

# High Dimensional Data Analysis With Dependency and Under Limited Memory

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# Outline

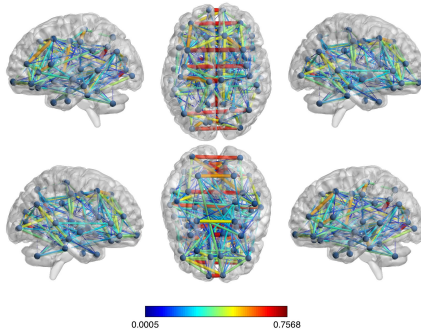
Multivariate Spectral Density Estimation Under Weak Sparsity

Low Rank Tucker Approximation of a Tensor from Streaming Data

A Small Example to Connect Two Researches

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# Motivation



**Figure:** Interactions Between Regions in Brain

## Weakly & strongly stationary time series

### Definition (Weak Stationarity)

$p$ -variate time series  $X$  is weakly stationary, if  $\mathbb{E}X_t = \mathbb{E}X_s$  for any  $t, s$  and  $\Gamma(\ell) := \mathbb{E}X_t X_{t-\ell}^\top$  only depends on the lag  $\ell$ .

### Definition (Strong Stationarity)

$p$ -variate time series  $X$  is strongly stationary, if for any sequence  $t_1, \dots, t_n$ ,  $X_{t_1} \cdots X_{t_n}$  has the same distribution of  $X_{t_1+\tau} \cdots X_{t_n+\tau}$  for any integer  $\tau$ ,

## Gaussian process

### Definition (Gaussian Process)

$p$ -variate time series  $X$  is Gaussian process if for any sequence  $t_1, \dots, t_n$ ,  $X_{t_1} \dots X_{t_n}$  are jointly Gaussian distributed.

For Gaussian process, weak stationarity is equivalent to strong stationarity.

## Spectral density

Given a weakly stationary  $p$ -variate time series  $X$ , the spectral density at frequency  $\omega \in [-\pi, \pi)$  is defined

$$f(\omega) = \sum_{\ell=-\infty}^{\infty} \Gamma(\ell) e^{-i\omega\ell}$$

where  $\Gamma(\ell) = \mathbb{E}X_0X_{-\ell}^\top$ .  $X_t$  is independent with  $X_s$ ,  $t \neq s$  iff  $f_{rs}(\omega) = 0$  for any  $\omega$ .

## Thresholding estimator under weak sparsity- An example

Suppose that we have  $n$  observation of  $p$ -variate Gaussian distribution as follows.

$$y_i \stackrel{i.i.d}{\sim} \mathcal{N} \left( \mu, \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & \sigma_p^2 \end{bmatrix} \right),$$

$$i = 1, \dots, n.$$

## An example

The maximum likelihood estimator for  $\mu_j$  is  $\bar{y}_j = \frac{1}{n} \sum_{i=1}^n y_{ij}$ .

**Does not Work Well Under Weak Sparsity**

$$\mu \in \left\{ \mu \in \mathbb{R}^p, \sum_{j=1}^p |\mu_j|^q \leq c_0(p) \right\}.$$

for some  $0 \leq q < 1$  and  $c_0(p)$  measures the weak sparsity.



## Solution : Thresholding

Suppose  $\sigma_i \leq B$ , define element-wise thresholding operator

$$T_\lambda(x) = \begin{cases} x & |x| \geq \lambda \\ 0 & \text{else} \end{cases}$$

. Hard thresholding estimator  $T_\lambda(\bar{y}_j)$  can be shown asymptotically consistent under weak sparsity where we set

$$\lambda \propto B \sqrt{\frac{\log p}{n}}$$

and assume  $\lambda \rightarrow 0$ .

## Two key ingredients for thresholding

Two key ingredients under above example assuming weak sparsity. Cai & Liu 2011

- ▶ An element-wise concentration inequality :

$$\mathbb{P}(|\bar{y}_j - \mu_j| \geq \eta) \leq 2 \exp(-n\eta^2/2\sigma_j^2).$$

- ▶  $\sigma_j$  are uniformly bounded.

## Shortcomings for hard thresholding

- ▶  $\sigma_j$  may vary much
- ▶  $B$  will appear in the thresholding value making convergence rate slow

## Solution: adaptive thresholding

Simply estimate  $\sigma_j$ , say with sample standard deviation:

$$\hat{\sigma}_j = \sqrt{1/(n-1) \sum_{i=1}^n (y_{ij} - \bar{y}_j)^2}$$

and replace  $B$ :  $\lambda_j \propto \hat{\sigma}_j \sqrt{\frac{\log p}{n}}$ . Now we can relax constraint in upper bound for  $\sigma_j$  and upper bound will not appear in rate of convergence.

## An Similar Example: Covariance Matrix

$$y_i \stackrel{i.i.d}{\sim} \mathcal{N}(0, \Sigma_{p \times p})$$

**Goal:** Estimate  $\Sigma$  assuming weak sparsity,  $\|\Sigma\|_1 \leq c_0(p)$  . Bickel & Levina 2008

## A similar example: covariance matrix

Estimate the expectation of a vector of length  $p^2$ :

$$[(y_1 y_1^\top)_{rs}, 1 \leq r, s \leq p].$$

**MLE:**  $\hat{\Sigma} = \frac{1}{n} \sum_{i=1}^n y_i y_i^\top$ . But we need to perform thresholding.  
Remember two ingredients:

- ▶  $\mathbb{P}(|\hat{\Sigma}_{rs} - \Sigma_{rs}| \geq \eta) \leq c_1 \exp(-c_2 n \eta^2)$
- ▶  $\text{var}((y_1 y_1^\top)_{rs}) = \Sigma_{rr} \Sigma_{ss} + \Sigma_{rs}^2 \leq 2 \max_{r=1}^p \Sigma_{rr}^2$

Thus Bickel & Levina 2008 presents an assumption  $\max_{r=1}^p \Sigma_{rr}$  is bounded.

## A similar example: covariance matrix

hard thresholding:  $\lambda_{rs} \propto (\max_{r=1}^p \Sigma_{rr}) \sqrt{\frac{\log p}{n}}$

adaptive thresholding:  $\lambda_{rs} \propto \sqrt{\widehat{\mathbf{var}}(y_1 y_1^\top)_{rs}} \sqrt{\frac{\log p}{n}}$  where

$$\widehat{\mathbf{var}}(y_1 y_1^\top)_{rs} = \frac{1}{n-1} \sum_{i=1}^n \left[ (y_i y_i^\top)_{rs} - \frac{1}{n} \sum_{i=1}^n (y_i y_i^\top)_{rs} \right]^2$$





# Outline

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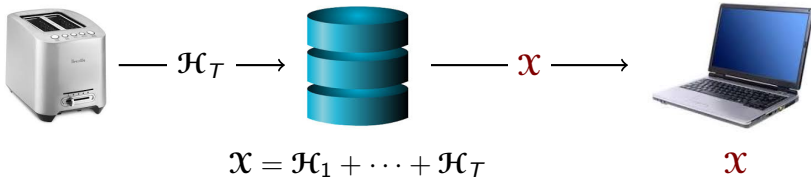
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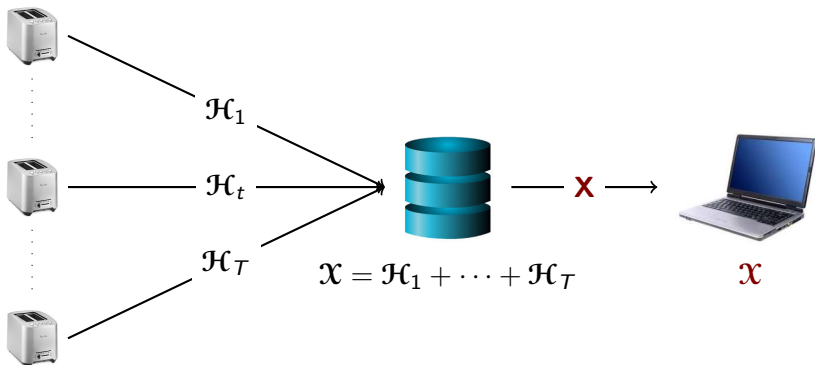
# Motivation

We listed three scenarios for Motivation Borrowed from Professor Udell's Recent Talk

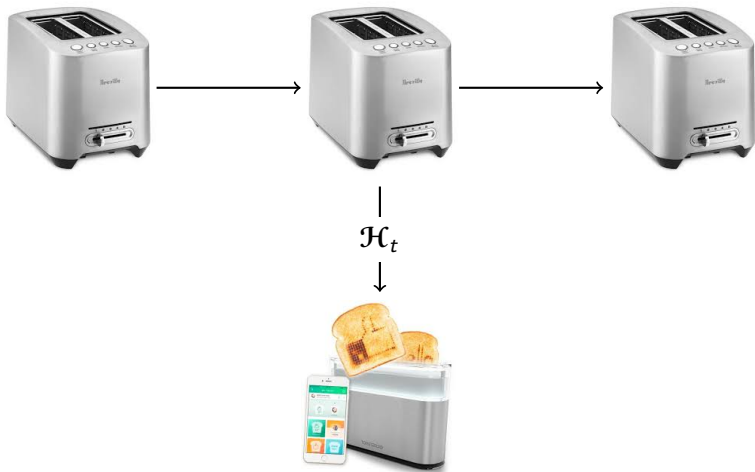
## Big data, small laptop



## Distributed data



## Streaming data



$$\mathcal{X}^{(t)} = \mathcal{H}_1 + \cdots + \mathcal{H}_t$$

## Notation

tensor to compress:

- ▶ tensor  $\mathcal{X} \in \mathbf{R}^{l_1 \times \cdots \times l_N}$  with  $N$  modes
- ▶ sometimes assume  $l_1 = \cdots = l_N = l$  for simplicity

indexing:

- ▶  $[N] = 1, \dots, N$
- ▶  $l_{(-n)} = l_1 \times \cdots \times l_{n-1} \times l_{n+1} \times \cdots \times l_N$

tensor operations:

- ▶ mode  $n$  product: for  $\mathcal{A} \in \mathbf{R}^{k \times l_n}$ ,  
 $\mathcal{X} \times_n \mathbf{A} \in \mathbf{R}^{l_1 \times \cdots \times l_{n-1} \times k \times l_{n+1} \times \cdots \times l_N}$
- ▶ unfolding  $\mathbf{X}^{(n)} \in \mathbf{R}^{l_n \times l_{(-n)}}$  stacks mode- $n$  fibers of  $\mathcal{X}$  as columns of matrix

## Review of Our Tool: Linear Sketch

A linear random projection can be represented as a random matrix  $\mathbf{\Omega} \in \mathbf{R}^{d \times k}$ , operating on a vector  $\mathbf{x} \in \mathbf{R}^d$  or a matrix  $\mathbf{X} \in \mathbf{R}^{m \times d}$  to reduce the dimension:

$$\begin{aligned}\mathbf{x} \in \mathbf{R}^n &\rightarrow \mathbf{\Omega}^\top \mathbf{x} \in \mathbf{R}^k \\ \mathbf{X} \in \mathbf{R}^{m \times d} &\rightarrow \mathbf{X}\mathbf{\Omega} \in \mathbf{R}^{m \times k}.\end{aligned}\tag{1}$$

## Properties Preserved after Projection

### Lemma (Arriaga & Vempala 2006)

Let  $\mathbf{x} \in \mathbf{R}^d$ , assume that the entries in  $\mathbf{\Omega} \in \mathbf{R}^{d \times k}$  are sampled independently from  $\mathcal{N}(0, 1)$ . Then

$$\mathbf{Prob} \left( (1 - \epsilon) \|\mathbf{x}\|^2 \leq \left\| \frac{1}{\sqrt{k}} \mathbf{\Omega}^\top \mathbf{x} \right\|^2 \leq (1 + \epsilon) \|\mathbf{x}\|^2 \right) \leq 1 - 2e^{-(\epsilon^2 - \epsilon^3)k/4}. \quad (2)$$

### Lemma (Halko, Martinsson & Tropp 2011)

Let  $\mathbf{X} \in \mathbf{R}^{m \times d}$ , assume that the entries in  $\mathbf{\Omega} \in \mathbf{R}^{d \times (k+p)}$  are sampled independently from  $\mathcal{N}(0, 1)$ . Then let  $\mathbf{Q}$  be the orthonormal matrix from QR factorization  $\mathbf{X}\mathbf{\Omega} = \mathbf{Q}\mathbf{R}$ , then

$$\|\mathbf{X} - \mathbf{Q}\mathbf{Q}^\top \mathbf{X}\|_F \leq \left(1 + \frac{k}{p-1}\right)^{1/2} \left(\sum_{j>k} \sigma_j^2\right)^{1/2}. \quad (3)$$



## Tucker factorization

rank  $\mathbf{r} = (r_1, \dots, r_N)$  **Tucker factorization** of  $\mathcal{X} \in \mathbf{R}^{I_1 \times \dots \times I_N}$ :

$$\mathcal{X} = \mathcal{G} \times_1 \mathbf{U}_1 \cdots \times_N \mathbf{U}_N =: \llbracket \mathcal{G}; \mathbf{U}_1, \dots, \mathbf{U}_N \rrbracket$$

where

- ▶  $\mathcal{G} \in \mathbf{R}^{r_1 \times \dots \times r_N}$  is the **core matrix**
- ▶  $\mathbf{U}_n \in \mathbf{R}^{I_n \times r_n}$  is the **factor matrix** for each mode  $n \in [N]$

(sometimes assume  $r_1 = \dots = r_N = r$  for simplicity)

Tucker is useful for compression: when  $N$  is small,

- ▶ Tucker stores  $O(rNI)$  numbers for rank  $r^3$  approximation
- ▶ CP stores  $O(rNI)$  numbers for rank  $r$  approximation

## The sketch

approximate factor matrices and core:

- ▶ **Factor sketch (k).** For each  $n \in [N]$ , fix random DRM  $\mathbf{\Omega}_n \in \mathbb{R}^{l_{(-n)} \times k_n}$  and compute the sketch

$$\mathbf{V}_n = \mathbf{X}^{(n)} \mathbf{\Omega}_n \in \mathbb{R}^{l_n \times k_n}.$$

- ▶ **Core sketch (s).** For each  $n \in [N]$ , fix random DRM  $\mathbf{\Phi}_n \in \mathbb{R}^{l_n \times s_n}$ . Compute the sketch

$$\mathcal{H} = \mathcal{X} \times_1 \mathbf{\Phi}_1^\top \cdots \times_N \mathbf{\Phi}_N^\top \in \mathbb{R}^{s_1 \times \cdots \times s_N}.$$

- ▶ *Rule of thumb.* Pick  $\mathbf{k}$  as big as you can afford, pick  $\mathbf{s} = 2\mathbf{k}$ .
- ▶ define  $(\mathcal{H}, \mathbf{V}_1, \dots, \mathbf{V}_N) = \text{SKETCH}(\mathcal{X}; \{\mathbf{\Phi}_n, \mathbf{\Omega}_n\}_{n \in [N]})$

## Low memory DRMs

factor sketch DRMs are big! Same size of the tensor

- ▶  $l_{(-n)} \times k_n$  for each  $n \in [N]$
- ▶ **Solution:** Generate random matrix  $\mathbf{A}_n \in \mathbb{R}^{l_n \times k}$  [Sun, Guo, Luo, Tropp & Udell 2019]

$$\boldsymbol{\Omega} := (\mathbf{A}_1 \odot \cdots \odot \mathbf{A}_N)$$

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} A_{11}\mathbf{B} & \cdots & A_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ A_{m1}\mathbf{B} & \cdots & A_{mn}\mathbf{B} \end{bmatrix}.$$

We let  $\mathbf{X} \odot \mathbf{Y}$  denotes the *Khatri-Rao product*,  $\mathbf{A} \in \mathbb{R}^{I \times K}$ ,  $\mathbf{B} \in \mathbb{R}^{J \times K}$ , i.e. the "matching column-wise" Kronecker product. The resulting matrix of size  $(IJ) \times K$  is given by:

$$\mathbf{A} \odot \mathbf{B} = [\mathbf{A}_{(1,.)} \otimes \mathbf{B}_{(1,.)}, \dots, \mathbf{A}_{(K,.)} \otimes \mathbf{B}_{(K,.)}]. \quad (4)$$

## Two pass algorithm

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**Algorithm** Two Pass Sketch and Low Rank Recovery

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**Given:** tensor  $\mathcal{X}$ , DRMs  $\{\Phi_n, \Omega_n\}_{n \in [N]}$  with parameters  $\mathbf{k}$  and  $\mathbf{s} \geq \mathbf{k}$

1. *Sketch.*  $(\mathcal{H}, \mathbf{V}_1, \dots, \mathbf{V}_N) = \text{SKETCH}(\mathcal{X}; \{\Phi_n, \Omega_n\}_{n \in [N]})$
2. *Recover factor matrices.* For  $n \in [N]$ ,

$$(\mathbf{Q}_n, \sim) \leftarrow \text{QR}(\mathbf{V}_n)$$

3. *Recover core.*

$$\mathcal{W} \leftarrow \mathcal{X} \times_1 \mathbf{Q}_1 \cdots \times_N \mathbf{Q}_N$$

**Return:** Tucker approximation  $\tilde{\mathcal{X}} = \llbracket \mathcal{W}; \mathbf{Q}_1, \dots, \mathbf{Q}_N \rrbracket$  with rank  $\leq \mathbf{k}$

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accesses  $\mathcal{X}$  twice: 1) to sketch 2) to recover core

## Intuition: one pass core recovery

- ▶ we want to know  $\mathcal{W}$ :  
compression of  $\mathcal{X}$  using factor range approximations  $\mathbf{Q}_n$
- ▶ we observe  $\mathcal{H}$ :  
compression of  $\mathcal{X}$  using random projections  $\Phi_n$

how to approximate  $\mathcal{W}$ ?

$$\begin{aligned}\mathcal{X} &\approx \mathcal{X} \times_1 \mathbf{Q}_1 \mathbf{Q}_1^\top \times \cdots \times_N \mathbf{Q}_N \mathbf{Q}_N^\top \\ &= \left( \mathcal{X} \times_1 \mathbf{Q}_1^\top \times_N \cdots \times \mathbf{Q}_N^\top \right) \times_1 \mathbf{Q}_1 \cdots \times_N \mathbf{Q}_N \\ &= \mathcal{W} \times_1 \mathbf{Q}_1 \cdots \times_N \mathbf{Q}_N \\ \underbrace{\mathcal{X} \times_1 \Phi_1^\top \cdots \times_N \Phi_N^\top}_{\mathcal{H}} &\approx \mathcal{W} \times_1 \Phi_1^\top \mathbf{Q}_1 \times \cdots \times_N \Phi_N^\top \mathbf{Q}_N\end{aligned}$$

we can solve for  $\mathcal{W}$ :  $s > k$ , so each  $\Phi_n^\top \mathbf{Q}_n$  has a left inverse (whp):

$$\mathcal{W} \approx \mathcal{H} \times_1 (\Phi_1^\top \mathbf{Q}_1)^\dagger \times \cdots \times_N (\Phi_N^\top \mathbf{Q}_N)^\dagger$$

## One pass algorithm

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**Algorithm** One Pass Sketch and Low Rank Recovery

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**Given:** tensor  $\mathcal{X}$ , rank  $\mathbf{r} = (r_1, \dots, r_N)$ , DRMs  $\{\Phi_n, \Omega_n\}_{n \in [N]}$

- ▶ *Sketch.*  $(\mathcal{H}, \mathbf{V}_1, \dots, \mathbf{V}_N) = \text{SKETCH}(\mathcal{X}; \{\Phi_n, \Omega_n\}_{n \in [N]})$
- ▶ *Recover factor matrices.* For  $n \in [N]$ ,

$$(\mathbf{Q}_n, \sim) \leftarrow \text{QR}(\mathbf{V}_n)$$

- ▶ *Recover core.*

$$\mathcal{W} \leftarrow \mathcal{H} \times_1 (\Phi_1^\top \mathbf{Q}_1)^\dagger \times \dots \times_N (\Phi_N^\top \mathbf{Q}_N)^\dagger$$

**Return:** Tucker approximation  $\hat{\mathcal{X}} = \llbracket \mathcal{W}; \mathbf{Q}_1, \dots, \mathbf{Q}_N \rrbracket$

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accesses  $\mathcal{X}$  only once, to sketch

Source: [Sun et al. 2019]

## Fixed rank approximation

to truncate reconstruction to rank  $\mathbf{r}$ , truncate core:

### Lemma

For a tensor  $\mathcal{W} \in \mathbb{R}^{k_1 \times \cdots \times k_N}$ , orthogonal matrices  $\mathbf{Q}_n \in \mathbb{R}^{k_n \times r_n}$ ,

$$[[\mathcal{W} \times_1 \mathbf{Q}_1 \cdots \times_N \mathbf{Q}_N]]_{\mathbf{r}} = [[\mathcal{W}]]_{\mathbf{r}} \times_1 \mathbf{Q}_1 \cdots \times_N \mathbf{Q}_N,$$

where  $[[\cdot]]$  denotes the best rank  $\mathbf{r}$  Tucker approximation.

$\implies$  compute fixed rank approximation using, e.g., HOOI on (small) core approximation  $\mathcal{W}$

## Tail Energy

For each unfolding  $\mathbf{X}^{(n)}$ , define its  $\rho$ th tail energy as

$$(\tau_{\rho}^{(n)})^2 := \sum_{k > \rho}^{\min(l_n, l_{(-n)})} \sigma_k^2(\mathbf{X}^{(n)}),$$

where  $\sigma_k(\mathbf{X}^{(n)})$  is the  $k$ th largest singular value of  $\mathbf{X}^{(n)}$ .



## Guarantees for two pass

**Theorem** ([Sun, Guo, Tropp & Udell 2018])

Sketch the tensor  $\mathcal{X}$  using a Tucker sketch with parameters  $\mathbf{k}$  using DRMs with i.i.d. Gaussian  $\mathcal{N}(0, 1)$  entries. Then the approximation  $\hat{\mathcal{X}}_2$  computed with the two pass method satisfies

$$\mathbb{E} \|\mathcal{X} - \hat{\mathcal{X}}_2\|_F^2 \leq \min_{1 \leq \rho_n < k_n - 1} \sum_{n=1}^N \left( 1 + \frac{\rho_n}{k_n - \rho_n - 1} \right) (\tau_{\rho_n}^{(n)})^2.$$

## Guarantees for one pass

**Theorem** ([Sun et al. 2018])

Sketch  $\mathcal{X}$  with Gaussian DRMs of parameters  $\mathbf{k}$ ,  $\mathbf{s} \geq 2\mathbf{k} + 1$ .  
Form a rank  $\mathbf{r}$  Tucker approximation  $\hat{\mathcal{X}}$  using the one pass algorithm. Then

$$\mathbb{E} \|\mathcal{X} - \hat{\mathcal{X}}\|_F^2 \leq (1 + \Delta) \min_{1 \leq \rho_n < k_n - 1} \sum_{n=1}^N \left( 1 + \frac{\rho_n}{k_n - \rho_n - 1} \right) (\tau_{\rho_n}^{(n)})^2$$

where  $\Delta = \max_{n=1}^N k_n / (s_n - k_n - 1)$

## Comparison to other methods in pseudo optimality

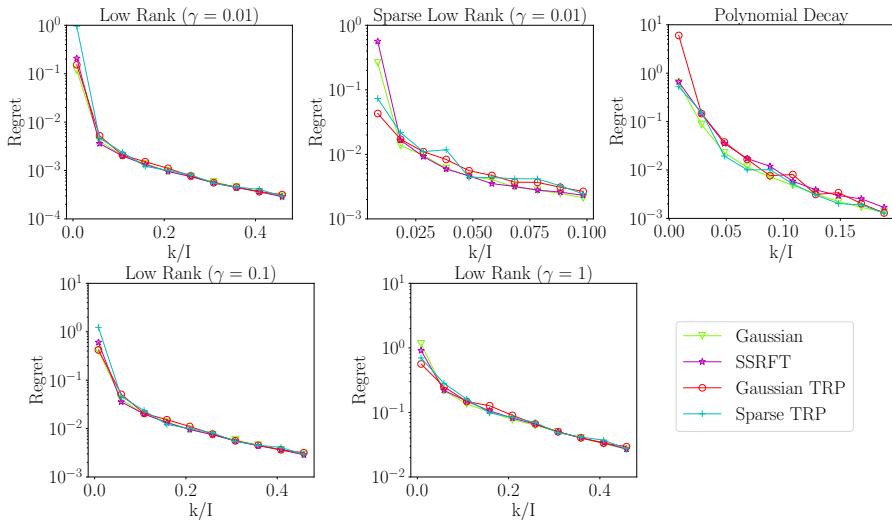
- ▶ HOSVD and ST-HOSVD is pseudo optimal with factor  $N$ :

$$\|\mathcal{X} - \llbracket \mathcal{X} \rrbracket_{\mathbf{ST-r}}\|_F \leq \sqrt{\sum_{n=1}^N (\tau_{r_n}^{(n)})^2} \leq \sqrt{N} \|\mathcal{X} - \llbracket \mathcal{X} \rrbracket_{\mathbf{r}}\|_F, \quad (5)$$

- ▶ Set  $\mathbf{k} = 2\mathbf{r} + 1$  and  $\mathbf{s} = 2\mathbf{k} + 1$ , and use truncated QR factorization to get  $\mathbf{Q} \in \mathbf{R}^{l_n \times r_n}$  from factor sketch.

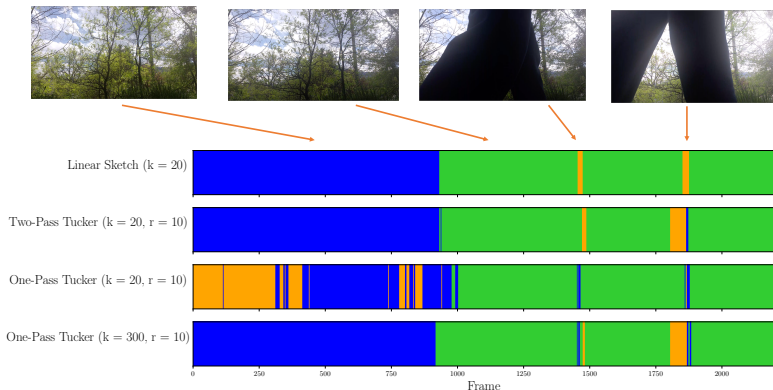
$$\|\mathcal{X} - \hat{\mathcal{X}}_2\|_F \leq \sqrt{\sum_{n=1}^N (\tau_{r_n}^{(n)})^2} \leq \sqrt{2N} \|\mathcal{X} - \llbracket \mathcal{X} \rrbracket_{\mathbf{r}}\|_F. \quad (6)$$

## Different DRMs perform similarly



Comments: Synthetic data,  $l = 600$  and  $\mathbf{r} = (5, 5, 5)$ .  $k/l = .4 \implies 20\times$  compression.

# Video scene classification



Comments: Video data  $2200 \times 1080 \times 1980$ . Classify scenes using  $k$ -means on: 1) linear sketch along the time dimension  $k = 20$  (Row 1); 2) The Tucker factor along the time dimension, computed via our two pass (Row 2) and one pass (Row 3) sketching algorithm  $(r, k, s) = (10, 20, 41)$ . 3) The Tucker factor along the time dimension, computed via our one pass (Row 4) sketching algorithm  $(r, k, s) = (10, 300, 601)$ .

## Property of tensor random projection

**Preserve Pair-wise Distance:** Fix  $\mathbf{x} \in \mathbb{R}^{\prod_{n=1}^N d_n}$ . Generate random matrix

$$\mathbf{\Omega} = (\mathbf{A}_1 \odot \cdots \odot \mathbf{A}_N)$$

### Theorem

Fix  $\mathbf{x} \in \mathbb{R}^{\prod_{n=1}^N d_n}$ . Form a TRP and  $\text{TRP}_T$  of order  $N$  with range  $k$  composed of independent matrices with independent columns whose entries are mean zero, variance one, fourth moment  $\Delta$ , and within each column every pair of elements has covariance zero. Then

$$\begin{aligned} \mathbb{E} \left\| \frac{1}{\sqrt{k}} \mathbf{\Omega}^\top \mathbf{x} \right\|^2 &= \|\mathbf{x}\|^2 \\ \text{var}(\| \frac{1}{\sqrt{k}} \mathbf{\Omega}^\top \mathbf{x} \|^2) &= \frac{1}{k} (\Delta^N - 3) \|\mathbf{x}\|_4^4 + \frac{2}{k} \|\mathbf{x}\|_2^4 \end{aligned} \tag{7}$$

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## A small example to connect two researches

Suppose we have a p-variate time series  $\{\mathcal{X}_t\}$  which reveal its column one by one.

- ▶ Before we know the exactly number of time points, we cannot calculate Fourier coefficients (depends in sample size)
- ▶ the data matrix  $\mathcal{X}$  is too big to store



## Sketching $\mathcal{X}$

Using sketching algorithm [Tropp, Yurtsever, Udell & Cevher 2017] to get an approximation of  $\mathcal{X} : \hat{\mathcal{X}}$ .  $\hat{d}_j = \hat{\mathcal{X}}^\top e_j$ . Simple observation:  $\|e_j\| = 1$ , which does averaging will not make error bound worse.

$$\|\hat{d}_j - d\| \leq \|\hat{\mathcal{X}} - \mathcal{X}\|.$$

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## References

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