Stochastic Calculus for Finance I, Solution for Exercises

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This is the solution for the textbook *Stochastic calculus for finance I*, by *Steven E. Shreve*. If you have any comments or suggestions, please email me at sunyufei814@gmail.com.

Chapter 2 Probability Theory on Coin Toss Space

Exercise 2.12

Proof

(i)

At time 0, the value of the chooser option is determined by the expected payoff at time m, discounted to the present value.

At time m, the chooser option holder selects the higher-value option:

$$V_m = max(C_m, P_m)$$

Thus, the chooser option value at time 0 is:

$$V_0 = \frac{E[V_m]}{(1+r)^m}$$

Substitute $V_m = max(C_m, P_m)$:

$$V_0 = \frac{E[max(C_m, P_m)]}{(1+r)^m}$$

Using put-call parity at time m:

$$C_m = P_m + S_m - K \cdot e^{-r(N-m)}$$

Substituting into the payoff function $max(C_m, P_m)$:

$$V_m = max\ (P_m + S_m - K \cdot e^{-r(N-m)}, P_m)$$

Simplify:

$$V_m = P_m + max\ (S_m - K \cdot e^{-r(N-m)}, 0)$$

At time 0, the value of the chooser option becomes:

$$V_0 = \frac{E[P_m + max\ (S_m - K \cdot e^{-r(N-m)}, 0)]}{(1+r)^m}$$

Split the expectation into two terms:

$$V_0 = \frac{E[P_m]}{(1+r)^m} + \frac{E[max\ (S_m - K \cdot e^{-r(N-m)}, 0)]}{(1+r)^m}$$

Term 1: $\frac{E[P_m]}{(1+r)^m}$

- This represents the value of a **put option** expiring at time N, with strike price K.

Term 2: $\frac{E[max\ (S_m - K \cdot e^{-r(N-m)}, 0)]}{(1+r)^m}$

This represents the value of a **call option** expiring at time m, but with a modified strike price: $K' = K \cdot (1+r)^{N-m}$

From the breakdown above, the chooser option value at time 0 is:

$$V_0 = P_0(K, N) + C_0(K \cdot (1+r)^{N-m}, m)$$

Where:

- $P_0(K, N)$: Price of a put option expiring at time N, with strike price K.

$C_0(K \cdot (1+r)^{N-m}, m)$: Price of a call option expiring at time m, with strike price $K' = K \cdot (1+r)^{N-m}$.