DPI Homework 3

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3.24

Show that the Laplacian defined in Eq. (3.63) is isotropic (invariant to rotation). You will need the following equations relating coordinates for axis rotation by an angle θ .

$$x = x' cos\theta - y' sin\theta \tag{1}$$

$$y = x' sin\theta + y' cos\theta \tag{2}$$

solution:

拉普拉斯算子是

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{3}$$

我们需要证明

$$\frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{4}$$

证明如下：

$$\frac{\partial f}{\partial x'} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x'} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial x'} = \frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta \tag{5}$$

$$\frac{\partial^2 f}{\partial x'^2} = \frac{\partial^2 f}{\partial x^2} cos^2\theta + \frac{\partial}{\partial x}(\frac{\partial f}{\partial y}) sin\theta cos\theta + \frac{\partial}{\partial y}(\frac{\partial f}{\partial x}) sin\theta cos\theta + \frac{\partial^2 f}{\partial y^2} sin^2\theta \tag{6}$$

同理可得:

$$\frac{\partial f}{\partial y'} = -\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta \tag{7}$$

$$\frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial y^2} sin^2\theta - \frac{\partial}{\partial x}(\frac{\partial f}{\partial y}) sin\theta cos\theta - \frac{\partial}{\partial y}(\frac{\partial f}{\partial x}) sin\theta cos\theta + \frac{\partial^2 f}{\partial y^2} cos^2\theta \tag{8}$$

因此:

$$\frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \tag{9}$$

3.27

Give a mask for performing unsharp masking in a single pass through an image. Assume that the average image is obtained using the filter in Fig. 3.32(a).

solution:

unsharp masking k 为 1，所以可以得到:

$$g(x,y) = f(x,y) + f(x,y) - \bar{f}(x,y) = 2f(x,y) - \bar{f}(x,y) \tag{10}$$

$\bar{f}(x,y)$ 的 mask 是

$$\begin{matrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{matrix}$$

f(x,y) 对应的 mask 是

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix}$$

上面两个 mask 根据公式(10) 进行运算可得 unsharp masking 的 mask 为

$$\begin{matrix} -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \\ -\frac{1}{9} & \frac{17}{9} & -\frac{1}{9} \\ -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \end{matrix}$$