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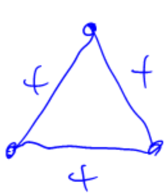
Pen & Paper Exercise 7

Social Networks

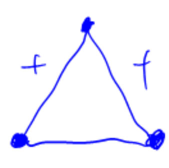
1 Signed Networks and Structural Balance

Assume that in a sparsely populated area, 50 farmers live along a 50km stretch of a river, each occupying exactly 1km of the river bank. After interviewing these farmers, you discover that each farmer is friends with all farmers that live at most 10km away from him or her, and enemies with every other farmer. Assuming you were to build the signed complete graph of this network, would it be structurally balanced (w.r.t. strong structural balance theory)? Explain your answer!

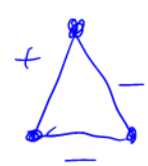
Signed networks, there are three kinds of triangles:



everyone is friends with each other
↳ balanced



one negative link
↳ unbalanced

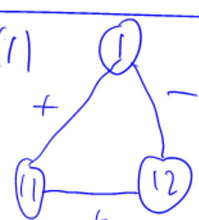


two negative relationships
↳ common enemy
↳ balanced

all negative
↳ could turn to "common enemy"
↳ unbalanced

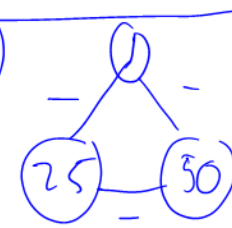
We have two kinds of unbalanced triangles in the network.
Letting the node ID, denote the km at the river bank, we for instance have:

(1)



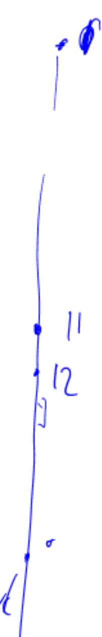
↳ unbalanced

(2)



unbalanced

→ the network is unbalanced



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2 Directed Acyclic Graphs

Consider a directed acyclic graph with node set $V = \{1, \dots, N\}$, in which a directed edge from node i to node j can only exist if $i > j$.

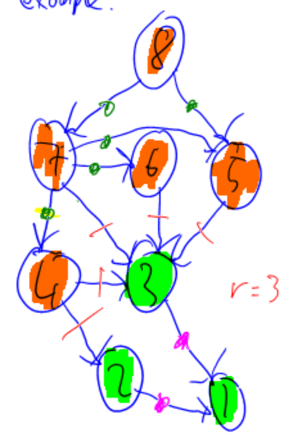
a) Write down an expression for the total number of ingoing edges at nodes $1, \dots, r$, and another for the total number of outgoing edges at nodes $1, \dots r$, both in terms of the in-degrees k_i^{in} and out-degrees k_i^{out} of the nodes.

b) Derive an expression for the total number edges running to nodes $1, \dots, r$ from nodes $r+1, \dots, n$.

c) Show that in any directed acyclic graph, the in- and out-degrees must satisfy

$$k_r^{in} \leq \sum_{r+1}^{n} (k_i^{out} - k_i^{in}), \quad k_{r+1}^{out} \leq \sum_{1}^{r} (k_i^{in} - k_i^{out})$$

example:



a) Let N_r^{in} denote # ingoing edges at nodes $1, \dots, r$

and N_r^{out} —"— # outgoing edges —"—

$$N_r^{in} = \sum_{i=1}^{r} k_i^{in} \qquad N_r^{out} = \sum_{i=1}^{r} k_i^{out}$$

b) Let N_r denote the searched-for value

$$N_r = \sum_{i=r+1}^{n} (k_i^{out} - k_i^{in}) = \sum_{i=1}^{r} (k_i^{in} - k_i^{out})$$

all outgoing edges from orange nodes

all orange → orange edges

all incoming edges at green nodes

all green → green edges

in example:
# of edges that point from orange nodes to green nodes
↳ edges with red marker

c) We have that

$$k_r^{in} \leq N_r \overset{b)}{=} \sum_{i=r+1}^{n} (k_i^{out} - k_i^{in}), \text{ and } \quad k_{r+1}^{out} \leq N_r \overset{b)}{=} \sum_{i=1}^{r} (k_i^{in} - k_i^{out})$$

# of all edges going from nodes $r+1, \dots, n$ to $1, \dots, r$

↳ # —— " —— to v

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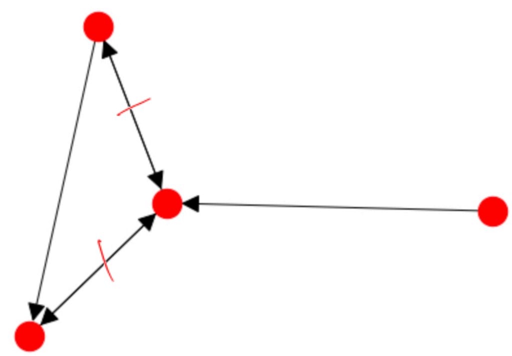
3 Reciprocity and Triadic Census

a) Calculate the *simple reciprocity* of the following directed graph.

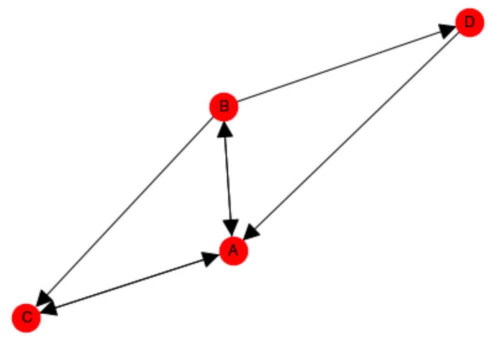


$$\text{reciprocity} = \frac{\#\text{ all edges } (i,j)\text{, where also } (j,i) \text{ is in the graph}}{\#\text{ all edges in the graph}}$$

→ A ⟷ B edges are TWO edges

$$= \frac{4}{6} = \frac{2}{3}$$

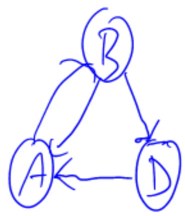
b) Carry out a *triadic census* of the following directed graph. Specifically, draw all triadic motifs that occur in the graph. Is there a pair of motifs among these that are equivalent?



number of triads: $\binom{4}{3} = 4$

↳ motifs in the graph:

210

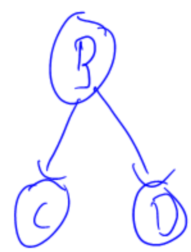


120C

C = circular

↳ "single edges go in cycles"

111D



021D

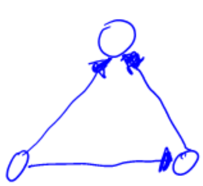
D ≙ down

↳ pairs with mutual or no edges drawn at the bottom by convention

other possible letters:

U = UP

T = transitive, as in here:



030T