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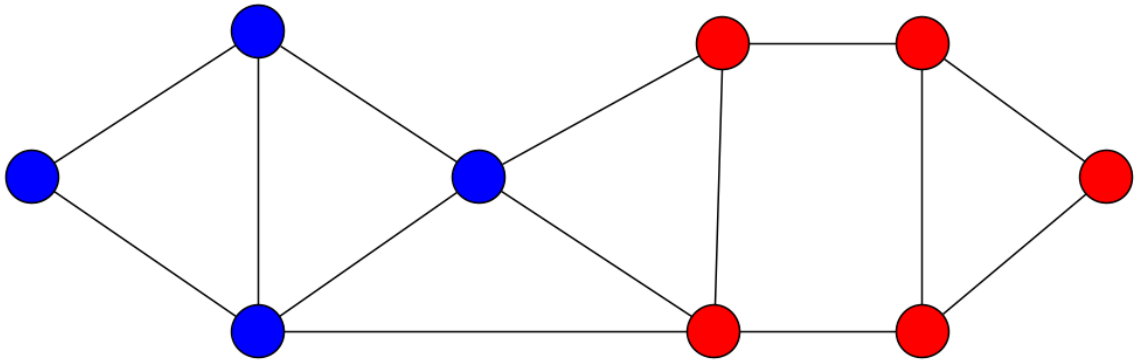
Pen & Paper Exercise 5

Social Networks

1 Evaluating the Quality of a Partition

Consider the directed graph that is presented in the following diagram:



The graph is split into the two communities that are indicated by the blue and red nodes.

a) Considering the given communities, compute the internal and external degrees of each given node!

b) Using your results from a), determine whether both communities are weak communities. Are they also strong communities?

2 Modularity of a Partition

Compute the modularity of the partition of the graph from task 1.

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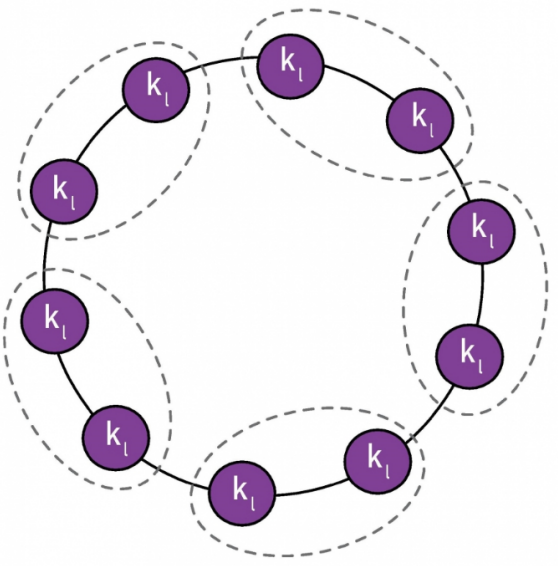
3 The Resolution Limit of Modularity

Consider a ring of communities, in which an even number n_c of node communities with l edges each are arranged in a ring, and from each community there are only two outgoing links, namely one to each of the neighbor communities. In this ring, we to want merge pairs of neighboring communities into a single community, as illustrated by the dotted lines in the following picture:



a) Compute the modularity M_s of the ring before merging the pairs!

b) Compute the modularity M_r of the ring after merging the pairs!

c) Show that there is a resolution limit to the modularity function by showing that for some values $l \geq 3$ and $n_c \geq 4$, the modularity M_p of the graph after pairing the communities is bigger than the modularity M_s before pairing. You may do this by either solving the corresponding inequality, or by directly giving an example graph.