Homework 3

1. The following example was taken from the book by John Hull (page 294, 7th ed.), which uses the Black-Scholes model to price a European call and a put. You can just ignore its calculation, but please use the parameters to compute the price for the **put option** using the **Monte Carlo** method (don't use Black-Scholes; the information below is simply background). Please also show a convergence table of the put prices and their standard errors by sampling 10, 100, 1000, 10000, 100000 and 1 million trajectories of the stock price. Note that your prices should converge to p = 0.81, as calculated with Black-Scholes below.

Example 13.6

The stock price 6 months from the expiration of an option is \$42, the exercise price of the option is \$40, the risk-free interest rate is 10% per annum, and the volatility is 20% per annum. This means that $S_0 = 42$, $K = 40$, $r = 0.1$, $\sigma = 0.2$, $T = 0.5$,

$$d_1 = \frac{\ln(42/40) + (0.1 + 0.2^2/2) \times 0.5}{0.2\sqrt{0.5}} = 0.7693$$

$$d_2 = \frac{\ln(42/40) + (0.1 - 0.2^2/2) \times 0.5}{0.2\sqrt{0.5}} = 0.6278$$

and

$$Ke^{-rT} = 40e^{-0.05} = 38.049$$

Hence, if the option is a European call, its value c is given by

$$c = 42N(0.7693) - 38.049N(0.6278)$$

If the option is a European put, its value p is given by

$$p = 38.049N(-0.6278) - 42N(-0.7693)$$

Using the polynomial approximation just given or the NORMSDIST function in Excel,

$$N(0.7693) = 0.7791, \qquad N(-0.7693) = 0.2209$$

$$N(0.6278) = 0.7349, \qquad N(-0.6278) = 0.2651$$

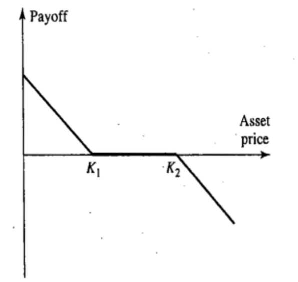
so that

$$c = 4.76, \qquad p = 0.81$$

Ignoring the time value of money, the stock price has to rise by \$2.76 for the purchaser of the call to break even. Similarly, the stock price has to fall by \$2.81 for the purchaser of the put to break even.

2. **Short Range Forwards** (John Hull, 7th ed. Chapter 15 Page 320)

   A *range forward contract* is a variation on a standard forward contract for hedging foreign exchange risk. In a ***short range forward contract***, a company buys a European put option with a strike price k1 and sells a European call option with a strike price k2, where k1 < k2. The payoff from a short range forward contract is a combination of the payoff from a long put and the payoff from a short call, as shown in the following figure.



In practice, a short range forward contract is set up so that the price of the put option equals the price of the call option. This means it costs nothing to set up a short range forward (buying a put and selling a call with the same price), just as it costs nothing to set up a regular forward contract.

Consider a US company that knows it will receive one million pounds sterling in three months. To hedge its risk, it signs a short range forward with an investment bank. The contract has k1 = 1.9, k2 = 1.9413, and T = 0.25 years. Suppose that the US and British interest rate are both 5%, the spot exchang rate Q0 = 1.92, and the exchange rate volatility is 0.14 per annum.

To summarize the parameters, Q0 (or s0) = 1.92, k_put = 1.9, k_call = 1.9413, r = 0.05, rf(or q) = 0.05, and T = 0.25

Please show that the price of this contract is zero using Monte Carlo. Note that we previously showed that for an underlying asset that pays no dividend,

$\Delta s = rs\Delta t + \sigma s\epsilon\sqrt{\Delta t}$ and $s_T = s_0 \exp\left(\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\epsilon\right)$.

However, for the exchange rate, the following similar equations apply,

$\Delta Q = \gamma Q\Delta t + \sigma Q\epsilon\sqrt{\Delta t}$ and $Q_T = Q_0 \exp\left(\left(\gamma - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\epsilon\right)$, where $\gamma = r - r_f$.

3. Lookback Options

The payoffs from lookback options depend on the maximum or minimum asset price reached during the life of the option. The payoff from a *floating lookback call* is the amount that the final asset price exceeds the minimum asset price achieved during the life of the option. At expiration T,

$$Payoff = S_T - S_{min,0\sim T}$$

Please use Monte Carlo methods to price a newly issued floating lookback call on a non-dividend-paying stock where the initial stock price is 50, its volatility is 0.4 per annum, the risk-free rate is 10% per annum, and the time to maturity is 0.25 years (or 3 months).