Stochastic Calculus for Finance I, Solution for Exercises

by Sun Yufei

Last updated: August 22, 2024

This is the solution for the textbook *Stochastic calculus for finance I*, by *Steven E. Shreve*. If you have any comments or suggestions, please email me at sunyufei814@gmail.com.

Chapter 2 Probability Theory on Coin Toss Space

Exercise 2.2

Proof

(i)

According to the Figure 2.3.1, $S_3(HHH) = 32$, $S_3(HHT) = S_3(HTH) = S_3(THH) = 8$, $S_3(HTT) = S_3(THT) = S_3(TTH) = 2$, $S_3(TTT) = 0.5$.

Thus, we can conclude that $P(S_3 = 32) = \tilde{p}^3 = \frac{1}{8}$, $P(S_3 = 8) = 3\tilde{p}^2\tilde{q}^1 = \frac{3}{8}$, $P(S_3 = 2) = 3\tilde{p}^1\tilde{q}^2 = \frac{3}{8}$, $P(S_3 = 0.5) = \tilde{q}^3 = \frac{1}{8}$.

(ii)

According to the Figure 2.3.1, At time 1, $S_1(H) = 8$, $S_1(T) = 2$. Thus, we have $\tilde{E}(S_1) = 8P(S_1 = 8) + 2P(S_1 = 2) = \frac{1}{2} * 8 + \frac{1}{2} * 2 = 5$.

At time 2, $S_2(HH) = 16$, $S_2(HT) = S_2(TH) = 4$, $S_2(TT) = 1$. Thus, we have $\tilde{E}(S_2) = 16P(S_2 = 16) + 4P(S_2 = 4) + 1P(S_2 = 1) = 16 * \frac{1}{4} + 4 * \frac{1}{2} + 1 * \frac{1}{4} = 6.25$.

At time 3, according to (i), we have $\tilde{E}(S_3) = 32P(S_3 = 32) + 8P(S_3 = 8) + 2P(S_3 = 2) + 0.5P(S_3 = 0.5) = 32 * \frac{1}{8} + 8 * \frac{3}{8} + 2 * \frac{3}{8} + 0.5 * \frac{1}{8} = 7.8125$.

the average rates of growth of the stock price under P are $r_0 = \left(\frac{\tilde{E}(S_1)}{S_0} - 1\right) * 100\% = \left(\frac{5}{4} - 1\right) * 100\% = 25\%$, $r_1 = \left(\frac{\tilde{E}(S_2)}{\tilde{E}(S_1)} - 1\right) * 100\% = \left(\frac{6.25}{5} - 1\right) * 100 = 25\%$, $r_1 = \left(\frac{\tilde{E}(S_3)}{\tilde{E}(S_2)} - 1\right) * 100\% = \left(\frac{7.8125}{6.25} - 1\right) * 100 = 25\%$.

(iii)

Recalculate (i)

$P(S_3 = 32) = \tilde{p}^3 = \frac{8}{27}$, $P(S_3 = 8) = 3\tilde{p}^2\tilde{q}^1 = \frac{4}{9}$, $P(S_3 = 2) = 3\tilde{p}^1\tilde{q}^2 = \frac{2}{9}$, $P(S_3 = 0.5) = \tilde{q}^3 = \frac{1}{27}$.

Recalculate (ii)

At time 1, $\tilde{E}(S_1) = 8P(S_1 = 8) + 2P(S_1 = 2) = \frac{2}{3} * 8 + \frac{1}{3} * 2 = 6$.

At time 2, $\tilde{E}(S_2) = 16P(S_2 = 16) + 4P(S_2 = 4) + 1P(S_2 = 1) = 16 * \frac{4}{9} + 4 * \frac{4}{9} + 1 * \frac{1}{9} = 9$.

At time 3, according to (i), we have $\tilde{E}(S_3) = 32P(S_3 = 32) + 8P(S_3 = 8) + 2P(S_3 = 2) + 0.5P(S_3 = 0.5) = 32 * \frac{8}{27} + 8 * \frac{4}{9} + 2 * \frac{2}{9} + 0.5 * \frac{1}{27} = 13.5$.

the average rates of growth of the stock price under P are $r_0 = \left(\frac{\tilde{E}(S_1)}{S_0} - 1\right) * 100\% = \left(\frac{6}{4} - 1\right) * 100\% = 50\%$, $r_1 = \left(\frac{\tilde{E}(S_2)}{\tilde{E}(S_1)} - 1\right) * 100\% = \left(\frac{9}{6} - 1\right) * 100 = 50\%$, $r_1 = \left(\frac{\tilde{E}(S_3)}{\tilde{E}(S_2)} - 1\right) * 100\% = \left(\frac{13.5}{9} - 1\right) * 100 = 50\%$.