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A new response surface method based on the adaptive bivariate cut-HDMR

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Abstract

Purpose – The purpose of this study is to improve the efficiency and accuracy of response surface method (RSM), as well as its robustness.

Design/methodology/approach – By introducing cut-high-dimensional representation model (HDMR), the delineation of cross terms and the constitution analysis of component function, a new adaptive RSM is presented for reliability calculation, where a sampling scheme is also proposed to help constructing response surface close to limit-state.

Findings – The proposed method has a more feasible process of evaluating undetermined coefficients of each component function than traditional RSM, and performs well in terms of balancing the efficiency and accuracy when compared to the traditional second-order polynomial RSM. Moreover, the proposed method is robust on the parameter in a wide range, indicating that it is able to obtain convergent result in a wide feasible domain of sample points.

Originality/value – This study constructed an adaptive bivariate cut-HDMR by introducing delineation of cross-terms and constitution of univariate component function; and a new sampling technique is proposed.

Keywords Dimension reduction, Cross terms, Iterative, Response surface method (RSM), Structure reliability

Paper type Research paper

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1. Introduction

In structural reliability analysis, response surface method (RSM) is a helpful technique when the limit-state function is implicit (Box and Wilson, 1954). In RSM, the limit-state function is approximated by explicit mathematical expression with undetermined coefficients. After the response surface function (RSF) is determined, many traditional reliability analysis or optimization methods, such as first-order reliability method (FORM) (Kang *et al.*, 2010), second-order reliability method (Zheng and Das, 2000), Monte–Carlo simulation (MCS) (Gupta and Manohar, 2004) and global optimization methods (Chen, 2020; Xu and Chen, 2014), can be incorporated with RSM to evaluate the reliability.

One of the main topics for RSM is to rationally determine the form of RSF. Although there are different kinds of RSFs, such as polynomial function (Bucher and Bourgund, 1990; Kaymaz and Mcmahon, 2005), artificial neural network (Deng *et al.*, 2005; Elhewy *et al.*, 2006) and radial basis function (Hardy, 1971; Deng, 2006), the polynomial function is generally widely used in RSM because of its simplicity. Wong approximated the limit-state function of soil-slope stability by a linear polynomial with cross terms, and the two-level factorial design is used to determine the unknown coefficients (Wong, 1984; Wong, 1985). Faravelli (1989) used a quadratic polynomial without cross terms as the RSF to approximate the nonlinearity, but because all the sample points are around the mean point, the precision of these two kinds of RSF near the design point may not be adequate. To improve the approximation accuracy of RSF around the design point, Bucher and Bourgund (1990) introduced an iterative technique to transit the center point of sample points from the mean point to a new point being closer to the design point, and a quadratic polynomial without cross terms is selected as the form of RSF. However, only one iterative step may not be able to guarantee that the new center point is close enough to the design point, and a sequence of iterations would be required to search for the design point. Liu and Moses (1994) suggested that iteration procedure should be introduced to find the design point until a convergence criterion is satisfied. On the other hand, neglecting the cross terms in RSM may not be sufficiently accurate in some cases, especially when the interactive effects are not negligible. For this purpose, Rajashekhar and Ellingwood (1993) took the cooperative effects of variables into consideration by adding the cross terms to quadratic polynomial RSF, whereas the computation efforts intensively increase because of the increase of undetermined coefficients of cross terms. Therefore, adaptively adding rational and necessary cross terms to the RSF becomes noticeable. Based on the sequential linear polynomial RSM with vector projection sampling technique (Kim and Na, 1997), Das and Zheng (2000) proposed an adaptive RSM to add square and cross terms to the linear RSF step by step in which the RSF has the same sign as the performance function at the sampling points. Because the adaptive iteration scheme is increasingly used in RSM, there is a growing concern for the convergence and robustness of RSM. Gayton *et al.* (2003) proposed a resampling technique with confidence interval to facilitate the convergence of RSM. Nguyen *et al.* (2009) approximated the limit-state with a double-weighted regression technique and adaptive grid size, resulting in the improvement of convergence speed and sensitivity of final outcome. Roussouly and Petitjean (2013) proposed an adaptive strategy where the cross-validation technique and statistical criteria are used to locate the response surface close to the design point. However, to the best of the authors' knowledge in RSM, there is no precise guideline or theory about ensuring the stability of convergence and robustness in RSM, and a parametric study carried out by Guan and Meichers (2001) implied that the convergence and robustness of RSM are greatly influenced by the selection of sample points.

Most of the polynomials RSM mentioned above inevitably involve the solution or optimization of high-dimensional equations to evaluate the underdetermined coefficients, which may be difficult to calculate if the coefficients matrix is large and asymmetric. To alleviate the effect of dimensional curse in high-dimensional calculating, one of the feasible ways is the dimensional decomposition method (DDM) which uses the low-dimensional model to approximate the real high-dimensional limit-state function (Rahman and Xu, 2004; Xu and Rahman, 2004; Rahman, 2013; Rabitz and Alis, 1999; Li *et al.*, 2001), and the conceptions of univariate or bivariate dimensional decomposition are implicitly involved in all the previously mentioned RSM. Recently, the DDM-based RSM is proposed and developed by many researchers where the dimensional-reduction method and the high-dimensional representation model (HDMR) are frequently used (Rao and Chowdhury, 2008; Chowdhury and Rao, 2009a, 2009b; Chowdhury *et al.*, 2010; Xu and Rahman, 2005). There are generally two types of combinations of DDM with RSM. The first one is to approximate the limit-state function in the neighborhood of the reference point (typically the mean value of random variable) directly; however, it is difficult to guarantee its computational efficiency because the iteration procedure is not considered. The second one is to approximate the component functions in the standard normal space, but because the determination of design point and the approximation of limit-state function are completely separated, it can be considered as higher order RSM which is complicated to implement (Gavin and Yau, 2008). Besides, all the variables in these two DDR-based RSM are equally treated, whereas the contribution of each variable is different in reality problem, and it is difficult to accurately approximate the nonlinearity of the component function because the number of sample points to approximate component function is artificially determined (Chowdhury *et al.*, 2010; Xu and Rahman, 2005).

As pointed out by Rahman that the ANOVA dimensional decomposition (ADD) is superior to cut-HDMR with respect to expected approximation error (Rahman, 2014); however, cut-HDMR is simpler to use and construct because it does not need to calculate the integral, and the required accuracy can be achieved with lower variate cut-HDMR model if the reference point is adaptively updated. Therefore, in this paper, we present a new RSM based on the adaptive bivariate cut-HDMR, and it differs from the adaptive polynomial dimensional decomposition model (Kattan, 2003; Xu, 2006) which can be regarded as the polynomial form of ADD. The rest of the paper is organized as follows. In Section 2, first, an adaptive bivariate cut-HDMR is proposed, in which the delineation of cross terms is used. Then, the polynomial approximation of component function is presented, and the criteria for the constitution analysis of univariate component function is proposed and proved. Finally, the global response surface is given together with its implementation. In Section 3, several mathematical and solid-mechanic examples are investigated to verify the accuracy, efficiency and robustness of proposed method, and the comparison of the proposed method and other existing RSM is given. At last, some conclusions are drawn in Section 4.

2. Adaptive response surface method

It is well known that the approximated RSF should be as close as possible to the actual limit-state surface around the design point. Even though the quadratic polynomial without cross terms is widely used in RSM, it will be more rational and helpful to improve the performance of RSM if the real existing cooperative effects among the random variables are involved.

2.1 Adaptive bivariate cut-high-dimensional representation model based on the delineation of cross terms

Without loss of generality, the performance function of a structure can be expressed as:

$$Z = g(\boldsymbol{X}) \tag{1}$$

where $g(\cdot)$ is the performance function, and $\boldsymbol{X} = (X_1, X_2, \ldots, X_n)$ is the random vector with n components. Introducing the cut-HDMR (Rahman and Xu, 2004; Xu and Rahman, 2004), equation (1) can be rewritten as:

$$\begin{aligned} g(\boldsymbol{X}) &= g_0 + \sum_{i=1}^{n} g_i(X_i) + \sum_{1 \le i<j \le n} g_{ij}(X_i, X_j) + \cdots \\ &+ \sum_{1 \le i_1<i_2<\ldots<i_l \le n} g_{i_1 i_2 \cdots i_l}(X_{i_1}, X_{i_2}, \cdots, X_{i_l}) \\ &+ \cdots + g_{12\cdots n}(X_1, X_2, \cdots, X_n) \end{aligned} \tag{2}$$

where g_0 is a constant, $g_i(\cdot)$ is the one-dimensional component function characterizing the effect of variable X_i on $g(\boldsymbol{X})$; $g_{i_1 i_2 \ldots i_l}(\cdot)$ is an l-dimension component function describing the cooperative effects of variables $X_{i_1}, X_{i_2}, \ldots, X_{i_l}$ on $g(\boldsymbol{X})$; in other words, $g_{i_1 i_2 \ldots i_l}(\cdot)$ comprises only the cross terms of $X_{i_1}, X_{i_2}, \ldots, X_{i_l}$ in the Taylor expansion of $g(\boldsymbol{X})$. The quantitative formula for $g_{i_1 i_2 \ldots i_l}(\cdot)$ is:

$$\begin{cases} g_0 = g(\boldsymbol{x_c}) \\ g_i(X_i) = g\left(X_i, \underline{\boldsymbol{x}}_{i,c}\right) - g_0 \\ g_{ij}(X_i, X_j) = g\left(X_i, X_j, \underline{\boldsymbol{x}}_{ij,c}\right) - g_i(X_i) - g_j(X_j) - g_0 \\ \vdots \\ g_{i_1 i_2 \cdots i_l}(X_{i_1}, X_{i_2}, \cdots, X_{i_l}) = g\left(X_{i_1}, X_{i_2}, \cdots, X_{i_l}, \underline{\boldsymbol{x}}_{i_1 i_2 \cdots i_l, c}\right) - \sum_{\{j_1 j_2 \cdots j_{l-1}\} \subset \{i_1 i_2 \cdots i_l\}} g_{j_1 j_2 \cdots j_{l-1}}(X_{j_1}, X_{j_2}, \cdots, X_{j_{l-1}}) - \\ \quad \sum_{\{j_1 j_2 \cdots j_{l-2}\} \subset \{i_1 i_2 \cdots i_l\}} g_{j_1 j_2 \cdots j_{l-2}}(X_{j_1}, X_{j_2}, \cdots, X_{j_{l-2}}) - \cdots - \sum_{j \subset \{i_1 i_2 \cdots i_l\}} g_j(X_j) - g_0 \end{cases} \tag{3}$$

where $\boldsymbol{x_c}$ is the reference point, and $\underline{\boldsymbol{x}}_{i_1 i_2 \cdots i_l c}$ is a sub-vector of $\boldsymbol{x}_c$ without the corresponding coordinates of $X_{i_1}, X_{i_2}, \ldots, X_{i_l}$. By truncating equation (2), $g(\boldsymbol{X})$ can be approximated as follows:

$$\begin{aligned} & g(\boldsymbol{X}) \approx g^{(s-HDMR)}(\boldsymbol{X}) \\ & = g_0 + \sum_{i=1}^{n} g_i(X_i) + \sum_{1 \le i<j \le n} g_{ij}(X_i, X_j) + \cdots + \sum_{1 \le i_1<i_2<\cdots<i_s \le n} g_{i_1 i_2 \cdots i_s}(X_{i_1}, X_{i_2}, \cdots X_{i_s}) \end{aligned} \tag{4}$$

where only cooperative effects with no more than s variables are involved. If $s = 2$, equation (4) becomes:

$$g(\boldsymbol{X}) \approx g^{(2-HDMR)}(\boldsymbol{X}) = g_0 + \sum_{i=1}^{n} g_i(X_i) + \sum_{1 \le i<j \le n} g_{ij}(X_i, X_j) \triangleq \tilde{g}(\boldsymbol{X}) \tag{5}$$

As mentioned above, $g_{ij}(X_i, X_j)$ represents the cooperative effects of X_i and X_j. However, not all the cooperative effects of each pair of (X_i, X_j) are certainly necessary in practice. Clearly, $g_{ij}(X_i, X_j)$ should disappear from equation (5) if the cross term of X_i and X_j does not exist in $g(\boldsymbol{X})$. By introducing an indication function $I(X_i, X_j)$ for the cooperative effects between X_i and X_j, say:

$$I(X_i, X_j) = \begin{cases} 1, & \text{if the cross terms of } X_i \text{ and } X_j \text{ exist} \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

which can be determined according to reference (Fan *et al.*, 2016a, 2016b), equation (5) is then further written as:

$$\tilde{g}(\boldsymbol{X}) = g_0 + \sum_{i=1}^{n} g_i(X_i) + \sum_{1 \le i<j \le n} I(X_i, X_j) g_{ij}(X_i, X_j) \tag{7}$$

Equation (7) is called the adaptive bivariate Cut-HDMR in this paper because of its rational involvement of $g_{ij}(X_i, X_j)$ by the indication function. It should be noted that because the structural reliability is calculated at the design point, the bivariate model is suitable to approximate the local property of the performance function at the reference point $\boldsymbol{x_c}$ in each iteration when it adaptively moves toward the design point.

2.2 Polynomial approximation for component function

In this section, each component function in equation (7) is approximated by the polynomial to form the adaptive response surface.

2.2.1 Univariate component function. In this work, a quadratic polynomial approximation is exploited for the univariate component function because of its simplicity, namely:

$$g_i(X_i) \approx \bar{g}_i(X_i) = a_{i,0} + a_{i,1}(X_i - x_{i,c}) + a_{i,2}(X_i - x_{i,c})^2 \tag{8}$$

where $a_{i,0}$, $a_{i,1}$ and $a_{i,2}$ are the undetermined coefficients, and can be determined by linear equations as follows:

$$\begin{bmatrix} 1 & x_{i,1} - x_{i,c} & (x_{i,1} - x_{i,c})^2 \\ 1 & x_{i,2} - x_{i,c} & (x_{i,2} - x_{i,c})^2 \\ 1 & x_{i,3} - x_{i,c} & (x_{i,3} - x_{i,c})^2 \end{bmatrix} \begin{Bmatrix} a_{i,0} \\ a_{i,1} \\ a_{i,2} \end{Bmatrix} = \begin{Bmatrix} g_{i,1} \\ g_{i,2} \\ g_{i,3} \end{Bmatrix} \tag{9}$$

in which $x_{i,q}$ $(q = 1, 2, 3)$ are the three coordinates along the corresponding axis of X_i, and $g_{i,q}$ $(q = 1, 2, 3)$ are the corresponding function values shown as follows:

$$\begin{cases} g_{i,1} = g(x_{1,c}, \cdots, x_{i-1,c}, x_{i,1}, x_{i+1,c}, \cdots, x_{n,c}) - g_0 \\ g_{i,2} = g(x_{1,c}, \cdots, x_{i-1,c}, x_{i,2}, x_{i+1,c}, \cdots, x_{n,c}) - g_0 \\ g_{i,3} = g(x_{1,c}, \cdots, x_{i-1,c}, x_{i,3}, x_{i+1,c}, \cdots, x_{n,c}) - g_0 \end{cases} \tag{10}$$

Substituting equation (10) into equation (9), the solutions for $a_{i,0}$, $a_{i,1}$ and $a_{i,2}$ are calculated as:

$$\begin{cases} a_{i,0} = [g_{i,1}\cdot(x_{i,1}-x_{i,c})(x_{i,2}-x_{i,c})(x_{i,3}-x_{i,c})(x_{i,3}-x_{i,2}) + g_{i,2}\cdot(x_{i,1}-x_{i,c})^2(x_{i,3}-x_{i,c})(x_{i,1}-x_{i,3}) \\ \qquad + g_{i,3}\cdot(x_{i,1}-x_{i,c})^2(x_{i,2}-x_{i,c})(x_{i,2}-X_{i,1})]/(x_{i,1}-x_{i,c})(x_{i,1}-x_{i,2})(x_{i,1}-x_{i,3})(x_{i,3}-x_{i,2}) \\ a_{i,1} = [g_{i,1}\cdot(x_{i,2}-x_{i,c})^2 - g_{i,2}\cdot(x_{i,1}-x_{i,c})^2 - g_{i,1}\cdot(x_{i,3}-x_{i,c})^2 + g_{i,3}\cdot(x_{i,1}-x_{i,c})^2 \\ \qquad + g_{i,2}\cdot(x_{i,3}-x_{i,c})^2 - g_{i,3}\cdot(x_{i,2}-x_{i,c})^2]/(x_{i,1}-x_{i,2})(x_{i,3}-x_{i,2})(x_{i,1}-x_{i,3}) \\ a_{i,2} = [g_{i,1}\cdot(x_{i,2}-x_{i,c}) - g_{i,2}\cdot(x_{i,1}-x_{i,c}) - g_{i,1}\cdot(x_{i,3}-x_{i,c}) + g_{i,3}\cdot(x_{i,1}-x_{i,c}) \\ \qquad + g_{i,2}\cdot(x_{i,3}-x_{i,c}) - g_{i,3}\cdot(x_{i,2}-x_{i,c})]/(x_{i,1}-x_{i,2})(x_{i,3}-x_{i,2})(x_{i,1}-x_{i,3}) \end{cases} \tag{11}$$

2.2.2 Constitution analysis of univariate component function. In equation (8), $g_i(X_i)$ is approximated by a general quadratic polynomial. However, for some special $g_i(X_i)$, either the linear or square term may be unnecessary and can be removed from equation (8).

2.2.2.1 Theoretical background

Lemma 1. If the solution of $a_{i,1}$ in equation (11) is always equal to 0 for an arbitrary set of coordinates $x_{i,q}$ ($q = 1, 2, 3$), then $g_i(X_i)$ must be a quadratic polynomial without the linear term.

Proof. Selecting arbitrary different three coordinates of X_i, namely, $x_{i,1}$, $x_{i,2}$ and $x_{i,3}$, it is easy to obtain the following equations according to equation (9), namely,

$$g_i(x_{i,1}) = a_{i,0} + a_{i,1}(x_{i,1} - x_{i,c}) + a_{i,2}(x_{i,1} - x_{i,c})^2 \tag{12a}$$

$$g_i(x_{i,2}) = a_{i,0} + a_{i,1}(x_{i,2} - x_{i,c}) + a_{i,2}(x_{i,2} - x_{i,c})^2 \tag{12b}$$

$$g_i(x_{i,3}) = a_{i,0} + a_{i,1}(x_{i,3} - x_{i,c}) + a_{i,2}(x_{i,3} - x_{i,c})^2 \tag{12c}$$

Based on equation (11), if $a_{i,1} = 0$, then

$$\begin{aligned} & g_{i,1}\cdot(x_{i,2}-x_{i,c})^2 + g_{i,2}\cdot(x_{i,3}-x_{i,c})^2 + g_{i,3}\cdot(x_{i,1}-x_{i,c})^2 = \\ & g_{i,1}\cdot(x_{i,3}-x_{i,c})^2 + g_{i,2}\cdot(x_{i,1}-x_{i,c})^2 + g_{i,3}\cdot(x_{i,2}-x_{i,c})^2 \end{aligned} \tag{13}$$

Substituting $g_i(x_{i,1})$, $g_i(x_{i,2})$ and $g_i(x_{i,3})$ by their Taylor's expansion at $x_i = x_{i,c}$ yields

$$\begin{aligned} & \sum_{\substack{k=1 \\ k\neq 2}}^{\infty} \frac{1}{k!} g_i^{(k)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^k(x_{i,2}-x_{i,c})^2 + (x_{i,2}-x_{i,c})^k(x_{i,3}-x_{i,c})^2 + (x_{i,3}-x_{i,c})^k(x_{i,1}-x_{i,c})^2\right] \\ & + 3g_i(x_{i,c}) + \frac{1}{2!} g_i^{(2)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^2(x_{i,2}-x_{i,c})^2 + (x_{i,2}-x_{i,c})^2(x_{i,3}-x_{i,c})^2 + (x_{i,3}-x_{i,c})^2(x_{i,1}-x_{i,c})^2\right] \\ & = \sum_{\substack{k=1 \\ k\neq 2}}^{\infty} \frac{1}{k!} g_i^{(k)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^k(x_{i,3}-x_{i,c})^2 + (x_{i,2}-x_{i,c})^k(x_{i,1}-x_{i,c})^2 + (x_{i,3}-x_{i,c})^k(x_{i,2}-x_{i,c})^2\right] \\ & + 3g_i(x_{i,c}) + \frac{1}{2!} g_i^{(2)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^2(x_{i,3}-x_{i,c})^2 + (x_{i,2}-x_{i,c})^2(x_{i,1}-x_{i,c})^2 + (x_{i,3}-x_{i,c})^2(x_{i,2}-x_{i,c})^2\right] \end{aligned}$$

$$\Longleftrightarrow \sum_{\substack{k=1\\k\neq 2}}^{\infty}\frac{1}{k!}g_i^{(k)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^k(x_{i,2}-x_{i,c})^2+(x_{i,2}-x_{i,c})^k(x_{i,3}-x_{i,c})^2+(x_{i,3}-x_{i,c})^k(x_{i,1}-x_{i,c})^2\right]$$

$$=\sum_{\substack{k=1\\k\neq 2}}^{\infty}\frac{1}{k!}g_i^{(k)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^k(x_{i,3}-x_{i,c})^2+(x_{i,2}-x_{i,c})^k(x_{i,1}-x_{i,c})^2+(x_{i,3}-x_{i,c})^k(x_{i,2}-x_{i,c})^2\right] \tag{14}$$

If equation (14) holds for all possible $x_{i,1}$, $x_{i,2}$ and $x_{i,3}$, there must exist

$$g_i^{(k)}(x_{i,c})=0 \quad k=1,3,4,\cdots,\infty \tag{15}$$

and $g_i(X_i)$ can be expressed as follows:

$$g_i(X_i)=g_i(x_{i,c})+\frac{1}{2!}g_i^{''}(x_{i,c})(X_i-x_{i,c})^2 \tag{16}$$

Therefore, *Lemma 1* is proved.

Lemma 2. If the solution of $a_{i,2}$ in equation (11) is always equal to 0 for an arbitrary set of coordinates $x_{i,q}$ (q = 1, 2, 3), then $g_i(X_i)$ must be a linear polynomial.

Proof. Similarly, $x_{i,1}$, $x_{i,2}$ and $x_{i,3}$ are selected. Based on equation (11) and if $a_{i,2}=0$, then,

$$g_{i,1}(x_{i,2}-x_{i,c})+g_{i,2}(x_{i,3}-x_{i,c})+g_{i,3}(x_{i,1}-x_{i,c})$$
$$=g_{i,1}(x_{i,3}-x_{i,c})+g_{i,2}(x_{i,1}-x_{i,c})+g_{i,3}(x_{i,2}-x_{i,c}) \tag{17}$$

Substituting $g_i(x_{i,1})$, $g_i(x_{i,2})$ and $g_i(x_{i,3})$ by their Taylor's expansion at $x_i=x_{i,c}$ yields:

$$\sum_{k=2}^{\infty}\frac{1}{k!}g_i^{(k)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^k(x_{i,2}-x_{i,c})+(x_{i,2}-x_{i,c})^k(x_{i,3}-x_{i,c})+(x_{i,3}-x_{i,c})^k(x_{i,1}-x_{i,c})\right]$$

$$=\sum_{k=2}^{\infty}\frac{1}{k!}g_i^{(k)}(x_{i,c})\left[(x_{i,1}-x_{i,c})^k(x_{i,3}-x_{i,c})+(x_{i,2}-x_{i,c})^k(x_{i,1}-x_{i,c})+(x_{i,3}-x_{i,c})^k(x_{i,2}-x_{i,c})\right] \tag{18}$$

If equation (18) holds for all possible $x_{i,1}$, $x_{i,2}$ and $x_{i,3}$, there must exist:

$$g_i^{(k)}(x_{i,c})=0 \quad k=2,3,4,\cdots,\infty \tag{19}$$

and $g_i(X_i)$ can be expressed as follows:

$$g_i(X_i)=g_i(x_{i,c})+g_i^{'}(x_{i,c})(X_i-x_{i,c}) \tag{20}$$

Therefore, *Lemma 2* is proved.

2.2.2.2 Practical criteria. It is worth noting that the value of $a_{i,1}$ or $a_{i,2}$ may not be equal to 0 because of the truncation error and structural analysis error, even if the linear or quadratic term of X_i does not exist in $g_i(X_i)$. Therefore, the practical substitution of $a_{i,1}=0$ or $a_{i,2}=0$ could be:

(a) For *Lemma 1*, the substitution of $a_{i,1} = 0$ is:

$$|a_{i,1}| \leq \varepsilon_1 \quad \cap \quad |a_{i,1}/a_{i,2}| \leq \varepsilon_2 \tag{21}$$

(b) For *Lemma 2*, the substitution of $a_{i,2} = 0$ is:

$$|a_{i,2}| \leq \varepsilon_1 \quad \cap \quad |a_{i,2}/a_{i,1}| \leq \varepsilon_2 \tag{22}$$

where both ε_1 and ε_2 are specific positive values, and $\varepsilon_1 = 10^{-6}$ and $\varepsilon_2 = 10^{-3}$ in this work.

In short, a quadratic polynomial without the linear term is noted as QP001, a linear polynomial is noted as QP010, and other functions are noted as QP011.

2.2.3 Bivariate component function. Because $g_{ij}(X_i, X_j)$ describes the cooperative effects of X_i and X_j, $g_{ij}(X_i, X_j)$ can be approximated as follows:

$$g_{ij}(X_i, X_j) \approx \bar{g}_{ij}(X_i, X_j) = b_{ij}(X_i - x_{i,c})^{m_i}(X_j - x_{j,c})^{m_j} \tag{23}$$

where b_{ij} is the undetermined coefficient, and m_i and m_j are the orders of variables X_i and X_j and determined by Table 1.

Because the reference point $\boldsymbol{x}_c$ satisfies equation (23) automatically, another additional point is required to solve b_{ij}. Clearly, based on the selected points in approximating univariate component functions $g_i(X_i)$ and $g_j(X_j)$, four possible points $(x_{i,ct}{}^{(1)}, x_{j,ct}{}^{(1)}, \underline{\boldsymbol{x}}_{ij,c})$, $(x_{i,ct}{}^{(2)}, x_{j,ct}{}^{(1)}, \underline{\boldsymbol{x}}_{ij,c})$, $(x_{i,ct}{}^{(1)}, x_{j,ct}{}^{(2)}, \underline{\boldsymbol{x}}_{ij,c})$ and $(x_{i,ct}{}^{(2)}, x_{j,ct}{}^{(2)}, \underline{\boldsymbol{x}}_{ij,c})$ can be chosen straightforward, where $x_{i,ct}{}^{(1)} = x_{i,1}$, $x_{i,ct}{}^{(2)} = x_{i,3}$, $x_{j,ct}{}^{(1)} = x_{j,1}$ and $x_{j,ct}{}^{(2)} = x_{j,3}$. To ensure that the sample point as close as possible to the limit-state, the additional point is chosen from the four possible points by the following rule:

$$(X_i, X_j) = \begin{cases} (x_{i,ct}{}^{(1)}, x_{j,ct}{}^{(1)}), & \text{if } \bar{g}'(x_{i,ct}{}^{(1)}, x_{j,ct}{}^{(1)}, \underline{x}_{ij,c}) = \min\limits_{k,q=1,2}\{\bar{g}'(x_{i,ct}{}^{(k)}, x_{j,ct}{}^{(q)}, \underline{x}_{ij,c})\} \\ (x_{i,ct}{}^{(2)}, x_{j,ct}{}^{(1)}), & \text{if } \bar{g}'(x_{i,ct}{}^{(2)}, x_{j,ct}{}^{(1)}, \underline{x}_{ij,c}) = \min\limits_{k,q=1,2}\{\bar{g}'(x_{i,ct}{}^{(k)}, x_{j,ct}{}^{(q)}, \underline{x}_{ij,c})\} \\ t \\ (x_{i,ct}{}^{(1)}, x_{j,ct}{}^{(2)}), & \text{if } \bar{g}'(x_{i,ct}{}^{(1)}, x_{j,ct}{}^{(2)}, \underline{x}_{ij,c}) = \min\limits_{k,q=1,2}\{\bar{g}'(x_{i,ct}{}^{(k)}, x_{j,ct}{}^{(q)}, \underline{x}_{ij,c})\} \\ (x_{i,ct}{}^{(2)}, x_{j,ct}{}^{(2)}), & \text{if } \bar{g}'(x_{i,ct}{}^{(2)}, x_{j,ct}{}^{(2)}, \underline{x}_{ij,c}) = \min\limits_{k,q=1,2}\{\bar{g}'(x_{i,ct}{}^{(k)}, x_{j,ct}{}^{(q)}, \underline{x}_{ij,c})\} \end{cases} \triangleq (x_{i,ct}, x_{j,ct}). \tag{24}$$

in which:

$$\bar{g}'(\boldsymbol{X}) = g_0 + \sum_{i=1}^{n} \bar{g}_i(X_i) \tag{25}$$

Then, b_{ij} can be determined by:

Constitution of $g_l(X_l)$	m_l
QP011 or QP010	1
QP001	2

Table 1. Determination of m_l $(l = i,j)$

$$b_{ij} = \left[g\left(x_{i,ct}, x_{j,ct}, \underline{\boldsymbol{x}}_{ij,c}\right) - g\left(x_{i,ct}, \underline{\boldsymbol{x}}_{i,c}\right) - g\left(x_{j,ct}, \underline{\boldsymbol{x}}_{j,c}\right) + g(\boldsymbol{x}_c)\right] / \left[(x_{i,ct} - x_{i,c})^{m_i}(x_{j,ct} - x_{j,c})^{m_j}\right] \tag{26}$$

2.3 Adaptive response surface method and its implementation

2.3.1 Adaptive response surface. Substituting the approximation of univariate component functions and bivariate ones into equation (7) yields the adaptive response surface as follows:

$$g(\boldsymbol{X}) \approx \tilde{g}(\boldsymbol{X}) \approx \bar{g}(\boldsymbol{X}) = g_0 + \sum_{i=1}^{n} \bar{g}_i(X_i) + \sum_{1 \le i < j \le n} I(X_i, X_j) \bar{g}_{ij}(X_i, X_j) \tag{27}$$

Based on equation (27), the reliability index β can be obtained easily by either FORM or MCS. Equation (27) is a quadratic polynomial response surface, and the undefined coefficients can be obtained separately for every component function as illustrated in Section 2.2, whereas in the traditional RSM, all coefficients are solved simultaneously. Therefore, the proposed RSM is much simpler than the traditional RSM with respect to evaluating unknown coefficients.

2.3.2 Numerical implementation. Combining with the constitution analysis of univariate component function and the delineation of cross terms, the procedure for a new RSM based on the adaptive bivariate cut-HDMR is shown as follows, and a brief flowchart illustrating the procedure is also presented in Figure 1.

(1) First iterative analysis:

- Set $k = 0$, $\boldsymbol{x}_c = \boldsymbol{\mu}_{\boldsymbol{X}}$, which is the mean value of $\boldsymbol{X}$.
- For $g_l(X_l)$ ($l = 1,\ldots,n$), by selecting $x_{l,1}{}^{(0)} = x_{l,c} - f_k\boldsymbol{\sigma}_l$, $x_{l,2}{}^{(0)} = x_{l,c}$ and $x_{l,3}{}^{(0)} = x_{l,c} + f_k\boldsymbol{\sigma}_l$, (if $k = 0$, then $f_k = 3$, else $f_k = 1$), $a_{l,0}{}^{(0)}$, $a_{l,1}{}^{(0)}$ and $a_{l,2}{}^{(0)}$ are calculated by:

$$\begin{cases} a_{l,0}{}^{(0)} = 0 \\ a_{l,1}{}^{(0)} = (g_{l,3} - g_{l,1})/f_k\boldsymbol{\sigma}_l \\ a_{l,2}{}^{(0)} = (g_{l,3} + g_{l,1})/f_k{}^2\boldsymbol{\sigma}_l{}^2 \end{cases} \tag{28}$$

which is derived from equation (11).

- Select an additional point ($x_i,1^{(0)}$, $x_j,1^{(0)}$, $\underline{\boldsymbol{x}}_{ij,c}$) and evaluate $\Delta_{g_{ij}}{}^{(0)}$ by:

$$\Delta g_{ij}{}^{(0)} = \frac{\left|\left[g\left(x_{i,1}{}^{(0)}, x_{j,2}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right) - g\left(x_{i,2}{}^{(0)}, x_{j,2}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right)\right] - \left[g\left(x_{i,1}{}^{(0)}, x_{j,1}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right) - g\left(x_{i,2}{}^{(0)}, x_{j,1}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right)\right]\right|}{\min\left\{\left|\left[g\left(x_{i,1}{}^{(0)}, x_{j,2}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right) - g\left(x_{i,2}{}^{(0)}, x_{j,2}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right)\right]\right|, \left|\left[g\left(x_{i,1}{}^{(0)}, x_{j,1}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right) - g\left(x_{i,2}{}^{(0)}, x_{j,1}{}^{(0)}, \underline{\boldsymbol{x}}_{ij,}c\right)\right]\right|\right\}} \tag{29}$$

Then, the indication functions $I^{(0)}(X_i, X_j)$ ($i = 1,\ldots,n-1; j > i$) can be determined by:

$$I^{(0)}(X_i, X_j) = \begin{cases} 1, \text{ if } \Delta g_{ij}{}^{(0)} > \varepsilon \\ 0, \text{ otherwise} \end{cases} \tag{30}$$

where ε is a specific positive value and $\varepsilon = 0.001$ in this work.

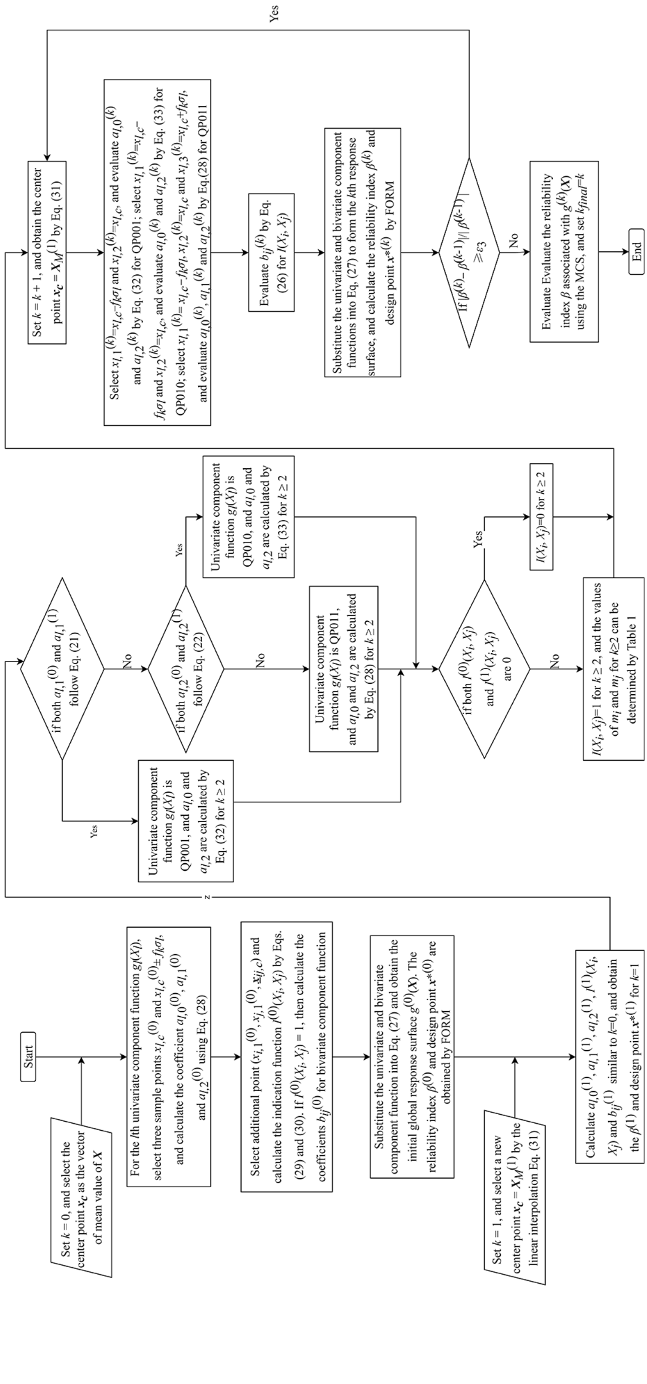

Figure 1.
Flowchart of the proposed method

- If $I^{(0)}(X_i, X_j)^{(0)}=1$, then calculate $b_{ij}^{(0)}$ through equation (26) with $mi=1$ and $mj=1$, in which points xc, $(x_{i,1}^{(0)}, x_{i,c})$, $(x_{j,1}^{(0)}, x_{j,c})$ are involved.
- Substituting the known $\bar{g}_1(X_1)$ and $\bar{g}_{ij}(X_i, X_j)$ into equation (27) yields the initial response surface, noted as $\bar{g}^{(0)}(X)$, and the reliability index $\beta^{(0)}$ of $\bar{g}^{(0)}(X)$ can be obtained by the FORM, together with the design point x*(0).
- Set $k = 1$, and select a new center point $X_M(1)$ for the first iteration using the linear interpolation (Bucher and Bourgund, 1990):

$$\boldsymbol{x}_M^{(k)} = \boldsymbol{\mu}_x + \left(\boldsymbol{x}^{*(k-1)} - \boldsymbol{\mu}_X\right) \frac{g(\boldsymbol{\mu}_x)}{g(\boldsymbol{\mu}_x) - g\left(\boldsymbol{x}^{*(k-1)}\right)} \tag{31}$$

- Let $x_c = x_M^{(1)}$. Being similar with the process of k=0, $a_{l,0}^{(1)}$, $a_{l,1}^{(1)}$, $a_{l,2}^{(1)}$, $I^{(1)}(X_i, X_j)$ and $b_{ij}^{(1)}$ (i=1,. . .,$n-1$;j >i), $\beta^{(1)}$ and $x^{*(1)}$ are available easily.

(2) Constitution analysis of the univariate component function. Based on the results of step (1):

- if both $a_{l,1}^{(0)}$ and $a_{l,1}^{(1)}$ follow equation (21), *gl(Xl)* is QP001, and the solutions for $a_{l,0}$ and $a_{l,2}$ in the subsequent iterative analysis are determined by:

$$\begin{cases} a_{l,0}{}^{(k)} = 0 \\ a_{l,2}{}^{(k)} = g_{l,1}/f_k{}^2\sigma_l{}^2 \end{cases} \qquad k \geq 2 \tag{32}$$

- if both $a_{l,2}^{(0)}$ and $a_{l,2}^{(1)}$ follow equation (22), *gl(Xl)* is QP010, and the solutions for $a_{l,0}$ and $a_{l,1}$ in the subsequent iterative analysis are determined by:

$$\begin{cases} a_{l,0}{}^{(k)} = 0 \\ a_{l,1}{}^{(k)} = g_{l,1}/f_k\sigma_l \end{cases} \qquad k \geq 2 \tag{33}$$

- if neither a) nor b) occurs, *gl(Xl)* is QP011, and the solutions for $al,_0$ $al,_1$ and $al,_2$ in the subsequent analysis are determined by equation (28).

(3) Delineating the existence of bivariate component function:
According to the results of Step (1), if both $I^{(0)}(X_i, X_j)$ and $I^{(1)}(X_i, X_j)$ are equal to 0, then $I(X_i, X_j) = 0$, otherwise $I(X_i, X_j) = 1$, which are effective in equation (27) for $k \geq 2$.

(4) Determination of m_i and m_j in subsequent iterative analysis:
According to the results of Step (2) and Step (3), if $I(X_i, X_j) = 1$ for $k \geq 2$, then the values of m_i and m_j for $k \geq 2$ can be determined by Table 1.

(5) Subsequent iterative analysis.

- If $|\beta^{(k)} - \beta^{(k-1)}|/|\beta^{(k-1)}| \leq \varepsilon_3$ ($\varepsilon_3=10^{-3}$ here), go to Sub-step (vi); or else, go to Sub-step (ii).
- Set $k = k+1$, and $\boldsymbol{x}_c = \boldsymbol{X}_M^{(k)}$, which is determined by equation (31).
- If $g_l(X_l)$ is QP001, select $x_{l,1}^{(k)} = x_{l,c} - f_k\sigma_l$ and $x_{l,2}^{(k)} = x_{l,c}$, and evaluate $a_{l,0}^{(k)}$ and $a_{l,2}^{(k)}$ by equation (32); if $g_l(X_l)$ is QP010, select $x_{l,1}^{(k)} = x_{l,c} - f_k\sigma_l$ and $x_{l,2}^{(k)} = x_{l,c}$, and evaluate $a_{l,0}^{(k)}$ and $a_{l,1}^{(k)}$ by equation (33); if $g_i(X_l)$ is QP011, select $x_{l,1}^{(k)} = x_{l,c} - f_k\sigma_l$, $x_{l,2}^{(k)} = x_{l,c}$ and $x_{l,3}^{(k)} = x_{l,c} + f_k\sigma_l$, and evaluate $a_{l,0}^{(k)}$, $a_{l,1}^{(k)}$ and $a_{l,2}^{(k)}$ by equation (28).
- If $I(X_i, X_j)=1$ for $k\geq 2$, evaluate $b_{ij}^{(k)}$ by equation (26) with m_i and m_j determined in Step (4), in which $x_{i,ct}$ and $x_{j,ct}$ are determined by equation (24).

- Substituting all of $\bar{g}_1(X_l)$ [i.e. $\bar{g}_i(X_i)$ in equation (27)] and $\bar{g}_{ij}(X_i, X_j)$ into equation (27) yields the kth response surface $\bar{g}^{(k)}(\boldsymbol{X})$, and the corresponding reliability index $\beta^{(k)}$ can be obtained by FORM, together with the corresponding design point $\boldsymbol{X}^{*(k)}$. And go to Sub-step (i).
- Evaluate the reliability index β associated with $\bar{g}^{(k)}(\boldsymbol{X})$ using the MCS, and set $k_{final} = k$.

Remark 1. Based on the constitution analysis, the constitution of every variable in the response surface may be different. Therefore, fewer points are required for special univariate component function, such as the quadratic polynomial without linear term and the linear polynomial, and the computational efficiency would be improved.

Remark 2. By introducing the delineation of cross terms into the RSF, some of the bivariate component functions disappear as $I(X_i, X_j) = 0$. Therefore, the RSF will become more concise and the computational efficiency would be also improved. It is worth noted that the delineation of cross terms has been proposed by the authors in point estimate method (Fan *et al.*, 2016a, 2016b; Liu *et al.*, 2019); however, in the present study, it is introduced in the RSM to further reduce the computational efforts by adaptively determining the existence of bivariate component function, and one should keep in mind that it can been seen as an extension of the previous work by the author and as one of the ingredients of the present study which is used to treat random variables differently and save computational efforts.

3. Numerical examples

In this section, seven numerical examples are illustrated in detail to investigate the performance of the proposed adaptive RSM; the first three examples are mathematical problems and the other four are mechanics problems, and the random variables are assumed to be independent. In each example, the precision, efficiency and robustness of the proposed method are compared with the RSM with quadratic polynomials (RSM with QP) (Liu and Moses, 1994) and the RSM with completely quadratic polynomials (RSM with CQP) (Rajashekhar and Ellingwood, 1993) and the MCS, where the efficiency is described by the number of function evaluation N, and the precision is reflected by the error estimation as follows:

$$\text{error}(\%) = \frac{|R - R_{MCS}|}{R_{MCS}} \times 100 \tag{34}$$

where R_{MCS} is the β calculated by MCS, and R is the β calculated by other three methods. Note that the number of sampling points is the same as the number of evaluation N. Moreover, the robustness is manifested by the convergence of all RSMs with respect to parameter f_k, and three strategies of f_k are considered in Examples 1–5, namely, all the f_k remain the same and change simultaneously; $f_0 = 3$ and $f_k (k > 0)$ remain the same and change simultaneously; and $f_k = 1$, and f_0 changes.

3.1 Set 1 – mathematical function

Example 1. Consider a limit-state function with three variables

$$g(\boldsymbol{X}) = 40 - X_1 + 1.2X_2{}^{p} + 5X_3 - X_1{}^2 + X_1X_2 + X_2X_3 \tag{35}$$

where X_1, X_2 and X_3 are independent standard normal variables. According to the specific value of p, there are two cases in this example: $p = 1$ and $p = 3$.

Case 1: $p = 1$

The adaptive response surface for equation (35) is:

$$g(\boldsymbol{X}) \approx g(\boldsymbol{x}_c) + \bar{g}_1(X_1) + \bar{g}_2(X_2) + \bar{g}_3(X_3) + I(X_1, X_2)\bar{g}_{12}(X_1, X_2) + I(X_1, X_3)\bar{g}_{13}(X_1, X_3) + I(X_2, X_3)\bar{g}_{23}(X_2, X_3) \tag{36}$$

in which $\boldsymbol{x}_c = \{0,0,0\}$, and

$$\begin{cases} \bar{g}_1(X_1) = a_{1,1}\left(X_1 - x_{1,c}\right) + a_{1,2}\left(X_1 - x_{1,c}\right)^2 \\ \bar{g}_2(X_2) = a_{2,1}\left(X_2 - x_{2,c}\right) + a_{2,2}\left(X_2 - x_{2,c}\right)^2 \\ \bar{g}_3(X_3) = a_{3,1}\left(X_3 - x_{3,c}\right) + a_{3,2}\left(X_3 - x_{3,c}\right)^2 \\ \bar{g}_{12}(X_1, X_2) = b_{12}\left(X_1 - x_{1,c}\right)\left(X_2 - x_{2,c}\right) \\ \bar{g}_{13}(X_1, X_3) = b_{13}\left(X_1 - x_{1,c}\right)\left(X_3 - x_{3,c}\right) \\ \bar{g}_{23}(X_2, X_3) = b_{23}\left(X_2 - x_{2,c}\right)\left(X_3 - x_{3,c}\right) \end{cases} \tag{37}$$

The constitution analysis of the univariate component function is shown in Table 2. According to the zeroth and first iteration, it is clear that $a_{2,2}{}^{(0)}$, $a_{2,2}{}^{(1)}$, $a_{3,2}{}^{(0)}$ and $a_{3,2}{}^{(1)}$ follow equation (22), $a_{1,2}{}^{(0)}$ and $a_{1,2}{}^{(1)}$ do not follow either equation (21) or equation (22). Therefore, $g_2(X_2)$ and $g_3(X_3)$ are QP010, and $g_1(X_1)$ is QP011, which agree well with the original function.

The processes and results of delineating the existence of bivariate component function are shown in Table 3, and $I^{(0)}(X_1, X_3) = I^{(1)}(X_1, X_3) = 0$ and $I^{(0)}(X_1, X_2) = I^{(1)}(X_1, X_2) = I^{(0)}(X_2, X_3) = I^{(1)}(X_2, X_3) = 1$, which also agree well with the limit-state function.

Further, all of m_1, m_2 and m_3 are 1 according to Table 1.

The convergence result is obtained after the first iteration, and the final response surface is:

$$\bar{g}^{(1)}(\boldsymbol{X}) = 40 - X_1 + 1.2X_2 + 5X_3 - X_1{}^2 + X_1X_2 + X_2X_3 \tag{38}$$

which is the same with the true limit-state function. Easily, β is 5.1993 by MCS, and the comparison among different methods is shown in Table 4. It is clear that the proposed method gives the "exact solution" for this case, and is more accurate and efficient than other two RSMs.

It can be observed from Table 4 that constitution analysis of univariate component function and delineating the existence of bivariate component function are unused because the proposed method is convergent with k_{final}=1. In other words, the proposed method approximates the limit-state by a completely quadratic polynomial for this case. However, the proposed method is more efficient than the one with CQP because of the selection of sample points. Moreover, the final response surface of RSM with CQP includes the non-existing cross term X_1X_3 which affects the accuracy. On the other hand, because k_{final} of the proposed method is less than the one with QP, the efficiency of the proposed method is improved even if more coefficients are solved in every iteration. By comparing with the other two RSMs, the proposed method is the easiest to implement in this example because all coefficients are available by the closed-form formula.

Iteration	0	1
Points	(0, 0, 0)	(4.6171, −1.6941, −1.2742)
	(−3, 0, 0)	(5.6171, −1.6941, −1.2742)
	(3, 0, 0)	(3.6171, −1.6941, −1.2742)
	(0, −3, 0)	(4.6171, −0.6941, −1.2742)
	(0, 3, 0)	(4.6171, −2.6941, −1.2742)
	(0, 0, −3)	(4.6171, −1.6941, −0.2742)
	(0, 0, 3)	(4.6171, −1.6941, −2.2742)
Coefficients	$a_{1,1}^{(0)} = -1$	$a_{1,1}^{(1)} = -11.9284$
	$a_{1,2}^{(0)} = -1$	$a_{1,2}^{(1)} = -1$
	$a_{2,1}^{(0)} = 1.2$	$a_{2,1}^{(1)} = 4.5429$
	$a_{2,2}^{(0)} = 0$	$a_{2,2}^{(1)} = -4.0.41 \times 10^{-39}$
	$a_{3,1}^{(0)} = 5$	$a_{3,1}^{(1)} = 3.3059$
	$a_{3,2}^{(0)} = 0$	$a_{3,2}^{(1)} = -3.67 \times 10^{-40}$
Center point	(4.6171, −1.6941, −1.2742)	

Table 2. Process of constitution analysis for Case 1 of Example 1

Iteration	0	1
Points for $g_{12}(X_1, X_2)$	(0, 0, 0)	(4.6171, −1.6941, −1.2742)
	(3, 0, 0)	(5.6171, −1.6941, −1.2742)
	(0, 3, 0)	(4.6171, −0.6941, −1.2742)
	(3, 3, 0)	(5.6171, −0.6941, −1.2742)
Points for $g_{13}(X_1, X_3)$	(0, 0, 0)	(4.6171, −1.6941, −1.2742)
	(3, 0, 0)	(5.6171, −1.6941, −1.2742)
	(0, 0, 3)	(4.6171, −1.6941, −0.2742)
	(3, 0, 3)	(5.6171, −1.6941, −0.2742)
Points for $g_{23}(X_2, X_3)$	(0, 0, 0)	(4.6171, −1.6941, −1.2742)
	(0, 3, 0)	(4.6171, −0.6941, −1.2742)
	(0, 0, 3)	(4.6171, −1.6941, −0.2742)
	(0, 3, 3)	(4.6171, −0.6941, −0.2742)
Value of $I(X_i, X_j)$	$I^{(0)}(X_1, X_2) = 1$	$I^{(1)}(X_1, X_2) = 1$
	$I^{(0)}(X_1, X_3) = 0$	$I^{(1)}(X_1, X_3) = 0$
	$I^{(0)}(X_2, X_3) = 1$	$I^{(1)}(X_2, X_3) = 1$

Table 3. Delineating the existence of bivariate component function for Case 1 of Example 1

Method	Constitution	k_{final}	N	β	Error (%)
MCS			10^7	5.1993	
RSM with QP (Liu and Moses, 1994)		3	31	5.0787	2.32
RSM with CQP (Rajashekhar and Ellingwood, 1993)		1	31	5.0799	2.29
Proposed method	$X_1, X_2, X_3, X_1^2,$	1	21	5.1993	0

Table 4. Results for Case 1 of Example 1

Case 2: $p = 3$:

According to the zeroth and first iteration, it can be founded that $g_1(X_1)$ and $g_2(X_2)$ are QP011 and $g_3(X_3)$ is QP010; $I^{(0)}(X_1, X_3) = I^{(1)}(X_1, X_3)=0$ and $I^{(0)}(X_1, X_2) = I^{(1)}(X_1, X_2)= I^{(0)}(X_2, X_3)=I^{(1)}(X_2, X_3)=1$.

According to Table 1, all of m_1, m_2 and m_3 are 1 in this case.

After the second iterations, the final response surface is available as follows:

$$\bar{g}^{(2)}(\boldsymbol{X}) = 5.952 - X_1 - 34.731X_2 + 5X_3 - X_1{}^2 - 11.373X_2{}^2 + X_1X_2 + X_2X_3 \tag{39}$$

and the β is 3.1896. Results obtained by different methods are shown in Table 5, and it can be observed that RSM with QP is unable to converge, whereas RSM with CQP and the proposed method can obtain convergent results. Further, because the constitution analysis of univariate component function and delineating the existence of bivariate component function reduce the number of function evaluations in kth iteration ($k > 2$), the efficiency of the proposed method is enhanced compared to the other two RSMs.

The influence of parameter f_k on precision and efficiency of different RSMs is shown in Figures 2–4, respectively. Because RSM with QP is unable to converge for all of the three strategies, only the proposed method and RSM with CQP are investigated for this case. As far as the precision is concerned, the proposed method and RSM with CQP are almost the same, and the results for Strategy 2 and Strategy 3 are also similar, whereas the one for Strategy 1 has slightly fluctuation. As for the efficiency, in this example, the proposed method is more efficient than RSM with CQP with less function evaluations, and the influence trend of f_k is similar for both RSMs in the three strategies, and Strategy 2 is more robust and efficient than the other two strategies for both RSMs in this example.

Example 2. Consider a limit-state function with exponential form:

$$g(\boldsymbol{X}) = \exp(0.4X_1 + 7) - \exp(0.3X_2 + 5) - 200 \tag{40}$$

where X_1 and X_2 are independent standard normal variables. In this example, the results that $I^{(0)}(X_1, X_2) = I^{(1)}(X_1, X_2) = 0$ are available by the delineation of the cross terms. Similarly, it can be found by the constitution analysis that the univariate component functions for X_1 and X_2 are QP011, and the values of m_1 and m_2 are 1 according to Table 1. Results obtained by different

Table 5.
Results summary for Case 2 of Example 1

Method	Constitution	k_{final}	N	β	Error (%)
Direct MCS			10^7	3.1526	
RSM with QP (Liu and Moses, 1994)		Not convergent			
RSM with CQP (Rajashekhar and Ellingwood, 1993)		2	47	3.1896	1.16
Proposed method	$X_1, X_2, X_3, X_1{}^2, X_2{}^2$	2	30	3.1896	1.16

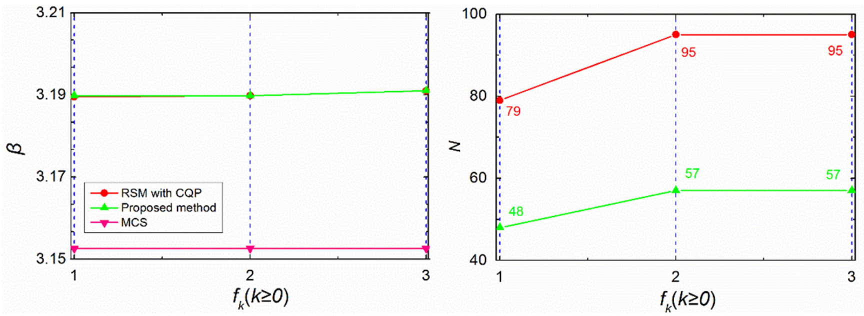

Figure 2.
Investigations of effect of parameter f_k for selection strategy 1 of case 2 of example 1

methods are shown in Table 6, and the investigations of the influence of parameter f_k on precision and efficiency for different RSMs are shown in Figures 5–7, respectively.

It can be observed from Table 6 that because the final response surface of RSM with CQP includes the non-existing cross term X_1X_2, the accuracy and efficiency of the proposed method are better than RSM with CQP. What is more, the β obtained by the proposed method is the same as that obtained by RSM with QP, whereas the proposed method is slightly less efficient than RSM with QP. The reason would be that in the proposed method, one additional function evaluation is required in the zeroth and the first iterations to delineate the existence of bivariate component function.

As shown in Figures 5–7, it is clear that both of the proposed method and RSM with QP have the same accuracy and robustness for all the strategies. However, because two additional function evaluations are necessary for delineating the existence of cross terms, the proposed method is less efficient than RSM with QP in this example. Further, RSM with CQP is the worst method in terms of accuracy, efficiency and robustness for this example.

Example 3. Consider a limit-state function with five independent variables:

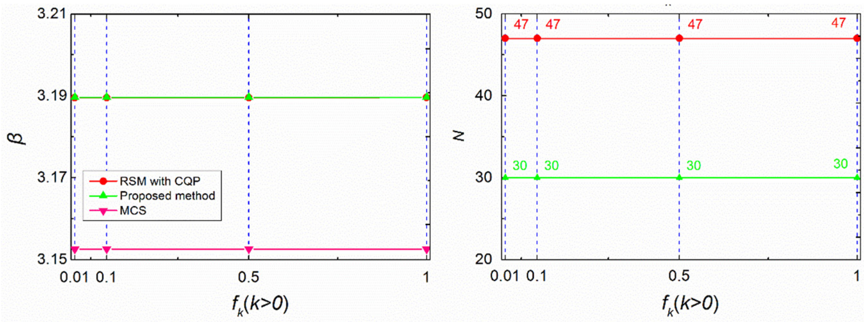

Figure 3. Investigations of effect of parameter f_k for selection strategy 2 of case 2 of example 1

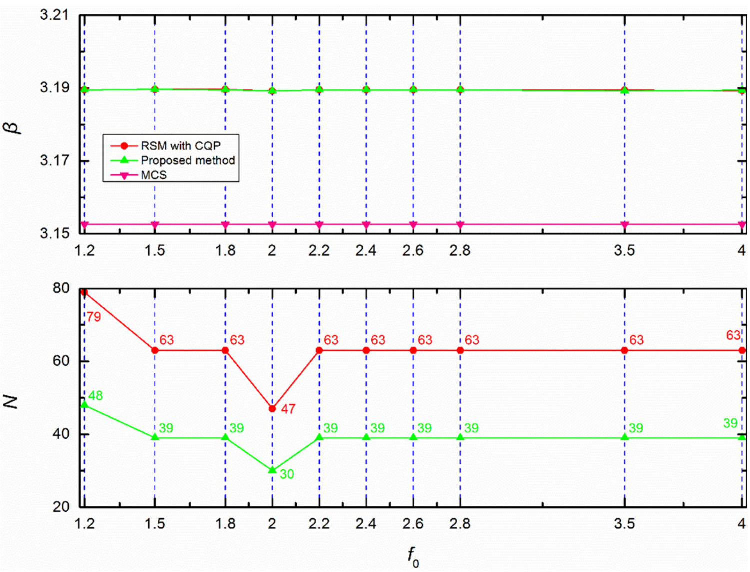

Figure 4. Investigations of effect of parameter f_k for selection strategy 3 of case 2 of example 1

$$g(\boldsymbol{X}) = X_1 - \frac{X_2{}^2 X_3{}^2}{X_4 X_5} \tag{41}$$

where $X_1 \sim N(5,1)$, $X_2 \sim N(1,1)$, $X_3 \sim N(1,1)$, $X_4 \sim N(6,1)$ and $X_5 \sim N(6,1)$.
In this example, the results that $I(X_1, X_2) = I(X_1, X_3) = I(X_1, X_4) = I(X_1, X_5) = 0$ and $I(X_2, X_3) = I(X_2, X_4) = I(X_2, X_5) = I(X_3, X_4) = I(X_3, X_5) = I(X_4, X_5) = 1$ are available by delineating the existence of cross terms. The univariate component functions for X_1 is QP010 and others are QP011 determined by constitution analysis, and thus all of m_1 m_2, m_3, m_4 and m_5 are equal to 1. Results obtained by different methods are shown in Table 7, and the influence of parameter f_k on precision and efficiency for different RSMs is investigated and shown in Figures 8–10.

It can be found from Table 7 that RSM with CQP is the most accurate but the lowest efficient in this example, and the proposed method and RSM with QP have nearly the same precision and efficiency, in which the proposed method is slightly better. As far as the

Table 6.
Results summary of Example 2

Method	Constitution	k_{final}	N	β	Error (%)
Direct MCS	–	–	10^7	2.725	–
RSM with QP (Liu and Moses, 1994)	–	3	23	2.710	0.6
RSM with CQP (Rajashekhar and Ellingwood, 1993)	–	3	39	2.619	3.9
Proposed method	X_1, X_2, X_1^2, X_2^2	3	25	2.710	0.6

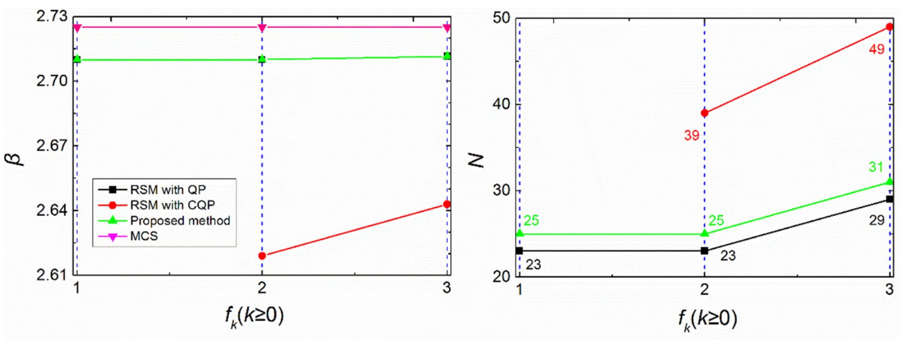

Figure 5.
Investigations of effect of parameter f_k for selection strategy 1 of example 2

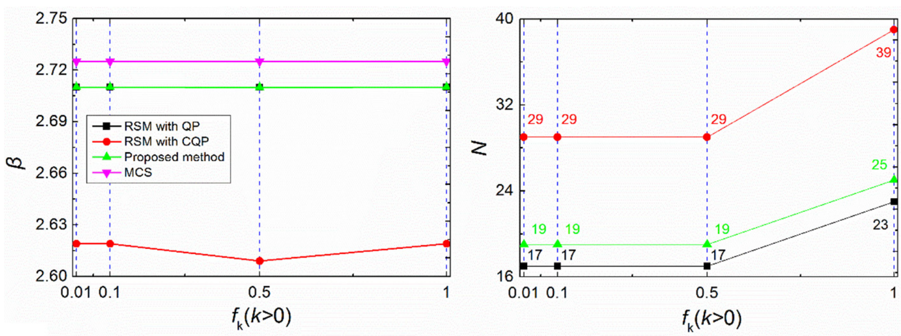

Figure 6.
Investigations of effect of parameter f_k for selection strategy 2 of example 2

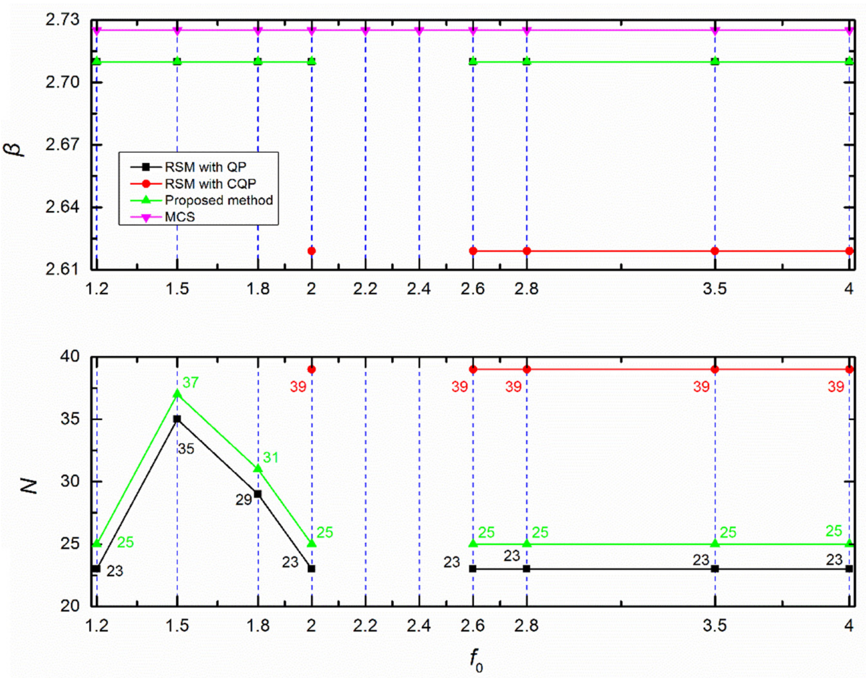

Figure 7. Investigations of effect of parameter f_k for selection strategy 3 of example 2

Table 7. Results summary of Example 3

Method	Constitution	k_{final}	N	β	Error (%)
Direct MCS	–	–	10^7	3.5073	–
RSM with QP (Liu and Moses, 1994)	–	6	83	3.4809	0.75
RSM with CQP (Rajashekhar and Ellingwood, 1993)	–	4	219	3.4951	0.35
Proposed method	$X_1, X_2, X_3, X_4 X_5, X_2^2, X_3^2, X_4^2, X_5^2$	3	77	3.4858	0.61

balance between precision and efficiency is concerned, the proposed method is the best choice to some extent for this example.

According to the robustness analyses for the three RSMs, which are shown in Figures 8–10, the proposed method and RSM with QP have the nearly similar performance on convergence, precision and efficiency, but RSM with CQP is of the worst convergence, which is unable to converge with f_k=1 as shown in Figure 8.

3.2 Set 2 – solid mechanic problems

Example 4. Consider a liner elastic cantilever beam subjected to one distributed load q and two concentrated loads F_1 and F_2 (Chowdhury and Rao, 2009a, 2009b), which is shown in Figure 11. The limit-state is defined that the displacement of Point B is equal to 15 mm in vertical direction and the limit-state function is then formulated by:

$$g(q, F_1, F_2, E) = 15 - \frac{qL^4}{8EI} - \frac{5F_1L^3}{48EI} - \frac{F_2L^3}{3EI} \tag{42}$$

where L is the length of the cantilever beam and is 3,000 mm, I is the cross-sectional moment of inertia of the cantilever beam with value of $9 \times 10^8 \text{mm}^4$, E is the elastic modules of material and is a lognormal variable with the mean value of 26,000 MPa and standard deviation of 39,000 MPa, q, F_1 and F_2 are Type I extreme values distributed with the mean values of 50 N/mm, 70,000 N, 100,000 N and standard deviations of 15 N/mm, 21,000 N and 25,000 N, respectively.

In this example, it is easy to obtain that $I(q, F_1) = I(F_1, F_2) = I(q, F_2) = 0$, $I(q, E) = I(F_1, E) = I(F_2, E) = 1$, and m_q, m_{F1}, m_{F2} and m_E are all equal to 1, and the univariate component functions of q, F_1, F_2 and E are all QP010. Table 8 shows the results obtained by different methods, and the robustness of different RSMs with parameters f_k are also investigated and the results are shown in Figures 12–14.

Based on the results in Table 8, it can be seen that the efficiency and accuracy of RSM with CQP are lower than other two RSMs, and the proposed method is more accurate in the RSM with QP, whereas RSM with QP is more efficient than the proposed method. Hence, the proposed method and RSM with QP are both appropriate for this example.

According to Figures 12–14, is the following are found: In Strategy 1, the proposed method has the same robustness with RSM with CQP, and RSM with QP is unable to converge for most cases; in Strategy 2, all the three methods have the similar robustness; and in Strategy 3, the proposed method has the best robustness.

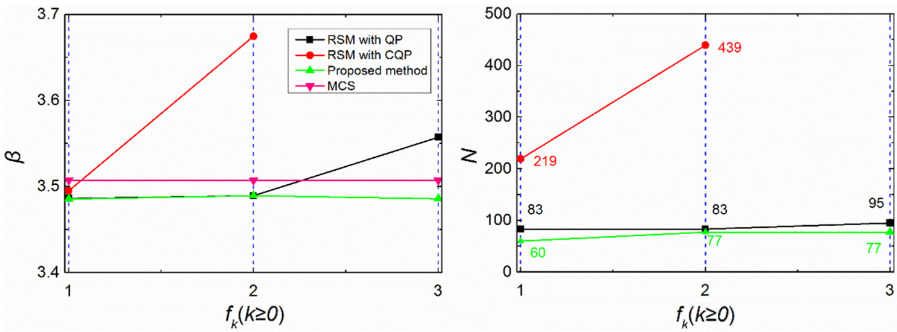

Figure 8.
Investigations of effect of parameter f_k for selection strategy 1 of example 3

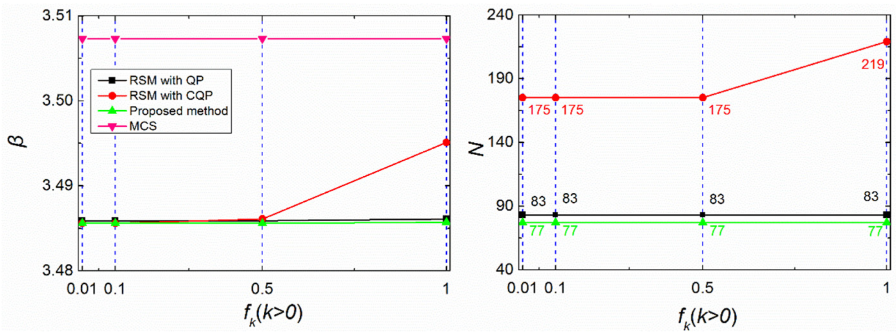

Figure 9.
Investigations of effect of parameter f_k for selection strategy 2 of example 3

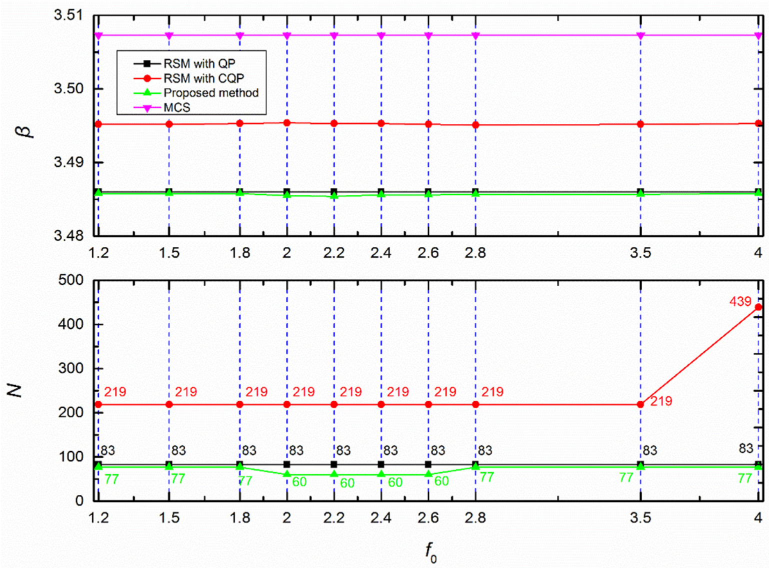

Figure 10. Investigations of effect of parameter f_k for selection strategy 3 of example 3

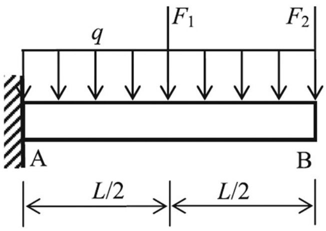

Figure 11. Model of cantilever beam

Method	Constitution	k_{final}	N	β	Error (%)
Direct MCS	–	–	10^7	3.3134	–
RSM with QP (Liu and Moses, 1994)	–	5	59	3.1208	5.8
RSM with CQP (Rajashekhar and Ellingwood, 1993)	–	3	103	3.1214	5.8
Proposed method	q, F_1, F_2, E	5	67	3.1324	5.4

Table 8. Results summary of Example 4

Example 5. Consider another cantilever beam subjected to distributed load, which is studied earlier by Rajashekhar and Ellingwood (1993). The limit-state function is:

$$g(\boldsymbol{X}) = 18.46154 - 74769.23\frac{X_1}{X_2{}^3} \tag{43}$$

where X_1 and X_2 are normal variable with the mean values of 1,000 N/mm^2, 250 mm and standard deviations of 200 N/mm^2 and 37.5 mm, respectively. The result of delineation of cross term is $I(X_1, X_2)$ =1, and the univariate component functions for X_1 are QP010 and for

X_2 are QP011, and m_1 and m_2 are 1, determined by constitution analysis. Table 9 presents the results obtained by MCS and three RSMs, and Figures 15–17 show the investigations of the robustness of RSMs with different strategies of f_k. It is worth noting that because RSM with QP is unable to converge for all of the three strategies, only the proposed method and RSM with CQP are illustrated.

It can be seen from Table 9 that the efficiency and accuracy of the proposed method are both better than RSM with CQP. Moreover, according to Figures 15–17, the robustness of the proposed method and RSM with CQP are nearly the same, but RSM with CQP is unable to converge with $f_k = 3$ in Figure 15 and $f_k = 0.01$ in Figure 16.

The next two examples, whose structural response is obtained using finite element method, are demonstrated to verify the efficiency and accuracy of the proposed method. For brevity, the strategy for parameter f_k is selected as $f_0 = 3$ and $f_k = 1$ ($k > 0$) for the three RSMs.

Example 6. An example of a two-bay and six-story steel frame building is presented under investigation, to which six horizontal loads are applied. The frame model is shown in Figure 18, with the span s = 5 m for each bay and the height h = 2.5 m for each floor. The beam and column elements of frame are both rectangular and denoted as B and C in Figure 18, respectively, and their cross-sectional parameters are listed in Table 10. The mean values and standard deviations of the 11 random variables, as well as the corresponding distribution types, are listed in Table 11. Note that the second inertial moment of cross section is calculated as (B1*B2^3)/12 and (C1*C2^3)/12 and cross-sectional area is calculated as B1*B2 and C1*C2 for beam and column, respectively.

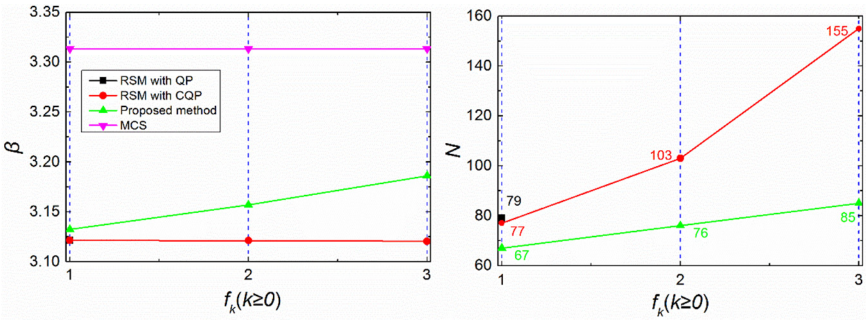

Figure 12.
Investigations of effect of parameter f_k for selection strategy 1 of example 4

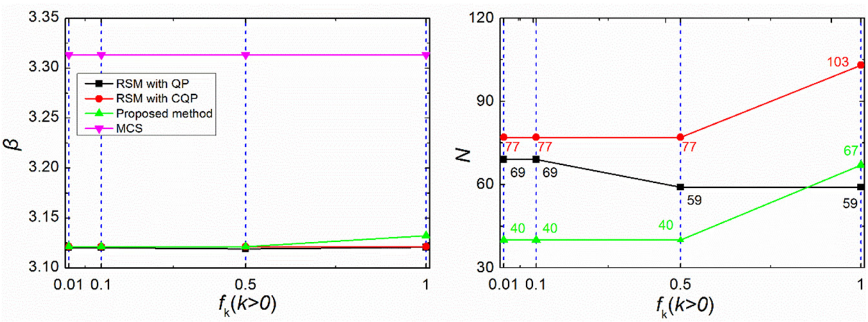

Figure 13.
Investigations of effect of parameter f_k for selection strategy 2 of example 4

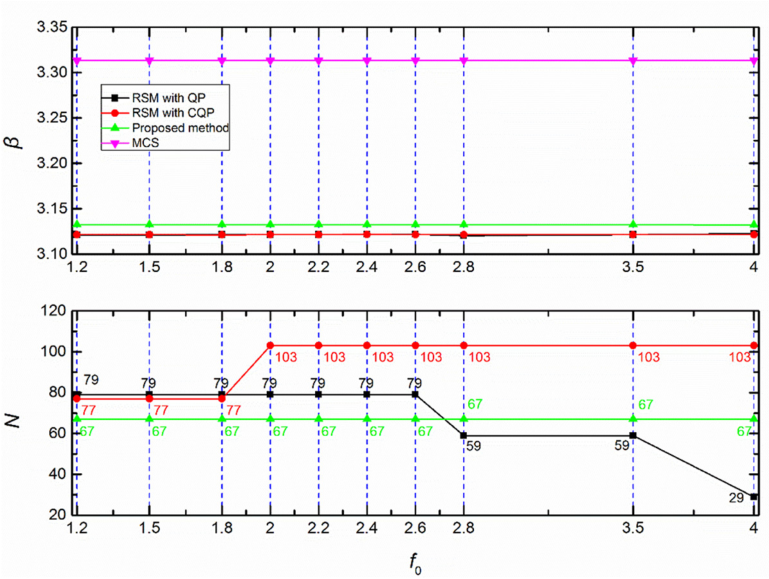

Figure 14. Investigations of effect of parameter f_k for selection strategy 3 of example 4

Table 9. Results summary of Example 5

Method	Constitution	k_{final}	N	β	Error (%)
Direct MCS	–	–	10^7	2.3450	–
RSM with QP (Liu and Moses, 1994)	–	Not convergent	–	–	–
RSM with CQP (Rajashekhar and Ellingwood, 1993)	–	5	59	2.3311	0.60
Proposed method	X_1, X_2, X_2^2	5	37	2.3337	0.48

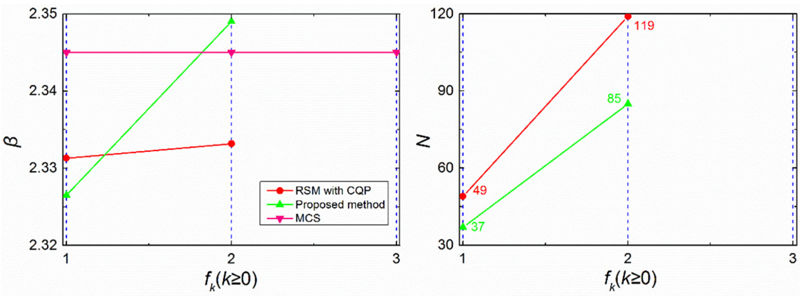

Figure 15. Investigations of effect of parameter f_k for selection strategy 1 of example 5

The reliability analysis is confined to serviceability limit-state as the exceedance of h/500 at Point a, and the limit-state function is defined as:

$$g(\boldsymbol{X}) = 0.03 - \text{displacement\_a}(\boldsymbol{X}) \tag{44}$$

where displacement_a($\boldsymbol{X}$) denotes the actual horizontal displacement which is a function of all the random variables. The structural response is obtained by finite element analysis which is implemented with MATLAB code (Kattan, 2003). Results obtained by different methods, as well as the constitution of univariate component function, are listed in Table 12. As can be seen from Table 12, the univariate component functions of Young's modulus E and cross-sectional parameters are QP011, and the rest are QP010, which is intuitive in linear finite element analysis. Furthermore, the values of delineation of cross terms are 0 for the 14 cross terms among P1–P6:

$$I(\mathrm{P}i, \mathrm{P}j) = 0, \text{ for } 1 \le i < j \le 6 \tag{44}$$

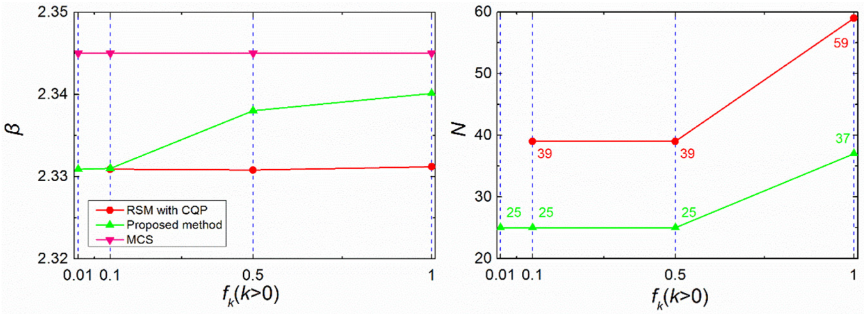

Figure 16.
Investigations of effect of parameter f_k for selection strategy 2 of example 5

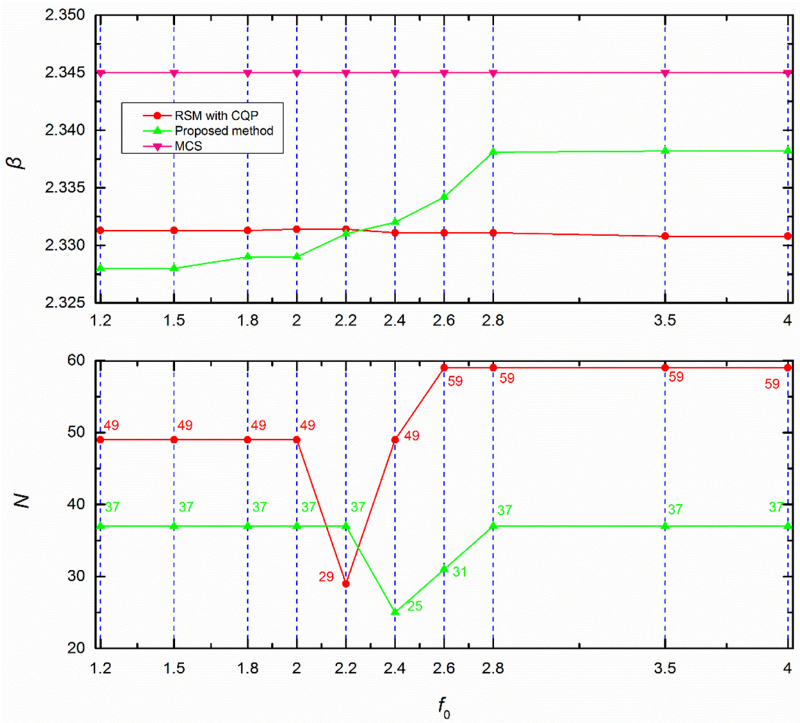

Figure 17.
Investigations of effect of parameter f_k for selection strategy 3 of example 5

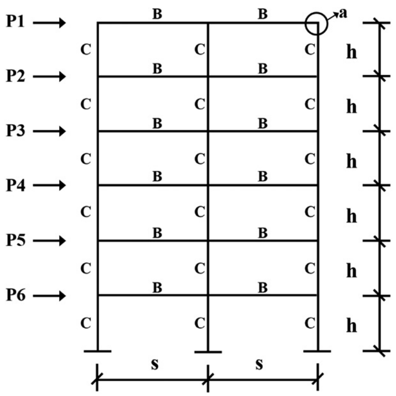

Figure 18. Model of steel frame of example 6

Table 10. Cross-sectional parameters of Example 6

Element type	Modulus of elasticity	Section width	Section depth	Shape
B	E	B1	B2	Rectangular
C	E	C1	C2	Rectangular

Table 11. Mean value and standard deviation of random variables of Example 6

Random variable	Type	Mean value	SD
P6	Rayleigh	70,000 N	21,000 N
P5	Rayleigh	90,000 N	27,000 N
P4	Rayleigh	110,000 N	33,000 N
P3	Rayleigh	130,000 N	39,000 N
P2	Rayleigh	150,000 N	45,000 N
P1	Rayleigh	170,000 N	51,000 N
E	Normal	3.1e11 Pa	3.1e10 Pa
B1	Normal	0.2 m	0.02 m
B2	Normal	0.5 m	0.05 m
C1	Normal	0.4 m	0.04 m
C2	Normal	0.4 m	0.04 m

Table 12. Results summary of Example 6

Method	Constitution	k_{final}	N	β	Error (%)
MCS	–	–	10^6	5.6874	–
RSM with QP (Liu and Moses, 1994)	–	1	47	9.9903	75.65
RSM with CQP (Rajashekhar and Ellingwood, 1993)	–	5	12431	5.1212	9.95
Proposed method	E, E^2, B1, $B1^2$, B2, $B2^2$, C1, $C1^2$, C2, $C2^2$, P1, P2, P3, P4, P5, P6	2	215	5.1264	9.86

and the others are equal to 1. This is because for structural analysis within the elastic domain and infinitesimal deformation, structural response under multiple loads is equivalent to the additive result of all the structural responses cause by each of the load, reflecting the principle of superposition. A similar result is also found in Example 4, and both of them show the rationality of delineation of cross terms in terms of physical intuition.

Comparison of adaptive RSM with other two RSMs shows the good quality of the proposed method in both accuracy and efficiency aspects, whereas RSM without QP is clearly inaccurate mainly because of the elimination of cross term and small number of sample points. Furthermore, although RSM with QP has the same accuracy as the proposed method, it requires a large number of sampling points to construct the response surface which is computational time-consuming and impractical. On the other hand, because the adaptive RSM characterizes the properties of limit-state function by the constitution analysis and delineation of cross terms, only crucial terms remain in the global RSF and therefore the computational efforts is saved.

Example 7. The last example consists in the reliability analysis of a two-dimensional tunnel subject to the weight load of the surrounding rock (Xu, 2006). There are 15 independent normal random variables representing the material properties of the tunnel and rock, including Young's modulus E, Poisson's ratio α and material volume weight γ, and their mean values and standard deviations are listed in Table 13. Note that three types of concrete C1, C2 and C3 are used for constructing the tunnel, and two types of rocks named R1 and R2 are considered in the analysis model, as shown in Figure 19. The finite element model has 2,729 nodes and 5,145 elements in total, and the corresponding finite element

Table 13. Mean value and standard deviation of random variables of Example 7

Random variable	Type	Mean value	SD
E of R1 (ER1)	Normal	2.0e9 Pa	5.0e8 Pa
α of R1 (αR1)	Normal	0.25	0.0125
γ of R1 (γR1)	Normal	2.2e4 kg/m^3	5,500 kg/m^3
E of R2 (ER2)	Normal	2.6e9 Pa	6.5e8 Pa
α of R2 (αR2)	Normal	0.2	0.01
γ of R2 (γR2)	Normal	2.3e4 kg/m^3	5,750 kg/m^3
E of C1 (EC1)	Normal	2.85e9 Pa	4.275e8 Pa
α of C1 (αC1)	Normal	2.85e10 Pa	0.01
γ of C1 (γC1)	Normal	2.5e4 kg/m^3	3,750 kg/m^3
E of C2 (EC2)	Normal	1.85e10 Pa	2.775e9 Pa
α of C2 (Ac2)	Normal	0.2	0.01
γ of C2 (γC2)	Normal	2.3e4 kg/m^3	3,450 kg/m^3
E of C3 (EC3)	Normal	2.85e10 Pa	4.275e9 Pa
α of C3 (αC3)	Normal	0.2	0.01
γ of C3 (γC3)	Normal	2.2e4 kg/m^3	3,300 kg/m^3

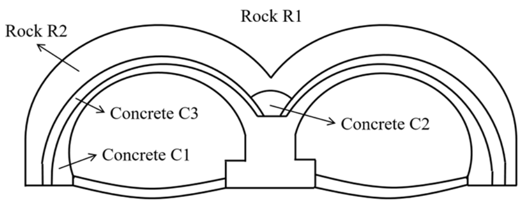

Figure 19. Model of tunnel with surrounding rock of example 7

model in Figure 20. For convenience, the abbreviations of each random variable are given in the first column of Table 13 within parenthesis.

Finite element method of plane strain problem is implemented for obtaining the structural response of the model, and the limit-state is defined as the exceedance of max principle stress in tunnel of a threshold of 9.5 MPa. The criteria is formulated as:

$$g(\boldsymbol{X}) = 9.5 - S(\boldsymbol{X}) \tag{45}$$

where $S(\boldsymbol{X})$ is the max principle stress in tunnel. Results of reliability index β obtained by different methods are listed in Table 14. According to the delineation of cross terms, it is shown in Table 13 that only 44 bivariate terms are necessary among all the 105 potential bivariate terms, and the constitution analysis indicates that all the univariate component functions are QP010 except for those of αR1, $\alpha R2$, αC1, αC2 and αC3, which are QP011, reflecting the fact that calculation of element stress in plain strain problem is a function linear to Young's modulus and volume weight and nonlinear to Poisson's ratio.

Comparison between the proposed method with MCS and the other two RSMs indicates that the proposed adaptive RSM is able to give a relatively accurate result with only 419 finite element analyses, whereas the number of function evaluations in RSM with QP grows explosively to achieve the similar accuracy of reliability index, indicating that for a reliability analysis with complicated finite element model and time consuming limit-state function, the proposed method is able to effectively reduce the number of evaluations without losing too much of the accuracy.

4. Conclusions

This paper proposed a new adaptive RSM based on the adaptive bivariate cut-HDMR model. The univariate component function is approximated by the second-order polynomial, and the constitution analysis is incorporated to determine the unnecessary terms in the univariate component functions with its practical application. The bivariate component function is constructed with the new sample techniques which chooses the sample points as close to the limit-state function as possible. The delineation of cross terms is introduced to construct the final global response surface for reliability analysis with iteration scheme. The mean values of random variables are selected as the initial point for the first iteration, and at the first two iterations, the coefficients of univariate component functions are calculated and used for constitution analysis, and the indication function of bivariate component function is also calculated for delineation of cross terms. After the first two steps, the unnecessary terms will be removed from the global response surface and thus the computational efficiency can be enhanced because of the reduction of unknown coefficients, which in return reduce the number of sample points and finite element analysis.

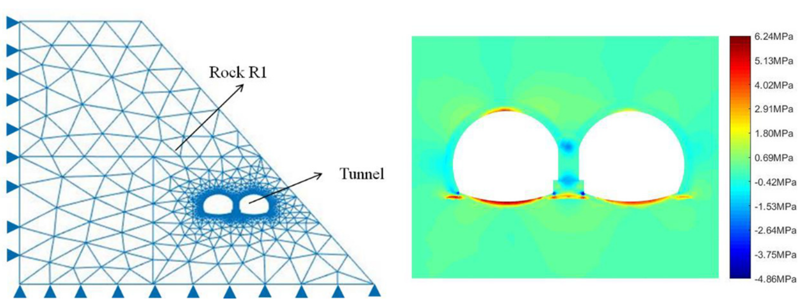

Figure 20. Finite element model of Tunnel of example 7 with boundary condition: (left) finite element discretization; (right) result with random variables at mean value around the tunnel

Table 14.
Results summary of Example 7

Method	Constitution	k_{final}	N	β	Error (%)
MCS	–	–	10^6	1.486	–
RSM with QP (Liu and Moses, 1994)	–	Not converge	–	–	–
RSM with CQP (Rajashekhar and Ellingwood, 1993)	–	3	131,196	1.4336	3.32
Proposed method	ER1, ER2, EC1, EC2 EC3, αR1, αR2, αC1, αC2, αC3, αR1^2, αR2^2, αC1^2, αC2^2, αC3^2, γR1, γR2, γC1, γC2, γC3	2	419	1.4752	0.73

Seven numerical examples are investigated in detail to discuss the accuracy and efficiency of the proposed method, including three mathematical examples and four mechanical examples. Two existing RSMs are also included in the examples for comparison purpose, and the reliability index obtained by the MCS is considered as the benchmark result for error analysis. Three strategies for selecting the values of parameter f_k are also proposed to investigate the robustness and sensitivity of the proposed method as well as the other two RSMs. The following main conclusions can be drawn from the results of numerical examples and comparisons between the proposed method and the other two RSMs:

- The proposed method has a more feasible process of evaluating undetermined coefficients of each component function than traditional RSM.
- The criterion of constitution analysis of univariate component function and delineation of bivariate component function are appropriate and practical.
- The proposed method performs well in terms of balancing the efficiency and accuracy when compared to the traditional second-order polynomial RSM, and this is because the proposed method is able to construct the response surface more close to the limit-state function without unnecessary terms.
- The proposed method is robust on the parameter f_k in a wide range, indicating that it is able to obtain convergent result in a wide feasible domain of sample points.

However, because the dependent variables can be transformed into independent variables by Rosenblatt or Nataf transformation, which will increase the nonlinearity in the performance function to some extent, further research is needed to investigate the effects of correlation between dependent variables on the accuracy, efficiency and robustness of the proposed method, and comparison should be made with the existing methods in uncertainty qualification and reliability analysis which can handle dependent random variables (Fan *et al.*, 2016a, 2016b).

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