

Q1.1

2 Points

$$f(n) = n^2 \ln n$$
$$g(n) = \sum_{i=1}^n i$$

Q1.2

2 Points

$$f(n) = 1.001^n$$
$$g(n) = n^{1.001}$$

Q1.3

2 Points

$$f(n) = 2n!$$
$$g(n) = (2n)!$$

Q1.4

2 Points

$$f(n) = \sum_{i=1}^{2n} i^2$$
$$g(n) = \sum_{i=1}^n (2i)^2$$

Q2

6 Points

This question tests your understanding of the Insertion Sort algorithm as stated in the textbook and the lecture slides. Assume that we use Insertion Sort to sort the array A with 5 elements where the initial values of the array elements (from A[1] to A[5]) are A :

| | | | | |
|---|---|---|---|---|
| 4 | 3 | 1 | 5 | 2 |
|---|---|---|---|---|

During the execution of the algorithm, we may have to write into one of the 5 memory locations of the array elements, i.e., write into A[i] for some $i \in \{1, 2, 3, 4, 5\}$. Every time we write into one of these locations, we say that A is overwritten. Check the corresponding box on the answer sheet to answer each of the following questions.

Q2.1

3 Points

(a) What is the array content immediately after A is overwritten the 1st time?

- ☐ A = 1 3 4 5 2
- ☐ A = 3 4 4 5 2
- ☐ A = 4 4 1 5 2
- ☐ None of the above

Save Answer

Q2.2

3 Points

(b) What is the array content immediately after A is overwritten the 3rd time?

- ☐ A = 1 3 4 5 2
- ☐ A = 3 4 4 5 2
- ☐ A = 4 4 1 5 2
- ☐ None of the above

Q3

3 Points

Let $f(n) = 3n^2 - 1000$. In order to prove that $f(n) \in \Omega(n^2)$, we need to find a positive constant $c > 0$ and an integer $N \geq 1$ such that

$$f(n) \geq c \times n^2, \forall n \geq N$$

Answer the following questions on the answer sheet.

- (a1) Will $c = 1$, $N = 8$ make the proof correct?
- (a2) Will $c = 3$, $N = 12$ make the proof correct?
- (a3) Will $c = 5$, $N = 13$ make the proof correct?

Q4

3 Points

Let $g(n) = 13n^2 + 1000$. In order to prove that $f(n) \in O(n^2)$, we need to find a positive constant $c > 0$ and an integer $N \geq 1$ such that

$$g(n) \leq c \times n^2, \forall n \geq N$$

Answer the following questions on the answer sheet.

- (b1) Will $c = 12$, $N = 20$ make the proof correct?
- (b2) Will $c = 13$, $N = 20$ make the proof correct?
- (b3) Will $c = 14$, $N = 10$ make the proof correct?

Q5

10 Points

For each of the following 5 statements, check the corresponding box on the answer sheet if and only if the statement is true.

(a) If $f(n) \in O(n)$ and $g(n) \in \Theta(n)$,
then $f(n) + n \times g(n) \in \Theta(n^2)$.

(b) If $f(n) \in O(n)$ and $g(n) \in \Theta(n)$,
then $n \times f(n) + g(n) \in \Theta(n^2)$.

(c) If $f(n) \in O(n)$,
then $n \times f(n) + 1000n \in \Theta(n^2)$.

(d) If $f(n) \in O(n)$,
then $n \times f(n) + 1000n^2 \in \Theta(n^2)$.

(e) $n! \in O(3^n)$.