ELEC207 - Instrumentation & Control

Part-A: Instrumentation

Problem Class 3 (Solution)

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Lecture schedule:
Monday 1700-1800 (CHAD-CHAD)
Thursday 1400-1500 (CTH-LTA)

Office Location: Room 513, Electrical Engineering

Outline

- Exam questions, solutions and interactive Qs:
  - Hamming code

ELEC226 Sept 2011
ELEC226 Jan 2012
ELEC207 May 2013
ELEC207 Sept 2013

Hamming code

a) What is a Hamming distance between two words of equal length?
Determine the Hamming distance between 1001110101 and 1101010111.

b) Given a 4 bit word determine the additional number of bits required to correct single bit error using Hamming code.
Derive the Hamming code, arranging the parity bits to locate the position of faulty bit.

Hamming code

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a) What is a Hamming distance between two words of equal length?
Determine the Hamming distance between 1001110101 and 1101010111.

The Hamming distance between two strings (data sets) of equal length is defined as the number of characters (bits) that are **different**. It is calculated by using **XOR** function on 2 sequences.

1 0 0 1 1 1 0 1 0 1
1 1 0 1 0 1 0 1 1 1
---------------------- XOR
0 1 0 0 1 0 0 0 1 0

See slide 21, Lecture 8

Therefore, hamming distance is **3**.

Hamming code

b) Given a 4 bit word determine the additional number of bits required to correct single bit error using Hamming code.
Derive the Hamming code, arranging the parity bits to locate the position of faulty bit.

$$2^r = n + 1$$

$$r = \log_2(n+1)$$

See slide 14, Lecture 8

$$r = \frac{1}{log_{10}(2)}\ log_{10}(n+1) = 3.322 * log_{10}(n+1)$$

Hamming code

b) Given a 4 bit word determine the additional number of bits required to correct single bit error using Hamming code.
Derive the Hamming code, arranging the parity bits to locate the position of faulty bit.

$$r = \frac{1}{log_{10}(2)}\ log_{10}(n+1) = 3.322 * log_{10}(n+1)$$

$$r = 3.322 * log_{10}(4+1)$$

$$r = 2.32$$

Therefore, additional parity bits required are 3.

Hamming (7, 4)

Hamming code (Exercise 1)

Determine the number of additional bits required to correct single bit error using Hamming code for:

1. n = 57
2. n = 26
3. n = 11

$$r = 3.322 * log_{10}(n + 1)$$

Hamming code (Exercise 1)

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Determine the number of additional bits required to correct single bit error using Hamming code for:

1. n = 57

   $r = 3.322 * log_{10}(57 + 1),$ $\boxed{r = 6}$ Hamming (63, 57)

2. n = 26

   $r = 3.322 * log_{10}(26 + 1)$ $\boxed{r = 5}$ Hamming (31, 26)

3. n = 11

   $r = 3.322 * log_{10}(11 + 1)$ $\boxed{r = 4}$ Hamming (15, 11)

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Hamming code

b) Given a 4 bit word determine the additional number of bits required to correct single bit error using Hamming code.
<u>Derive the Hamming code, arranging the parity bits to locate the position of faulty bit.</u>

See slides 15-16, Lecture 8

R_1	R_2	D_1	R_3	D_2	D_3	D_4
1	2	3	4	5	6	7

1. Number the bits starting from 1: bit 1, 2, 3, 4, 5, 6 ….
2. Create the code word, leaving spaces for the parity bits.
3. Parity bits takes up positions in the following order: 1, 2, 4, 8 …..

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Hamming code

R_1	R_2	D_1	R_3	D_2	D_3	D_4
1	2	3	4	5	6	7

- R_1 points to bit positions (1, 3, 5, 7)
  - $R_1 \rightarrow D_1$, D_2 and D_4
- R_2 points to bit positions (2, 3, 6, 7)
  - $R_2 \rightarrow D_1$, D_3 and D_4
- R_3 points to bit positions (4, 5, 6, 7)
  - $R_3 \rightarrow D_2$, D_3 and D_4
- The position of faulty bit is calculated by the sum of incorrect parity bit positions.

Hamming Code (Exercise 2)

A byte of data 10111010 has to be encoded using Hamming code. Using **even parity**, determine the **code word**, including the **interleaved parity bits**.

Hamming Code (Exercise 2)

Create <u>even parity</u> Hamming code for: 10111010 (information)

$$r = 3.322 * log_{10}(8+1), \qquad \boxed{r = 4}$$

- Create the data word, leaving spaces for the parity bits:

  _ _ 1 _ 0 1 1 _ 1 0 1 0
- Calculate the parity for each parity bit (a ? represents the bit position being set):

1. Position 1 checks bits 1,3,5,7,9,11:

   ? _ **1** _ **0** 1 **1** _ **1** 0 **1** 0

   0 _ **1** _ **0** 1 **1** _ **1** 0 **1** 0
2. Position 2 checks bits 2,3,6,7,10,11:

   0 **?** **1** _ 0 **1** **1** _ 1 **0** **1** 0

   0 **0** **1** _ 0 **1** **1** _ 1 **0** **1** 0

Hamming Code (Exercise 2)

2. Position 2 checks bits 2,3,6,7,10,11:

   0 **?** **1** _ 0 **1** **1** _ 1 **0** **1** 0

   0 **0** **1** _ 0 **1** **1** _ 1 **0** **1** 0

3. Position 4 checks bits 4,5,6,7,12:

   0 0 1 **?** **0** **1** **1** _ 1 0 1 **0**

   0 0 1 **0** **0** **1** **1** _ 1 0 1 **0**

4. Position 8 checks bits 8,9,10,11,12:

   0 0 1 0 0 1 1 **?** **1** **0** **1** **0**

   0 0 1 0 0 1 1 **0** **1** **0** **1** **0**

Code word: 001001101010.

Hamming code (Exercise 3)

Determine if the following code words are correct, assuming they were created using **even parity** Hamming Code. If a code word is found to be incorrect, <u>determine the correct code word and extract out the original data.</u>

1. 010101100011
2. 111110001100
3. 011100101110

Hamming Code (Exercise 3)

1. The code word received is 010101100011.

Position 1 checks bits 1,3,5,7,9,11:

0_0_0_1_0_1_ Even parity check, correct!

Position 2 checks bits 2,3,6,7,10,11:

_10_ _11_ _ 01_ Even parity check, correct!

Position 4 checks bits 4,5,6,7,12:

_ _ _1011_ _ _ _1 Even parity check, correct!

Position 8 checks bits 8,9,10,11,12:

_ _ _ _ _ _ _00011. Even parity check, correct!

Received data is correct, original data word is 00110011

Hamming Code (Exercise 3)

2. The code word received is 111110001100.

Position 1 checks bits 1,3,5,7,9,11:

1_1_1_0_1_0_ Even parity check, correct!

Position 2 checks bits 2,3,6,7,10,11:

_11_ _00_ _ 10 _ Even parity check, INCORRECT!

Position 4 checks bits 4,5,6,7,12:

_ _ _1100_ _ _ _0 Even parity check, correct!

Position 8 checks bits 8,9,10,11,12:

_ _ _ _ _ _ _01100. Even parity check, correct!

Hamming Code (Exercise 3)

The code word received is 1**1**1110001100.

Parity bit at position 2 is faulty. Single parity error indicates parity bit error.

Correct code word is 1**0**1110001100.

See slides 12-13, Lecture 8

Original data word is 11001100.

Hamming Code (Exercise 3)

3. The code word received is 011100101110.

Position 1 checks bits 1,3,5,7,9,11:

0_1_0_1_1_1_ Parity check, correct!

Position 2 checks bits 2,3,6,7,10,11:

_11_ _01_ _ 11_ Parity check, INCORRECT!

Position 4 checks bits 4,5,6,7,12:

_ _ _1001_ _ _ _0 Parity check, correct!

Position 8 checks bits 8,9,10,11,12:

_ _ _ _ _ _ _01110. Even parity check, INCORRECT!

Hamming Code (Exercise 3)

Parity bits at position 2 and 8 are faulty. Double parity error indicates data bit error.

Add the positions of two faulty parity bits: 2 + 8 = 10. This means that bit position 10 is faulty.

Received code word is 011100101110.

Correct code word is 011100101010.

Original data word is 10011010.

See slides 13 and 20, Lecture 8

Summary

Topics covered:

- Exam questions:
  - ✓ Hamming code.

Next Lecture:

- Monday, 30th Nov, 2015 (CHAD-CHAD) 1700-1800