Stochastic Calculus for Finance I, Solution for Exercises

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This is the solution for the textbook *Stochastic calculus for finance I*, by *Steven E. Shreve*. If you have any comments or suggestions, please email me at sunyufei814@gmail.com.

Chapter 2 Probability Theory on Coin Toss Space

Exercise 2.7

Proof

Process Definition:

- Gambling Outcomes: Each round of the game results in X_i, which are independent and fair (like coin tosses). Each X_i takes values of +1 (win) or −1 (loss), both with a probability of 1/2.
- Cumulative Gains S_n: The cumulative gain after the $n-th$ round is defined recursively by:

$$S_{n+1} = S_n + b_n(X_1 + X_2, \dots, X_n)X_{n+1}$$

where S_n represents the cumulative gain after n rounds, and $b_n(X_1 + X_2, \dots, X_n)$ is the bet size for the next round, which is a function based on the outcomes of all previous rounds.

Martingale Property:

- This process S_n is a martingale because, given the past outcomes, the expected future gain is zero:

$$E[S_{n+1}|S_1, S_2, \dots, S_n] = S_n$$

This is due to the fair nature of the game, where the expected outcome of X_{n+1} is zero.

Non-Markov Property:

- However, S_n is not a Markov process because the next state S_{n+1} depends not only on the current state S_n but also on the entire history of past outcomes through $b_n(X_1 + X_2, \dots, X_n)$. In a Markov process, the future state should depend only on the current state, not the full sequence of past states.