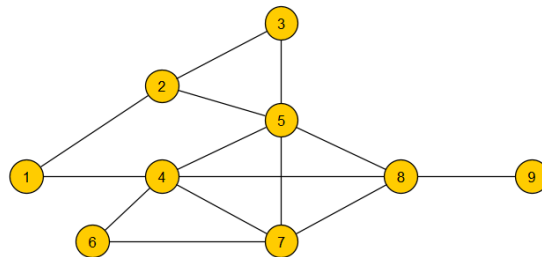


# Pen & Paper Exercise 2

## Social Networks

### 1 Local Clustering and Centralities

Consider the following graph



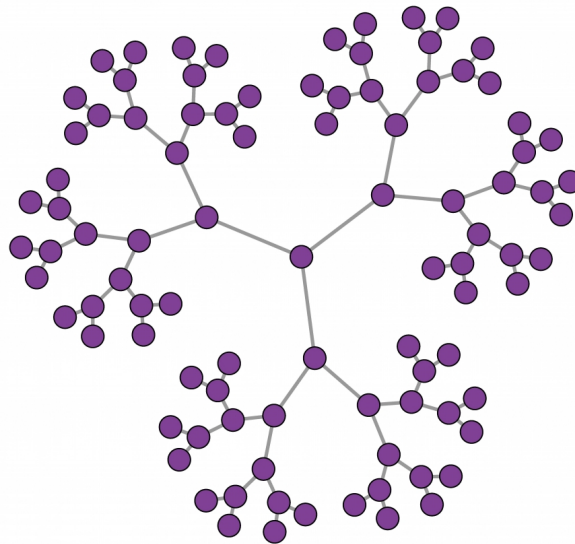
- Compute the local clustering coefficient for all nodes in the network. What is the average local clustering?
- Determine the coreness of each node in the network!
- Calculate the closeness centralities of nodes 1 and 5!

### 2 Betweenness Centrality

Consider a network of  $2n$  nodes arranged in a ring. Each node has two neighbors. In such a network all nodes have the same betweenness centrality. Derive the formula for this value as a function of  $n$ .

### 3 Small Worlds

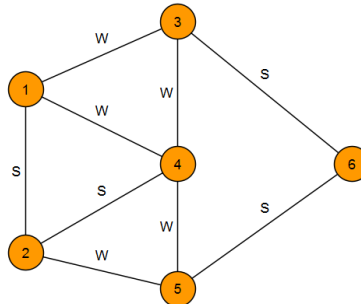
A Cayley tree is a symmetric regular tree in which each node is connected to the same number of  $k$  others, until we get out to the leaves. A Cayley tree with  $k = 3$  looks like this:



Show that the number of nodes which are reachable in exactly  $d$  steps from the central node is  $k(k-1)^{d-1}$ . Further, find an expression for the diameter of the network in terms of  $k$  and the number of nodes  $n$ . Does the Cayley tree display the small-world effect? Explain your answer.

### 4 Weak Ties and Strong Triadic Closure

Consider the following labeled graph, in which the are labeled as weak (W) or strong (S):



Do all nodes in this graph fulfill the strong triadic closure property? Provide an explanation to your answer!