Tutorial #9

Monday, November 13, 2017 8:22 PM

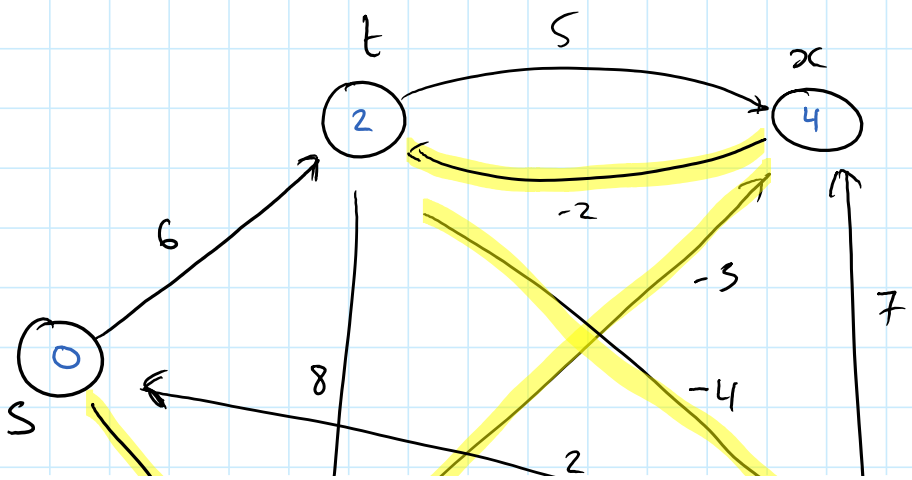
Single Source Shortest Path

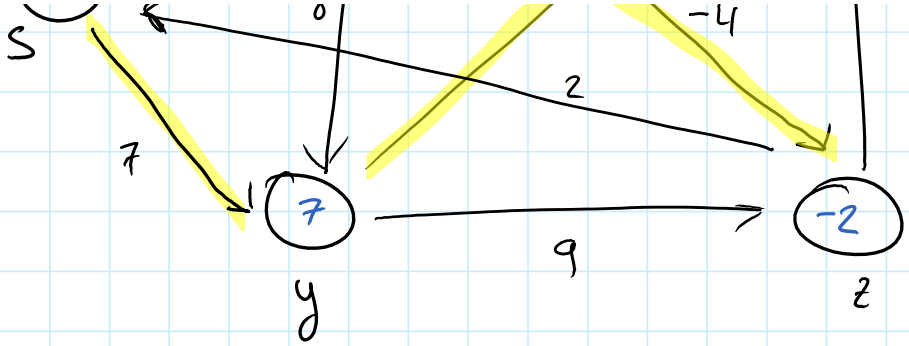
- Given graph $G = (V, E)$
- want to find the shortest path from a given source vertex $s \in V$ to each vertex $v \in V$

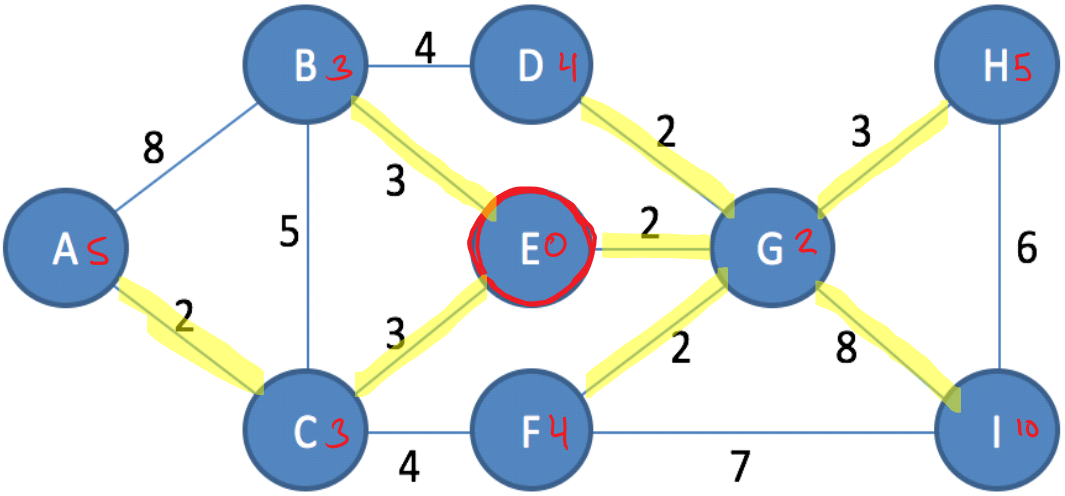
Bellman-Ford

- Returns boolean indicating whether or not there is a negative-weight cycle that is reachable from the source.
- If neg-weight cycle ⇒ False (no soln exists)
- If no such cycle ⇒ True (+ shortest paths & weights)

```
BELLMAN-FORD(G, w, s)
1  INITIALIZE-SINGLE-SOURCE(G, s)
2  for i = 1 to |G.V| − 1
3       for each edge (u, v) ∈ G.E
4            RELAX(u, v, w)
5  for each edge (u, v) ∈ G.E
6       if v.d > u.d + w(u, v)
7            return FALSE
8  return TRUE
```







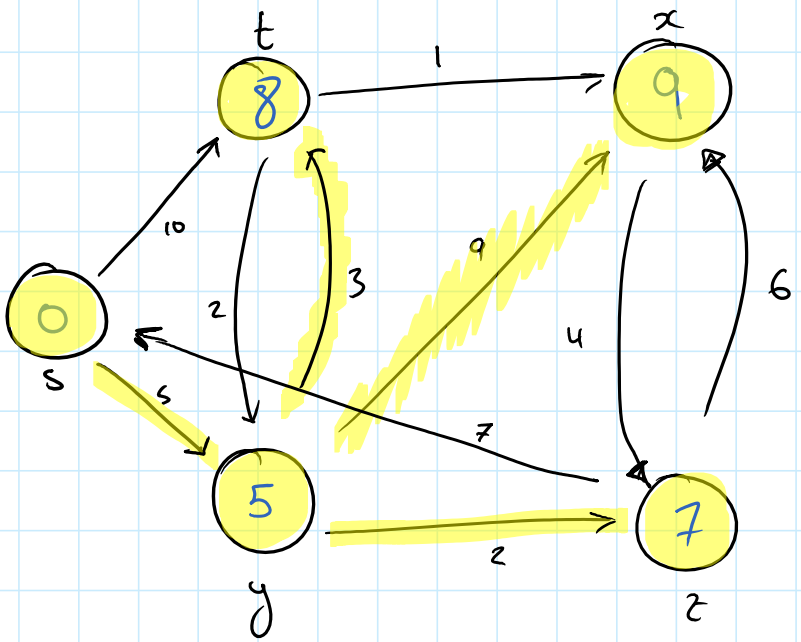
Vertex	Distance	Path
A	5	E, C, A
B	3	E, B
C	3	E, C
D	4	E, G, D
E	0	E
F	4	E, G, F
G	2	E, G
H	5	E, G, H
I	10	E, G, I

Dijkstra's algorithm

- Single-source shortest paths on weighted directed Graph

- weights are non-negative
- lower running time than Bellman-Ford.

```
DIJKSTRA(G, w, s)
1  INITIALIZE-SINGLE-SOURCE(G, s)
2  S = ∅
3  Q = G.V
4  while Q ≠ ∅
5      u = EXTRACT-MIN(Q)
6      S = S ∪ {u}
7      for each vertex v ∈ G.Adj[u]
8          RELAX(u, v, w)
```



$S = \{\}$

$Q = \{(s)_0, t_\infty, x_\infty, y_\infty, z_\infty\}$

$S = \{s\}$

$Q = \{(y)_5, t_{10}, x_\infty, z_\infty\}$

$S = \{s, y\}$

$Q = \{(z)_7, t_8, x_{14}\}$

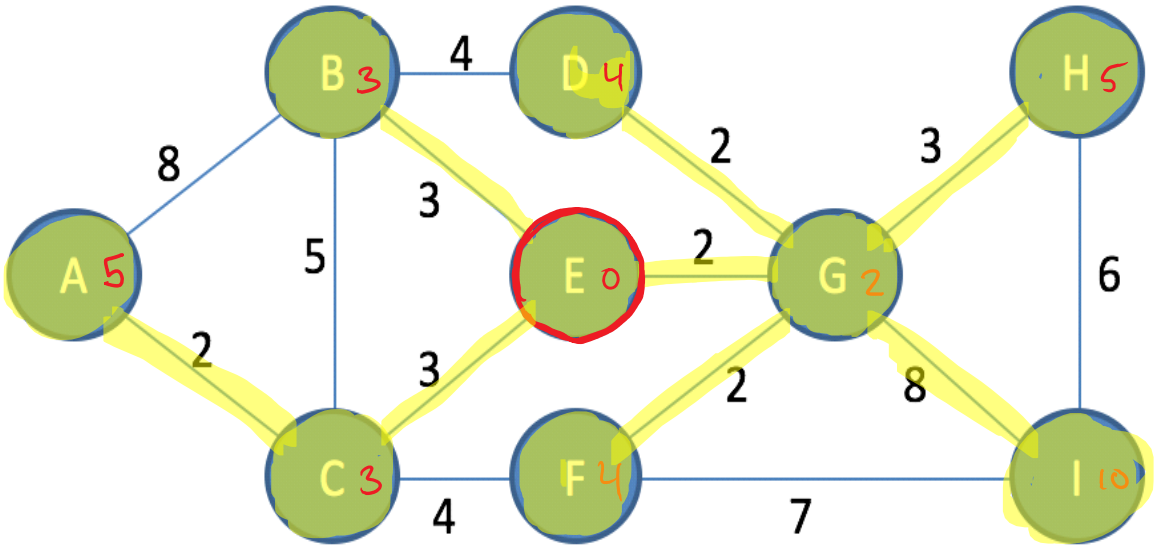
$S = \{s, y, z\}$

$Q = \{(t)_8, x_{14}\}$

$S = \{t, s, y, z\}$

$Q = \{x\}$

$S = \{t, s, y, z, x\}$



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