

积分表

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有理函数积分表

- ▶ $\int k dx = kx + C$
- ▶ $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C (\alpha \neq -1)$
- ▶ $\int \frac{dx}{x} = \ln |x| + C$
- ▶ $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + C (a \neq 0)$
- ▶ $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C$



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有理函数积分表

- ▶ $\int (ax + b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$
- ▶ $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax + b| + C$
- ▶ $\int x(ax + b)^n dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)} (ax + b)^{n+1} + C$
- ▶ $\int \frac{x}{ax+b} dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax + b| + C$
- ▶ $\int \frac{x}{(ax+b)^2} dx = \frac{b}{a^2(ax+b)} + \frac{1}{a^2} \ln |ax + b| + C$



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有理函数积分表

- ▶ $\int \frac{x}{(ax+b)^n} dx = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} + C (n \notin \{1, 2\})$
- ▶ $\int \frac{x^2}{ax+b} dx = \frac{1}{a^3} \left[\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2 \ln |ax+b| \right] + C$
- ▶ $\int \frac{x^2}{(ax+b)^2} dx = \frac{1}{a^3} (ax+b - 2b \ln |ax+b| - \frac{b^2}{ax+b}) + C$
- ▶ $\int \frac{x^2}{(ax+b)^3} dx = \frac{1}{a^3} \left[\ln |ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2} \right] + C$
- ▶ $\int \frac{x^2}{(ax+b)^n} dx = \frac{1}{a^3} \left[-\frac{1}{(n-3)(ax+b)^{n-3}} + \frac{2b}{(n-2)(ax+b)^{n-2}} - \frac{b^2}{(n-1)(ax+b)^{n-1}} \right] + C (n \notin \{1, 2, 3\})$



无理函数积分表

- ▶ $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
- ▶ $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln \left| \frac{x+\sqrt{x^2-a^2}}{a} \right| + C$
- ▶ $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln |x + \sqrt{x^2+a^2}| + C$
- ▶ $\int \frac{dx}{x\sqrt{ax+b}} = -\frac{2}{\sqrt{b}} \tanh^{-1} \sqrt{\frac{ax+b}{b}} + C$
- ▶ $\int \frac{\sqrt{ax+b}}{x} dx = 2(\sqrt{ax+b} - \sqrt{b} \tanh^{-1} \sqrt{\frac{ax+b}{b}}) + C$



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指数函数积分表

- ▶ $\int e^{cx} dx = \frac{e^{cx}}{c} + C$
- ▶ $\int a^{cx} dx = \frac{a^{cx}}{c \ln a} + C$
- ▶ $\int x e^{cx} dx = \frac{e^{cx}(cx-1)}{c^2} + C$
- ▶ $\int x^2 e^{cx} dx = e^{cx} \left(\frac{x^2}{c} - \frac{2x}{c^2} + \frac{2}{c^3} \right) + C$



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对数函数积分表

- ▶ $\int \ln x dx = x \ln x - x + C$
- ▶ $\int \ln^2 x dx = x \ln^2 x - 2x \ln x + 2x + C$
- ▶ $\int \frac{x}{x \ln x} = \ln |\ln x| + C$
- ▶ $\int \sin \ln x dx = \frac{x(\sin \ln x - \cos \ln x)}{2} + C$
- ▶ $\int \cos \ln x dx = \frac{x(\sin \ln x + \cos \ln x)}{2} + C$



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三角函数积分表

- ▶ $\int \sin x dx = -\cos x + C$
- ▶ $\int \cos x dx = \sin x + C$
- ▶ $\int x \sin cx dx = -\frac{\sin cx}{c^2} - \frac{x \cos cx}{c} + C$
- ▶ $\int x \cos cx dx = \frac{\cos cx}{c^2} + \frac{x \sin cx}{c} + C$
- ▶ $\int \tan x dx = -\frac{\ln |\cos x|}{1} + C = \frac{\ln |\sec x|}{1} + C$



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三角函数积分表

- ▶ $\int \sec x dx = \ln |\sec x + \tan x| + C$
- ▶ $\int \csc x dx = \ln |\csc x - \cot x| + C$
- ▶ $\int \sec^2 x dx = \tan x + C$
- ▶ $\int \csc^2 x dx = -\cot x + C$
- ▶ $\int \sec x \tan x dx = \sec x + C$
- ▶ $\int \csc x \cot x dx = -\csc x + C$



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反三角函数积分表

- ▶ $\int \arcsin \frac{x}{c} dx = x \arcsin \frac{x}{c} + \sqrt{c^2 - x^2} + C$
- ▶ $\int x \arcsin \frac{x}{c} dx = (\frac{x^2}{2} - \frac{c^2}{4}) \arcsin \frac{x}{c} + \frac{x}{4} \sqrt{c^2 - x^2} + C$
- ▶ $\int x^2 \arcsin \frac{x}{c} dx = \frac{x^3}{3} \arcsin \frac{x}{c} + \frac{x^2 + 2c^2}{9} \sqrt{c^2 - x^2} + C$
- ▶ $\int \arctan \frac{x}{c} dx = x \arctan \frac{x}{c} - \frac{c}{2} \ln(c^2 + x^2) + C$
- ▶ $\int x \arctan \frac{x}{c} dx = \frac{c^2 + x^2}{2} \arctan \frac{x}{c} - \frac{cx^2}{6} + \frac{c^3}{6} \ln c^2 + x^2 + C$



双曲函数积分表

- ▶ $\int \sinh cx dx = \frac{\cosh cx}{c} + C$
- ▶ $\int \cosh cx dx = \frac{\sinh cx}{c} + C$
- ▶ $\int \sinh^2 cx dx = \frac{\sinh 2cx}{4c} - \frac{x}{2} + C$
- ▶ $\int \cosh^2 cx dx = \frac{\sinh 2cx}{4c} + \frac{x}{2} + C$
- ▶ $\int \tanh cx dx = \frac{1}{c} \ln |\cosh cx| + C$
- ▶ $\int \coth cx dx = \frac{1}{c} \ln |\sinh cx| + C$



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第一换元法

设函数 $F(u)$ 是 $f(u)$ 在区间 I 上的一个原函数, 即 $\int f(u)du = F(u) + C$, 又 $u = \phi(x)$ 在区间 I 上可导且其值域 $R(\phi) \subset I$, 则有:

$$\int f[\phi(x)]\phi'(x)dx = F[\phi(x)] + C$$



第二换元法

就是把第一换元法倒过来用.



分部积分法

根据求导法则，有：

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$



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三角换元法

无理式 $\sqrt{x^2 + a^2}$, $\sqrt{x^2 - a^2}$, $\sqrt{a^2 - x^2}$, 分别采用 $x = a \tan t$, $x = a \sec t$, $x = a \sin t$ 代换.

