the suitable **triplet** to perform computations.

Calling (τ_1, τ_3) the time gaps between first and second observation $(t_1 - t_2)$ and third and second $(t_3 - t_2)$, respectively, we define the parameters (a_1, a_3) as follows:

$$\tau_1 = (t_1 - t_2) \cdot kgauss \qquad \tau_3 = (t_3 - t_2) \cdot kgauss$$

$$a_1 \simeq \frac{\tau_3}{\tau_3 - \tau_1} \qquad a_3 \simeq -\frac{\tau_1}{\tau_3 - \tau_1}$$

[5]
$$\begin{aligned} Being\ |L| = & \ \ell_1(m_2 n_3 - m_3 n_2) - \ell_2(m_1 n_3 - n_1 m_3) + \ell_3(m_1 n_2 - n_1 m_2) \\ \overline{\ell_1} = & \ (m_2 n_3 - m_3 n_2)/|L| \qquad \overline{m_1} = -(\ell_2 n_3 - \ell_3 n_2)/|L| \qquad \overline{n_1} = (\ell_2 m_3 - \ell_3 m_2)/|L| \\ \overline{\ell_2} = & \ -(m_1 n_3 - m_3 n_1)/|L| \qquad \overline{m_2} = (\ell_1 n_3 - \ell_3 n_1)/|L| \qquad \overline{n_2} = -(\ell_1 m_3 - \ell_3 m_1)/|L| \\ \overline{\ell_3} = & \ (m_1 n_2 - m_2 n_1)/|L| \qquad \overline{m_3} = -(\ell_1 n_2 - \ell_2 n_1)/|L| \qquad \overline{n_3} = (\ell_1 m_2 - \ell_2 m_1)/|L| \end{aligned}$$

Besides, we need to consider the approximate parameters (a_{1u}, a_{3u}), developed in series of powers up to second term.

$$a_{1u} \simeq \frac{\tau_3 \cdot [(\tau_3 - \tau_1)^2 - \tau_3^2]}{6\,(\tau_3 - \tau_1)} \qquad\qquad a_{3u} \simeq -\frac{\tau_1 \cdot [(\tau_3 - \tau_1)^2 - \tau_1^2]}{6\,(\tau_3 - \tau_1)}$$

Next step regards the determination of a new couple (d_1, d_2) of parameters

$$d_1 = M_{21} \cdot a_1 - M_{22} + M_{23} \cdot a_3 \qquad\qquad d_1 = M_{21} \cdot a_{1u} + M_{23} \cdot a_{3u}$$

alongwith the dot product (C) involving the middle observation

$$C = \vec{L_2} \cdot \vec{R_2}$$

Thus, we obtain the coefficients (L_t, M_t, K_t) of **Lagrange 8th degree equation**, which admits two real roots [6] of the unknown r_2, one negative (discarted) and one positive, whose solution is commonly computed per successive approximations starting from a suitable value (Newton-Raphson method).

$$r_2^8 - L_t \cdot r_2^6 - \mu M_t \cdot r_2^3 - \mu^2 K_t = 0 \tag{2}$$

$$\begin{array}{ll} \text{being} & L_t = d_1^2 + 2 \cdot C \cdot d_1 + R_2^2 \\ & M_t = 2\,(C \cdot d_2 + d_1 \cdot d_2) \\ & K_t = d_2^2 \\ \text{and} & \\ & \mu = 1 \quad \text{(canonical units)} \end{array}$$

2.5 Topocentric Vectors $(\vec{\rho})$ - Components

Being computed r_2 through the solution of Lagrange equation, we may proceed to compute the three **topocentric vectors** by means of coefficients (c_1, c_3) and $c_2 = -1$ always constant.

The dynamical parameter u is used

$$u = \frac{\mu}{r_2^3}$$

in order to determine next *still approximate* parameters

$$f_1 \simeq 1 - \frac{u \cdot \tau_1^2}{2} \qquad\qquad f_3 \simeq 1 - \frac{u \cdot \tau_3^2}{2}$$

$$g_1 \simeq \tau_1 \left(1 - \frac{u \cdot \tau_1^2}{6}\right) \qquad\qquad g_3 \simeq \tau_3 \left(1 - \frac{u \cdot \tau_3^2}{6}\right)$$

$$f_g = f_1\, g_3 - f_3\, g_1$$

and then (c_1, c_2) become as follows; added also two useful parameters (d_{n1}, d_{n3})

$$c_1 \simeq \frac{g_3}{f_g} \qquad\qquad c_3 \simeq -\frac{g_1}{f_g}$$

$$d_{n1} \simeq -\frac{f_3}{f_g} \qquad\qquad d_{n3} \simeq \frac{f_1}{f_g}$$

[6] See typical function in **Appendix**

In matrix form, the determination of *topocentric vectors* can be expressed as shown here

$$\begin{pmatrix} c_1\,\rho_1 \\ \\ -\rho_2 \\ \\ c_3\,\rho_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ \\ M_{21} & M_{22} & M_{23} \\ \\ M_{31} & M_{32} & R_{33} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ \\ 1 \\ \\ c_3 \end{pmatrix}$$

Explicitally, we obtain

$$\begin{cases} \vec{\rho}_1 = & M_{11}\,\vec{i} \ + \dfrac{M_{12}}{c_1}\,\vec{j} \ + \dfrac{M_{13}\,c_3}{c_1}\,\vec{k} & = \rho_{1x}\,\vec{i} + \rho_{1y}\,\vec{j} + \rho_{1z}\,\vec{k} \\ \\ \vec{\rho}_2 = & -M_{21}\,c_1\,\vec{i} \ - M_{22}\,\vec{j} \ - M_{23}\,c_3\,\vec{k} & = \rho_{2x}\,\vec{i} + \rho_{2y}\,\vec{j} + \rho_{2z}\,\vec{k} \\ \\ \vec{\rho}_3 = & \dfrac{M_{31}\,c_1}{c_3}\,\vec{i} \ + \dfrac{M_{32}}{c_3}\,\vec{j} \ + M_{33}\,\vec{k} & = \rho_{3x}\,\vec{i} + \rho_{3y}\,\vec{j} + \rho_{3z}\,\vec{k} \end{cases} \tag{3}$$

being $(\vec{i}, \vec{j}, \vec{k})$ the unit vectors of Earth centered system (x, y, z) and modula of topocentric vectors computed by the pithagorean formula

$$\rho_1 = \sqrt{\rho_{1x}^2 + \rho_{1y}^2 + \rho_{1z}^2}$$

$$\rho_2 = \sqrt{\rho_{2x}^2 + \rho_{2y}^2 + \rho_{2z}^2}$$

$$\rho_3 = \sqrt{\rho_{3x}^2 + \rho_{3y}^2 + \rho_{3z}^2}$$

2.6 Geocentric Vectors ($\vec{r}$)

In harmony with Fig. 2 and by calling (x_1, y_1, z_1) the components of vector $\vec{r_1}$, and similarly (x_2, y_2, z_2) for $\vec{r_2}$, (x_3, y_3, z_3) for $\vec{r_3}$, we obtain

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ \\ y_1 & y_2 & y_3 \\ \\ z_1 & z_2 & z_3 \end{pmatrix} = \begin{pmatrix} \rho_1 \\ \\ \rho_2 \\ \\ \rho_3 \end{pmatrix} \cdot \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ \\ m_1 & m_2 & m_3 \\ \\ n_1 & n_2 & n_3 \end{pmatrix} + \begin{pmatrix} R_{1x} & R_{2x} & R_{3x} \\ \\ R_{1y} & R_{2y} & R_{3y} \\ \\ R_{1z} & R_{2z} & R_{3z} \end{pmatrix}$$

2.7 Velocity Vector ($\vec{V_2}$)

The **Gibbs theorem** permits to know the components of velocity regarding the second satellite point, throughout the up-to-now *rough parameters* early computed.

We have

$$\begin{cases} V_{2x} = & d_{n1}\,x_1 + d_{n3}\,x_3 \\ \\ V_{2y} = & d_{n1}\,y_1 + d_{n3}\,y_3 \\ \\ V_{2z} = & d_{n1}\,z_1 + d_{n3}\,z_3 \\ \\ V_2 = & \sqrt{V_{2x}^2 + V_{2y}^2 + V_{2z}^2} \end{cases} \tag{4}$$

Thus, the **energy integral** of a central motion can be found in order to have the first approach of the orbital *semimajor axis* (a)

$$\begin{cases} a = \dfrac{1}{\dfrac{2}{r_2} - \dfrac{V_2^2}{\mu}} \\ \\ \text{being} \quad \mu = 1 \qquad \text{(in canonical units)} \end{cases} \tag{5}$$

Afterwards, the traditional way of *Gauss theorem* spreadly applied during the centuries to solar preliminary orbits, ends on computing eccentricity and angular elements.

Here we would enforce the method by pushing the refination of parameters (c_1, c_3) in order to reach the wished accuracy on determining the semimajor axis a, by means of **Universal Variables** described in next chapter.

3 Refining (a) with Universal Variables

The formulation due to Everhart and Pitkin [7] deals with the following main equations

$$\begin{cases} \tau = & r_0 \cdot S_1 + \sigma_0 \cdot S_2 + \mu \cdot S_3 \\ \\ r = & r_0 \cdot S_0 + \sigma_0 \cdot S_1 + \mu \cdot S_2 \end{cases} \tag{6}$$

being (r, r_0) the initial/final position radii, $\sigma_0 = \vec{r_0} \cdot \vec{V_0}$ the dot product of starting position and velocity vectors, $\tau = (t - t_0) \cdot kgauss$ the flight time of conic arc.

The quadruple values (S_0, S_1, S_2, S_3) are called **Conic Functions** and depend on *generalized eccentric anomaly* ψ defined by

$$\beta = \alpha \cdot \psi^2 \qquad \text{being } \alpha = -\frac{\mu}{\text{a}}$$

and next recursive formula (S_n) is used for conic functions determination

$$S_n = \psi^n \left(\frac{1}{n!} + \frac{\beta}{(n+2)!} + \frac{\beta^2}{(n+4)!} + \frac{\beta^3}{(n+6)!} + \ldots + \frac{\beta^k}{(2k+n)!} \right)$$

The first equation of (6) represents the Kepler equation, which should be solved per successive approximations of the unknown ψ, by means of Newton-Raphson iterative process, starting from

$$\psi_a = \frac{\tau}{r_0}$$

so that

$$\psi = \quad \psi_a - \frac{F(\psi_a)}{F'(\psi_a)}$$

$$\text{until} \quad |\psi - \psi_a| < \epsilon \quad \text{(accuracy desired)}$$

coming $F(\psi_a)$ from the first equation of (6) and $F'(\psi_a)$ from the second one, as follows

$$F(\psi) = \quad r_0 \cdot S_1 + \sigma_0 \cdot S_2 + \mu \cdot S_3 - \tau$$

$$F'(\psi) = \quad r_0 \cdot S_0 + \sigma_0 \cdot S_1 + \mu \cdot S_2 - r$$

[7] E.Everhart, E.T.Pitkin - **Universal Variables in the Two Body Problem** - American Association of Physics Teachers - Aug.1983 - p.712-717

The final values of *conic functions* allow to compute the *refined parameters* (**f,g**):

$$f = 1 - \frac{S_2}{r_0} \qquad g = \tau - S_3$$

Such a process is applied twice, so that both couples (f_1, g_1) and (f_3, g_3) are determined, and consequently new refined values of (d_{n1}, d_{n3}) be obtained.

So, the semimajor axis (a) is also more precise and the iteration will flow until

$$|a - a_{previous}| < \epsilon \quad \text{(accuracy required)}$$

Besides, the UV's process determines the couple of coefficients $(\mathbf{f}', \mathbf{g}')$

$$f' = -\frac{\mu S_1}{r\, r_0} \qquad g' = 1 - \frac{\mu S_2}{r}$$

Thus, the final position/velocity vectors, starting from the initial (and rough) $(\vec{r_0}, \vec{V_0})$, become

$$\begin{cases} \vec{r} = & f \cdot \vec{r}_0 + g \cdot \vec{V}_0 \\ \vec{V} = & f' \cdot \vec{r}_0 + g' \cdot \vec{V}_0 \end{cases} \tag{7}$$

4 Orbital Elements

Since the large diffusion of personal computers, the trigonometric way for finding the *keplerian classic elements* of an orbit has been abandoned.
Nevertheless, carefullness should be done for **satellites orbits** in order to avoid indeterminacies when inclination and eccentricity approach zero $(i \simeq 0, e \simeq 0)$. In such circumstances the process of conversion from $(\vec{r}, \vec{V})$ vectors has to involve the so-called **equinoctial elements**: (a) semimajor axis, (h, k) components of eccentricity vector, (p, q) components of ascending node vector, (λ) mean longitude.
In our Fortran code listing we have applied the formulation of this paper [8] and transferred to a devoted subroutine.

5 Fortran program ORBIT6F.f90

The variety of algorithms for the precise calculations presented in this paper has been possible thanks to the **excellent** package SOFA, early mentioned.
The compilation can be carried out with the opensource *gfortran*, freely available in any Linux distribution and should embed program code, Sofa libraries (222.for), support routine (iers.for) for interpolation of (X,Y,DUT1) parameters and two subroutines to perform universal variables algorithms and equinoctial elements.

See below how it appears its *cover*

```
! ---------------------------------------------------------------------------------
!  PROGRAM orbit6F.f90  -> Satellite Orbital Elements through 3 optical observations
!                         refined through Universal Variables - Canonical Units used
!
!  Code in Fortran 90 + SOFA library written in Fortran 77 (222.for) + iers.for routine
!          (working folder should contain dbase files: iers.for, finals-iers.txt,
!                                                      obs.dbase like 447.dat)
```

[8] D.A. Danielson + alias, **Semianalytic Satellite Theory** - pp. 4-9,
pages available here: http://astrodinamica.altervista.org/PDF/Equinoctials.pdf

```
!
!          Observation data taken from p.447-449 of the book:
!          "Fundamentals of Astrodynamics and Applications" - 3rd ed. (2007)
!          by David A. Vallado (ISBN 978-1-881883-14-2)
!
!          Code Authors: Giuseppe Matarazzo, Aldo Nicola
!
!  Compilation:
!          gfortran orbit6F.f90 222.for iers.for twosubs.f90 -o orbit6F  (Linux)
!                   (F suffix stands for Final)
!
!  Starting date: Apr.21,2013
!  Work in progress: Apr.25  (ended)
! ------------------------------------------------------------------------------------
```

The example presented in Table 1 has the following output:

```
     ----- DATE and LOCATION -----
 Date (YYYY:MM:DD)=  2012  8 20
 Longit(degs), Latit(degs), Height(m)= -110.000000  40.000000   2000.

       ----- 1st OBSERVATION -----
 Time (HH:MM:SS.SS)=  11  40  28.00
   RA (HH:MM:SS.SS)=   0   3  45.58
  Dec (dd:mm:ss.ss)=  18  40   3.78

       ----- 2nd OBSERVATION -----
 Time (HH:MM:SS.SS)=  11  48  28.00
   RA (HH:MM:SS.SS)=   3   0   6.18
  Dec (dd:mm:ss.ss)=  35  39  53.07

       ----- 3rd OBSERVATION -----
 Time (HH:MM:SS.SS)=  11  52  28.00
   RA (HH:MM:SS.SS)=   4  31  32.80
  Dec (dd:mm:ss.ss)=  36  59  47.70

                  ---------  IERS EOP DATA INPUT -------

TT(1)56159.487212785672    TT(2)56159.492768341224    TT(3)56159.495546119004
DUT1(1) 0.4048588 sec.     DUT1(2) 0.4048533 sec.     DUT1(3) 0.4048505 sec.

LOD(1)  0.6820    msec.    LOD(2)  0.6818    msec.    LOD(3)  0.6817    msec.

XP(1)   0.171071  arcsec.  XP(2)   0.171060  arcsec.  XP(3)   0.171054  arcsec.
YP(1)   0.386175  arcsec.  YP(2)   0.386190  arcsec.  YP(3)   0.386197  arcsec.

 MU =    1.0000000000000000      Geocentric gravitational constant
                                 (=1, in Canonical Units)
 Lagrange equation ->  x^8 - Lt*x^6 - MU*Mt*x^3 - MU^2*Kt = 0
 (Lt)=    1.0710489647314285
 (mu*Mt)=    5.4774111558750356
 (mu^2*Kt)=    7.1359019777520434

 r2=    1.6385492565015218
```

```
------------ Phase of parameters iteration --------
  (f3,g3)=   0.98994297169473999       0.29646998825432597
  (f1,g1)=   0.95977188677896019      -0.58695670440260805


      (u)=   0.22731154626166913
  (c1,c3)=   0.34250339529911589       0.67809448549988049
     (c2)=   -1.0000000000000000

--------- Final iteration up to desired accuracy of (a) -----
               a=    12480.890848086192       km
  (delta-a)=  4.16106910831559603E-006  km

    (f3,g3)=   0.99044471466969486       0.29653016542371313
    (f1,g1)=   0.96283418375997831      -0.58772105096716976
    (c1,c3)=   0.34177637096318852       0.67739876532024212


   --- Given Position Vector [km] & Velocity Vector [km/s] ---
   6366.6974156853048        5301.3792474541997        6522.0645828661536
  -4.1653974156783944        4.8200830876264646        1.7567488730256118

  ---------- Output Satellite Orbital Elements ---------------------------
      Semi-major axis=    12480.885312846240       km
    1-   Eccentricity=   0.21495404498185380       dimensionless
    2-   Eccentricity=   0.21495404498185380       dimensionless
          Inclination=    39.997318123483097       degs
          Node (RAAN)=    330.02090257804031       degs
  Argument of Perigee=    21.107108569104970       degs
         Mean anomaly=    35.262696814790701       degs
         True anomaly=    53.121497114179412       degs
  ----------------------------------------------------------------------

  *** The numerical results are given to an unrealistic precision,
      for comparison purposes ***
```

6 Conclusion

The astrodynamical culture, in the current internet era, suffers of the lack of deep investigations on different aspect of problems; everything is taken *as is* by supposing that *thruth* is in because it comes from publications issued in the web.
This paper represents a way to see **satellites preliminary orbits** *rigorously* computed throughout the IAU specifications and rules.

(May.01, 2013)

References

[1] David A. Vallado
Fundamentals of Astrodynamics and Applications - 3rd Edition - (2007) - pp. 447-449

Appendix

The diagram shows both roots, but only the positive one (point A) is acceptable for dynamical purposes.

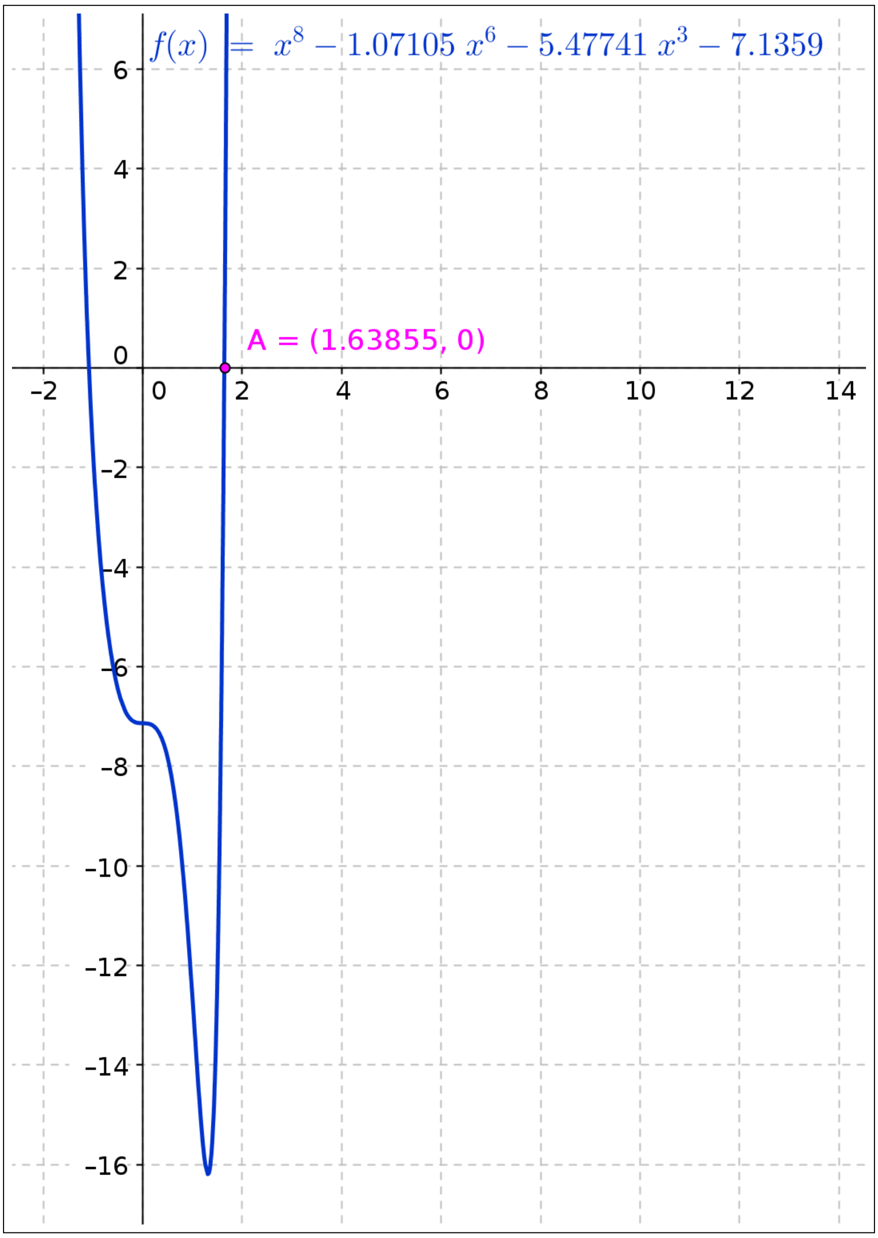

Figure 3: 8th degree Lagrange equation