Context-based Pedestrian Path Prediction

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Abstract

This project focus on intention prediction of pedestrians when they cross the road through Dynamical Bayesian Network. The insight is that the futrue path of a pedestrian is usually relevant to the environment layout around him. Hence his model incorporates situation awareness of pedestrian and the environment layout as latent states to infer the changes of pedestrian dynamic state, which determines the trajectories of this pedestrian. This work involves a particular scenario where a crossing pedestrian might cross or stop at the curb. Experiment shows that we can anticipate the dynamic changes of this pedestrian ahead of event and therefore predicts a more accurate path with this model. All code are publicly available on my github https://github.com/sunyasheng/Bayesian-Statistics-Assignment.

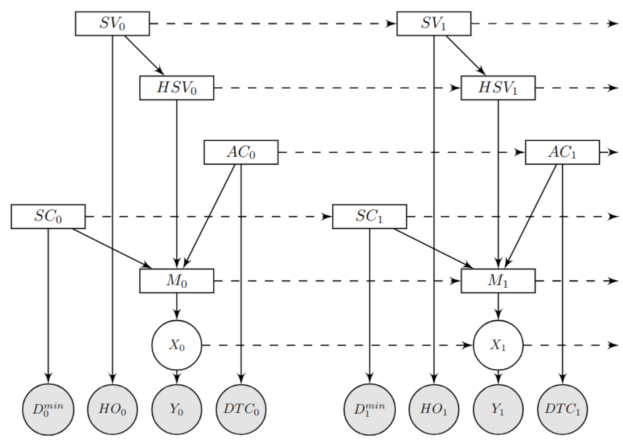

Figure 1. The DBN framework unrolled in two time slices, rectangle/circle/shaded nodes are discret/continous/observed variables.

1. Introduction

Self-driving has made a significant progress in these years while many automated car has traveled on the road for a tested driving. The safety of pedestrians plays an important role in the decision making process of self-driving car. Accurate pedestrian path prediction is a prerequisite for the car to navigate through the crowd without threatening the safety of pedestrians. Accurate trajectory prediction is a challenging problem, since the dynamics of a pedestiran highly variable.

An insight in this work is that the environment layout surrounding him will affect his intention, which might be reflected in his following action. We assume that the pedestrian decision is to large extent affected by three key parts: the existence of an approaching vehicle on collision course, the pedestrian's awareness and the spatial layout of environment. Therefore, the Dynamic Bayesian Network(DBN) is proposed to integrate these parts into an unified framework. The DBN captures these three factors as latent states working on top of a Switching Linear Dynamic System(SLDS), hence controlling the dynamics of the pedestiran.

Bayesian Network models the causal relationship by a directed graph, which can be utilized to describe the cause and affect of pedestrian action or the environment context in a probabilistic fashion. For instance, the pedestrian is less likely to cross the road if the car drives with a high speed in this critical situation. Or the pedestrian tends to cross if he does not even notice the driving car. Those relationships can be modeled by a conditional probability which is presented as an edge in the Bayesian Network. Situation criticality is estimated by the expected minimum distance of the pedestrian and the vechicle at the closest approach in future. Situation awareness represents whether or not this pedestrian has seen the vehicle up to now, which is estimated through his head orientation of past time. Spatial layout is described by the distance to the curbside. The observables(see the shaded node in Fig. 1) i.e. expected distance at closest approach, pedestrian position, head orientation, curb location, are measured by the external sensors.

Since the state of pedestrian changes evolving with time slices, the Dynamic Bayesian Network(DBN) is utilized here to reflect the state evolution with timestamps. All the parameters in DBN are learned by the annotated data in a supervised learning fashion.

2. Methodology

Our goal is to devise a model that can successfully anticipate the dynamic of the pedestrian through the context information. The dynamic is the switching state in SLDS, which indicates the basical motion model used at each time instance. We argue that the decision of pedestrian to stop at the curb or cross the street is highly influenced by his awareness of situation, his position with respect to the curbside and the existence of an approaching vehicle on collision course. Hence, the DBN is proposed to capture these three factors as latent state which affects the switching state in Switching Linear Dynamic System(SLDS) as shown in Figure 1.

2.1. Graphical Model

The proposed DBN is shown in Fig. 1. Those variables can be split into two sets: one set related to SLDS that consists of switching state M, latent position state X and associated observation Y while another one relating to the scene context information including spatial layout, situation criticality and the pedestrian's awareness($Z = \{SV, HSV, SC, AC\}$), which influences the switching state M in SLDS and associated observables $E = \{HO, D^{min}, DTC\}$.

SLDS. The SLDS consists of a continous hidden state X_t, a discret switching state M_t and the observation of the state Y_t with observation noise $N(0, R)$. Two motion types are involved at any time instances in this secenario, including walking($M_t = m_w$) and standing($M_t = m_s$). The velocity in standing motion type is zero while the velocity in walking motion type is v_t. A person's lateral position at time t is denoted by x_t. Formally, the motion dynamics is illustrated as

$$x_t = x_{t-1} + v_t + \epsilon$$
$$v_t = \begin{cases} 0 & M_t = m_s \\ v^{m_w} & m_t = m_w \end{cases}$$

Here we assume zero-mean process noise $\epsilon \sim N(0, Q)$ and also the fixed time interval $\Delta t = 1$. The velocity can also be included in this dynamic system such that the state can be described as $X_t = [x_t, v_t^{m_w}]^T$.

$$X_t = A^{(M_t)} X_{t-1} + \begin{bmatrix} \epsilon \\ 0 \end{bmatrix} \quad \epsilon_t \sim N(0, Q)$$
$$Y_t = CX_t + \mu_t \quad \mu_t \sim N(0, R)$$

where the switching state determines the linear state transformation matrix $A^{(m)}$,

$$A^{m_s} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{m_w} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

The lateral position $Y_t \in R$ is the observation with the observation matrix C = [1 0]. Therefore, the conditional probability distributions of this graphical model can be described as $P(X_t|X_{t-1}, M_t) = N(X_t|A^{(M_t)}X_{t-1}, Q)$ and $P(Y_t|X_t) = N(Y_t|CX_t, R)$.

Context. The transition probability of the switching state M_t in SLDS is conditioned on the current context information SC_t, HSV_t, AC_t and previous switching state M_{t-1}. Formally,

$$P(M_t|M_{t-1}, SC_t, HSV_t, AC_t)$$

Essentially, this is a switching state transition table parameterized by SC_t, HSV_t, AC_t. This temporal transition of context state SC, HSV, AC can be factorized to

$$\begin{aligned} &P(SC_t, HSV_t, AC_t|SC_{t-1}, HSV_{t-1}, AC_{t-1}) = \\ &\quad P(HSV_t|HSV_{t-1}, SV_t) \times P(SV_t|SV_{t-1}) \\ &\quad \times P(SC_t|SC_{t-1}) \times P(AC_t|AC_{t-1}) \end{aligned}$$

where the lalent variable SV indicates whether the pedestiran sees the vehicles at current time step. HSV indicates whether the pedestrian is aware that the vehicle approaches. HSV should be true if the pedestrian sees the vechicle for some $t' \leq t$.

$$P(HSV_t|HSV_{t-1}, SV_t) = \begin{cases} 1 & HSV_t = (HSV_{t-1} V SV_t) \\ 0 & otherwise \end{cases}$$

The transition probability of HSV_t can be defined simply the logical OR between the Boolean HSV_{t-1} and SV_t variables.

The latent variable SC indicates the criticality of the situation if both the pedestrian and the vehicle continue approaching with their current velocities. AC indicates whether the pedestrian is currently on a position near the curbside since the curbside is usually where he chooses to wait or delay his crossing behavior. Similarly, the SV, SC and AC varaibles also depend on their previous value at last time instance, which is designed for the smoothness of those latent variables.

The observations obtained by sensors serve as evidence for the latent context variables SV, SC, AC. The observation probability distribution can also be factorized to

$$P(HO_t|SV_t) \times P(D_t^{min}|SC_t) \times P(DTC|AC_t)$$

The head orientation observation variable HO_t provides evidence for the latent SV variable. HO_t is a vector which captures the classifier responses in certain looking direction. Multiple classifiers has been trained to identify the looking direction of the pedestrian through the image of his head. The values conform to different distributions across classes conditioned on whether the pedestiran sees the vechicle

SV. The HO_t here is hence modeled as a Multinomial distribution conditioned SV_t with p_{sv} as parameter vector.

$$P(HO_t|SV_t = sv) = Mult(HO_t|p_{sv})$$

The expected minimum distance between the pedestiran and vehicle is taken as the evidence to estimate the situation criticality SC, if they continue approaching with their current velocities. It seems that the assumption is navie, but it provides the information to measure how critical this situation is. The gamma distribution is used here to describe D^{min} conditioned on SC with the shape parameter a and scale parameter b.

$$P(D_t^{min}|SC_t = sc) = \Gamma(D_t^{min}|a_{sc}, b_{sc})$$

The At-Curb variable AC is measured by the distance between the lateral position of the pedestrian and the position of the curb ridge. The position of curb is obtained by the image processing procedure. And the position of the pedestiran is the filtered expected position $E[x_t]$ from the Kalman Filter. Therefore, the Distance-to-Curb, DTC, is computed as the difference between the filtered position of Kalman and and the curb position. Also it is worthy to mention that the DTC can be estimated even at future time instances using the predicted position. The distribution of DTC conditioned on AC is modeled as a Normal distribution,

$$P(DTC_t|AC_t = ac) = N(DTC_t|\mu_{ac}, \sigma_{ac})$$

2.2. Inference

The DBN model works in a forward fashion to predict and filter the state when it receives the measurement from sensors at each time step. Exact inference in this model is intractable since the exact posterior distribution will have $|M|^T$ modes after T time steps. Therefore here we use the Expectation Propogation [4] technique to do approximate inference, where the posterior distribution is approximated by a simple distribution through minimizing the KL divergence of the approximated and exact posterior distribution. Hence DBN consists of three main procedure for each time instance: predict, update and collapse.

A prediction for time t is denoted by $\bar{P}_t(\cdot) = P(\cdot|O_{1:t-1})$ before receiving the observation O_t, and updated estimate after observation receiving is denoted by $\hat{P}_t(\cdot) = P(\cdot|O_{1:t})$. $\tilde{P}(\cdot)$ represents the approximated updated estimate which collapsed at each time instance t and will be passed to the predict module as input at time step $t+1$.

Predict The predicted posterior distribution$\tilde{P}_{t-1}(\cdot)$ after collapse serves as input of this time instance, including the joint distribution over latent motion and context nodes $\tilde{P}_{t-1}(X_{t-1}, Z_{t-1})$ and the conditional Normal distribution $\tilde{P}_{t-1}(X_{t-1}|M_{t-1}) = N(X_{t-1}|\tilde{\mu}_{t-1}^{(M_{t-1})}, \Sigma_{t-1}^{\tilde{M}_{t-1}})$. Here we use Z_t to represent HSV_t, SV_t, SC_t, AC_t. The joint probability of latent variables in previous and current time steps is computed based on the transition probability we trained.

$$\bar{P}_t(M_t, M_{t-1}, Z_t, Z_{t-1}) = P(M_t|M_{t-1}, Z_t)P(Z_t|Z_{t-1})\tilde{P}_{t-1}(M_{t-1}, Z_{t-1})$$

For continous state X_t we condition on all the possible models M_t to predict the effect of models M_t. Formally,

$$\bar{P}_t(X_t|M_t, M_{t-1}) = \int P(X_t|X_{t-1}, M_t) \times \tilde{P}_{t-1}(X_{t-1}|M_{t-1})dX_{t-1}$$

Update The observation is incorporated in the update step to obtain the joint posterior. The likelihood term of observation follows the Kalman Filtering setting.

$$\begin{aligned} P(Y_t|M_t, M_{t-1}) &= \int P(Y_t|X_t) \times \bar{P}(X_t|M_t, M_{t-1})dX_t \\ &= N(Y_t|C\bar{\mu}^{(M_t, M_{t-1})}, \bar{\Sigma}_t^{(M_t, M_{t-1})} + R) \end{aligned}$$

And the updated X_t also follows the standard Kalman Filtering setting conditioned on $|M|^2$ transition conditions.

$$\hat{P}_t(X_t|M_t, M_{t-1}) \propto P(Y_t|X_t) \times \bar{P}(X_t|M_t, M_{t-1})$$

The joint posterior of latent discret nodes are given by

$$\hat{P}_t(M_t, M_{t-1}, Z_t, Z_{t-1}) \propto P(Y_t|M_t, M_{t-1}) P(E_t|Z_t)\bar{P}_t(M_t, M_{t-1}, Z_t, Z_{t-1})$$

Here the observation E_t represents the $\{DTC_t, HO_t, D_t^{min}\}$

Collapse To make this model tractable, the state of previous state need to be maraginalized out and only the varaibles at current time step are kept, which will be carried over to the next prediction step.

$$\tilde{P}_t(M_t, Z_t) = \sum_{M_{t-1}} \sum_{Z_{t-1}} \hat{P}_t(M_t, M_{t-1}, Z_t, Z_{t-1})$$

The continous variable X_t is also approximated by marginalizing the variable from previous time step.

$$\begin{aligned} \tilde{P}_t(X_t|M_t) &= \sum_{M_{t-1}} \hat{P}_t(X_t|M_t, M_{t-1}) \times P(M_{t-1}|M) \\ &= N(X_t|\tilde{\mu}_t^{(M_t)}, \tilde{\Sigma}_t^{(M_t)}) \end{aligned}$$

The parameters $(\tilde{\mu}_t^{(M_t)}, \tilde{\Sigma}_t^{(M_t)})$ are calculated through minimizing the KL divergence between exact positerior and approximate posterior [4] under the constraint that the approximate posterior distribution is Normal Distribution. The $P(M_{t-1}|M_t)$ is obtained by marginalizing and normalizing $\hat{P}_t(M_t, M_{t-1}, Z_t, Z_{t-1})$

Context	Sequence Number
Not SC, SV, Crossing	14
Not SC, not SV, Crossing	9
SC, not SV, Crossing	11
SC,SV,Stopping	14
SC,SV,Crossing	10

Table 1. The detail of dataset statistics.

3. Experiment

3.1. Dataset Description

The dataset comes from a publicly available dataset [1, 2] which is annotated by Daimler Company for self-driving research.

This dataset [2] includes 58 sequences recorded by a stereo camera involving single pedestrian with intention to cross the road, which features different situation criticalities(critical, vs. non-critical), pedestrian behavior(crossing vs. stopping at the curb) and pedestrian situational awareness(seen vehicle vs. not seen vehicle). The detail of this dataset statistics is shown in Table. 1. There are five subscenarios in this dataset, four of which represent normal pedestrian behaviros while the last sub-secenario is anomalous since the pedestrian choose to cross even though he is aware of the critical situation. For framework synchronization sequences are labeled with time-to-event(TTE) values. TTE is defined to be 0 when the pedestrians step on the ground at the curbside. Frames before/after TTE = 0 have negative/positive values.

Position ground truth(GT) comes from manual labeling of the pedestrian bounding boxes and the median disparity of the upper pedestiran body area is calculated. A mean gait cycle consumes 17.3 frames(1.0s) with 1.6 frames(0.1s) standard deviation from the analysis of the trajectories.

The positional observation Y_t is estimated by the resulting bounding boxes extracted from HOG/linSVM detector and has compensated the vehicle ego-motion. The detector takes as input region of interests provided by the obstacle detection module using dense stereo data. Then the bounding box is extracted by the detector and the median disparity is calculated to estimate the position of the pedestiran.

The angle of observed head orientation HO_t is discretized to eight orientation classes of $0°, 45°, \cdots, 315°$. The detector for each class is trained, i.e. $f_0, \cdots, f_{315}$, so the response of the detector $f_o(I_t)$ serves the evidence that the region of interest contains the head orientation of orientation class o. In more detail, every detector is a neural network trained in a $one - vs - rest$ fashion. For detection, regions of interest are proposed from disparity based image segmentation and the most likely head image region $I^\star$ is selected to rescale to $16 \times 16px$ before classification. The output of classifier is the response strength $HO_t = [f_0(I_t^\star), \cdots, f_{315}(I_t^\star)]$ over different discretized head orientation.

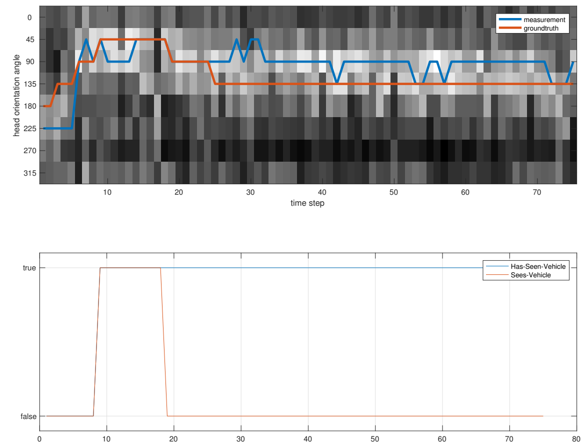

Figure 2. The head measurement and groundtruth of a crossing pedestrian.

Fig. 2 shows a sequence where the pedestrian crosses the road when the situation is not critical. From the figure we can see that the pedestrian sees the approaching vechicle($SV = True$) when his head angle is in the range of $\pm 45°$ at about $t = 10$ hence the $HSV = True$ since that time.

The expected minimum distance D^{min} between vehicle and pedestrian is calculated at each time step based on the current position and velocity. Vehicle speed is obtained through on-borad sensors and pedestrian velocity is approximated by the first order of derivative over the last 10 frames. For distance to curb(DTC), the curbside is detected by Hough transform. Y_t^{curb} is then estimated by the mean lateral position of detected line.

The ground truth GT switching state $M_t = m_s$ when TTE $>= 0$, and $M_t = m_w$ at all other moments for the stopping sequences. For the crossing sequences, $M_t = m_w$ at all time steps.

For head obaservation HO, we assume that the pedestrian is aware of($SV = True$) an approaching vehicle if the GT head orientation is in a range of $\pm 45°$ centered angle $0°$(the direction of head pointing towards the camera) and does not see($SV = False$) the vechicle for angles outside this range. The distribution $\Gamma(D^{min}|a_{sc}, b_{sc})$ of observation D^{min} is estimated over situation critical SC labels, which is set once for all time instances per sequence. The distribution $N(DTC_t|\mu_{ac}, \sigma_{ac})$ is estimated from GT curb positions and GT pedestrian positions over at curb AC labels, which are set to AC_t = true if $-1 \leq TTE \leq 1$ in crossing sequences or $TTE \geq -1$ in stopping sequences.

3.2. Training Details

In this section, we introduce training details of the conditional probability distribution computation. Two kinds of probability distribution is required to be estimated, including the transition probability between adjacent time slices and the emission probability at the same time instance.

Transition Probability

$$P(SV_t|SV_{t-1}) = \frac{\sharp(SV_t, SV_{t-1})}{\sharp(SV_{t-1})}$$

$$P(SC_t|SC_{t-1}) = \frac{\sharp(SC_t, SC_{t-1})}{\sharp(SC_{t-1})}$$

$$P(AC_t|SC_{t-1}) = \frac{\sharp(AC_t, AC_{t-1})}{\sharp(AC_{t-1})}$$

$$P(M_t|M_{t-1}, SC_t, HSV_t, AC_t) = \frac{\sharp(M_t, M_{t-1}, SC_t, HSV_t, AC_t)}{\sharp(M_{t-1}, SC_t, HSV_t, AC_t)}$$

Those conditional probability distribution is approximated by the fraction of existence frequency of the joint variables over marginal variables. Note that the SC labels are only set once in real dataset, so we fix the SC situation probability to 1/100 for the changing state.

Emission Probability

$$P(HO_t|SV_t) = Multi(HO_t|p_{sv})$$

The Head-Orientation HO_t serves as evidence for Sees-Vehicle(SV_t). HO_t is a vector which is the response over different classes of orientation. Hence HO_t is modeled as a sample from a Multinomial distribution conditioned on SV_t, with parameter vector p_{sv}. The parameter vector p_{sv} is estimated by the average of responses over different classes.

$$P(D_t^{min}|SC_t = sc) = \Gamma(D_t^{min}|a_{sc}, b_{sc})$$

Situation-Critical SC is infered by the minimum distance D^{min} between the pedestrian and vehicle under the constant velocity assumption, which indicates how critical the situation is and therefore lead to more accurate path prediction. The parameter is estimated by maximum likelihood estimation following [3]. Here we show the minimum distance D^{min} distribution over the Situation-Critical(SC) variable after MLE in Figure. 3. The figure implies that usually the expected minimum distance is small under the critical situation, which conforms to our intuition.

$$P(DTC_t|AC_t) = N(DTC_t|\mu_{ac}, \sigma_{ac})$$

For At-Curb (AC_t), we consider the Distance-To-Curb DTC_t. The expected lateral position of the pedestrian is obtained by the Kalman Filter in switching Linear Dynamic System while the location measure of the curb ridge is also filtered but by the constant position Kalman filter with zero mean process noise. The distribution is modeled as a Normal distribution with the mean μ and variance σ needing to be learned. After MLE, the distribution is shown in Figure 4. The mean of distance shows no difference in at-curb or not-at-curb situation, both located near 0. But the variance is smaller in at-curb situation compared with the not-at-curb situation.

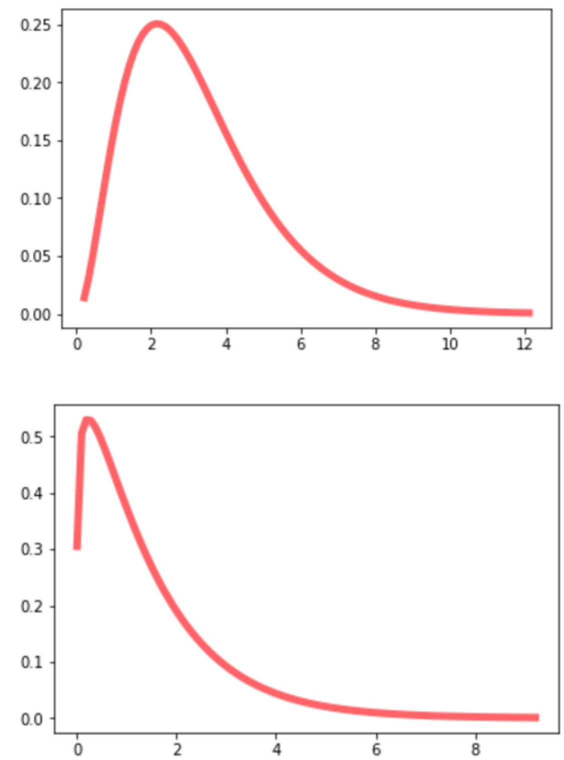

Figure 3. Probability distribution of minimum distance between pedestrian and vehicle. Upper figure is $\Gamma(D_t^{min}|a_0, b_0)$ and Lower $\Gamma(D_t^{min}|a_1, b_1)$.

3.3. Evaluation

Fig. 5 illustrates a sequence from the stopping scenario in this fourth row of Table.1 where this pedestrian stops when he become aware of the critical situation. This pedestrian has not seen the vechicle and realize the critical situation at TTE = -18(t=13), hence the DBN model predicts that he will continuing walking. It is reasonable that this model anticipate that the pedestrian will continue walking at this time step since there is no enough evidence showing the likelihood of stopping so far. But at TTE = -4(t=27), the strong evidence of having seen the vechicle, at curb and situation criticality is shown infered by the obaservation hence the

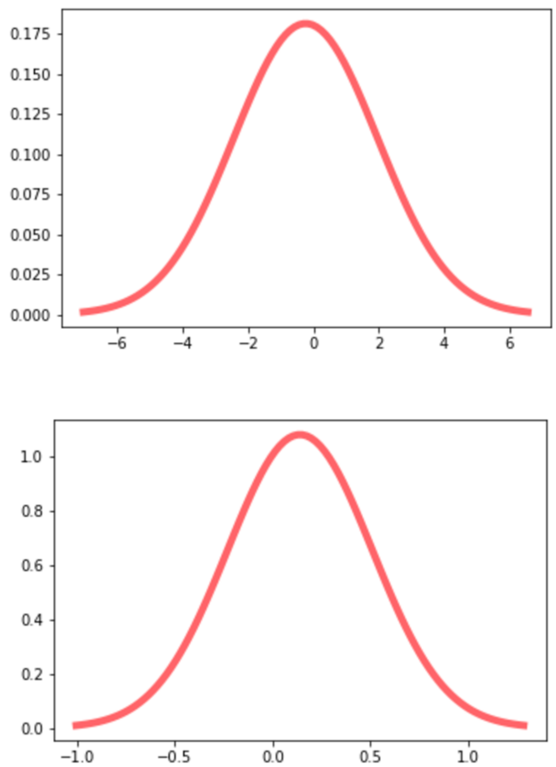

Figure 4. Probability distribution of distance between curb and pedestrian position. Upper figure is $N(DTC|\mu_0, \sigma_0)$ and Lower $N(DTC|\mu_1, \sigma_1)$.

model predicts that the pedestrian is very likely to stop. So the predicted path is very close to groundtruth benefitting from the correctly estimated motion m_s state. The DBN enables more accurate and resonable path prediction using the information from the context.

4. Summary

A novel model is proposed for pedestrian path prediction, which takes the context surrounding an ego-pedestrian into consideration. This DBN based model predicts the pedestrian path through SLDS with a latent state controlling the dynamics of pedestrian, which is determined by the situation awareness, environmental spatial layout and situation criticality. Experiment shows that the presented model can predict the intention or dynamic of a pedestrian ahead of time, thus we can predict the pedestrian path more accurately.

This model can scale to more complicated scenario. Future work will involve incorporation of of other secene context(e.g. interaction between pedestrians) and more flexible dynamics of pedestrians(e.g. acceration) instead of constant velocity assumption.

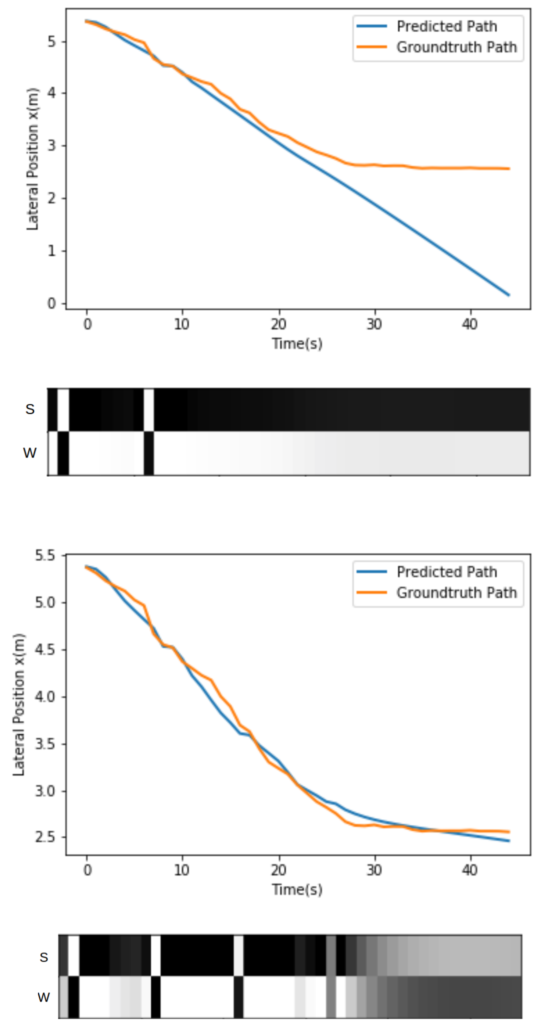

Figure 5. The predicted lateral position and the groundtruth lateral position. Upper figure shows the result predicted at TTE = -18(t=13) and lower figure shows the result predicted at TTE = -4(t=27). The bar below represents the probability of switching state in SLDS. The brighter the color of bar is, the higher probability it has.

References

[1] Kooij, Julian Francisco Pieter, et al. Context-Based Pedestrian Path Prediction. Computer Vision ECCV 2014. Springer International Publishing, 2014:618-633. 4

[2] http://www.lookingatpeople.com/download-daimler-ped-predict-2014/index.html 4

[3] http://tminka.github.io/papers/minka-gamma.pdf 5

[4] Minka, and P. Thomas. ”Expectation Propagation for approximate Bayesian inference.” 7.7(2013):362-369. 3

[5] https://en.wikipedia.org/wiki/Metropolis-Hastings_algorithm 7

[6] https://sunyasheng.github.io/ 7

5. Appendix

In this section, we show another strategy MCMC(Markov Chain Mento Carlo) to calculate the posterior probability distribution of expected minimum distance(D_t^{min}) between pedestrian and vechicle. The new particle is obtained through random walk in traditional Metropolis-Hastings algorithm [5]. Since the dimension of varaibles is not independent and not isotropic, the random walk may behave poorly [6]. Here Adaptive Metropolis algorithm is introduced to illustrate the covariance of the variables for a better proposed paritcle. Suppose that the probability distribution of the proposed state is Normal distribution.

$$p(x^\star \mid x_n) = N(x_n, \sigma^2 C_t)$$

The covariance can be illustrated as

$$Cov(X_0, X_1, ..., X_t) = \frac{\sum_{i=1}^{t-1}(x_i x_i^T - ((t-1)+1)\bar{x}_{t-1}\bar{x}_{t-1}^T)}{t}$$

where

$$\bar{x}_{t-1} = \frac{\sum_{i=0}^{t-1} x_i}{t}$$

The C_t can be updated at every time t if we write the formula recursively.

$$C_{t+1} = \frac{(t-1)C_t}{t} + \frac{t\bar{x}_{t-1}\bar{x}_{t-1}^T - (t+1)\bar{x}_t\bar{x}_t^T + x_t x_t^T}{t}$$

Here the likelihood function is

$$P(D_t^{min}|SC_t = sc) = \Gamma(D_t^{min}|a_{sc}, b_{sc})$$

It is conditioned on the Situation-Critical SC. There are two parameters in Gamma distribution a_{sc} and b_{sc}. a_{sc} is the location parameter and b_{sc} is scale parameter. And we assume the prior probability of a_{sc} and b_{sc} are positive numbers, which is subject to uniform distribution. Formally,

$$a_{sc} \sim U(0, \infty)$$
$$b_{sc} \sim U(0, \infty)$$

The posterior distribution of a_{sc} and b_{sc} after Adaptive Metropolis algorithm is shown in Figure. 6.

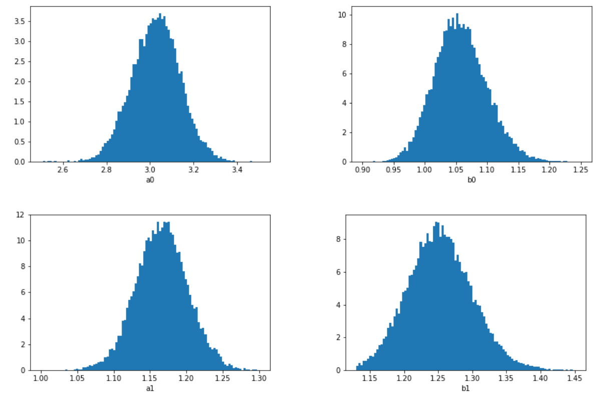

Figure 6. Posterior probability distribution of a_{sc} and b_{sc}.

Typically, the mean is ab and the mode is $(a-1)b$ in Gamma density. The mode is 2.371 when $sc = 0$ and is 0.348 when $sc = 1$, which implies that expected distance D_t^{min} between pedestrian and vechicle is closer when the situation is critical. And this posterior probability distribution is close to the distribution obtained by the Maximum Likelihood Estimation(MLE). For more implementation detail, please check my github https://github.com/sunyasheng/Bayesian-Statistics-Assignment.