Stochastic Calculus for Finance I, Solution for Exercises

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This is the solution for the textbook *Stochastic calculus for finance I*, by *Steven E. Shreve*. If you have any comments or suggestions, please email me at sunyufei814@gmail.com.

Chapter 2 Probability Theory on Coin Toss Space

Exercise 2.3

Proof

The key to this problem lies in applying Jensen's inequality, which states that if φ is a convex function and X is a random variable, then:

$$\varphi(E[X]) \leq E[\varphi(X)]$$

This inequality holds with equality if φ is linear or if X is constant.

Since M_n is a martingale, we know that $M_n = E_n[M_{n+1}]$.

Now consider $\varphi(M_{n+1})$. Using Jensen's inequality, we have:

$$\varphi(E_n[M_{n+1}]) \leq E[\varphi(M_{n+1})]$$

Since $M_n = E_n[M_{n+1}]$,

$$\boldsymbol{\varphi(M_n) \leq E[\varphi(M_{n+1})]}$$

Thus, $\varphi(M_n)$ is a submartingale.