DPI Homework 4

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4.2

Show that $\tilde{F}(\mu)$ in Eq. (4.4-2) is infinitely periodic in both directions, with period $1/\Delta T$

SOLUTION:

先证明：

$$\tilde{F}(\mu + k[1/\Delta T]) = \tilde{F}(\mu)$$

证明如下：

根据公式 4.3-5

$$\begin{aligned}\tilde{F}(\mu + k[1/\Delta T]) &= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} F(\mu + \frac{k}{\Delta T} - \frac{n}{\Delta T}) \\ &= \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty} F(\mu + \frac{k-n}{\Delta T}) \\ &= \frac{1}{\Delta T}\sum_{m=-\infty}^{\infty} F(\mu - \frac{m}{\Delta T}) \\ &= \tilde{F}(\mu)\end{aligned}$$

再证明：

$$\tilde{F}(\mu + k\Delta T) = \tilde{F}(\mu)$$

证明如下：

根据公式 4.4-2

$$\begin{aligned}\tilde{F}(\mu + k\Delta T) &= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi(\mu + k/\Delta T)n\Delta T} \\ &= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T} e^{-j2\pi kn} \\ &= \sum_{n=-\infty}^{\infty} f_n e^{-j2\pi\mu n\Delta T} \\ &= \tilde{F}(\mu)\end{aligned}$$