High Dimensional Data Analysis With Dependency and Under Limited Memory

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Outline

Multivariate Spectral Density Estimation Under Weak Sparsity

Low Rank Tucker Approximation of a Tensor from Streaming Data

A Connection of These Two

Motivation

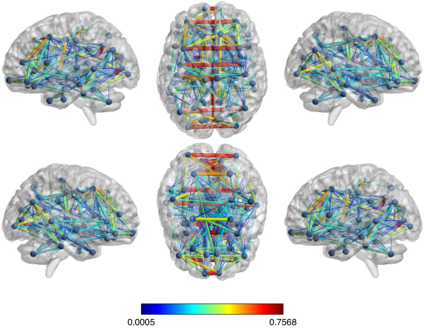

Figure: Interactions Between Regions in Brain

Weakly & Strongly Stationary Time

Definition (Weak Stationarity)

p-variate time series X is weakly stationary, if $\mathbb{E}X_t = \mathbb{E}X_s$ for any t, s and $\Gamma(\ell) := \mathbb{E}X_t X_{t-\ell}^\top$ only depends on the lag ℓ.

Definition (Strong Stationarity)

p-variate time series X is strongly stationary, if for any sequence $t_1, \cdots, t_n$, $X_{t_1} \cdots X_{t_n}$ has the same distribution of $X_{t_1+\tau} \cdots X_{t_n+\tau}$ for any integer τ,

Gaussian Process

Definition (Gaussian Process)

p-variate time series X is Gaussian process if for any sequence $t_1, \cdots, t_n$, $X_{t_1} \cdots X_{t_n}$ are jointly Gaussian distributed.

For Gaussian process, weak stationarity is equivalent to strong stationarity.

Spectral Density

Given a weakly stationary p-variate time series X, the spectral density at frequency $\omega \in [-\pi, \pi)$ is defined

$$f(\omega) = \sum_{\ell=-\infty}^{\infty} \Gamma(\ell) e^{-i\omega\ell}$$

where $\Gamma(\ell) = \mathbb{E} X_0 X_{-\ell}^\top$. X_t is independent with X_s, $t \neq s$ iff $f_{rs}(\omega) = 0$ for any ω.

Thresholding Estimator Under Weak Sparsity- A Example

Suppose that we have n observation of p-variate Gaussian distribution as follows.

$$y_i \overset{i.i.d}{\sim} \mathcal{N}\left(\mu, \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & 0 & \sigma_p^2 \end{bmatrix}\right),$$

$i = 1, \cdots, n$.

A Example

The maximum likelihood estimator for μ_j is $\bar{y}_j = \frac{1}{n}\sum_{i=1}^{n} y_{ij}$.
Does not Work Well Under Weak Sparsity

$$\mu \in \left\{ \mu \in \mathbb{R}^p, \sum_{j=1}^{p} |\mu_j|^q \leq c_0(p) \right\}.$$

for some $0 \leq q < 1$ and $c_0(p)$ measures the weak sparsity.

Solution : Thresholding

Suppose $\sigma_i \leq B$, define element-wise thresholding operator

$$T_\lambda(x) = \begin{cases} x & |x| \geq \lambda \\ 0 & \text{else} \end{cases}$$

. Hard thresholding estimator $T_\lambda(\bar{y}_j)$ can be shown asymptotically consistent under weak sparsity where we set

$$\lambda \propto B\sqrt{\frac{\log p}{n}}.$$

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