ELEC207 - Instrumentation & Control

Part-A: Instrumentation

Problem Class 1

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Lecture schedule:

Monday 1700-1800 (CHAD-CHAD)
Thursday 1400-1500 (CTH-LTA)

Office Location: Room 513, Electrical Engineering

- Feedback on HW # 1
  - See VITAL for HW #2, due on 28th Oct (midnight)
- Exam questions on
  - Transducer specification
    - Random Errors
  - Strain Gauge

Feedback (HW # 1)

a) Define 'gauge factor', G, of a strain gauge and explain why the value for a metallic resistance gauge is normally greater than unity. (6marks)

- Gauge factor is the ratio of fractional change in resistance to applied strain

$$\mathrm{G} = \frac{\frac{\Delta R}{R}}{\mathrm{e}}; \qquad \mathrm{e} = \frac{\Delta \mathrm{L}}{\mathrm{L}}$$

Slide 19, Lecture 2.pdf

Feedback (HW # 1)

a) Define ‘gauge factor’, G, of a strain gauge and <u>explain why the value for a metallic resistance gauge is normally greater than unity.</u> (6marks)

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + (1 + 2v)\frac{\Delta L}{\mathrm{L}}$$

Slides 14-23, Lecture 2

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + (1 + 2v).e$$

$$\frac{{\Delta R}/{R}}{e} = 1 + 2v + \frac{1}{e}\frac{\Delta \rho}{\rho}$$

$$G = 1 + 2v + \frac{1}{e}\frac{\Delta \rho}{\rho}$$

Feedback (HW # 1)

a) Define 'gauge factor', G, of a strain gauge and explain why the value for a metallic resistance gauge is normally greater than unity. (6marks)

$$G = 1 + 2v + \frac{1}{e}\frac{\Delta\rho}{\rho}$$

- v (Poisson's ratio) is typically between 0.25 to 0.4 for most materials (about 0.3 for metals).

- $\frac{1}{e}\frac{\Delta\rho}{\rho}$ (Strain induced change in resistivity, Piezoresistive effect) is about 0.4.

This is why, G is greater than unity (about 2) for most metals.

Feedback (HW # 1)

Q. A half bridge that employs two active strain gauges is shown in Fig. 3. Assuming G, represents the gauge factor, e is applied strain, Vs is the supply voltage.

Derive the following relationship, $V_{out} = \frac{1}{2} V_s. \mathrm{G}. \mathrm{e}$

$$V_{out} = V_B - V_A$$

$$= \frac{R + \Delta R}{(R + \Delta R) + (R - \Delta R)} V_S - \frac{R}{R + R} V_S$$

$$= \left(\frac{R + \Delta R}{2R} - \frac{1}{2} \right) V_S$$

$$= \frac{\Delta R}{2R} V_S$$

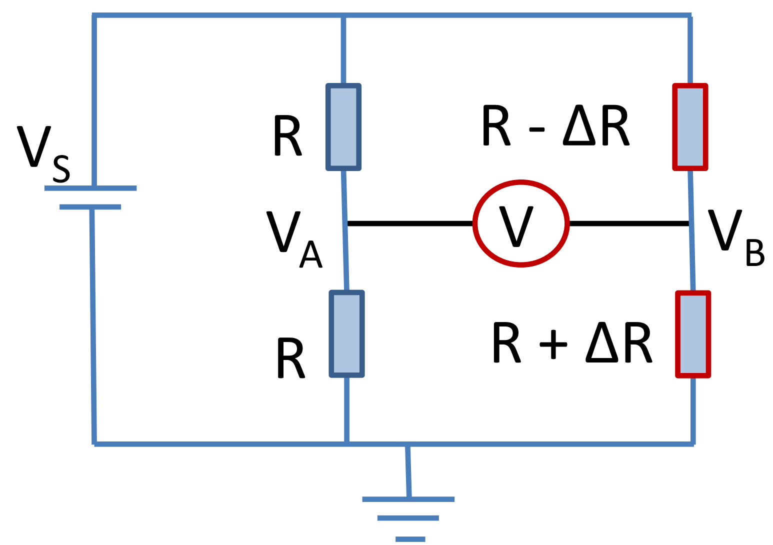


Fig. 3

Feedback (HW # 1)

$$V_{out} = \frac{\Delta R}{2R} V_S$$

$$\frac{\Delta R}{R} = G \cdot e$$

$$= \frac{1}{2} G \cdot e \cdot V_S$$

$$V_{out} = \frac{V_S \cdot G \cdot e}{2}$$

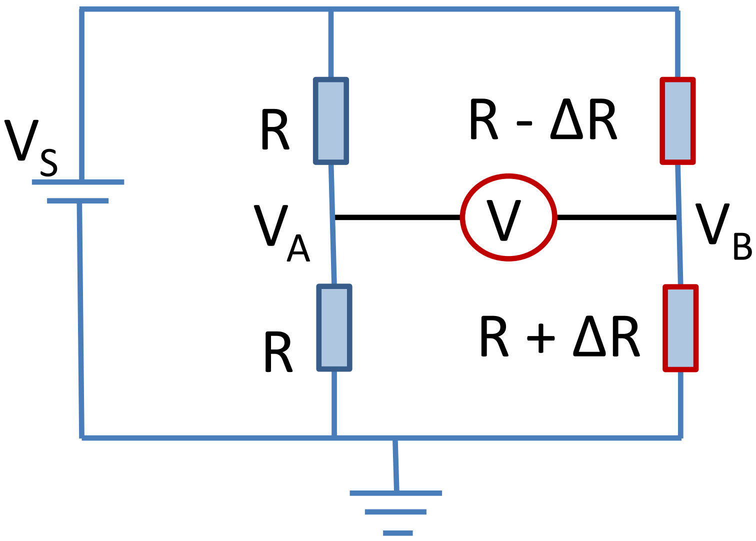


Fig. 3

Define sensitivity, resolution, linearity (Nonlinearity), hysteresis of a transducer, and MEMS (? marks each)

Sensitivity is a measure of the change in the output of an instrument for a change in the measured input variable. (Slides 3-13, Lecture 2)

Resolution is the smallest increment of the measurand, which can be measured by the instrument. (Slides 3-13, Lecture 2)

Nonlinearity is defined as the maximum deviation of any of the output readings from the approximate transfer function. (Slides 3-13, Lecture 2)

ELEC207: 2013, 2014, 2015

Define sensitivity, resolution, linearity (Nonlinearity), hysteresis of a transducer, and MEMS (2 marks each)

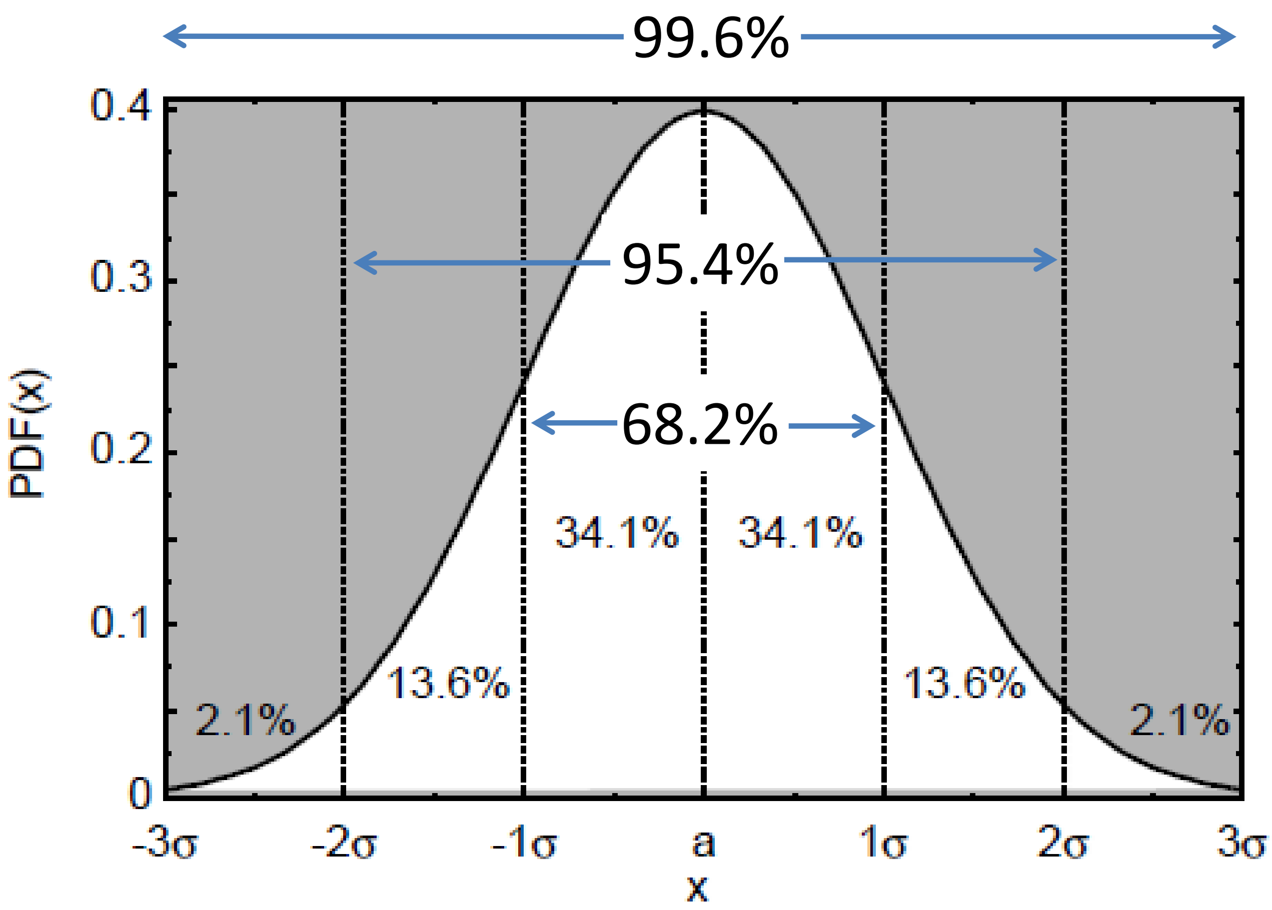
Hysteresis is the deviation of sensor's output at a specified point of the input signal, when this input signal is approached from opposite directions. It is expressed as maximum hysteresis, in % of full scale reading. (Slides 3-13, Lecture 2)

MEMS stands for Micro-Electro-Mechanical Systems. It is the integration of a number of micro-components on a single chip, which allows microsystem to (measure and) control the system. Typical dimensions are in microns (10^{-6} m or µm). (Slides 17-21, Lecture 1)

A constant acceleration is measured by a well calibrated and aligned MEMS (Micro-Electro-Mechanical Systems) accelerometer. The measurement (only affected by random vibrations) produced the following values of acceleration: 4.1 g, 3.8 g, 3.9 g, 4.3 g, 4.0 g,3.9 g, 4.2 g, 3.8 g. Write the formulas for the average acceleration and its standard deviation and calculate their values. What is the confidence interval for 95.4% confidence level? (8 Marks)

- Mean?
- Standard deviation?
- Confidence interval for 95.4% confidence level?

Random error



Random errors have a Gaussian (bell-shaped), also called normal probability density function (PDF).

Random error

For 'N' measurements of x_i , the average value (Mean) is given by:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

The standard deviation (σ) is given by:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Random error

- $\bar{x} \pm 2\sigma$ is used to determine interval for confidence level of 95.4%.

- This means that the actual value of the measurand (which is never known exactly!) is within the interval $(\bar{x} \pm 2\sigma)$ with the probability of 95.4%.

Random error: Mean

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \text{Mean}$$

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i$$

$$\bar{x} = \frac{1}{8}(4.1 + 3.8 + 3.9 + 4.3 + 4.0 + 3.9 + 4.2 + 3.8)$$

$$\boxed{\bar{x} = \frac{1}{8}(32) = 4\ (g)}$$

Random error: Standard Deviation

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

$$\sigma = \sqrt{\frac{1}{8}\sum_{i=1}^{8}(x_i - 4)^2}$$

$$\sigma = \sqrt{\frac{1}{8}\left[(4.1-4)^2 + (3.8-4)^2 + (3.9-4)^2 + (4.3-4)^2 + (4-4)^2 + (3.9-4)^2 + (4.2-4)^2 + (3.8-4)^2\right]}$$

$$\sigma = \sqrt{\frac{1}{8}\left[(0.1)^2 + (0.2)^2 + (0.1)^2 + (0.3-4)^2 + (0)^2 + (0.1)^2 + (0.2)^2 + (0.2)^2\right]}$$

Random error: Standard Deviation

$$\sigma = \sqrt{\frac{1}{8}\left[(0.1)^2 + (0.2)^2 + (0.1)^2 + (0.3)^2 + (0)^2 + (0.1)^2 + (0.2)^2 + (0.2)^2\right]}$$

$$\sigma = \sqrt{\frac{1}{8}\left[0.01 + 0.04 + 0.01 + 0.09 + 0.01 + 0.04 + 0.04\right]}$$

$$\sigma = \sqrt{\frac{1}{8}\left[0.24\right]}$$

$$\boxed{\sigma = 0.1732\ (g)}$$

Random error: Confidence interval

Confidence interval of 95.4% is given by $(\bar{x} \pm 2\sigma)$

$$(\bar{x} \pm 2\sigma) = 4 \pm 2(0.1732)$$

$$(\bar{x} \pm 2\sigma) = 4 \pm 0.3464$$

$$\boxed{\begin{aligned}(\bar{x} \pm 2\sigma) = 3.6535\ (g) \\ 4.3464\ (g)\end{aligned}}$$

Strain measurement

- (b) Four identical strain gauges are available to design a system to measure the strain in a beam (Figure) subject to bending. Sketch a diagram to show how the gauges should be mounted on this beam to provide maximum measurement sensitivity together with temperature compensation. (5 marks)

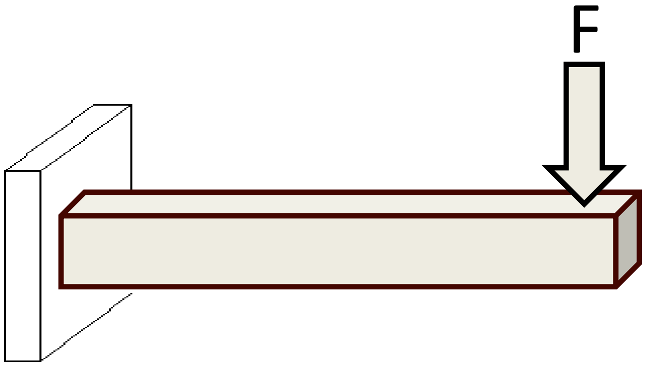
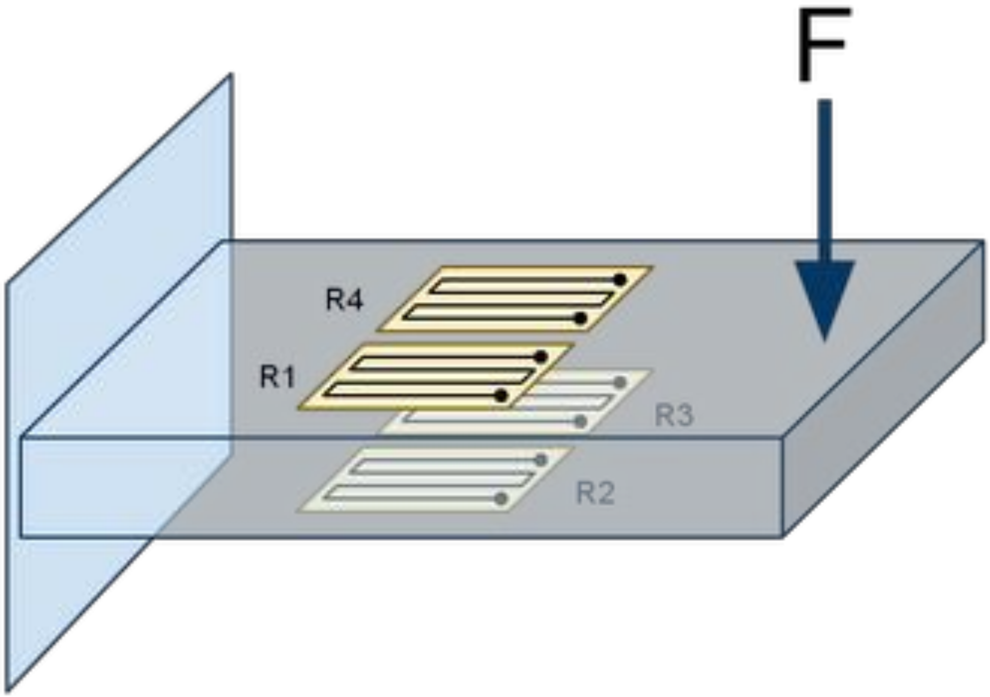


Figure: A beam is subjected to strain, which has to be determined using strain gauges.

Strain measurement



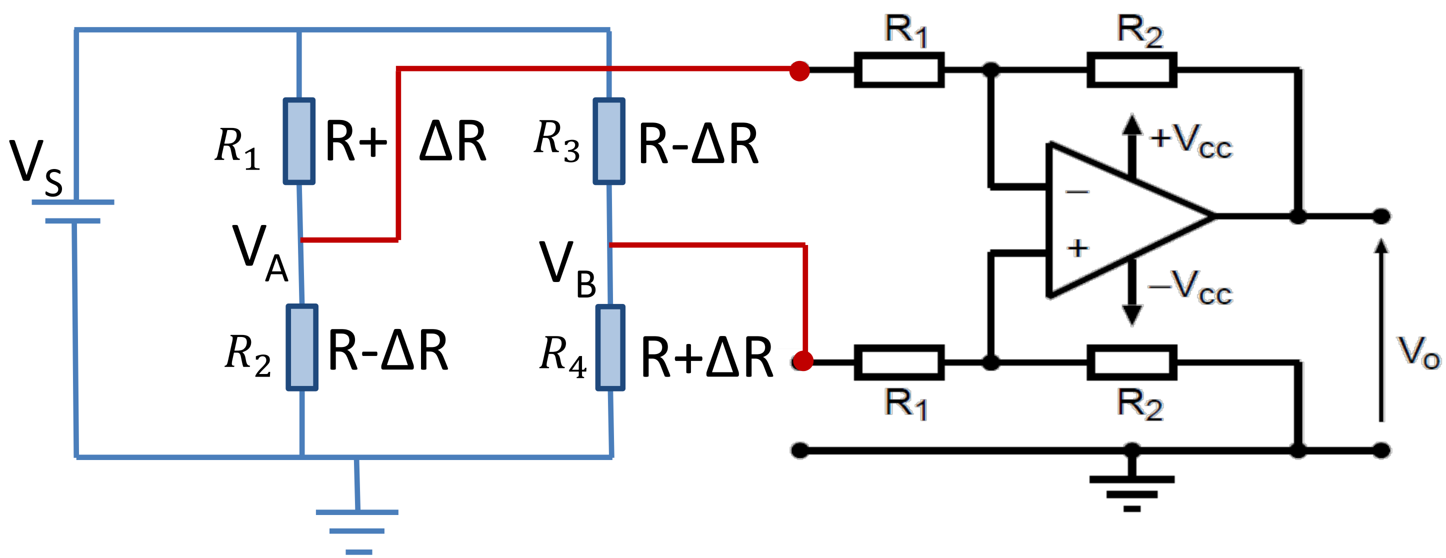
R+ΔR → expansion for R1 and R4

R-ΔR → compression for R2 and R3

Strain measurement

- (c) Design a circuit using these four gauges, a bridge circuit and a difference amplifier. Clearly show how the strain gauges shown in your answer to part (b) should be arranged in the bridge. The circuit diagram should clearly show the relation between the position of each gauge on the beam and its corresponding position in the circuit. (5 marks)

Strain measurement

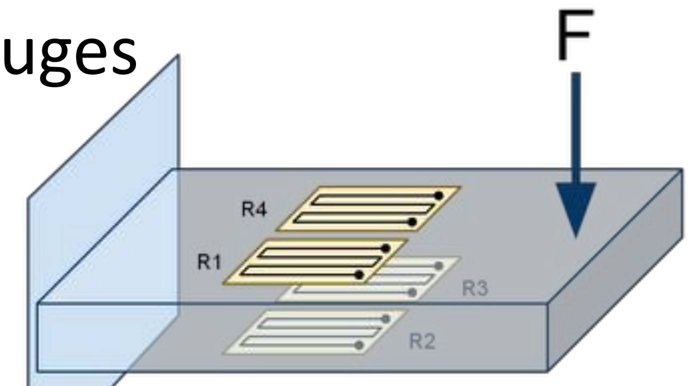


Full bridge

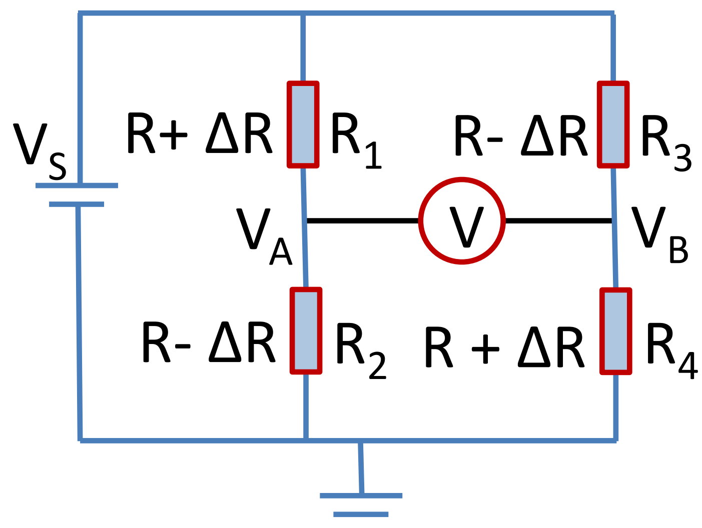
- Full bridge arrangement quadruples (4-times) the output (compared with quarter bridge) by employing four strain gauges.

$$V_{out} = V_s.\mathrm{G.e}$$

- Temperature compensation is achieved through active gauges

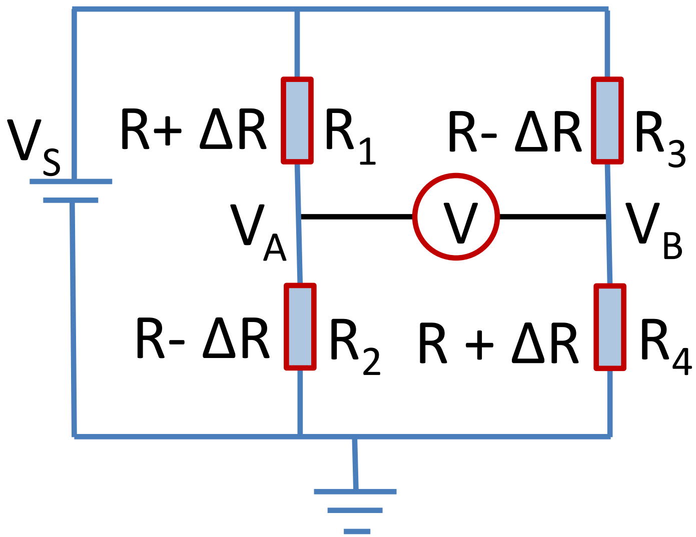


R+ΔR → expansion
R-ΔR → compression



Strain measurement

- (a) A full bridge that employs four active strain gauges (Fig below) is working with operating voltage (Vs) of 10V. The resistances of unloaded gauges are 300 Ω and gauge factor of strain gauges is 2.1. Find V_{AB} (Vout), if the measured strain is 3,000 microstrain.



Strain measurement

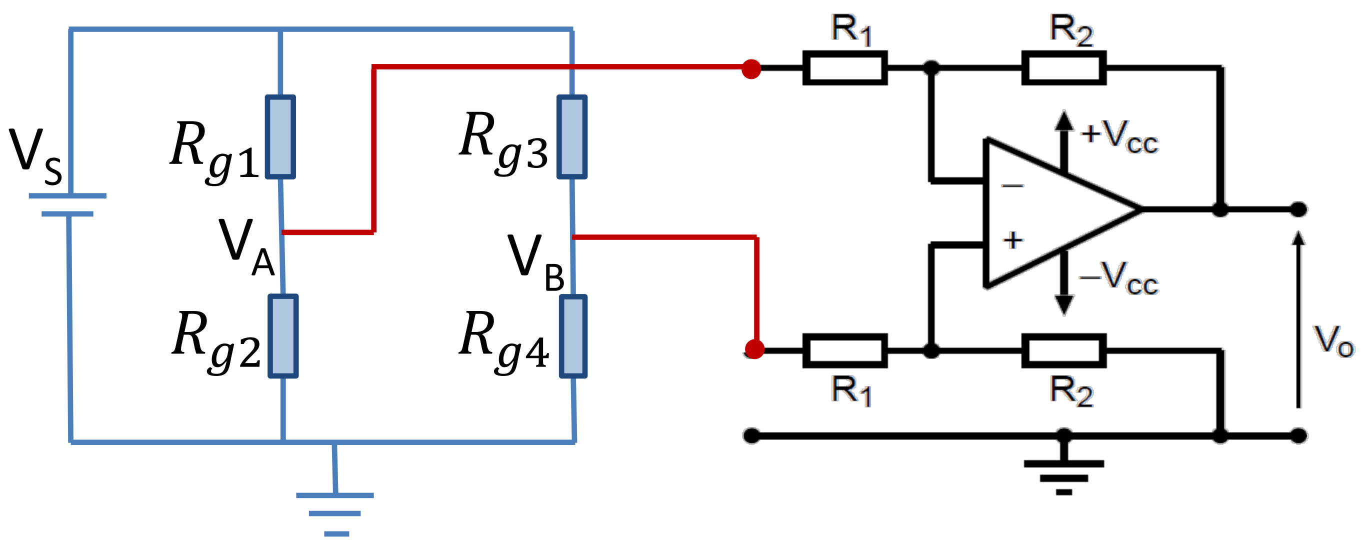
$$V_{out} = V_s.\mathrm{G}.\mathrm{e}$$

$$V_{out} = (10).(2.1).(3000 * 10^{-6})$$

$$\boxed{V_{out} = 63\ mV}$$

Strain measurement

b) The bridge output is amplified to give an output of 5 V. Sketch an amplifier circuit, including the power supply, to show how the amplifier is to be connected to the bridge. Determine the value of the resistors required to give this output. (6marks)



Strain measurement

$$V_o = \frac{R_2}{R_1}(V_B - V_A)$$
$$= \frac{R_2}{R_1}(V_s . G . e)$$

$$5 = \frac{R_2}{R_1}(63 * 10^{-3})$$

$$\boxed{\frac{R_2}{R_1} = 79.365}$$

Assuming, $R_1 = R_g = 300\Omega$

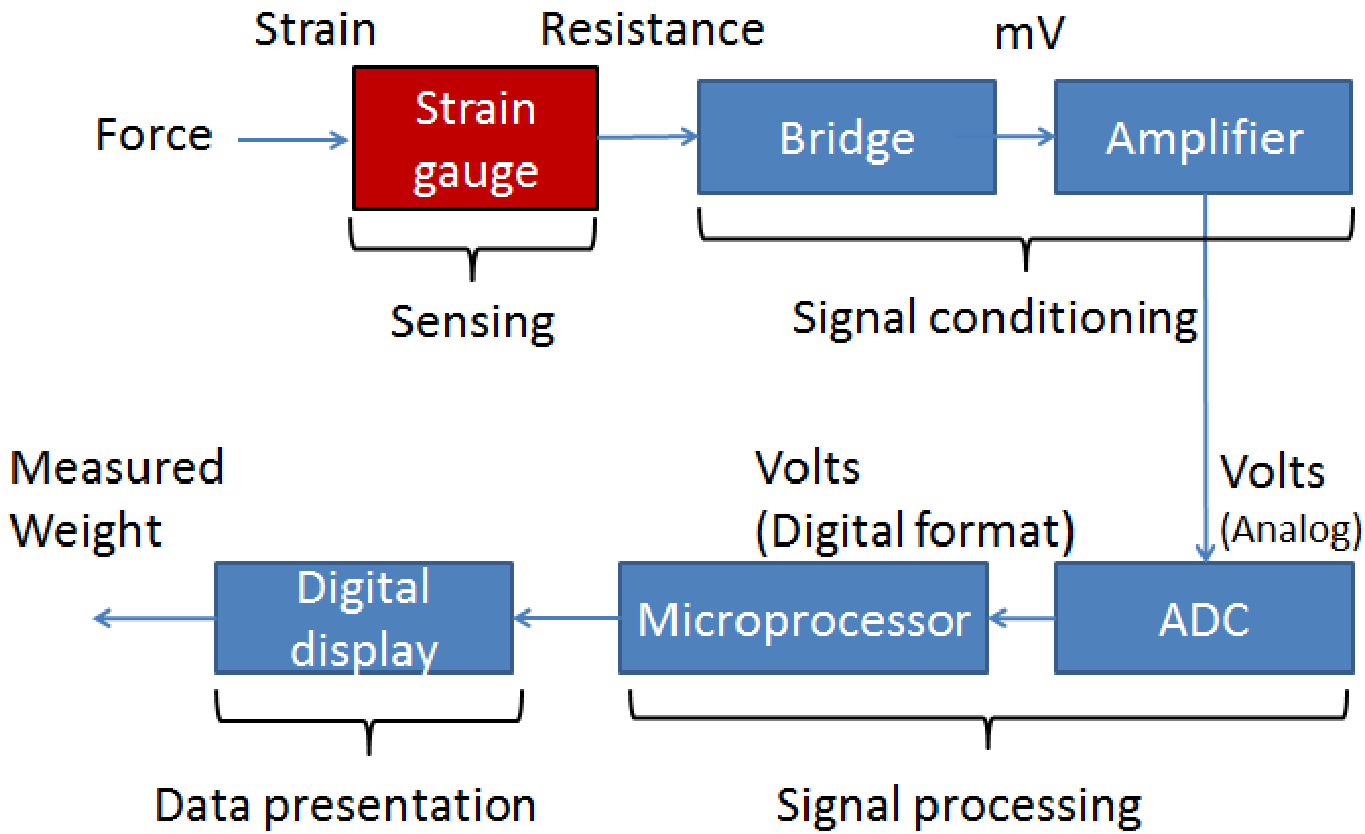
$$R_2 = 300 * 79.365 = 23.81\ K\Omega$$

Strain measurement

Q. With the aid of a block diagram, show how a strain gauge is used in a measurement system. The block diagram should clearly show the sensing element, the required signal conditioning and signal processing elements.

4 marks

Strain measurement



Summary

Topics covered:

- ✓ Feedback on HW-1
- ✓ Exam questions on
  - Transducer specification
    - Random Errors
  - Strain gauge

Next Lecture:

- Monday, 26th Oct, 2015 (CHAD-CHAD), 1700-1800
- Displacement measurement