

PS3 -Ying Sun

January 30, 2019

1 Problem Set 3

1.1 Ying Sun

5.1

If the individual lives for one period, the optimal problem is:

$$\max_{W_2 \in [0, W_1]} u(W_1 - W_2)$$

The optimal decision is characterized by:

$$W_2 = 0$$

5.2

The condition that characterizes the optimal amount of cake to leave for the next period W_3 in period 2 is:

$$\max_{W_3 \in [0, W_2]} u(W_2 - W_3)$$

The condition that characterizes the optimal amount of cake to leave for the next period W_2 in period 1 is:

$$\max_{W_2 \in [0, W_1]} [u(W_1 - W_2) + \max_{W_3 \in [0, W_2]} \beta u(W_2 - W_3)]$$

The optimal decision is characterized by:

$$W_3 = 0$$

$$-u'(W_1 - W_2) + \beta u'(W_2 - W_3) = 0$$

5.3

If the individual lives for three period, the conditions that characterizes the optimal amount of cake to leave for the next period in period W_2 :

$$\max_{W_2 \in [0, W_1]} \{u(W_1 - W_2) + \max_{W_3 \in [0, W_2]} \beta [u(W_2 - W_3) + \max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4)]\}$$

in period W_3 :

$$\max_{W_3 \in [0, W_2]} \beta [u(W_2 - W_3) + \max_{W_4 \in [0, W_3]} \beta u(W_3 - W_4)]$$

in period W_4 :

$$\max_{W_4 \in [0, W_3]} u(W_3 - W_4)$$

The optimal decision is characterized by:

$$W_4 = 0$$

$$-u'(W_1 - W_2) + \beta u'(W_2 - W_3) = 0$$

$$-\beta u'(W_2 - W_3) + \beta^2 u'(W_3 - W_4) = 0$$

Assume that $u(x) = \ln(x)$, $W_1 = 1$ and $W_4 = 0$ Then we can solve the equations to get the following consequences:

$$W_2 = 0.631$$

$$W_3 = 0.299$$

$$c_1 = W_1 - W_2 = 0.369$$

$$c_2 = W_2 - W_3 = 0.332$$

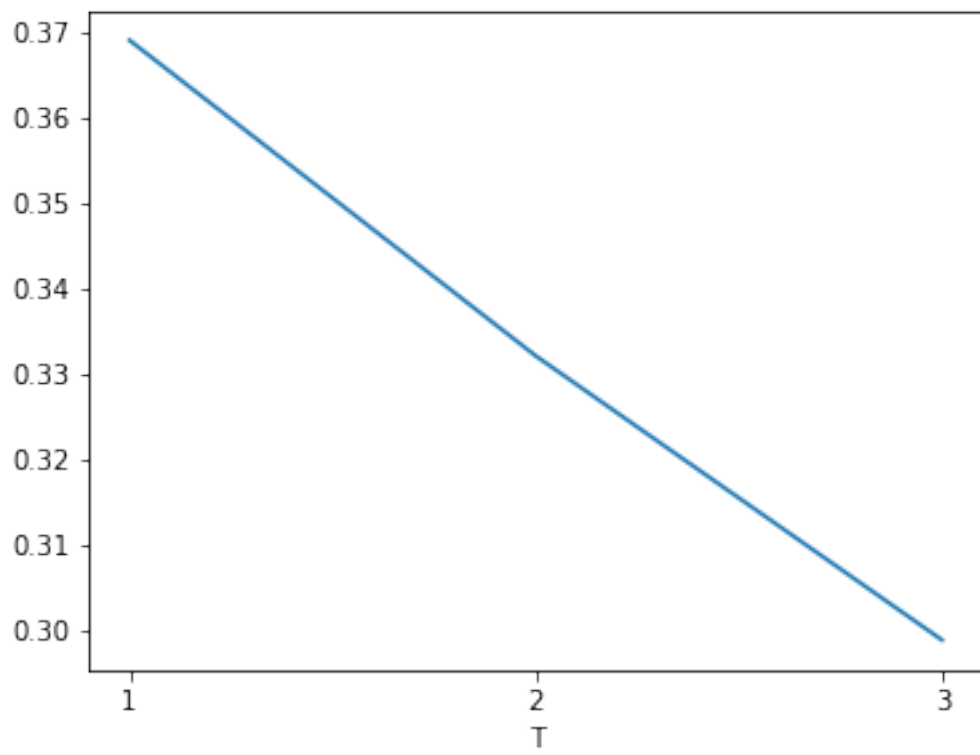
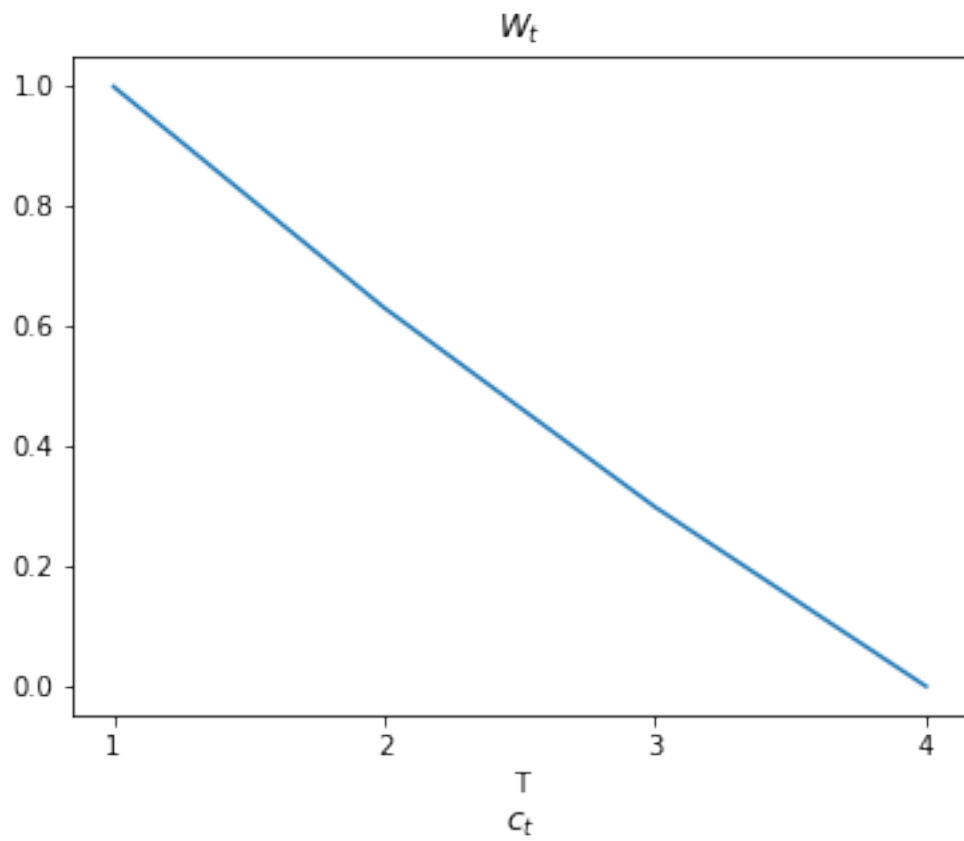
$$c_3 = W_3 - W_4 = 0.299$$

The evolution of $\{c_t\}_{t=1}^3$ and $\{W_t\}_{t=1}^4$ is as follows:

In [2]: `import matplotlib.pyplot as plt`

```
W = [1, 1-1/(1+0.9+0.81), 1-1.9/(1+0.9+0.81), 0]
c = [1/(1+0.9+0.81), 0.9/(1+0.9+0.81), 0.81/(1+0.9+0.81)]
T = [1,2,3,4]
fig, ax=plt.subplots(2,1,figsize=(6,10))
ax[0].plot(T, W)
ax[1].plot(T[:-1], c)
ax[0].set_title(r"$W_t$")
ax[0].set_xlabel('T')
ax[0].set_xticks([1,2,3,4])
ax[1].set_title(r"$c_t$")
ax[1].set_xlabel('T')
ax[1].set_xticks([1,2,3])
```

Out[2]: [`<matplotlib.axis.XTick at 0x1574a16e278>`,
`<matplotlib.axis.XTick at 0x1574a167b70>`,
`<matplotlib.axis.XTick at 0x1574a167a20>`]



5.4

The condition that characterizes the optimal choice in period T-1 is:

$$-u'(W_{T-1} - \psi_{T-1}(W_{T-1})) + \beta u'(\psi_{T-1}(W_{T-1})) = 0$$

Then we can express V_{T-1} as follows:

$$V_{T-1}(W_{T-1}) = u(W_{T-1} - \psi_{T-1}(W_{T-1})) + \beta u(\psi_{T-1}(W_{T-1}))$$

5.5

According to 5.4,

$$\psi_T(\bar{W}) = W_{T+1} = 0$$

$$\psi_{T-1}(\bar{W}) = W_T \neq 0$$

then we get

$$\psi_T \neq \psi_{T-1}$$

$$V_T(\bar{W}) = \ln(\bar{W})$$

$$V_{T-1}(\bar{W}) = \ln\left(\frac{\bar{W}}{1+\beta}\right) + \beta \ln\left(\frac{\beta \bar{W}}{1+\beta}\right)$$

So

$$V_T(\bar{W}) \neq V_{T-1}(\bar{W})$$

5.6

The finite horizon Bellman equation for the value function at time T-2 is:

$$V_{T-2}(W_{T-2}) = \max_{W_{T-1}} \ln(W_{T-2} - W_{T-1}) + \beta \ln\left(\frac{W_{T-1}}{1+\beta}\right) + \beta^2 \ln\left(\frac{\beta W_{T-1}}{1+\beta}\right)$$

The condition that characterizes the optimal choice in period T-2 is:

$$-\frac{1}{(W_{T-2} - \psi_{T-2}(W_{T-2}))} + (\beta + \beta^2) \frac{1}{\psi_{T-2}(W_{T-2})} = 0$$

The analytical solution for $\psi_{T-2}(W_{T-2})$ and $V_{T-2}(W_{T-2})$ is:

$$\psi_{T-2}(W_{T-2}) = \frac{\beta + \beta^2}{1 + \beta + \beta^2} W_{T-2}$$

$$V_{T-2}(W_{T-2}) = \ln\left(\frac{W_{T-2}}{1 + \beta + \beta^2}\right) + \beta \ln\left(\frac{\beta W_{T-2}}{1 + \beta + \beta^2}\right) + \beta^2 \ln\left(\frac{\beta^2 W_{T-2}}{1 + \beta + \beta^2}\right)$$

5.7

Based on mathematical induction, the analytical solution for $\psi_{T-s}(W_{T-s})$ and $V_{T-s}(W_{T-s})$ is:

$$\psi_{T-s}(W_{T-s}) = \frac{\sum_{i=1}^s \beta^i}{1 + \sum_{i=1}^s \beta^i} W_{T-s}$$

$$V_{T-s}(W_{T-s}) = \left[\sum_{i=0}^s \beta^i \ln \left(\frac{\beta^i W_{T-s}}{1 + \sum_{i=1}^s \beta^i} \right) \right]$$

When the horizon becomes infinite:

$$\psi(W_{T-s}) = \beta W_{T-s}$$

$$V(W_{T-s}) = \left(\frac{1}{1-\beta} \right) \ln((1-\beta)W_{T-s}) + \frac{\beta}{(1-\beta)^2} \ln(\beta)$$

5.8

$$V(W) = \max_{w \in [0, W]} u(W - w) + \beta V(w)$$

5.9

In [3]: `import numpy as np`

```
W_lb = 1e-2
W_ub = 1.0
N = 100
W_vec = np.linspace(W_lb, W_ub, N)
W_vec
```

Out[3]: array([0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1 , 0.11, 0.12, 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, 0.19, 0.2 , 0.21, 0.22, 0.23, 0.24, 0.25, 0.26, 0.27, 0.28, 0.29, 0.3 , 0.31, 0.32, 0.33, 0.34, 0.35, 0.36, 0.37, 0.38, 0.39, 0.4 , 0.41, 0.42, 0.43, 0.44, 0.45, 0.46, 0.47, 0.48, 0.49, 0.5 , 0.51, 0.52, 0.53, 0.54, 0.55, 0.56, 0.57, 0.58, 0.59, 0.6 , 0.61, 0.62, 0.63, 0.64, 0.65, 0.66, 0.67, 0.68, 0.69, 0.7 , 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77, 0.78, 0.79, 0.8 , 0.81, 0.82, 0.83, 0.84, 0.85, 0.86, 0.87, 0.88, 0.89, 0.9 , 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99, 1.])

5.10

```
In [4]: beta = 0.9
def utility(c):
    util = np.log(c)
    return util
V_init = np.zeros_like(W_vec)
W_prime = np.zeros_like(W_vec)
u_vec = utility(W_vec-W_prime)
V_T = (u_vec + beta * V_init)
print("The policy function is:\n", W_prime)
print("The value function is:\n", V_T)
```

[illegible]

```
[ -4.60517019 -3.91202301 -3.5065579 -3.21887582 -2.99573227 -2.81341072
-2.65926004 -2.52572864 -2.40794561 -2.30258509 -2.20727491 -2.12026354
-2.04022083 -1.96611286 -1.89711998 -1.83258146 -1.77195684 -1.71479843
-1.66073121 -1.60943791 -1.56064775 -1.51412773 -1.46967597 -1.42711636
-1.38629436 -1.34707365 -1.30933332 -1.27296568 -1.23787436 -1.2039728
-1.17118298 -1.13943428 -1.10866262 -1.07880966 -1.04982212 -1.02165125
-0.99425227 -0.96758403 -0.94160854 -0.91629073 -0.89159812 -0.86750057
-0.84397007 -0.82098055 -0.7985077 -0.77652879 -0.75502258 -0.73396918
-0.71334989 -0.69314718 -0.67334455 -0.65392647 -0.63487827 -0.61618614
-0.597837 -0.5798185 -0.56211892 -0.54472718 -0.52763274 -0.51082562
-0.49429632 -0.4780358 -0.46203546 -0.4462871 -0.43078292 -0.41551544
-0.40047757 -0.38566248 -0.37106368 -0.35667494 -0.34249031 -0.32850407
-0.31471074 -0.30110509 -0.28768207 -0.27443685 -0.26136476 -0.24846136
-0.23572233 -0.22314355 -0.21072103 -0.19845094 -0.18632958 -0.17435339
-0.16251893 -0.15082289 -0.13926207 -0.12783337 -0.11653382 -0.10536052
-0.09431068 -0.08338161 -0.07257069 -0.0618754 -0.05129329 -0.04082199
-0.03045921 -0.02020271 -0.01005034 0. ]
```

```
In [5]: def distance(V_new, V_init):
        distance_metric = ((V_new-V_init)**2).sum()
        return distance_metric

delta1 = distance(V_T, V_init)
print("The distance metric of T is:", delta1)
```

5.12

6

```

W_index = np.argmax(u_mat + beta*V_prime, axis=1)
W_T = W_vec[W_index]
print("The policy function is:\n", W_T)
print("The value function is:\n", V_Tminus1)

```

The policy function is:

```

[0.01 0.01 0.01 0.02 0.02 0.03 0.03 0.04 0.04 0.05 0.05 0.06 0.06 0.07
 0.07 0.08 0.08 0.09 0.09 0.09 0.1  0.1  0.11 0.11 0.12 0.12 0.13 0.13
 0.14 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.18 0.18 0.19 0.19 0.2
 0.2  0.21 0.21 0.22 0.22 0.23 0.23 0.24 0.24 0.25 0.25 0.26 0.26 0.27
 0.27 0.27 0.28 0.28 0.29 0.29 0.3  0.3  0.31 0.31 0.32 0.32 0.33 0.33
 0.34 0.34 0.35 0.35 0.36 0.36 0.36 0.37 0.37 0.38 0.38 0.39 0.39 0.4
 0.4  0.41 0.41 0.42 0.42 0.43 0.43 0.44 0.44 0.45 0.45 0.45 0.46 0.46
 0.47 0.47]

```

The value function is:

```

[-8.10161181e+04 -8.74982335e+00 -8.05667617e+00 -7.43284371e+00
 -7.02737860e+00 -6.66246000e+00 -6.37477793e+00 -6.11586407e+00
 -5.89272052e+00 -5.69189132e+00 -5.50956976e+00 -5.34548036e+00
 -5.19132968e+00 -5.05259407e+00 -4.91906268e+00 -4.79888442e+00
 -4.68110139e+00 -4.57509666e+00 -4.46973614e+00 -4.37442596e+00
 -4.27960150e+00 -4.19259012e+00 -4.10681096e+00 -4.02676825e+00
 -3.94845801e+00 -3.87435004e+00 -3.80231160e+00 -3.73331873e+00
 -3.66662156e+00 -3.60208303e+00 -3.53998945e+00 -3.47936483e+00
 -3.42128016e+00 -3.36412175e+00 -3.30955959e+00 -3.25549236e+00
 -3.20404979e+00 -3.15275650e+00 -3.10396633e+00 -3.05530583e+00
 -3.00878582e+00 -2.96262185e+00 -2.91817009e+00 -2.87425894e+00
 -2.83169933e+00 -2.78983132e+00 -2.74900932e+00 -2.70900273e+00
 -2.66978202e+00 -2.63147837e+00 -2.59373804e+00 -2.55699824e+00
 -2.52063060e+00 -2.48533196e+00 -2.45024064e+00 -2.41627434e+00
 -2.38237279e+00 -2.34958297e+00 -2.31685209e+00 -2.28510339e+00
 -2.25352120e+00 -2.22274954e+00 -2.19223815e+00 -2.16238519e+00
 -2.13287434e+00 -2.10388681e+00 -2.07531298e+00 -2.04714210e+00
 -2.01944761e+00 -1.99204864e+00 -1.96518097e+00 -1.93851272e+00
 -1.91242394e+00 -1.88644845e+00 -1.86109466e+00 -1.83577685e+00
 -1.81108424e+00 -1.78642517e+00 -1.76232761e+00 -1.73832619e+00
 -1.71479569e+00 -1.69141776e+00 -1.66842824e+00 -1.64564221e+00
 -1.62316935e+00 -1.60094600e+00 -1.57896710e+00 -1.55727930e+00
 -1.53577310e+00 -1.51459565e+00 -1.49354224e+00 -1.47285167e+00
 -1.45223238e+00 -1.43200681e+00 -1.41180411e+00 -1.39200148e+00
 -1.37222046e+00 -1.35280238e+00 -1.33344679e+00 -1.31439860e+00]

```

```

In [7]: delta2 = distance(V_Tminus1, V_T)
print("The distance metric of T-1 is:", delta2)
diff1 = delta2-delta1
print("The difference between delta_{T-1} and delta_T is: ", diff1)

```

The distance metric of T-1 is: 6562865744.5285635

The difference between δ_{T-1} and δ_T is: 6562865565.602453

5.13

```
In [8]: V_prime = np.tile(V_Tminus1.reshape((1,N)), (N,1))
        V_prime[~c_pos] = -9e+4
        V_Tminus2 = (u_mat + beta * V_prime).max(axis = 1)
        W_index = np.argmax(u_mat + beta*V_prime, axis=1)
        W_Tminus1 = W_vec[W_index]
        print("The policy function is:\n", W_Tminus1)
        print("The value function is:\n", V_Tminus2)
```

The policy function is:

```
[0.01 0.01 0.02 0.02 0.03 0.04 0.04 0.05 0.06 0.06 0.07 0.08 0.08 0.09
 0.09 0.1  0.11 0.11 0.12 0.13 0.13 0.14 0.15 0.15 0.16 0.17 0.17 0.18
 0.18 0.19 0.19 0.2  0.21 0.21 0.22 0.23 0.23 0.24 0.25 0.25 0.26 0.27
 0.27 0.28 0.28 0.29 0.3  0.3  0.31 0.32 0.32 0.33 0.34 0.34 0.35 0.35
 0.36 0.36 0.37 0.38 0.38 0.39 0.4  0.4  0.41 0.42 0.42 0.43 0.44 0.44
 0.45 0.45 0.46 0.47 0.47 0.48 0.49 0.49 0.5  0.51 0.51 0.52 0.52 0.53
 0.54 0.54 0.55 0.55 0.56 0.57 0.57 0.58 0.59 0.59 0.6  0.61 0.61 0.62
 0.63 0.63]
```

The value function is:

```
[-8.10161181e+04 -7.29191115e+04 -1.24800112e+01 -1.17868640e+01
 -1.11630316e+01 -1.06015823e+01 -1.01961172e+01 -9.83119864e+00
 -9.50277190e+00 -9.21508983e+00 -8.95617596e+00 -8.72315349e+00
 -8.50000993e+00 -8.29918074e+00 -8.11685918e+00 -7.93611290e+00
 -7.77202350e+00 -7.61787282e+00 -7.47019236e+00 -7.33145675e+00
 -7.19792536e+00 -7.07306331e+00 -6.95288505e+00 -6.83510202e+00
 -6.72694159e+00 -6.62093686e+00 -6.51557634e+00 -6.42017208e+00
 -6.32486190e+00 -6.23003744e+00 -6.14302606e+00 -6.05724690e+00
 -5.97190488e+00 -5.89186218e+00 -5.81355194e+00 -5.73635069e+00
 -5.66224272e+00 -5.59020428e+00 -5.51972507e+00 -5.45073219e+00
 -5.38403502e+00 -5.31920043e+00 -5.25466191e+00 -5.19256832e+00
 -5.13194370e+00 -5.07191624e+00 -5.01383157e+00 -4.95667316e+00
 -4.90078893e+00 -4.84622677e+00 -4.79215955e+00 -4.73988335e+00
 -4.68844078e+00 -4.63714748e+00 -4.58804154e+00 -4.53925138e+00
 -4.49059088e+00 -4.44407086e+00 -4.39777255e+00 -4.35160858e+00
 -4.30715682e+00 -4.26324567e+00 -4.21945122e+00 -4.17689161e+00
 -4.13502359e+00 -4.09347602e+00 -4.05265403e+00 -4.01264744e+00
 -3.97312741e+00 -3.93390670e+00 -3.89560304e+00 -3.85786272e+00
 -3.82018150e+00 -3.78344171e+00 -3.74707406e+00 -3.71106814e+00
 -3.67576949e+00 -3.64067817e+00 -3.60620489e+00 -3.57223859e+00
 -3.53833704e+00 -3.50527122e+00 -3.47248140e+00 -3.43975052e+00
 -3.40798174e+00 -3.37623305e+00 -3.34465086e+00 -3.31387920e+00
 -3.28330953e+00 -3.25279814e+00 -3.22294517e+00 -3.19343433e+00
 -3.16397654e+00 -3.13498901e+00 -3.10641518e+00 -3.07799121e+00
 -3.04982033e+00 -3.02212584e+00 -2.99466558e+00 -2.96726661e+00]
```



```
In [9]: delta3 = distance(V_Tminus2, V_Tminus1)
print("The distance metric of T-2 is:", delta3)
diff2 = delta3-delta2
print("The difference between delta_{T-2} and delta_{T-1} is: ", diff2)
diff3 = abs(delta3-delta1)
print("The difference between delta_{T-2} and delta_{T} is: ", diff3)
```

The distance metric of T-2 is: 5315921432.356884

The difference between delta_{T-2} and delta_{T-1} is: -1246944312.1716795

The difference between delta_{T-2} and delta_{T} is: 5315921253.430774

5.14

```
In [10]: maxiters = 500
toler = 1e-9
dist = 10.0
VF_iter = 0
W_prime = W_vec
V_init = np.zeros_like(W_vec)
while dist >= toler and VF_iter < maxiters:
    VF_iter += 1
    W = W_prime
    V_prime = np.tile(V_init.reshape((1,N)), (N,1))
    V_prime[~c_pos] = -9e+4
    V_new = (u_mat + beta * V_prime).max(axis = 1)
    W_index = np.argmax(u_mat + beta*V_prime, axis=1)
    W_prime = W_vec[W_index]
    dist = ((V_new - V_init)**2).sum()
    V_init = V_new
    print("Iteration=", VF_iter, ", distance=", dist)

print("Converged.")
print("psi(W) is", W_prime)
print("V(W) is", V_init)
print("After {} iterations, V(W) converged.".format(VF_iter))
```

```
Iteration= 1 ,distance= 6563611570.214573
Iteration= 2 ,distance= 5316525743.271798
Iteration= 3 ,distance= 4306386030.006323
Iteration= 4 ,distance= 3488172794.5714226
Iteration= 5 ,distance= 2825420037.630621
Iteration= 6 ,distance= 2288590282.5590844
Iteration= 7 ,distance= 1853758166.5375955
Iteration= 8 ,distance= 1501544142.7426846
Iteration= 9 ,distance= 1216250776.642259
Iteration= 10 ,distance= 985163145.1419044
Iteration= 11 ,distance= 797982160.0373642
Iteration= 12 ,distance= 646365559.4266962
```

Iteration= 13 ,distance= 523556110.9935629
Iteration= 14 ,distance= 424080456.20615387
Iteration= 15 ,distance= 343505174.74340343
Iteration= 16 ,distance= 278239195.8852569
Iteration= 17 ,distance= 225373752.31221965
Iteration= 18 ,distance= 182552742.55996224
Iteration= 19 ,distance= 147867724.27005062
Iteration= 20 ,distance= 119772859.10622115
Iteration= 21 ,distance= 97016018.0385953
Iteration= 22 ,distance= 78582976.63315375
Iteration= 23 ,distance= 63652212.98545916
Iteration= 24 ,distance= 51558294.32776982
Iteration= 25 ,distance= 41762220.1200653
Iteration= 26 ,distance= 33827399.92370578
Iteration= 27 ,distance= 27400195.48398699
Iteration= 28 ,distance= 22194159.813998673
Iteration= 29 ,distance= 17977270.853317253
Iteration= 30 ,distance= 14561590.732883928
Iteration= 31 ,distance= 11794889.778709196
Iteration= 32 ,distance= 9553861.95285677
Iteration= 33 ,distance= 7738629.36668709
Iteration= 34 ,distance= 6268290.928397754
Iteration= 35 ,distance= 5077316.754233369
Iteration= 36 ,distance= 4112627.637429302
Iteration= 37 ,distance= 3331229.420618224
Iteration= 38 ,distance= 2698296.8357309587
Iteration= 39 ,distance= 2185621.415306089
Iteration= 40 ,distance= 1770354.30067605
Iteration= 41 ,distance= 1433987.914944458
Iteration= 42 ,distance= 1161531.1223381404
Iteration= 43 ,distance= 940841.1015289264
Iteration= 44 ,distance= 762082.1677779292
Iteration= 45 ,distance= 617287.4156153646
Iteration= 46 ,distance= 500003.6522140527
Iteration= 47 ,distance= 405003.7907942261
Iteration= 48 ,distance= 328053.89114983805
Iteration= 49 ,distance= 265724.4605370663
Iteration= 50 ,distance= 215237.6113282474
Iteration= 51 ,distance= 174343.25337503717
Iteration= 52 ,distance= 141218.81426884216
Iteration= 53 ,distance= 114388.00935499243
Iteration= 54 ,distance= 92655.04917523317
Iteration= 55 ,distance= 75051.34382720977
Iteration= 56 ,distance= 60792.33427200552
Iteration= 57 ,distance= 49242.52936902187
Iteration= 58 ,distance= 39887.180071882685
Iteration= 59 ,distance= 32309.340384184605
Iteration= 60 ,distance= 26171.282961856

```

Iteration= 61 ,distance= 21199.44996270106
Iteration= 62 ,distance= 17172.25765403326
Iteration= 63 ,distance= 13910.225445246653
Iteration= 64 ,distance= 11267.97247896265
Iteration= 65 ,distance= 9127.741149256373
Iteration= 66 ,distance= 7394.146525049693
Iteration= 67 ,distance= 5989.928390551506
Iteration= 68 ,distance= 4852.503735570291
Iteration= 69 ,distance= 3931.1818695174757
Iteration= 70 ,distance= 3184.904097831992
Iteration= 71 ,distance= 2580.410948027704
Iteration= 72 ,distance= 2090.7641764207274
Iteration= 73 ,distance= 1694.1410904514291
Iteration= 74 ,distance= 1372.8672014517142
Iteration= 75 ,distance= 1112.627119310548
Iteration= 76 ,distance= 901.8230243640993
Iteration= 77 ,distance= 731.0600844858621
Iteration= 78 ,distance= 592.7310334744678
Iteration= 79 ,distance= 480.6745763286175
Iteration= 80 ,distance= 389.89717447236984
Iteration= 81 ,distance= 316.3533259689117
Iteration= 82 ,distance= 256.76931150496404
Iteration= 83 ,distance= 208.4941502491576
Iteration= 84 ,distance= 169.37141215479707
Iteration= 85 ,distance= 137.665420034222
Iteration= 86 ,distance= 111.96869169941148
Iteration= 87 ,distance= 91.12990180160644
Iteration= 88 ,distance= 74.23008056632061
Iteration= 89 ,distance= 60.52291228596271
Iteration= 90 ,distance= 49.38428817377354
Iteration= 91 ,distance= 40.24315102906038
Iteration= 92 ,distance= 32.126682064026454
Iteration= 93 ,distance= 25.541799163125216
Iteration= 94 ,distance= 19.767115537050348
Iteration= 95 ,distance= 15.155025726039097
Iteration= 96 ,distance= 11.174263919022518
Iteration= 97 ,distance= 8.02000552946439
Iteration= 98 ,distance= 5.341082016591021
Iteration= 99 ,distance= 3.2347254868217576
Iteration= 100 ,distance= 1.4634529907462
Iteration= 101 ,distance= 0.0

```

Converged.

```

psi(W) is [0.01 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1 0.11 0.12 0.13
0.14 0.15 0.16 0.17 0.18 0.19 0.2 0.21 0.22 0.23 0.24 0.25 0.26 0.27
0.28 0.29 0.3 0.31 0.32 0.33 0.34 0.35 0.36 0.37 0.38 0.39 0.4 0.41
0.42 0.43 0.44 0.45 0.46 0.47 0.48 0.49 0.5 0.51 0.52 0.53 0.54 0.55
0.56 0.57 0.58 0.59 0.6 0.61 0.62 0.63 0.64 0.65 0.66 0.67 0.68 0.69
0.7 0.71 0.72 0.73 0.74 0.75 0.76 0.77 0.78 0.79 0.8 0.81 0.82 0.83

```

```
0.84 0.85 0.86 0.87 0.88 0.89 0.89 0.9 0.91 0.92 0.93 0.94 0.95 0.96
0.97 0.98]
```

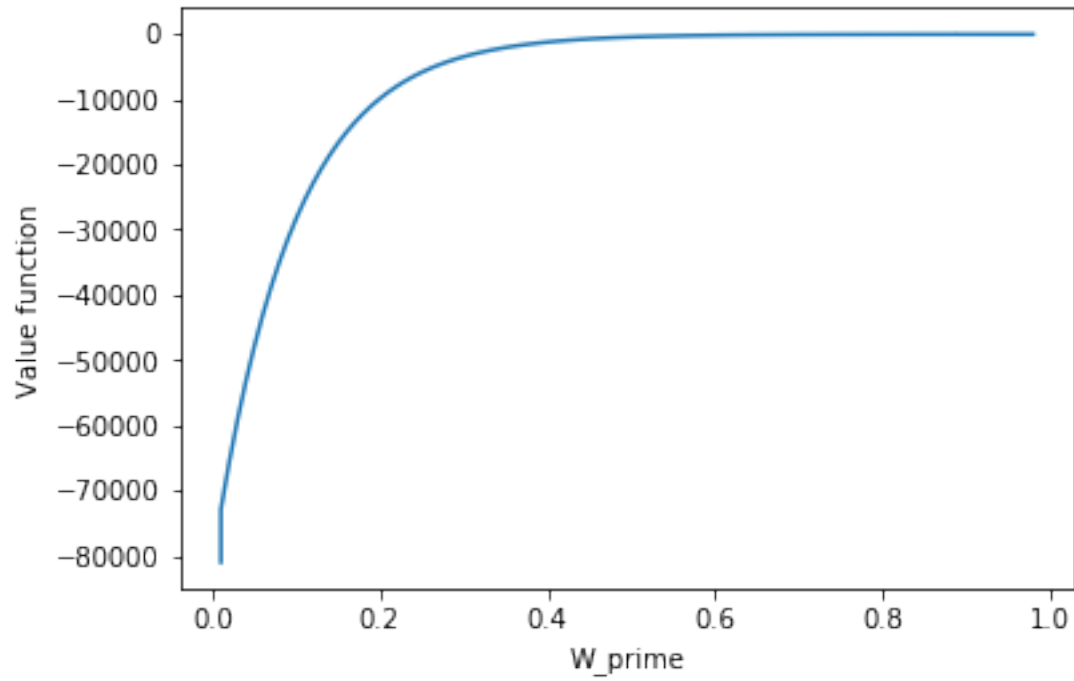
```
V(W) is [-8.10161181e+04 -7.29191115e+04 -6.56318055e+04 -5.90732301e+04
-5.31705123e+04 -4.78580662e+04 -4.30768648e+04 -3.87737835e+04
-3.49010103e+04 -3.14155144e+04 -2.82785681e+04 -2.54553165e+04
-2.29143900e+04 -2.06275562e+04 -1.85694057e+04 -1.67170703e+04
-1.50499685e+04 -1.35495768e+04 -1.21992243e+04 -1.09839070e+04
-9.89012150e+03 -8.90571452e+03 -8.01974824e+03 -7.22237858e+03
-6.50474589e+03 -5.85887647e+03 -5.27759400e+03 -4.75443977e+03
-4.28360096e+03 -3.85984604e+03 -3.47846660e+03 -3.13522511e+03
-2.82630777e+03 -2.54828216e+03 -2.29805912e+03 -2.07285838e+03
-1.87017771e+03 -1.68776511e+03 -1.52359377e+03 -1.37583956e+03
-1.24286077e+03 -1.12317987e+03 -1.01546705e+03 -9.18525516e+02
-8.31278135e+02 -7.52755491e+02 -6.82085112e+02 -6.18481771e+02
-5.61238764e+02 -5.09720058e+02 -4.63353223e+02 -4.21623071e+02
-3.84065934e+02 -3.50264510e+02 -3.19843230e+02 -2.92464077e+02
-2.67822839e+02 -2.45645726e+02 -2.25686323e+02 -2.07722861e+02
-1.91555745e+02 -1.77005341e+02 -1.63909977e+02 -1.52124149e+02
-1.41516905e+02 -1.31970384e+02 -1.23378516e+02 -1.15645835e+02
-1.08686421e+02 -1.02422949e+02 -9.67858247e+01 -9.17124124e+01
-8.71463414e+01 -8.30368774e+01 -7.93383599e+01 -7.60096941e+01
-7.30138948e+01 -7.03176755e+01 -6.78910782e+01 -6.57071405e+01
-6.37415967e+01 -6.19726072e+01 -6.03805167e+01 -5.89476352e+01
-5.76580418e+01 -5.64974078e+01 -5.54528372e+01 -5.45127237e+01
-5.36666215e+01 -5.29051296e+01 -5.22119824e+01 -5.15266396e+01
-5.09028071e+01 -5.02859987e+01 -4.97245494e+01 -4.91694218e+01
-4.86641175e+01 -4.81645026e+01 -4.77097288e+01 -4.72600754e+01]
```

After 101 iterations, V(W) converged.

```
In [11]: import matplotlib.pyplot as plt
```

```
plt.plot(W_prime, V_init)
plt.xlabel("W_prime")
plt.ylabel("Value function")
```

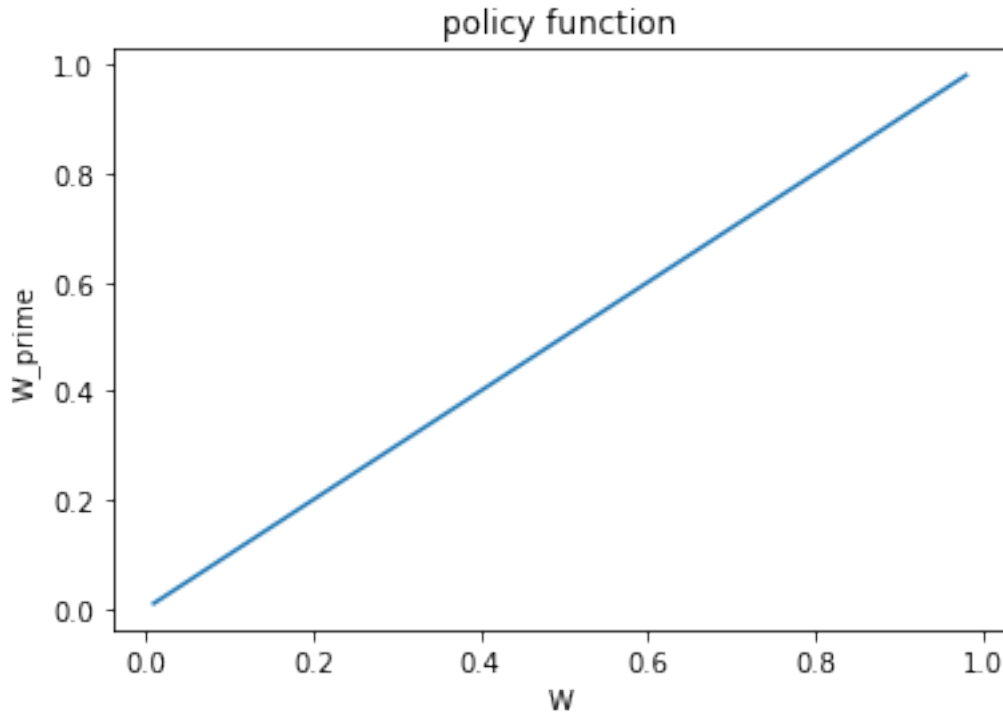
```
Out[11]: Text(0, 0.5, 'Value function')
```



5.15

```
In [12]: plt.plot(W, W_prime)
plt.title("policy function")
plt.xlabel('W')
plt.ylabel("W_prime")
```

```
Out[12]: Text(0, 0.5, 'W_prime')
```



5.16

```
In [13]: from scipy.stats import norm
```

```
sigma = 0.5
mu = 4*sigma
M = 7
epsilon = np.linspace(mu - 3*sigma, mu + 3*sigma, M)
def gamma(x):
    gamma = norm(loc=mu, scale=sigma).pdf(x)
    return gamma
gamma = gamma(epsilon)
print("The epsilon is:\n", epsilon)
print("The probability distribution is:\n", gamma)
```

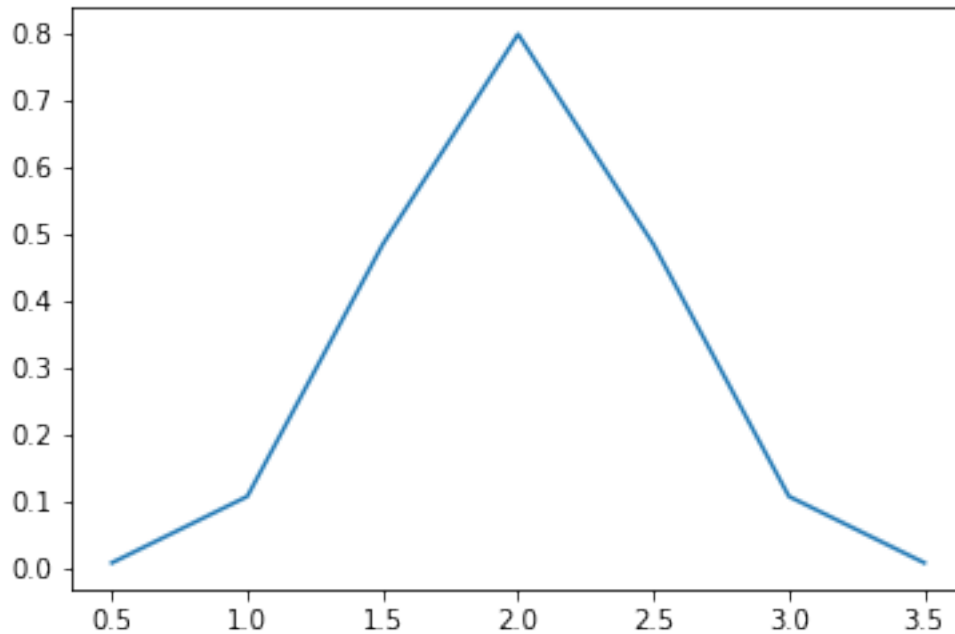
The epsilon is:

```
[0.5 1.  1.5 2.  2.5 3.  3.5]
```

The probability distribution is:

```
[0.0088637  0.10798193 0.48394145 0.79788456 0.48394145 0.10798193
 0.0088637 ]
```

```
In [14]: plt.plot(epsilon,gamma)
plt.show()
```



5.17

```
In [15]: N = 100
         W_vec = np.linspace(1e-2,1.0, N)
         W_vec

Out[15]: array([0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1 , 0.11,
                0.12, 0.13, 0.14, 0.15, 0.16, 0.17, 0.18, 0.19, 0.2 , 0.21, 0.22,
                0.23, 0.24, 0.25, 0.26, 0.27, 0.28, 0.29, 0.3 , 0.31, 0.32, 0.33,
                0.34, 0.35, 0.36, 0.37, 0.38, 0.39, 0.4 , 0.41, 0.42, 0.43, 0.44,
                0.45, 0.46, 0.47, 0.48, 0.49, 0.5 , 0.51, 0.52, 0.53, 0.54, 0.55,
                0.56, 0.57, 0.58, 0.59, 0.6 , 0.61, 0.62, 0.63, 0.64, 0.65, 0.66,
                0.67, 0.68, 0.69, 0.7 , 0.71, 0.72, 0.73, 0.74, 0.75, 0.76, 0.77,
                0.78, 0.79, 0.8 , 0.81, 0.82, 0.83, 0.84, 0.85, 0.86, 0.87, 0.88,
                0.89, 0.9 , 0.91, 0.92, 0.93, 0.94, 0.95, 0.96, 0.97, 0.98, 0.99,
                1.  ])
```

```
In [16]: # Create utility matrix
         c_mat = (np.tile(W_vec.reshape((N,1)), (1,N))
                  - np.tile(W_vec.reshape((1,N)), (N,1)))
         c_pos = c_mat>0
         c_mat[~c_pos] = 1e-7
         u_mat = utility(c_mat)
         D3_array = np.array([u_mat*e for e in epsilon])
         V_init = np.zeros((N,M))
         EV = (V_init @ gamma).reshape((N,1))
         EV_mat = np.tile(EV.reshape((1,N)), (N,1))
```

The policy function is:

16

[illegible]

```

[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
[0.01 0.01 0.01 0.01 0.01 0.01 0.01]

```

The value function is:

```

[[-8.10080590e+04 -8.10161181e+04 -8.10241771e+04 -8.10322362e+04
  -8.10402952e+04 -8.10483543e+04 -8.10564133e+04]
[-2.30258509e+00 -4.60517019e+00 -6.90775528e+00 -9.21034037e+00
  -1.15129255e+01 -1.38155106e+01 -1.61180957e+01]
[-1.95601150e+00 -3.91202301e+00 -5.86803451e+00 -7.82404601e+00
  -9.78005751e+00 -1.17360690e+01 -1.36920805e+01]
[-1.75327895e+00 -3.50655790e+00 -5.25983685e+00 -7.01311579e+00
  -8.76639474e+00 -1.05196737e+01 -1.22729526e+01]
[-1.60943791e+00 -3.21887582e+00 -4.82831374e+00 -6.43775165e+00
  -8.04718956e+00 -9.65662747e+00 -1.12660654e+01]
[-1.49786614e+00 -2.99573227e+00 -4.49359841e+00 -5.99146455e+00
  -7.48933068e+00 -8.98719682e+00 -1.04850630e+01]
[-1.40670536e+00 -2.81341072e+00 -4.22011608e+00 -5.62682143e+00
  -7.03352679e+00 -8.44023215e+00 -9.84693751e+00]
[-1.32963002e+00 -2.65926004e+00 -3.98889006e+00 -5.31852007e+00
  -6.64815009e+00 -7.97778011e+00 -9.30741013e+00]
[-1.26286432e+00 -2.52572864e+00 -3.78859297e+00 -5.05145729e+00
  -6.31432161e+00 -7.57718593e+00 -8.84005026e+00]
[-1.20397280e+00 -2.40794561e+00 -3.61191841e+00 -4.81589122e+00
  -6.01986402e+00 -7.22383683e+00 -8.42780963e+00]
[-1.15129255e+00 -2.30258509e+00 -3.45387764e+00 -4.60517019e+00
  -5.75646273e+00 -6.90775528e+00 -8.05904783e+00]
[-1.10363746e+00 -2.20727491e+00 -3.31091237e+00 -4.41454983e+00
  -5.51818728e+00 -6.62182474e+00 -7.72546220e+00]
[-1.06013177e+00 -2.12026354e+00 -3.18039530e+00 -4.24052707e+00
  -5.30065884e+00 -6.36079061e+00 -7.42092238e+00]
[-1.02011041e+00 -2.04022083e+00 -3.06033124e+00 -4.08044166e+00
  -5.10055207e+00 -6.12066249e+00 -7.14077290e+00]

```

[-9.83056428e-01 -1.96611286e+00 -2.94916928e+00 -3.93222571e+00
 -4.91528214e+00 -5.89833857e+00 -6.88139500e+00]
 [-9.48559992e-01 -1.89711998e+00 -2.84567998e+00 -3.79423997e+00
 -4.74279996e+00 -5.69135995e+00 -6.63991995e+00]
 [-9.16290732e-01 -1.83258146e+00 -2.74887220e+00 -3.66516293e+00
 -4.58145366e+00 -5.49774439e+00 -6.41403512e+00]
 [-8.85978421e-01 -1.77195684e+00 -2.65793526e+00 -3.54391368e+00
 -4.42989210e+00 -5.31587053e+00 -6.20184895e+00]
 [-8.57399214e-01 -1.71479843e+00 -2.57219764e+00 -3.42959686e+00
 -4.28699607e+00 -5.14439528e+00 -6.00179450e+00]
 [-8.30365603e-01 -1.66073121e+00 -2.49109681e+00 -3.32146241e+00
 -4.15182802e+00 -4.98219362e+00 -5.81255922e+00]
 [-8.04718956e-01 -1.60943791e+00 -2.41415687e+00 -3.21887582e+00
 -4.02359478e+00 -4.82831374e+00 -5.63303269e+00]
 [-7.80323874e-01 -1.56064775e+00 -2.34097162e+00 -3.12129550e+00
 -3.90161937e+00 -4.68194324e+00 -5.46226712e+00]
 [-7.57063866e-01 -1.51412773e+00 -2.27119160e+00 -3.02825547e+00
 -3.78531933e+00 -4.54238320e+00 -5.29944706e+00]
 [-7.34837985e-01 -1.46967597e+00 -2.20451396e+00 -2.93935194e+00
 -3.67418993e+00 -4.40902791e+00 -5.14386590e+00]
 [-7.13558178e-01 -1.42711636e+00 -2.14067453e+00 -2.85423271e+00
 -3.56779089e+00 -4.28134907e+00 -4.99490724e+00]
 [-6.93147181e-01 -1.38629436e+00 -2.07944154e+00 -2.77258872e+00
 -3.46573590e+00 -4.15888308e+00 -4.85203026e+00]
 [-6.73536824e-01 -1.34707365e+00 -2.02061047e+00 -2.69414730e+00
 -3.36768412e+00 -4.04122094e+00 -4.71475777e+00]
 [-6.54666660e-01 -1.30933332e+00 -1.96399998e+00 -2.61866664e+00
 -3.27333330e+00 -3.92799996e+00 -4.58266662e+00]
 [-6.36482838e-01 -1.27296568e+00 -1.90944851e+00 -2.54593135e+00
 -3.18241419e+00 -3.81889703e+00 -4.45537987e+00]
 [-6.18937178e-01 -1.23787436e+00 -1.85681153e+00 -2.47574871e+00
 -3.09468589e+00 -3.71362307e+00 -4.33256025e+00]
 [-6.01986402e-01 -1.20397280e+00 -1.80595921e+00 -2.40794561e+00
 -3.00993201e+00 -3.61191841e+00 -4.21390482e+00]
 [-5.85591491e-01 -1.17118298e+00 -1.75677447e+00 -2.34236596e+00
 -2.92795745e+00 -3.51354894e+00 -4.09914044e+00]
 [-5.69717142e-01 -1.13943428e+00 -1.70915142e+00 -2.27886857e+00
 -2.84858571e+00 -3.41830285e+00 -3.98801999e+00]
 [-5.54331312e-01 -1.10866262e+00 -1.66299394e+00 -2.21732525e+00
 -2.77165656e+00 -3.32598787e+00 -3.88031919e+00]
 [-5.39404831e-01 -1.07880966e+00 -1.61821449e+00 -2.15761932e+00
 -2.69702415e+00 -3.23642898e+00 -3.77583381e+00]
 [-5.24911062e-01 -1.04982212e+00 -1.57473319e+00 -2.09964425e+00
 -2.62455531e+00 -3.14946637e+00 -3.67437744e+00]
 [-5.10825624e-01 -1.02165125e+00 -1.53247687e+00 -2.04330250e+00
 -2.55412812e+00 -3.06495374e+00 -3.57577937e+00]
 [-4.97126137e-01 -9.94252273e-01 -1.49137841e+00 -1.98850455e+00
 -2.48563068e+00 -2.98275682e+00 -3.47988296e+00]

[-4.83792013e-01 -9.67584026e-01 -1.45137604e+00 -1.93516805e+00
 -2.41896007e+00 -2.90275208e+00 -3.38654409e+00]
 [-4.70804270e-01 -9.41608540e-01 -1.41241281e+00 -1.88321708e+00
 -2.35402135e+00 -2.82482562e+00 -3.29562989e+00]
 [-4.58145366e-01 -9.16290732e-01 -1.37443610e+00 -1.83258146e+00
 -2.29072683e+00 -2.74887220e+00 -3.20701756e+00]
 [-4.45799060e-01 -8.91598119e-01 -1.33739718e+00 -1.78319624e+00
 -2.22899530e+00 -2.67479436e+00 -3.12059342e+00]
 [-4.33750284e-01 -8.67500568e-01 -1.30125085e+00 -1.73500114e+00
 -2.16875142e+00 -2.60250170e+00 -3.03625199e+00]
 [-4.21985035e-01 -8.43970070e-01 -1.26595511e+00 -1.68794014e+00
 -2.10992518e+00 -2.53191021e+00 -2.95389525e+00]
 [-4.10490276e-01 -8.20980552e-01 -1.23147083e+00 -1.64196110e+00
 -2.05245138e+00 -2.46294166e+00 -2.87343193e+00]
 [-3.99253848e-01 -7.98507696e-01 -1.19776154e+00 -1.59701539e+00
 -1.99626924e+00 -2.39552309e+00 -2.79477694e+00]
 [-3.88264395e-01 -7.76528789e-01 -1.16479318e+00 -1.55305758e+00
 -1.94132197e+00 -2.32958637e+00 -2.71785076e+00]
 [-3.77511292e-01 -7.55022584e-01 -1.13253388e+00 -1.51004517e+00
 -1.88755646e+00 -2.26506775e+00 -2.64257904e+00]
 [-3.66984588e-01 -7.33969175e-01 -1.10095376e+00 -1.46793835e+00
 -1.83492294e+00 -2.20190753e+00 -2.56889211e+00]
 [-3.56674944e-01 -7.13349888e-01 -1.07002483e+00 -1.42669978e+00
 -1.78337472e+00 -2.14004966e+00 -2.49672461e+00]
 [-3.46573590e-01 -6.93147181e-01 -1.03972077e+00 -1.38629436e+00
 -1.73286795e+00 -2.07944154e+00 -2.42601513e+00]
 [-3.36672277e-01 -6.73344553e-01 -1.01001683e+00 -1.34668911e+00
 -1.68336138e+00 -2.02003366e+00 -2.35670594e+00]
 [-3.26963234e-01 -6.53926467e-01 -9.80889701e-01 -1.30785293e+00
 -1.63481617e+00 -1.96177940e+00 -2.28874264e+00]
 [-3.17439136e-01 -6.34878272e-01 -9.52317409e-01 -1.26975654e+00
 -1.58719568e+00 -1.90463482e+00 -2.22207395e+00]
 [-3.08093070e-01 -6.16186139e-01 -9.24279209e-01 -1.23237228e+00
 -1.54046535e+00 -1.84855842e+00 -2.15665149e+00]
 [-2.98918500e-01 -5.97837001e-01 -8.96755501e-01 -1.19567400e+00
 -1.49459250e+00 -1.79351100e+00 -2.09242950e+00]
 [-2.89909248e-01 -5.79818495e-01 -8.69727743e-01 -1.15963699e+00
 -1.44954624e+00 -1.73945549e+00 -2.02936473e+00]
 [-2.81059459e-01 -5.62118918e-01 -8.43178377e-01 -1.12423784e+00
 -1.40529730e+00 -1.68635675e+00 -1.96741621e+00]
 [-2.72363588e-01 -5.44727175e-01 -8.17090763e-01 -1.08945435e+00
 -1.36181794e+00 -1.63418153e+00 -1.90654511e+00]
 [-2.63816371e-01 -5.27632742e-01 -7.91449113e-01 -1.05526548e+00
 -1.31908186e+00 -1.58289823e+00 -1.84671460e+00]
 [-2.55412812e-01 -5.10825624e-01 -7.66238436e-01 -1.02165125e+00
 -1.27706406e+00 -1.53247687e+00 -1.78788968e+00]
 [-2.47148161e-01 -4.94296322e-01 -7.41444483e-01 -9.88592644e-01
 -1.23574080e+00 -1.48288897e+00 -1.73003713e+00]

[-2.39017900e-01 -4.78035801e-01 -7.17053701e-01 -9.56071602e-01
 -1.19508950e+00 -1.43410740e+00 -1.67312530e+00]
 [-2.31017730e-01 -4.62035460e-01 -6.93053189e-01 -9.24070919e-01
 -1.15508865e+00 -1.38610638e+00 -1.61712411e+00]
 [-2.23143551e-01 -4.46287103e-01 -6.69430654e-01 -8.92574205e-01
 -1.11571776e+00 -1.33886131e+00 -1.56200486e+00]
 [-2.15391458e-01 -4.30782916e-01 -6.46174374e-01 -8.61565832e-01
 -1.07695729e+00 -1.29234875e+00 -1.50774021e+00]
 [-2.07757722e-01 -4.15515444e-01 -6.23273166e-01 -8.31030888e-01
 -1.03878861e+00 -1.24654633e+00 -1.45430405e+00]
 [-2.00238783e-01 -4.00477567e-01 -6.00716350e-01 -8.00955133e-01
 -1.00119392e+00 -1.20143270e+00 -1.40167148e+00]
 [-1.92831240e-01 -3.85662481e-01 -5.78493721e-01 -7.71324962e-01
 -9.64156202e-01 -1.15698744e+00 -1.34981868e+00]
 [-1.85531841e-01 -3.71063681e-01 -5.56595522e-01 -7.42127363e-01
 -9.27659203e-01 -1.11319104e+00 -1.29872288e+00]
 [-1.78337472e-01 -3.56674944e-01 -5.35012416e-01 -7.13349888e-01
 -8.91687360e-01 -1.07002483e+00 -1.24836230e+00]
 [-1.71245154e-01 -3.42490309e-01 -5.13735463e-01 -6.84980618e-01
 -8.56225772e-01 -1.02747093e+00 -1.19871608e+00]
 [-1.64252033e-01 -3.28504067e-01 -4.92756100e-01 -6.57008134e-01
 -8.21260167e-01 -9.85512201e-01 -1.14976423e+00]
 [-1.57355372e-01 -3.14710745e-01 -4.72066117e-01 -6.29421490e-01
 -7.86776862e-01 -9.44132235e-01 -1.10148761e+00]
 [-1.50552546e-01 -3.01105093e-01 -4.51657639e-01 -6.02210186e-01
 -7.52762732e-01 -9.03315278e-01 -1.05386782e+00]
 [-1.43841036e-01 -2.87682072e-01 -4.31523109e-01 -5.75364145e-01
 -7.19205181e-01 -8.63046217e-01 -1.00688725e+00]
 [-1.37218423e-01 -2.74436846e-01 -4.11655269e-01 -5.48873691e-01
 -6.86092114e-01 -8.23310537e-01 -9.60528960e-01]
 [-1.30682382e-01 -2.61364764e-01 -3.92047146e-01 -5.22729528e-01
 -6.53411910e-01 -7.84094292e-01 -9.14776674e-01]
 [-1.24230680e-01 -2.48461359e-01 -3.72692039e-01 -4.96922719e-01
 -6.21153398e-01 -7.45384078e-01 -8.69614758e-01]
 [-1.17861167e-01 -2.35722334e-01 -3.53583500e-01 -4.71444667e-01
 -5.89305834e-01 -7.07167001e-01 -8.25028167e-01]
 [-1.11571776e-01 -2.23143551e-01 -3.34715327e-01 -4.46287103e-01
 -5.57858878e-01 -6.69430654e-01 -7.81002430e-01]
 [-1.05360516e-01 -2.10721031e-01 -3.16081547e-01 -4.21442063e-01
 -5.26802578e-01 -6.32163094e-01 -7.37523610e-01]
 [-9.92254694e-02 -1.98450939e-01 -2.97676408e-01 -3.96901877e-01
 -4.96127347e-01 -5.95352816e-01 -6.94578286e-01]
 [-9.31647891e-02 -1.86329578e-01 -2.79494367e-01 -3.72659156e-01
 -4.65823945e-01 -5.58988735e-01 -6.52153524e-01]
 [-8.71766936e-02 -1.74353387e-01 -2.61530081e-01 -3.48706774e-01
 -4.35883468e-01 -5.23060161e-01 -6.10236855e-01]
 [-8.12594647e-02 -1.62518929e-01 -2.43778394e-01 -3.25037859e-01
 -4.06297324e-01 -4.87556788e-01 -5.68816253e-01]

```

[-7.54114449e-02 -1.50822890e-01 -2.26234335e-01 -3.01645779e-01
 -3.77057224e-01 -4.52468669e-01 -5.27880114e-01]
[-6.96310337e-02 -1.39262067e-01 -2.08893101e-01 -2.78524135e-01
 -3.48155168e-01 -4.17786202e-01 -4.87417236e-01]
[-6.39166858e-02 -1.27833372e-01 -1.91750057e-01 -2.55666743e-01
 -3.19583429e-01 -3.83500115e-01 -4.47416800e-01]
[-5.82669081e-02 -1.16533816e-01 -1.74800724e-01 -2.33067633e-01
 -2.91334541e-01 -3.49601449e-01 -4.07868357e-01]
[-5.26802578e-02 -1.05360516e-01 -1.58040773e-01 -2.10721031e-01
 -2.63401289e-01 -3.16081547e-01 -3.68761805e-01]
[-4.71553397e-02 -9.43106795e-02 -1.41466019e-01 -1.88621359e-01
 -2.35776699e-01 -2.82932038e-01 -3.30087378e-01]
[-4.16908045e-02 -8.33816089e-02 -1.25072413e-01 -1.66763218e-01
 -2.08454022e-01 -2.50144827e-01 -2.91835631e-01]
[-3.62853464e-02 -7.25706928e-02 -1.08856039e-01 -1.45141386e-01
 -1.81426732e-01 -2.17712079e-01 -2.53997425e-01]
[-3.09377019e-02 -6.18754037e-02 -9.28131056e-02 -1.23750807e-01
 -1.54688509e-01 -1.85626211e-01 -2.16563913e-01]
[-2.56466472e-02 -5.12932944e-02 -7.69399416e-02 -1.02586589e-01
 -1.28233236e-01 -1.53879883e-01 -1.79526530e-01]
[-2.04109973e-02 -4.08219945e-02 -6.12329918e-02 -8.16439890e-02
 -1.02054986e-01 -1.22465984e-01 -1.42876981e-01]
[-1.52296037e-02 -3.04592075e-02 -4.56888112e-02 -6.09184150e-02
 -7.61480187e-02 -9.13776225e-02 -1.06607226e-01]
[-1.01013537e-02 -2.02027073e-02 -3.03040610e-02 -4.04054146e-02
 -5.05067683e-02 -6.06081220e-02 -7.07094756e-02]
[-5.02516793e-03 -1.00503359e-02 -1.50755038e-02 -2.01006717e-02
 -2.51258396e-02 -3.01510076e-02 -3.51761755e-02]]

```

5.18

```

In [17]: delta1 = distance(V_new, V_init)
         print("The distance metric is:\n", delta1)

```

```

The distance metric is:
45963571196.10551

```

5.19

```

In [18]: V_init = V_new
         EV = (V_init @ gamma).reshape((N,1))
         EV_mat = np.tile(EV.reshape((1,N)), (N,1))
         EV_mat[~c_pos] = -9e+4
         EV_D3array = np.array([EV_mat for e in range(M)])
         V_new_D3array = D3_array + beta*EV_D3array
         V_new = np.zeros((N,M))
         W_prime = np.zeros((N,M))

```

```

for i in range(N):
    arr = V_new_D3array[:, i, :]
    V_new[i] = arr.max(axis=1)
    W_index = np.argmax(arr, axis=1)
    W_prime[i] = W_vec[W_index]

print("The policy function is\n", W_prime)
print("The value function is:\n", V_new)

```

The policy function is

```

: [[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.04 0.04 0.04 0.04 0.03 0.03 0.03]
 [0.05 0.05 0.04 0.04 0.04 0.04 0.04]
 [0.06 0.06 0.05 0.05 0.05 0.04 0.04]
 [0.07 0.06 0.06 0.05 0.05 0.05 0.05]
 [0.08 0.07 0.07 0.06 0.06 0.05 0.05]
 [0.09 0.08 0.07 0.07 0.06 0.06 0.06]
 [0.1  0.09 0.08 0.07 0.07 0.06 0.06]
 [0.11 0.1  0.09 0.08 0.07 0.07 0.07]
 [0.11 0.1  0.09 0.09 0.08 0.08 0.07]
 [0.12 0.11 0.1  0.09 0.09 0.08 0.08]
 [0.13 0.12 0.11 0.1  0.09 0.09 0.08]
 [0.14 0.13 0.12 0.11 0.1  0.09 0.09]
 [0.15 0.14 0.12 0.11 0.1  0.1  0.09]
 [0.16 0.14 0.13 0.12 0.11 0.1  0.1 ]
 [0.17 0.15 0.14 0.13 0.12 0.11 0.1 ]
 [0.18 0.16 0.14 0.13 0.12 0.11 0.11]
 [0.19 0.17 0.15 0.14 0.13 0.12 0.11]
 [0.19 0.17 0.16 0.14 0.13 0.12 0.12]
 [0.2  0.18 0.17 0.15 0.14 0.13 0.12]
 [0.21 0.19 0.17 0.16 0.15 0.14 0.13]
 [0.22 0.2  0.18 0.16 0.15 0.14 0.13]
 [0.23 0.21 0.19 0.17 0.16 0.15 0.14]
 [0.24 0.21 0.19 0.18 0.16 0.15 0.14]
 [0.25 0.22 0.2  0.18 0.17 0.16 0.15]
 [0.26 0.23 0.21 0.19 0.18 0.16 0.15]
 [0.26 0.24 0.21 0.2  0.18 0.17 0.16]
 [0.27 0.24 0.22 0.2  0.19 0.17 0.16]
 [0.28 0.25 0.23 0.21 0.19 0.18 0.17]
 [0.29 0.26 0.24 0.22 0.2  0.18 0.17]
 [0.3  0.27 0.24 0.22 0.2  0.19 0.18]
 [0.31 0.28 0.25 0.23 0.21 0.2  0.18]
 [0.32 0.28 0.26 0.23 0.22 0.2  0.19]
 [0.33 0.29 0.26 0.24 0.22 0.21 0.19]
 [0.33 0.3  0.27 0.25 0.23 0.21 0.2 ]

```

[0.34 0.31 0.28 0.25 0.23 0.22 0.2]
 [0.35 0.32 0.29 0.26 0.24 0.22 0.21]
 [0.36 0.32 0.29 0.27 0.25 0.23 0.21]
 [0.37 0.33 0.3 0.27 0.25 0.23 0.22]
 [0.38 0.34 0.31 0.28 0.26 0.24 0.22]
 [0.39 0.35 0.31 0.29 0.26 0.24 0.23]
 [0.4 0.35 0.32 0.29 0.27 0.25 0.23]
 [0.4 0.36 0.33 0.3 0.28 0.26 0.24]
 [0.41 0.37 0.33 0.31 0.28 0.26 0.24]
 [0.42 0.38 0.34 0.31 0.29 0.27 0.25]
 [0.43 0.39 0.35 0.32 0.29 0.27 0.25]
 [0.44 0.39 0.36 0.32 0.3 0.28 0.26]
 [0.45 0.4 0.36 0.33 0.31 0.28 0.26]
 [0.46 0.41 0.37 0.34 0.31 0.29 0.27]
 [0.47 0.42 0.38 0.34 0.32 0.29 0.27]
 [0.48 0.42 0.38 0.35 0.32 0.3 0.28]
 [0.48 0.43 0.39 0.36 0.33 0.3 0.28]
 [0.49 0.44 0.4 0.36 0.33 0.31 0.29]
 [0.5 0.45 0.41 0.37 0.34 0.32 0.29]
 [0.51 0.46 0.41 0.38 0.35 0.32 0.3]
 [0.52 0.46 0.42 0.38 0.35 0.33 0.3]
 [0.53 0.47 0.43 0.39 0.36 0.33 0.31]
 [0.54 0.48 0.43 0.4 0.36 0.34 0.31]
 [0.55 0.49 0.44 0.4 0.37 0.34 0.32]
 [0.55 0.5 0.45 0.41 0.38 0.35 0.32]
 [0.56 0.5 0.45 0.41 0.38 0.35 0.33]
 [0.57 0.51 0.46 0.42 0.39 0.36 0.33]
 [0.58 0.52 0.47 0.43 0.39 0.36 0.34]
 [0.59 0.53 0.48 0.43 0.4 0.37 0.34]
 [0.6 0.53 0.48 0.44 0.41 0.38 0.35]
 [0.61 0.54 0.49 0.45 0.41 0.38 0.35]
 [0.62 0.55 0.5 0.45 0.42 0.39 0.36]
 [0.62 0.56 0.5 0.46 0.42 0.39 0.36]
 [0.63 0.57 0.51 0.47 0.43 0.4 0.37]
 [0.64 0.57 0.52 0.47 0.43 0.4 0.38]
 [0.65 0.58 0.53 0.48 0.44 0.41 0.38]
 [0.66 0.59 0.53 0.49 0.45 0.41 0.39]
 [0.67 0.6 0.54 0.49 0.45 0.42 0.39]
 [0.68 0.6 0.55 0.5 0.46 0.42 0.4]
 [0.69 0.61 0.55 0.5 0.46 0.43 0.4]
 [0.69 0.62 0.56 0.51 0.47 0.44 0.41]
 [0.7 0.63 0.57 0.52 0.48 0.44 0.41]
 [0.71 0.64 0.57 0.52 0.48 0.45 0.42]
 [0.72 0.64 0.58 0.53 0.49 0.45 0.42]
 [0.73 0.65 0.59 0.54 0.49 0.46 0.43]
 [0.74 0.66 0.6 0.54 0.5 0.46 0.43]
 [0.75 0.67 0.6 0.55 0.51 0.47 0.44]
 [0.76 0.68 0.61 0.56 0.51 0.47 0.44]


```

[0.77 0.68 0.62 0.56 0.52 0.48 0.45]
[0.77 0.69 0.62 0.57 0.52 0.48 0.45]
[0.78 0.7 0.63 0.58 0.53 0.49 0.46]
[0.79 0.71 0.64 0.58 0.54 0.5 0.46]
[0.8 0.71 0.65 0.59 0.54 0.5 0.47]
[0.81 0.72 0.65 0.59 0.55 0.51 0.47]
[0.82 0.73 0.66 0.6 0.55 0.51 0.48]
[0.83 0.74 0.67 0.61 0.56 0.52 0.48]
[0.84 0.75 0.67 0.61 0.56 0.52 0.49]
[0.84 0.75 0.68 0.62 0.57 0.53 0.49]
[0.85 0.76 0.69 0.63 0.58 0.53 0.5 ]
[0.86 0.77 0.69 0.63 0.58 0.54 0.5 ]
[0.87 0.78 0.7 0.64 0.59 0.54 0.51]
[0.88 0.78 0.71 0.65 0.59 0.55 0.51]]

```

The value function is:

```

[[-8.10080590e+04 -8.10161181e+04 -8.10241771e+04 -8.10322362e+04
-8.10402952e+04 -8.10483543e+04 -8.10564133e+04]
[-8.10080590e+04 -8.10161181e+04 -8.10241771e+04 -8.10322362e+04
-8.10402952e+04 -8.10483543e+04 -8.10564133e+04]
[-1.88767109e+01 -2.11792960e+01 -2.34818811e+01 -2.57844662e+01
-2.80870513e+01 -3.03896364e+01 -3.26922215e+01]
[-1.63820564e+01 -1.86846415e+01 -2.09872266e+01 -2.32898117e+01
-2.55923968e+01 -2.78949819e+01 -3.01975670e+01]
[-1.49227771e+01 -1.72253622e+01 -1.95279472e+01 -2.18305323e+01
-2.38595288e+01 -2.58155403e+01 -2.77715518e+01]
[-1.38874019e+01 -1.61899870e+01 -1.84882265e+01 -2.04442380e+01
-2.24002495e+01 -2.43562610e+01 -2.63122725e+01]
[-1.30843025e+01 -1.53868876e+01 -1.74528513e+01 -1.94088628e+01
-2.13648743e+01 -2.31398657e+01 -2.48931446e+01]
[-1.24281225e+01 -1.46937404e+01 -1.66497519e+01 -1.85979326e+01
-2.03512115e+01 -2.21044905e+01 -2.38577694e+01]
[-1.18733303e+01 -1.40375605e+01 -1.59935720e+01 -1.77948332e+01
-1.95481122e+01 -2.12414443e+01 -2.28508822e+01]
[-1.13927474e+01 -1.34827682e+01 -1.53853743e+01 -1.71386533e+01
-1.88289070e+01 -2.04383449e+01 -2.20477828e+01]
[-1.09688432e+01 -1.30021853e+01 -1.48305820e+01 -1.65632891e+01
-1.81727270e+01 -1.97689142e+01 -2.12667804e+01]
[-1.05896480e+01 -1.25782811e+01 -1.43499991e+01 -1.60084968e+01
-1.76148681e+01 -1.91127343e+01 -2.06106004e+01]
[-1.02430744e+01 -1.21728160e+01 -1.39184760e+01 -1.55279139e+01
-1.70600759e+01 -1.85579420e+01 -1.99724750e+01]
[-9.90005063e+00 -1.17936208e+01 -1.34945719e+01 -1.50816268e+01
-1.65794930e+01 -1.80109773e+01 -1.94176827e+01]
[-9.58689445e+00 -1.14505970e+01 -1.31153766e+01 -1.46577227e+01
-1.61236891e+01 -1.75303944e+01 -1.88781553e+01]
[-9.29881868e+00 -1.11374408e+01 -1.27723529e+01 -1.42785275e+01
-1.56997849e+01 -1.70679424e+01 -1.83975724e+01]
[-9.03210219e+00 -1.08493651e+01 -1.24376375e+01 -1.39138843e+01

```

-1.53144082e+01 -1.66440382e+01 -1.79302125e+01]
 [-8.78379507e+00 -1.05616830e+01 -1.21244814e+01 -1.35708606e+01
 -1.49352130e+01 -1.62434440e+01 -1.75063084e+01]
 [-8.55151928e+00 -1.02949665e+01 -1.18364056e+01 -1.32577044e+01
 -1.45921892e+01 -1.58642488e+01 -1.70940677e+01]
 [-8.33332970e+00 -1.00466594e+01 -1.15629233e+01 -1.29494030e+01
 -1.42583607e+01 -1.55108997e+01 -1.67148725e+01]
 [-8.12761510e+00 -9.81438360e+00 -1.12962068e+01 -1.26613273e+01
 -1.39452046e+01 -1.51678760e+01 -1.63461107e+01]
 [-7.92488255e+00 -9.59124005e+00 -1.10478996e+01 -1.23942645e+01
 -1.36507470e+01 -1.48517944e+01 -1.60030870e+01]
 [-7.73029323e+00 -9.37305048e+00 -1.08156238e+01 -1.21275480e+01
 -1.33626712e+01 -1.45386382e+01 -1.56695013e+01]
 [-7.54568735e+00 -9.16733588e+00 -1.05843978e+01 -1.18792409e+01
 -1.30959547e+01 -1.42505625e+01 -1.53563451e+01]
 [-7.37009029e+00 -8.97274656e+00 -1.03662083e+01 -1.16436748e+01
 -1.28325534e+01 -1.39646319e+01 -1.50518053e+01]
 [-7.20266356e+00 -8.78814067e+00 -1.01604937e+01 -1.14113990e+01
 -1.25842463e+01 -1.36979154e+01 -1.47637296e+01]
 [-7.04268053e+00 -8.60581912e+00 -9.96019657e+00 -1.11932094e+01
 -1.23459709e+01 -1.34368813e+01 -1.44835801e+01]
 [-6.88950738e+00 -8.43022206e+00 -9.76560725e+00 -1.09824884e+01
 -1.21136951e+01 -1.31885742e+01 -1.42168636e+01]
 [-6.74258797e+00 -8.26279533e+00 -9.58100137e+00 -1.07767738e+01
 -1.18955055e+01 -1.29484461e+01 -1.39574857e+01]
 [-6.59874694e+00 -8.10281229e+00 -9.40432681e+00 -1.05821845e+01
 -1.16779770e+01 -1.27161703e+01 -1.37091786e+01]
 [-6.45759058e+00 -7.94866161e+00 -9.22872976e+00 -1.03915641e+01
 -1.14722624e+01 -1.24938463e+01 -1.34677035e+01]
 [-6.32176217e+00 -7.79548847e+00 -9.06130303e+00 -1.02069582e+01
 -1.12721557e+01 -1.22756568e+01 -1.32354277e+01]
 [-6.19087409e+00 -7.64856906e+00 -8.90131999e+00 -1.00313612e+01
 -1.10775664e+01 -1.20686782e+01 -1.30095429e+01]
 [-6.06457952e+00 -7.50741271e+00 -8.74327922e+00 -9.85733842e+00
 -1.08922964e+01 -1.18629636e+01 -1.27913533e+01]
 [-5.94256697e+00 -7.37158430e+00 -8.59010607e+00 -9.68991169e+00
 -1.07076905e+01 -1.16683742e+01 -1.25791671e+01]
 [-5.82455555e+00 -7.23805290e+00 -8.44318666e+00 -9.52982627e+00
 -1.05320935e+01 -1.14747587e+01 -1.23734525e+01]
 [-5.71029117e+00 -7.10716482e+00 -8.30022139e+00 -9.36984323e+00
 -1.03596113e+01 -1.12901528e+01 -1.21733981e+01]
 [-5.59871940e+00 -6.98087026e+00 -8.15906504e+00 -9.21667009e+00
 -1.01921846e+01 -1.11082789e+01 -1.19788088e+01]
 [-5.48797141e+00 -6.85885770e+00 -8.02323663e+00 -9.06845414e+00
 -1.00308383e+01 -1.09326819e+01 -1.17895735e+01]
 [-5.38052983e+00 -6.74084629e+00 -7.89234854e+00 -8.92153474e+00
 -9.87085524e+00 -1.07612066e+01 -1.16049676e+01]
 [-5.27620294e+00 -6.62306325e+00 -7.76183148e+00 -8.78037838e+00

-9.71768209e+00 -1.05937799e+01 -1.14254411e+01]
 [-5.17481523e+00 -6.50879887e+00 -7.63553692e+00 -8.64239264e+00
 -9.56612054e+00 -1.04315782e+01 -1.12498440e+01]
 [-5.07620561e+00 -6.39805088e+00 -7.51352436e+00 -8.50656423e+00
 -9.41920113e+00 -1.02715952e+01 -1.10790785e+01]
 [-4.98022591e+00 -6.29060930e+00 -7.39346030e+00 -8.37567614e+00
 -9.27630509e+00 -1.01177153e+01 -1.09116517e+01]
 [-4.88673946e+00 -6.18524878e+00 -7.27544888e+00 -8.24659910e+00
 -9.13514874e+00 -9.96454217e+00 -1.07488317e+01]
 [-4.79557869e+00 -6.08092189e+00 -7.16118450e+00 -8.12030454e+00
 -8.99932033e+00 -9.81762276e+00 -1.05888486e+01]
 [-4.70445925e+00 -5.97953418e+00 -7.05002255e+00 -7.99829198e+00
 -8.86415228e+00 -9.67125227e+00 -1.04332675e+01]
 [-4.61558990e+00 -5.88092457e+00 -6.93927456e+00 -7.87704274e+00
 -8.73326419e+00 -9.53009591e+00 -1.02800943e+01]
 [-4.52886219e+00 -5.78494486e+00 -6.83183297e+00 -7.75903133e+00
 -8.60503096e+00 -9.39053587e+00 -1.01311357e+01]
 [-4.44417533e+00 -5.68963468e+00 -6.72750608e+00 -7.64471450e+00
 -8.47873639e+00 -9.25470746e+00 -9.98421627e+00]
 [-4.36143546e+00 -5.59614824e+00 -6.62401678e+00 -7.53045012e+00
 -8.35672384e+00 -9.12135217e+00 -9.84133929e+00]
 [-4.28055507e+00 -5.50502880e+00 -6.52262907e+00 -7.41970213e+00
 -8.23474843e+00 -8.99046408e+00 -9.70018293e+00]
 [-4.20145242e+00 -5.41615945e+00 -6.42401945e+00 -7.31156768e+00
 -8.11673701e+00 -8.86278524e+00 -9.56291044e+00]
 [-4.12405104e+00 -5.32914808e+00 -6.32721167e+00 -7.20412610e+00
 -8.00043697e+00 -8.73649068e+00 -9.42708203e+00]
 [-4.04697570e+00 -5.24242037e+00 -6.23123197e+00 -7.09979921e+00
 -7.88617259e+00 -8.61402470e+00 -9.29499088e+00]
 [-3.97120394e+00 -5.15773350e+00 -6.13774552e+00 -6.99721262e+00
 -7.77504319e+00 -8.49201214e+00 -9.16410279e+00]
 [-3.89699459e+00 -5.07499364e+00 -6.04662608e+00 -6.89582491e+00
 -7.66429520e+00 -8.37400073e+00 -9.03681604e+00]
 [-3.82428453e+00 -4.99411325e+00 -5.95568915e+00 -6.79721530e+00
 -7.55685361e+00 -8.25633859e+00 -8.91052148e+00]
 [-3.75301437e+00 -4.91407054e+00 -5.86681980e+00 -6.69963497e+00
 -7.45045458e+00 -8.14207421e+00 -8.78770186e+00]
 [-3.68312818e+00 -4.83496789e+00 -5.78009210e+00 -6.60365527e+00
 -7.34612769e+00 -8.02885322e+00 -8.66568930e+00]
 [-3.61457323e+00 -4.75756651e+00 -5.69435447e+00 -6.51016882e+00
 -7.24407270e+00 -7.91810523e+00 -8.54703387e+00]
 [-3.54729977e+00 -4.68179475e+00 -5.60966761e+00 -6.41712879e+00
 -7.14268499e+00 -7.80900230e+00 -8.42902246e+00]
 [-3.48053407e+00 -4.60758540e+00 -5.52692774e+00 -6.32600935e+00
 -7.04407538e+00 -7.70156072e+00 -8.31425808e+00]
 [-3.41449505e+00 -4.53347743e+00 -5.44582691e+00 -6.23710583e+00
 -6.94602360e+00 -7.59628676e+00 -8.19999370e+00]
 [-3.34964598e+00 -4.46076737e+00 -5.36494653e+00 -6.14823648e+00

-6.85004389e+00 -7.49195987e+00 -8.08887325e+00]
 [-3.28594475e+00 -4.38949720e+00 -5.28584388e+00 -6.06150877e+00
 -6.75569307e+00 -7.39025521e+00 -7.97812526e+00]
 [-3.22335142e+00 -4.31961101e+00 -5.20844249e+00 -5.97638954e+00
 -6.66220663e+00 -7.28886750e+00 -7.87042446e+00]
 [-3.16182812e+00 -4.25061814e+00 -5.13150255e+00 -5.89170268e+00
 -6.57108719e+00 -7.19025789e+00 -7.76298288e+00]
 [-3.10133887e+00 -4.18206320e+00 -5.05573079e+00 -5.80896281e+00
 -6.48016808e+00 -7.09188842e+00 -7.65849750e+00]
 [-3.04184948e+00 -4.11478973e+00 -4.98152144e+00 -5.72731882e+00
 -6.39129873e+00 -6.99590872e+00 -7.55417061e+00]
 [-2.98295797e+00 -4.04875071e+00 -4.90833620e+00 -5.64643844e+00
 -6.30357043e+00 -6.90066262e+00 -7.45271424e+00]
 [-2.92443593e+00 -3.98390165e+00 -4.83562613e+00 -5.56733579e+00
 -6.21684272e+00 -6.80717618e+00 -7.35132653e+00]
 [-2.86685029e+00 -3.91936313e+00 -4.76435597e+00 -5.48889436e+00
 -6.13208884e+00 -6.71486120e+00 -7.25271691e+00]
 [-2.81017155e+00 -3.85566189e+00 -4.69446978e+00 -5.41149298e+00
 -6.04740198e+00 -6.62374176e+00 -7.15411884e+00]
 [-2.75437159e+00 -3.79306856e+00 -4.62468976e+00 -5.33572122e+00
 -5.96466211e+00 -6.53418287e+00 -7.05813914e+00]
 [-2.69942356e+00 -3.73154526e+00 -4.55613481e+00 -5.26024056e+00
 -5.88268755e+00 -6.44531352e+00 -6.96224273e+00]
 [-2.64530186e+00 -3.67092064e+00 -4.48886135e+00 -5.18603121e+00
 -5.80180717e+00 -6.35835091e+00 -6.86875628e+00]
 [-2.59198198e+00 -3.61043139e+00 -4.42218370e+00 -5.11329593e+00
 -5.72243542e+00 -6.27162321e+00 -6.77541742e+00]
 [-2.53930172e+00 -3.55094200e+00 -4.35614468e+00 -5.04058586e+00
 -5.64333277e+00 -6.18693634e+00 -6.68429798e+00]
 [-2.48676027e+00 -3.49241997e+00 -4.29129562e+00 -4.96931570e+00
 -5.56593139e+00 -6.10242371e+00 -6.59338378e+00]
 [-2.43497483e+00 -3.43483433e+00 -4.22745620e+00 -4.89913306e+00
 -5.48900224e+00 -6.01968384e+00 -6.50451443e+00]
 [-2.38392397e+00 -3.37767592e+00 -4.16375496e+00 -4.82924687e+00
 -5.41323048e+00 -5.93748692e+00 -6.41590210e+00]
 [-2.33358712e+00 -3.32099718e+00 -4.10116163e+00 -4.76069193e+00
 -5.33859807e+00 -5.85660654e+00 -6.32917439e+00]
 [-2.28394460e+00 -3.26519721e+00 -4.03963833e+00 -4.69288882e+00
 -5.26438873e+00 -5.77660179e+00 -6.24275025e+00]
 [-2.23497751e+00 -3.21024919e+00 -3.97840534e+00 -4.62561536e+00
 -5.19167867e+00 -5.69749914e+00 -6.15806338e+00]
 [-2.18666772e+00 -3.15612748e+00 -3.91791609e+00 -4.55957633e+00
 -5.11920982e+00 -5.61957268e+00 -6.07372195e+00]
 [-2.13899781e+00 -3.10206026e+00 -3.85842670e+00 -4.49399669e+00
 -5.04793966e+00 -5.54217130e+00 -5.99098209e+00]
 [-2.09134272e+00 -3.04874038e+00 -3.79959563e+00 -4.42914762e+00
 -4.97751247e+00 -5.46621788e+00 -5.90862535e+00]
 [-2.04429596e+00 -2.99619893e+00 -3.74107360e+00 -4.36544639e+00

```

-4.90762628e+00 -5.39044611e+00 -5.82774496e+00]
[-1.99785627e+00 -2.94441349e+00 -3.68348796e+00 -4.30194900e+00
-4.83907133e+00 -5.31623677e+00 -5.74728165e+00]
[-1.95200819e+00 -2.89312020e+00 -3.62680922e+00 -4.23935567e+00
-4.77057390e+00 -5.24215893e+00 -5.66817900e+00]
[-1.90673683e+00 -2.84206933e+00 -3.57019873e+00 -4.17781235e+00
-4.70330043e+00 -5.16944887e+00 -5.58952400e+00]
[-1.86202786e+00 -2.79173249e+00 -3.51439876e+00 -4.11628904e+00
-4.63662981e+00 -5.09715622e+00 -5.51212262e+00]
[-1.81786748e+00 -2.74208997e+00 -3.45945074e+00 -4.05579979e+00
-4.57059079e+00 -5.02588605e+00 -5.43519644e+00]
[-1.77424240e+00 -2.69312288e+00 -3.40489927e+00 -3.99609387e+00
-4.50565208e+00 -4.95529456e+00 -5.35942468e+00]
[-1.73073671e+00 -2.64433271e+00 -3.35077757e+00 -3.93660449e+00
-4.44080301e+00 -4.88540837e+00 -5.28415296e+00]
[-1.68763409e+00 -2.59602292e+00 -3.29745769e+00 -3.87808245e+00
-4.37710178e+00 -4.81643981e+00 -5.20994362e+00]
[-1.64504157e+00 -2.54835301e+00 -3.24482071e+00 -3.82010738e+00
-4.31380726e+00 -4.74788487e+00 -5.13625669e+00]
[-1.60294722e+00 -2.50130625e+00 -3.19227926e+00 -3.76252174e+00
-4.25121393e+00 -4.68046630e+00 -5.06354662e+00]
[-1.56133953e+00 -2.45478623e+00 -3.14049382e+00 -3.70584300e+00
-4.18948240e+00 -4.61319284e+00 -4.99137912e+00]]

```

```

In [19]: delta2 = distance(V_init, V_new)
         print("The distance metric is:\n", delta2)
         diff2= delta2-delta1
         print("The difference between delta_{T-1} and delta_T is: \n", diff2)

```

The distance metric is:

45953155651.859604

The difference between delta_{T-1} and delta_T is:

-10415544.245903015

5.20

```

In [20]: V_init = V_new
         EV = (V_init @ gamma).reshape((N,1))
         EV_mat = np.tile(EV.reshape((1,N)), (N,1))
         EV_mat[~c_pos] = -9e+4
         EV_D3array = np.array([EV_mat for i in range(M)])
         V_new_D3array = D3_array + beta*EV_D3array
         V_new = np.zeros((N,M))
         W_prime = np.zeros((N,M))
         for i in range(N):
             arr = V_new_D3array[:, i, :]
             V_new[i] = arr.max(axis=1)

```

```

W_index = np.argmax(arr, axis=1)
W_prime[i] = W_vec[W_index]

print("The policy function is:\n", W_prime)
print("The value function is:\n", V_new)

```

The policy function is:

```

[[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.04 0.04 0.04 0.04 0.04 0.04 0.04]
 [0.05 0.05 0.05 0.05 0.05 0.05 0.05]
 [0.06 0.06 0.06 0.06 0.06 0.06 0.05]
 [0.07 0.07 0.07 0.07 0.07 0.06 0.06]
 [0.08 0.08 0.08 0.08 0.07 0.07 0.07]
 [0.09 0.09 0.09 0.08 0.08 0.08 0.08]
 [0.1  0.1  0.1  0.09 0.09 0.09 0.08]
 [0.11 0.11 0.11 0.1  0.1  0.09 0.09]
 [0.12 0.12 0.11 0.11 0.11 0.1  0.1 ]
 [0.13 0.13 0.12 0.12 0.11 0.11 0.11]
 [0.14 0.14 0.13 0.13 0.12 0.12 0.11]
 [0.15 0.15 0.14 0.14 0.13 0.13 0.12]
 [0.16 0.15 0.15 0.14 0.14 0.13 0.13]
 [0.17 0.16 0.16 0.15 0.15 0.14 0.14]
 [0.18 0.17 0.17 0.16 0.15 0.15 0.14]
 [0.19 0.18 0.18 0.17 0.16 0.16 0.15]
 [0.2  0.19 0.18 0.18 0.17 0.16 0.16]
 [0.21 0.2  0.19 0.18 0.18 0.17 0.17]
 [0.22 0.21 0.2  0.19 0.19 0.18 0.17]
 [0.23 0.22 0.21 0.2  0.19 0.19 0.18]
 [0.24 0.23 0.22 0.21 0.2  0.2  0.19]
 [0.25 0.24 0.23 0.22 0.21 0.2  0.2 ]
 [0.26 0.25 0.24 0.23 0.22 0.21 0.2 ]
 [0.27 0.26 0.24 0.24 0.23 0.22 0.21]
 [0.28 0.26 0.25 0.24 0.23 0.23 0.22]
 [0.29 0.27 0.26 0.25 0.24 0.23 0.23]
 [0.3  0.28 0.27 0.26 0.25 0.24 0.23]
 [0.3  0.29 0.28 0.27 0.26 0.25 0.24]
 [0.31 0.3  0.29 0.28 0.27 0.26 0.25]
 [0.32 0.31 0.3  0.29 0.27 0.26 0.25]
 [0.33 0.32 0.31 0.29 0.28 0.27 0.26]
 [0.34 0.33 0.31 0.3  0.29 0.28 0.27]
 [0.35 0.34 0.32 0.31 0.3  0.29 0.28]
 [0.36 0.35 0.33 0.32 0.31 0.29 0.28]
 [0.37 0.36 0.34 0.33 0.31 0.3  0.29]
 [0.38 0.36 0.35 0.34 0.32 0.31 0.3 ]
 [0.39 0.37 0.36 0.34 0.33 0.32 0.31]

```

[0.4 0.38 0.37 0.35 0.34 0.33 0.31]
 [0.41 0.39 0.38 0.36 0.35 0.33 0.32]
 [0.42 0.4 0.38 0.37 0.35 0.34 0.33]
 [0.43 0.41 0.39 0.38 0.36 0.35 0.34]
 [0.44 0.42 0.4 0.39 0.37 0.36 0.34]
 [0.45 0.43 0.41 0.39 0.38 0.36 0.35]
 [0.46 0.44 0.42 0.4 0.39 0.37 0.36]
 [0.47 0.45 0.43 0.41 0.39 0.38 0.37]
 [0.48 0.46 0.44 0.42 0.4 0.39 0.37]
 [0.49 0.46 0.45 0.43 0.41 0.4 0.38]
 [0.5 0.47 0.45 0.44 0.42 0.4 0.39]
 [0.51 0.48 0.46 0.44 0.43 0.41 0.4]
 [0.51 0.49 0.47 0.45 0.43 0.42 0.4]
 [0.52 0.5 0.48 0.46 0.44 0.43 0.41]
 [0.53 0.51 0.49 0.47 0.45 0.43 0.42]
 [0.54 0.52 0.5 0.48 0.46 0.44 0.43]
 [0.55 0.53 0.51 0.49 0.47 0.45 0.43]
 [0.56 0.54 0.51 0.49 0.48 0.46 0.44]
 [0.57 0.55 0.52 0.5 0.48 0.46 0.45]
 [0.58 0.56 0.53 0.51 0.49 0.47 0.46]
 [0.59 0.56 0.54 0.52 0.5 0.48 0.46]
 [0.6 0.57 0.55 0.53 0.51 0.49 0.47]
 [0.61 0.58 0.56 0.54 0.52 0.49 0.48]
 [0.62 0.59 0.57 0.54 0.52 0.5 0.48]
 [0.63 0.6 0.58 0.55 0.53 0.51 0.49]
 [0.64 0.61 0.58 0.56 0.54 0.52 0.5]
 [0.65 0.62 0.59 0.57 0.55 0.53 0.51]
 [0.66 0.63 0.6 0.58 0.55 0.53 0.52]
 [0.67 0.64 0.61 0.59 0.56 0.54 0.52]
 [0.68 0.65 0.62 0.59 0.57 0.55 0.53]
 [0.69 0.66 0.63 0.6 0.58 0.56 0.54]
 [0.7 0.66 0.64 0.61 0.59 0.56 0.54]
 [0.71 0.67 0.65 0.62 0.59 0.57 0.55]
 [0.71 0.68 0.65 0.63 0.6 0.58 0.56]
 [0.72 0.69 0.66 0.64 0.61 0.59 0.57]
 [0.73 0.7 0.67 0.64 0.62 0.6 0.57]
 [0.74 0.71 0.68 0.65 0.63 0.6 0.58]
 [0.75 0.72 0.69 0.66 0.63 0.61 0.59]
 [0.76 0.73 0.7 0.67 0.64 0.62 0.6]
 [0.77 0.74 0.71 0.68 0.65 0.63 0.6]
 [0.78 0.75 0.71 0.69 0.66 0.63 0.61]
 [0.79 0.76 0.72 0.69 0.67 0.64 0.62]
 [0.8 0.76 0.73 0.7 0.68 0.65 0.63]
 [0.81 0.77 0.74 0.71 0.68 0.66 0.63]
 [0.82 0.78 0.75 0.72 0.69 0.66 0.64]
 [0.83 0.79 0.76 0.73 0.7 0.67 0.65]
 [0.84 0.8 0.77 0.74 0.71 0.68 0.66]
 [0.85 0.81 0.78 0.74 0.72 0.69 0.66]

```

[0.86 0.82 0.79 0.75 0.72 0.7 0.67]
[0.87 0.83 0.79 0.76 0.73 0.7 0.68]
[0.88 0.84 0.8 0.77 0.74 0.71 0.69]
[0.89 0.85 0.81 0.78 0.75 0.72 0.69]
[0.9 0.86 0.82 0.79 0.76 0.73 0.7 ]
[0.91 0.87 0.83 0.79 0.76 0.73 0.71]
[0.91 0.87 0.84 0.8 0.77 0.74 0.72]
[0.92 0.88 0.85 0.81 0.78 0.75 0.72]
[0.93 0.89 0.85 0.82 0.79 0.76 0.73]
[0.94 0.9 0.86 0.83 0.79 0.76 0.74]
[0.95 0.91 0.87 0.84 0.8 0.77 0.74]]

```

The value function is:

```

[[-8.10080590e+04 -8.10161181e+04 -8.10241771e+04 -8.10322362e+04
-8.10402952e+04 -8.10483543e+04 -8.10564133e+04]
[-8.10080590e+04 -8.10161181e+04 -8.10241771e+04 -8.10322362e+04
-8.10402952e+04 -8.10483543e+04 -8.10564133e+04]
[-8.10080590e+04 -8.10161181e+04 -8.10241771e+04 -8.10322362e+04
-8.10402952e+04 -8.10483543e+04 -8.10564133e+04]
[-4.87020633e+01 -5.10046483e+01 -5.33072334e+01 -5.56098185e+01
-5.79124036e+01 -6.02149887e+01 -6.25175738e+01]
[-4.42129004e+01 -4.65154855e+01 -4.88180706e+01 -5.11206557e+01
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[-3.90535002e+01 -4.13560853e+01 -4.36586704e+01 -4.59612555e+01
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[-2.72684189e+01 -2.95283886e+01 -3.14844001e+01 -3.33173217e+01
-3.50706007e+01 -3.67157623e+01 -3.83252002e+01]

```


[-2.66482914e+01 -2.88778568e+01 -3.08338683e+01 -3.26294814e+01
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 [-2.05800445e+01 -2.24731935e+01 -2.41476267e+01 -2.56767857e+01
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 [-1.58429837e+01 -1.75333623e+01 -1.89837855e+01 -2.02838854e+01
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 -1.81816317e+01 -1.91673699e+01 -2.00888212e+01]

[-1.29008259e+01 -1.44525210e+01 -1.57676420e+01 -1.69302165e+01
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 [-1.06042515e+01 -1.20493187e+01 -1.32533579e+01 -1.43095284e+01
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 [-1.03496022e+01 -1.17824493e+01 -1.29733937e+01 -1.40175806e+01
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 -1.46603658e+01 -1.55068190e+01 -1.62875374e+01]
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 [-9.73849894e+00 -1.11403037e+01 -1.23057260e+01 -1.33198828e+01
 -1.42294416e+01 -1.50587874e+01 -1.58223712e+01]

```

[-9.62063755e+00 -1.10168444e+01 -1.21752089e+01 -1.31842986e+01
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[-8.61307430e+00 -9.96107355e+00 -1.10715309e+01 -1.20341881e+01
 -1.28902277e+01 -1.36657485e+01 -1.43772391e+01]]

```

```

In [21]: delta3 = distance(V_init, V_new)
         print("The distance metric is:", delta3)
         diff3 = delta3-delta2
         print("The difference between delta_{T-2} and delta_{T-1} is: ", diff3)
         diff4 = delta3-delta1
         print("The difference between delta_{T-2} and delta_{T-1} is: ", diff4)

```

The distance metric is: 45934472596.166245

The difference between delta_{T-2} and delta_{T-1} is: -18683055.693359375

The difference between delta_{T-2} and delta_{T-1} is: -29098599.93926239

5.21

```

In [23]: toler = 1e-9
         delta = 10.0
         VF_iter = 0
         V_init = np.zeros((N,M))

         while delta>toler and VF_iter<maxiters:
             VF_iter += 1
             EV = (V_init @ gamma).reshape((N,1))
             EV_mat = np.tile(EV.reshape((1,N)), (N,1))
             EV_mat[~c_pos] = -9e+4
             EV_D3array = np.array([EV_mat for e in range(M)])
             V_new_D3array = D3_array + beta*EV_D3array
             V_new = np.zeros((N,M))

```

```

W_prime = np.zeros((N,M))
for i in range(N):
    arr = V_new_D3array[:, i, :]
    V_new[i] = arr.max(axis=1)
    W_index = np.argmax(arr, axis=1)
    W_prime[i] = W_vec[W_index]
delta = distance(V_init, V_new)
V_init = V_new
print('Iteration=', VF_iter, ', distance= ', delta)

print("Converged.")
print("psi(W) is", W_prime)
print("V(W) is", V_init)
print("After {} iterations, V(W) converged.".format(VF_iter))

Iteration= 1 , distance= 45963571196.10551
Iteration= 2 , distance= 45953155651.859604
Iteration= 3 , distance= 45934472596.166245
Iteration= 4 , distance= 45901033774.70888
Iteration= 5 , distance= 45841440579.44991
Iteration= 6 , distance= 45736060684.31199
Iteration= 7 , distance= 45552402370.20611
Iteration= 8 , distance= 45241165831.95568
Iteration= 9 , distance= 44743295702.61597
Iteration= 10 , distance= 44048017695.89585
Iteration= 11 , distance= 43443854297.730034
Iteration= 12 , distance= 44442974185.644424
Iteration= 13 , distance= 52967967667.04762
Iteration= 14 , distance= 89997648384.71588
Iteration= 15 , distance= 226554825804.06644
Iteration= 16 , distance= 457375601904.8917
Iteration= 17 , distance= 192167632805.63055
Iteration= 18 , distance= 0.0
Converged.
psi(W) is [[0.01 0.01 0.01 0.01 0.01 0.01 0.01]
 [0.02 0.02 0.02 0.02 0.02 0.02 0.02]
 [0.03 0.03 0.03 0.03 0.03 0.03 0.03]
 [0.04 0.04 0.04 0.04 0.04 0.04 0.04]
 [0.05 0.05 0.05 0.05 0.05 0.05 0.05]
 [0.06 0.06 0.06 0.06 0.06 0.06 0.06]
 [0.07 0.07 0.07 0.07 0.07 0.07 0.07]
 [0.08 0.08 0.08 0.08 0.08 0.08 0.08]
 [0.09 0.09 0.09 0.09 0.09 0.09 0.09]
 [0.1 0.1 0.1 0.1 0.1 0.1 0.1]
 [0.11 0.11 0.11 0.11 0.11 0.11 0.11]
 [0.12 0.12 0.12 0.12 0.12 0.12 0.12]
 [0.13 0.13 0.13 0.13 0.13 0.13 0.13]
 [0.14 0.14 0.14 0.14 0.14 0.14 0.14]]

```

[0.15 0.15 0.15 0.15 0.15 0.15 0.15]
[0.16 0.16 0.16 0.16 0.16 0.16 0.16]
[0.17 0.17 0.17 0.17 0.17 0.17 0.17]
[0.18 0.18 0.18 0.18 0.18 0.18 0.18]
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[illegible]

[illegible]

[illegible]

[illegible]

5.22

```
In [24]: from mpl_toolkits import mplot3d

X, Y = np.meshgrid(W_vec, epsilon)
fig = plt.figure(figsize=(16, 12))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(X.T, Y.T, W_prime)
ax.set_xlabel('W_vec')
ax.set_ylabel('epsilon')
ax.set_title('Converged Policy Function')
ax.view_init(elev=30,azim=30)
plt.show()
```

