积分表

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有理函数积分表

- $\int k \mathrm{d}x = kx + C$
- $\int x^{\alpha} \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + C(\alpha \neq 1)$
- $\int \frac{\mathrm{d}x}{x} = \ln|x| + C$
- $\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + C(a \neq 0)$
- $\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

有理函数积分表

- $\int (ax+b)^n \mathrm{d}x = \frac{(ax+b)^{n+1}}{a(n+1)} + C$
- $\int \frac{1}{ax+b} \mathrm{d}x = \frac{1}{a} \ln|ax+b| + C$
- $\int x(ax+b)^n \mathrm{d}x = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} + C$
- $\int \frac{x}{ax+b} \mathrm{d}x = \frac{x}{a} - \frac{b}{a^2} \ln|ax+b| + C$
- $\int \frac{x}{(ax+b)^2} \mathrm{d}x = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln|ax+b| + C$

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有理函数积分表

- $\int \frac{x}{(ax+b)^n}\mathrm{d}x = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} + C(n \notin \{1,2\})$
- $\int \frac{x^2}{ax+b}\mathrm{d}x = \frac{1}{a^3}[\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2 \ln|ax+b|] + C$
- $\int \frac{x^2}{(ax+b)^2}\mathrm{d}x = \frac{1}{a^3}(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}) + C$
- $\int \frac{x^2}{(ax+b)^3}\mathrm{d}x = \frac{1}{a^3}[\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}] + C$
- $\int \frac{x^2}{(ax+b)^n}\mathrm{d}x = \frac{1}{a^3}[-\frac{1}{(n-3)(ax+b)^{n-3}} + \frac{2b}{(n-2)(ax+b)^{n-2}} - \frac{b^2}{(n-1)(ax+b)^{n-1}}] + C(n \notin \{1,2,3\})$

无理函数积分表

- $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + C$
- $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2+a^2}\right| + C$
- $\int \frac{dx}{x\sqrt{ax+b}} = -\frac{2}{\sqrt{b}}\tanh^{-1}\sqrt{\frac{ax+b}{b}} + C$
- $\int \frac{\sqrt{ax+b}}{x}dx = 2(\sqrt{ax+b} - \sqrt{b}\tanh^{-1}\sqrt{\frac{ax+b}{b}}) + C$

指数函数积分表

- $\int e^{cx} dx = \frac{e^{cx}}{c} + C$
- $\int a^{cx} dx = \frac{a^{cx}}{c \ln a} + C$
- $\int x e^{cx} dx = \frac{e^{cx}(cx-1)}{c^2} + C$
- $\int x^2 e^{cx} dx = e^{cx}\left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3}\right) + C$

对数函数积分表

- $\int \ln x dx = x \ln x - x + C$
- $\int \ln^2 x dx = x \ln^2 x - 2x \ln x + 2x + C$
- $\int \frac{x}{x \ln x} = \ln|\ln x| + C$
- $\int \sin \ln x dx = \frac{x(\sin \ln x - \cos \ln x)}{2} + C$
- $\int \cos \ln x dx = \frac{x(\sin \ln x + \cos \ln x)}{2} + C$

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三角函数积分表

- $\int \sin x dx = \cos x + C$
- $\int \cos x dx = -\sin x + C$
- $\int x \sin cx dx = \frac{\sin cx}{c^2} - \frac{x \cos cx}{c} + C$
- $\int x \cos cx dx = \frac{\cos cx}{c^2} + \frac{x \sin cx}{c} + C$
- $\int \tan x dx = -\frac{\ln |cos cx|}{c} + C = \frac{\ln |\sec cx|}{c} + C$

三角函数积分表

- $\int \sec x dx = \ln|\sec x + \tan x| + C$
- $\int \csc x dx = \ln|\csc x - \cot x| + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int \csc^2 x dx = -\cot x + C$
- $\int \sec x \tan x dx = \sec x + C$
- $\int \csc x \cot x dx = -\csc x + C$

反三角函数积分表

- $\int \arcsin \frac{x}{c} dx = x \arcsin \frac{x}{c} + \sqrt{c^2 - x^2} + C$
- $\int x \arcsin \frac{x}{c} dx = (\frac{x^2}{2} - \frac{c^2}{4}) \arcsin \frac{x}{c} + \frac{x}{4}\sqrt{c^2 - x^2} + C$
- $\int x^2 \arcsin \frac{x}{c} dx = \frac{x^3}{3} \arcsin xc + \frac{x^2+2c^2}{9}\sqrt{c^2 - x^2} + C$
- $\int \arctan \frac{x}{c} dx = x \arctan x - \frac{c}{2} \ln(c^2x^2 + 1) + C$
- $\int x \arctan \frac{x}{c} dx = \frac{c^2+x^2}{2} \arctan xc - \frac{cx^2}{6} + \frac{c^3}{6} \ln c^2 + x^2 + C$

双曲函数积分表

- $\int \sinh cx dx = \frac{\cosh cx}{c} + C$
- $\int \cosh cx dx = \frac{\sinh cx}{c} + C$
- $\int \sinh^2 cx dx = \frac{\sinh 2cx}{4c} - \frac{x}{2} + C$
- $\int \cosh^2 cx dx = \frac{\sinh 2cx}{4c} + \frac{x}{2} + C$
- $\int \tanh cx dx = \frac{1}{c} \ln |\cosh cx| + C$
- $\int \coth cx dx = \frac{1}{c} \ln |\sinh cx| + C$

第一换元法

设函数 $F(u)$ 是 $f(u)$ 在区间 I 上的一个原函数，即 $\int f(u)\mathrm{d}u = F(u) + C$，又 $u = \phi(x)$ 在区间 I 上可导且其值域 $R(\phi) \subset I$，则有：

$$\int f[\phi(x)]\phi'(x)\mathrm{d}x = F[\phi(x)] + C$$

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第二换元法

就是把第一换元法倒过来用.

分部积分法

根据求导法则，有：

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

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三角换元法

无理式 $\sqrt{x^2+a^2}$，$\sqrt{x^2-a^2}$，$\sqrt{a^2-x^2}$，分别采用 $x=a\tan t$，$x=a\sec t$，$x=a\sin t$ 代换.