Stochastic Calculus for Finance I, Solution for Exercises

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This is the solution for the textbook *Stochastic calculus for finance I*, by *Steven E. Shreve*. If you have any comments or suggestions, please email me at sunyufei814@gmail.com.

Chapter 1 The Binomial No-Arbitrage Pricing Model

Exercise 1.5

Analysis of the exercise

Consider an agent who initially sells a lookback option for $V_0 = 1.376$ and purchases $\Delta_0 = 0.1733$ shares of stock at time zero. At the first time point, if the stock price increases, the portfolio will be valued at $V_1(H) = 2.24$. Now, assume she adjusts her position in the stock by holding $\Delta_1(H) = \frac{V_2(HH)-V_2(HT)}{S_2(HH)-S_2(HT)}$shares. Demonstrate that, at time two, if the stock price rises again, her portfolio will be valued at $V_2(HH) = 3.2$. Conversely, if the stock price falls, her portfolio will be worth $V_2(HT) = 2.4$. Finally, assuming the stock price rises in the first period and falls in the second period, suppose the agent adjusts her position to $\Delta_2(HT) = \frac{V_3(HTH)-V_3(HTT)}{S_3(HTH)-S_3(HTT)}$ shares. Show that at time three, if the stock price rises in the third period, her portfolio will be valued at $V_3(HTH) = 0$, whereas if the stock price falls, her portfolio will be worth $V_3(HTT) = 6$. This demonstrates that she has successfully hedged her short position in the option.

We need to approve:

$$X_2(HH) = V_2(HH) = 3.20$$

$$X_2(HT) = V_2(HT) = 2.40$$

$$X_3(HTH) = V_3(HTH) = 0$$

$$X_3(HTT) = V_3(HTT) = 6$$

Proof

The agent adjusts her position based on the new information:

- After observing the stock price at time 1, she adjusts her position by buying $\Delta_1(H)$ shares if the stock price goes up.

Given:

$$\Delta_1(H) = \frac{V_2(HH) - V_2(HT)}{S_2(HH) - S_2(HT)} = \frac{3.20 - 2.40}{16 - 4} = \frac{1}{15}$$

Thus, this will leave her to invest $X_1(H) - \Delta_1(H)S_1(H) = 2.24 - \frac{1}{15} * 8 = \frac{128}{75}$ in the money market.

According to the formula 1.2.14, at the time two, if the stock goes up again, she will have a portfolio valued at

$X_2(HH)$

$= \Delta_1(H)S_2(HH) + (1 + r)[X_1(H) - \Delta_1(H)S_1(H)]$

$=\frac{1}{15} * 16 + (1 + 0.25) * \frac{128}{75} = \mathbf{3.2}$

If the stock goes down, she will have a portfolio valued at

$X_2(HT)$

$= \Delta_1(H)S_2(HT) + (1 + r)[X_1(H) - \Delta_1(H)S_1(H)]$

$=\frac{1}{15} * 4 + (1 + 0.25) * \frac{128}{75} = \mathbf{2.4}$

- After observing the stock price at time 2, she adjusts her position by buying $\Delta_1(HT)$ shares if the stock price goes up first then goes down.

Given:

$$\Delta_2(HT) = \frac{V_3(HTH) - V_3(HTT)}{S_3(HTH) - S_3(HTT)} = \frac{0 - 6}{8 - 2} = -1$$

Thus, this will leave her to invest $X_2(HT) - \Delta_2(HT)S_2(HT) = 2.4 - (-1) * 4 = 6.4$ in the money market.

According to the formula 1.2.14, at the time three, if the stock goes up again, she will have a portfolio valued at

$X_3(HTH)$

$= \Delta_2(HT)S_3(HTH) + (1 + r)[X_2(HT) - \Delta_2(HT)S_2(HT)]$

$= -1 * 8 + (1 + 0.25) * 6.4 = \mathbf{0}$

If the stock goes down, she will have a portfolio valued at

$X_3(HTT)$

$= \Delta_2(HT)S_3(HTT) + (1 + r)[X_2(HT) - \Delta_2(HT)S_2(HT)]$

$= -1 * 2 + (1 + 0.25) * 6.4 = \mathbf{6}$

PROOF COMPLETED.