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Pen & Paper Exercise 8

Social Networks

1 Temporal Networks

Consider the following temporal graph, in which every edge is labeled with the timestep $t \in \{1,2,3,4,5\}$ that it has been added to the graph. Note that these edges persist after they were added, i.e. they do not disappear in the next timestep!

↳ Ex.

$\lambda_{I,1}(A) = 3$,

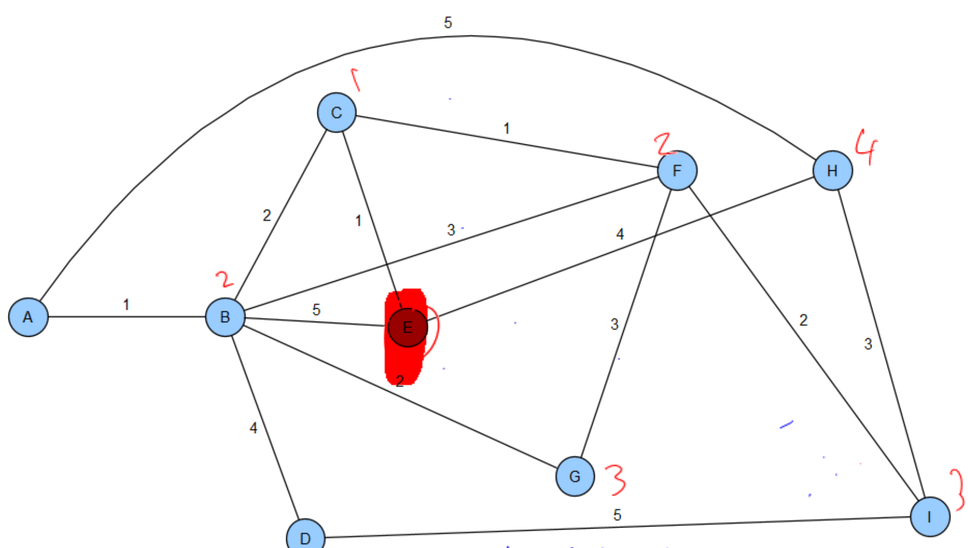
as we can reach

F at $t_1 = 2$,

B at $t_2 = 3$,

A at $t_3 = 4$

→ duration is $t_3 - t_0 = 4 - 1 = 3$



$\lambda_{I,2}(A) = 4 - 2 = 2$

edge labels correspond to time steps in which the edges are added to the graph, e.g. edge (H, I) is traversable from time $t = 3$

Compute the closeness centralities of nodes E and I with respect to shortest paths and fastest paths, i.e., compute

(i) the closeness centralities of these nodes assuming the network to be static,

(ii) the closeness centralities $C_c(i,t)$ of nodes E and I, choosing $t = 1$!

(i) $C(i) = \frac{N-1}{\sum_{j \neq i} d(i,j)}$ → $C(E) = \frac{8}{2+1+1+2+2+2+1+2} = \frac{8}{13}$

$$c(I) = \frac{8}{2+2+2+1+2+1+2+1} = \frac{8}{13}$$

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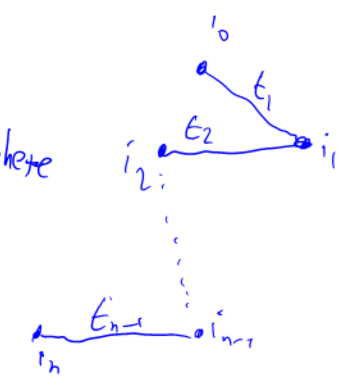
(ii) $$C_c(i,t) = \frac{N-1}{\sum_{j \neq i} \lambda_{i,t}(j)}$$

→ $\lambda_{i,t}(j)$ represents the duration of the fastest path from i to j starting at time point t

↳ recall, a path is modeled as a sequence

$$p = \{(i_0, i_1, t_1), (i_1, i_2, t_2), \dots, (i_{n-1}, i_n, t_n)\}, \text{ where } \boxed{t_1 < t_2 < \dots < t_n}$$

$\to t_1 = t_n$



↳ the duration is $\boxed{t_n - t_1}$

↳ in $\lambda_{i,t}(j)$, we always assume $\boxed{t_0 = t}$, and make "self-loops" if nothing is possible in the beginning

↳ we want to measure earliest arrival starting at given t, rather than shortest duration

j	A	B	C	D	E	F	G	H
$\lambda_{I,1}(j)$	3	2	2	3	3	1	2	2

$$\to C_c(I) = \frac{8}{3+2+2+3+3+1+2+2} = \frac{8}{18} = \frac{4}{9}$$

j	A	B	C	D	F	G	H	I
$\lambda_{E,1}(j)$	2	1	0	3	1	2	3	2

$$\to C_c(E) = \frac{8}{2+1+0+3+1+2+3+2} = \frac{8}{14} = \frac{4}{7}$$

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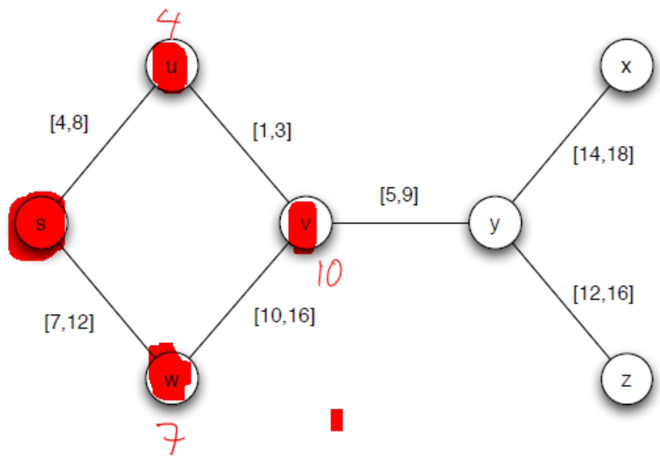
2 Spreading in a Temporal Network

Consider the following temporal network, where the edge labels indicate at which time intervals two individuals had contact.

Let's mark every node with the earliest time, that the disease could have reached it



Now assume that at time $t = 0$, individual s has gotten infected, and that the disease is spreading according to the SI model with $\beta = 1$, i.e., if an infected individual had contact with a susceptible individual at any specific time, the disease will be transmitted.

a) Suppose that s is the only individual who had the disease at time 0. Which nodes will have acquired the disease by the end of the observation period, at time 20?

b) Suppose that you find, in fact, that all nodes have the disease at time 20. You're fairly certain that the disease couldn't have been introduced into this group from other sources, and so you suspect instead that a value you're using as the start or end of one of the time intervals is incorrect. Can you find a single number, designating the start or end of one of the time intervals, that you could change so that in the resulting network, it's possible for the disease to have flowed from s to every other node?

a) only nodes s, u, v, w would be infected in the end

b) change interval at edge (v,y) to [5,11]

or ——— " ——— (u,v) to [1,5]