

Pen & Paper Exercise 3

Social Networks

1 Giant Components in Large Random Graphs

Consider a random graph $G(n, p)$ for a large n .

- a) If a network has a giant component that fills exactly half of the network, what is the average degree of a node?
- b) For this same random graph, what is the probability that a node belongs to the giant component if it has degree exactly 5?
- c) Calculate the fraction of nodes in the giant component that have degree exactly 5.

2 The Circle Model

Consider the ring lattice, i.e., a Watts-Strogatz graph with $p = 0$.

- a) Prove that the clustering coefficient of every node in this network indeed equals

$$C = \frac{3(k-2)}{4(k-1)}$$

where k is the degree of each node.

- b) Compute the average path length of such a graph. For simplicity, you may assume a special case in which the number of nodes n is uneven, and $n - 1 \bmod k = 0$.

3 Matchings in a Configuration Model

Recall the edge stub matching algorithm to create a random graph according to the configuration model. How many possible matchings are there for a configuration of a graph with m links?