Stochastic Calculus for Finance I, Solution for Exercises

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This is the solution for the textbook *Stochastic calculus for finance I*, by *Steven E. Shreve*. If you have any comments or suggestions, please email me at sunyufei814@gmail.com.

Chapter 2 Probability Theory on Coin Toss Space

Exercise 2.11

Proof

(i)

When $S_N = K$, then $C_N = (S_N - K)^+ = 0$, $P_N = (K - S_N)^+ = 0$, $F_N = S_N - K = 0$.

When $S_N > K$, then $P_N = (K - S_N)^+ = 0$, $C_N = F_N = S_N - K$. Therefore, $C_N = F_N + P_N$.

When $S_N < K$, then $C_N = (S_N - K)^+ = 0$, $P_N = -(S_N - K)$, $F_N = S_N - K$, Therefore, $C_N = F_N + P_N$.

(ii)

$$C_n = \tilde{E}_n\left[\frac{C_N}{(1+r)^{N-n}}\right] = \tilde{E}_n\left[\frac{F_N + P_N}{(1+r)^{N-n}}\right] = \tilde{E}_n\left[\frac{F_N}{(1+r)^{N-n}}\right] + \tilde{E}_n\left[\frac{P_N}{(1+r)^{N-n}}\right] = F_n + P_n$$

(iii)

$$F_0 = \tilde{E}_n\left[\frac{F_N}{(1+r)^N}\right] = \tilde{E}_n\left[\frac{S_N - K}{(1+r)^N}\right] = \tilde{E}_n\left[\frac{S_N}{(1+r)^N}\right] - \tilde{E}_n\left[\frac{K}{(1+r)^N}\right] = S_0 - \frac{K}{(1+r)^N}$$

(iv)

At time 0, the one has F_0 and one share of stock. $F_0 = S_0 + (F_0 - S_0)$.

At time N, the one has $S_N + (F_0 - S_0)(1+r)^N = S_N + \left(-\frac{K}{(1+r)^N}\right)(1+r)^N = S_N - K = F_N$.

(v)

If $K = (1+r)^N S_0$, then according to (iii), $F_0 = 0$. According to (ii), $C_n = F_n + P_n$, thus, $C_0 = F_0 + P_0$. Thus, $C_0 = P_0$.

(vi)

According to (ii), if $C_n = P_n$, then $F_n = 0$. Note $F_n = \tilde{E}_n\left[\frac{S_N - K}{(1+r)^{N-n}}\right] = S_n - \frac{K}{(1+r)^{N-n}} = S_n - \frac{(1+r)^N S_0}{(1+r)^{N-n}} = S_n - (1+r)^n S_0$ is not zero for all n.