

Pen & Paper Exercise 7

Social Networks

1 Signed Networks and Structural Balance

Assume that in a sparsely populated area, 50 farmers live along a 50km stretch of a river, each occupying exactly 1km of the river bank. After interviewing these farmers, you discover that each farmer is friends with all farmers that live at most 10km away from him or her, and enemies with every other farmer. Assuming you were to build the signed complete graph of this network, would it be structurally balanced (w.r.t. strong structural balance theory)? Explain your answer!

2 Directed Acyclic Graphs

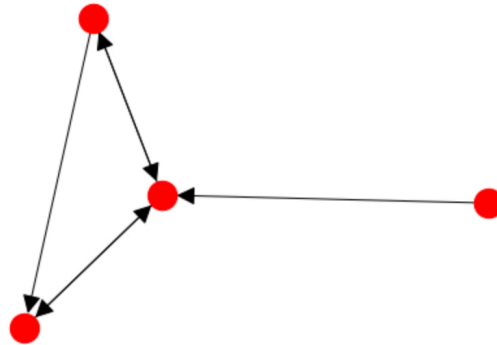
Consider a directed acyclic graph with node set $V = \{1, \dots, N\}$, in which a directed edge from node i to node j can only exist if $i > j$.

- Write down an expression for the total number of ingoing edges at nodes $1, \dots, r$, and another for the total number of outgoing edges at nodes $1, \dots, r$, both in terms of the in-degrees k_i^{in} and out-degrees k_i^{out} of the nodes.
- Derive an expression for the total number edges running to nodes $1, \dots, r$ from nodes $r + 1, \dots, n$.
- Show that in any directed acyclic graph, the in- and out-degrees must satisfy

$$k_r^{in} \leq \sum_{r+1}^n (k_i^{out} - k_i^{in}), \quad k_{r+1}^{out} \leq \sum_1^r (k_i^{in} - k_i^{out})$$

3 Reciprocity and Triadic Census

a) Calculate the *simple reciprocity* of the following directed graph.



b) Carry out a *triadic census* of the following directed graph. Specifically, draw all triadic motifs that occur in the graph. Is there a pair of motifs among these that are equivalent?

