Stochastic Calculus for Finance I, Solution for Exercises

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This is the solution for the textbook *Stochastic calculus for finance I*, by *Steven E. Shreve*. If you have any comments or suggestions, please email me at sunyufei814@gmail.com.

Chapter 2 Probability Theory on Coin Toss Space

Exercise 2.6

Proof

To show that InI_nIn is a martingale, we must show that:

$$E[I_{n+1}] = In$$

Let's compute I_{n+1}:

$$I_{n+1} = \sum_{j=0}^{n} \Delta_j (M_{j+1} - M_j)$$

We can rewrite this as:

$$I_{n+1} = \sum_{j=0}^{n-1} \Delta_j (M_{j+1} - M_j) + \Delta_n (M_{n+1} - M_n)$$

Notice that the first part of this sum is I_n:

$$I_{n+1} = I_n + \Delta_n (M_{n+1} - M_n)$$

$$E[I_{n+1}] = E[I_n + \Delta_n (M_{n+1} - M_n)]$$

$$E[I_{n+1}] = I_n + \Delta_n E[(M_{n+1} - M_n)]$$

Because M_n is a martingale, we have:

$$E[(M_{n+1} - M_n)] = 0$$

Substitute this back into the equation:

$$E[I_{n+1}] = I_n + \Delta_n * 0 = I_n$$

Thus, we have shown that:

$$E[I_{n+1}] = I_n$$