Second Coursework
(Component Analysis and Optimisation)

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General instructions: Regarding the coding part, you should only complete the matlab functions required for both parts as described below. No other matlab file will be taken into account during marking. Your code should contain comments about the mathematical methodology you used in your implementation. Regarding the written part, you should produce a pdf named `report.pdf`, where you will include your answers. No aspect of your submission may be hand-drawn. You are strongly encouraged to use LaTeX to create the written component.

Part I: You are given a facial dataset with identity information per sample (the dataset is part of PIE facial database). You are also given six Matlab code files. In particular, `demo_PCA.m`, `demo_wPCA.m` and `demo_LDA.m` run a face recognition protocol and report the results in the end (in the form of recognition error). Demo files may not be modified. PCA.m, wPCA.m and LDA.m are utilised by `demo_PCA.m`, `demo_wPCA.m` and `demo_LDA.m`, respectively, and are to be completed matlab functions that should perform dimensionality reduction techniques (PCA, whitened PCA and LDA, respectively) on Small Sample Sized (SSS) problems (number of samples significantly less that the number of features), according to the notes. You should attach in your report plots of the recognition error versus the number of components kept for each of the methods. Briefly explain which method is the best and why.

(10 marks)

Part II:

Assume you are given a set of n data samples $\mathbf{x}_1, \ldots, \mathbf{x}_n$. The minimum enclosing hyper-sphere of the above data samples can be found by the following constrained optimisation problem

$$\begin{array}{ll} \min_{R,\mathbf{a},\xi_i} & R^2 + C\sum_{i=1}^{n}\xi_i \\ \text{subject to} & (\mathbf{x}_i - \mathbf{a})^T(\mathbf{x}_i - \mathbf{a}) \leq R^2 + \xi_i, \forall \xi_i \geq 0. \end{array} \tag{1}$$

where R is the radius of the hyper-sphere, $\mathbf{a}$ is the center of the hyper-sphere and the variable C gives the trade-of between simplicity (or volume of the sphere) and the number of errors.

You should perform the following tasks

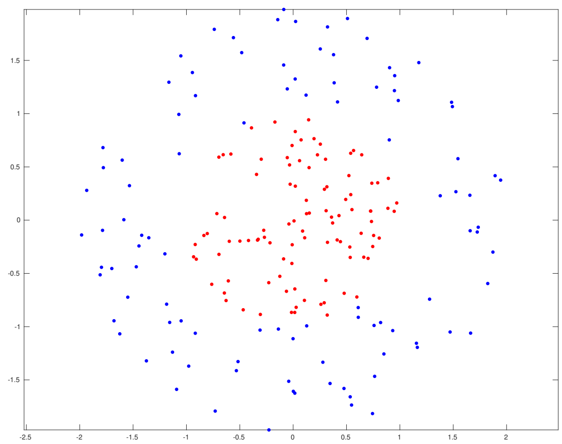

Figure 1: The two classes of simulated data (class one with red, class two with blue)

1. Formulate the Lagrangian of the above optimisation problem and derive its dual. Attach your answers in the report. Show the steps that you followed, do not only report the final results.

2. Perform the above when using arbitrary positive definite kernels (i.e., $k(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_j)$) , i.e. find the dual of the following optimisation problem

$$\begin{array}{ll} \min_{R,\mathbf{a},\xi_i} & R^2 + C\sum_{i=1}^{n} \xi_i \\ \text{subject to} & (\phi(\mathbf{x}_i) - \mathbf{a})^T(\phi(\mathbf{x}_i) - \mathbf{a}) \leq R^2 + \xi_i, \forall \xi_i \geq 0. \end{array} \tag{2}$$

   Attach your answers in the report. Show the steps that you followed, do not only report the final results.

3. Assume you are given the following code to simulate data. Code to produce data for class one (with red in Figure 1)

```
rng(1); % For reproducibility
r = sqrt(rand(100,1)); % Radius
t = 2*pi*rand(100,1);  % Angle
data1 = [r.*cos(t), r.*sin(t)]; % Points
```

   Code to produce data for class two (with blue in Figure 1)

```
r2 = sqrt(3*rand(100,1)+1); % Radius
t2 = 2*pi*rand(100,1);      % Angle
data2 = [r2.*cos(t2), r2.*sin(t2)]; % points
```

You can plot the data using the following code.

```
figure;
plot(data1(:,1),data1(:,2),'r.','MarkerSize',15)
hold on
plot(data2(:,1),data2(:,2),'b.','MarkerSize',15)
axis equal
hold off
```

Produce data from the two classes and find their optimal enclosing hyper-sphere (i.e., calculate the centres and the radii) by solving the dual optimisation problem of (2) in Matlab (to solve the quadratic constrained optimisation problem you may use `quadprog()`). To solve this coding part, you should fill in the function `calcRandCentre` in the Matlab file `partII.m`. Attach the final plot with the centres and the radii in your report.

(10 marks)