A survey of statistical arbitrage pairs trading strategies with non-machine learning methods, 2016-2023

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Abstract

This review examines the growing body of literature on the paired trading framework, which involves relative value arbitrage strategies between two or more securities. The existing research is categorized into five groups: the distance method uses non-parametric distance measures to identify paired trading opportunities; the cointegration method relies on formal cointegration tests to reveal stationary spread time series; the time series method focuses on finding optimal trading rules for mean-reverting spreads; the stochastic control method aims to determine the optimal portfolio holdings in paired trading relative to other available securities; the "other methods" category includes additional related paired trading frameworks, albeit with limited supporting literature. By thoroughly evaluating the extensive body of research, comprising over 100 papers from 2016 to 2023, we ultimately reveal the strengths and weaknesses associated with further research and implementation.

Keywords

Statistical arbitrage, pairs trading, distance method, cointegration method, time series method, stochastic control method, mean-reversion

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1. Introduction

The reason for selecting papers from 2016 to 2023 in this review is that there already exists a review by Krauss, C. (2017) that covers this topic from 2006 to 2015.Although this review primarily examines the literature from 2016 to 2023, it begins with a discussion of the earliest paper, Gatev et al. (2006), to provide a comprehensive perspective.

Based on the work of Gatev et al. (2006), pairs trading is conceptually straightforward and involves two main steps. First, identify two securities that have historically shown a synchronized price movement during a formation period. Then, in a subsequent trading period, observe the spread between their prices. If the prices deviate and the spread increases, the strategy involves shorting the security that has gained more value and buying the one that has lost value. Assuming the two securities maintain an equilibrium relationship, the spread is expected to revert to its historical average. Once this reversion occurs, the positions are closed to realize a profit. The basic idea of univariate pairs trading can be expanded to more complex scenarios. In quasi-multivariate models, a single security is traded against a weighted portfolio of other correlated securities. In fully multivariate models, entire groups of stocks are traded against other groups. These sophisticated strategies can be collectively referred to as (quasi-)multivariate pairs trading, generalized pairs trading, or statistical arbitrage. All these methods are encompassed under the broader category of "statistical arbitrage pairs trading" (or simply "pairs trading"), as they represent the foundation for more advanced techniques (Vidyamurthy, 2004; Avellaneda and Lee, 2010). Pairs trading is also related to other long-short strategies, such as those exploiting discrepancies from the law of one price, lead-lag effects, and return reversals. For a detailed exploration of these and other long-short return phenomena, refer to Jacobs (2015).

The seminal paper in the pairs trading field, authored by Gatev et al. (2006), referred to as GGR, has received significant attention. The paper presents a straightforward yet powerful algorithm applied to a broad dataset of U.S. equities, with careful adjustments to mitigate data snooping biases. The results demonstrate annualized excess returns of up to 11 percent, with minimal exposure to systematic risk factors. Crucially, these returns cannot be attributed to previously recognized sources of profit, such as the reversal profits identified by Jegadeesh (1990) and Lehmann (1990) or the momentum profits described by Jegadeesh and Titman (1993). The persistence of these unexplained excess returns has cemented GGR's pairs trading strategy as a notable anomaly in capital markets, enduring through time and receiving validation from subsequent research, including studies by Do and Faff (2010, 2012).

Despite these insights, it is important to note that academic research on pairs trading remains relatively limited compared to studies on contrarian and momentum strategies. However, recent years have seen a notable increase in interest, resulting in a growing body of theoretical frameworks and empirical applications across various asset classes. The apparent simplicity of

GGR's strategy becomes less evident with these recent advancements. In this review, we categorize the literature on pairs trading into five distinct streams:

Distance Approach: This is the most extensively studied framework within pairs trading research. During the formation period, various distance metrics are utilized to identify pairs of securities that move together. In the trading period, simple non-parametric threshold rules trigger trading signals. The strengths of this strategy lie in its simplicity and transparency, making it suitable for large-scale empirical applications. Research consistently shows that distance-based pairs trading can be profitable across different markets, asset classes, and time frames.

Cointegration Approach: This approach involves applying cointegration tests during the formation period to identify co-moving securities. Simple algorithms, often based on GGR's threshold rule, are used to generate trading signals during the trading period. The main advantage of this method is the more statistically reliable equilibrium relationship identified between the pairs.

Time Series Approach: Unlike other approaches, the formation period is generally disregarded here. Researchers assume that co-moving securities have been identified through previous analyses. The focus is on the trading period, specifically on generating optimized trading signals using various time series analysis methods, such as modeling the spread as a mean-reverting process.

Stochastic Control Approach: Similar to the time series approach, this method also overlooks the formation period. This stream of literature focuses on determining the optimal portfolio holdings for pairs trades compared to other available assets. Stochastic control theory is employed to identify the value and optimal policy functions for the portfolio in question.

Other Approaches: This category includes additional pairs trading frameworks that have relatively limited supporting literature and are not closely related to the previously discussed approaches. Among these are also the non-machine learning approaches, the copula approach, and the Hurst exponent approach. These methods explore alternative techniques and models to identify trading opportunities, often incorporating more complex statistical and computational tools.

Table 1 offers a summary of representative studies for each approach, detailing the data samples used and the performance evaluation metrics as reported in the respective studies.

Table 1. Overview pairs trading approaches

Approach	Representative articles	Data Sample	Performance Evaluation Metrics
Distance	Bowen and Hutchinson	London Stock	Sharpe ratio,

	(2016) Zhang and Urquhart (2019)	Exchange stocks 1979-2012 CSI300, HSHKI, HSAHP stocks 1996-2017	Annualized returns Average excess returns, Standard deviation, Abnormal returns
Cointegration	Ekkarntrong et al. (2017) Figuerola-Ferretti et al. (2018)	U.S. The Global Dow stocks 2002-2012 STOXX Europe 600 stocks 2000-2017	Excess returns Sharpe ratio, Standard deviation
Time series	Chen et al. (2017) Kim et al. (2017)	U.S. stocks 2006-2014 KOSPI 100 stocks 2005-2015	Round-trip trades, Annualized returns Cumulative returns, Annualized Sharpe Ratio
Stochastic control	Göncü and Akyildirim (2016) Liu et al. (2017)	U.S. U.K. commodity futures 1997-2015 U.S. Oil stocks 2007-2008, 2013-2015	Cumulative returns, Annualized Returns Annualized Sharpe ratio, Annualized returns
Others: Copula	Xie et al. (2016) Krauss and Stübinger (2017)	U.S. stocks 2003-2012 U.S. S&P 100 stocks 1990-2014	Excess returns, Cumulative returns Annualized Sharpe ratio, Excess returns
Others: Hurst exponent	Ramos-Requena et al. (2017) Ramos-Requena et al. (2021)	U.S. The Global Dow stocks 2000-2015 U.S. Nasdaq Inc stocks 2000-2021	WO, LO, PAOW, PAOL Average annualized returns, Sharpe ratio
Others: Entropy	Amer and Islam (2023)	PSX stocks 2017-2019	Annualized returns

Considering the diversity within the mentioned categories, this survey makes two key contributions: First, it provides a comprehensive review of the literature on pairs trading across five main approaches. Second, it discusses in detail the most significant contributions within each category. Drawing from an extensive collection of over 100 papers, the survey offers an in-depth analysis of each approach, highlighting strengths and weaknesses that are pertinent for further research and practical implementation. This makes the survey valuable for both researchers and practitioners. The structure of the remainder of this document is outlined as follows: Section 1 introduces the relevance of the research topic within the field of finance and defines the primary objectives of the study, outlining the key research questions and the unique contributions made to existing literature on pairs trading strategies. Section 2 reviews related work, providing an overview of foundational research, key findings, methodologies, and gaps that this study aims to address. Section 3 presents a brief overview of the scope of the study, detailing the main areas of investigation and offering a roadmap for the subsequent sections. Section 4 focuses on data analysis, describing the data sources, processing techniques, and preliminary analysis steps that form the basis for the methodological exploration. Section 5 examines non-machine learning models in pairs trading, including distance methods, cointegration methods, stochastic control methods, time series methods, and other approaches such as the Copula, Hurst exponent, and entropic methods, providing a comprehensive analysis of each model's theoretical foundation and empirical applications. Section 6 presents the conclusions of the study, summarizing the key findings and their implications, and outlines potential directions for future research.

2. Related Work

In this section, I explore the diverse applications of distance, time series, cointegration, and stochastic control methods in pairs trading, illustrating how these approaches can be tailored to enhance trading strategies. This literature review outlines the progressive integration of these methods into pairs trading, highlighting the shift from traditional techniques to advanced computational models.

The distance method was the earliest approach used to identify trading pairs based on market data. Due to its simplicity and ease of understanding, this method selects pairs by calculating the distance between historical prices, making it a fundamental tool in pairs trading for finding asset pairs with similar price movements. Time series methods were subsequently developed to build on the distance method, analyzing the time series data of asset prices using statistical and mathematical models, such as mean reversion models. This approach focuses on the dynamic changes in asset prices over time to capture arbitrage opportunities, refining both the timing and selection of trades.

As the study of pairs trading evolved, cointegration methods were introduced to identify asset pairs that maintain a certain equilibrium relationship in the long run, despite short-term divergences. This method is particularly valuable for capturing long-term arbitrage opportunities. Stochastic control methods represent the latest frontier in this field, involving sophisticated mathematical and computational models. These methods employ stochastic processes and optimal control theory to dynamically adjust trading strategies in response to market changes, aiming to maximize returns or minimize risks. This approach is at the cutting edge of pairs trading research.

Throughout these developments, these methods have significantly enhanced the effectiveness and profitability of pairs trading, underscoring their importance in shaping future financial market strategies.

2.1 Distance Methods

The exploration begins with a look at the distance method, where algorithms calculate the distance between historical price data to identify trading pairs. This method is pivotal for discovering naturally occurring correlations in financial markets and has been instrumental in the development of pair trading strategies.

Bowen, D.A. and Hutchinson, M.C. (2016) provide the first comprehensive analysis of the profitability and risk characteristics of the pairs trading strategy in the UK equity market. The study employs a statistical arbitrage approach, particularly focusing on short-term price reversal strategies, by matching pairs of stocks that exhibit similar price movements in the past and trading them when their prices deviate beyond a specific threshold. The results indicate that the strategy performs well during financial crises, exhibits market-neutral characteristics, and has limited exposure to known equity risk factors. Despite accounting for transaction costs, the strategy consistently shows high returns across different market and economic states.

Chen, H. et al. (2019) empirically investigate the profitability of a pairs trading strategy based on historical return correlations and short-term price reversal. The method involves selecting highly correlated stock pairs and taking opposite long and short positions when prices deviate beyond a threshold. The findings reveal significant abnormal returns in the US market, particularly driven by short-term reversal and industry momentum.

Zhang, H. and Urquhart, A. (2019) investigate the profitability of a distance-based pairs trading strategy across mainland China and Hong Kong stock markets. The method involves selecting stock pairs by calculating the Sum of Squared Deviations of their price series and trading when the deviation from the historical mean exceeds two standard deviations. The findings indicate that the strategy is more effective in less integrated markets, especially during periods of market turbulence.

In the paper by Ramos-Requena, J.P., Trinidad-Segovia, J.E., and Sánchez-Granero, M.Á. (2020), various methods for pairs trading are explored, including minimal distance, cointegration, correlation, and Hurst exponent approaches. The study focuses on identifying stock pairs using these methods and trading based on price deviations, with particular emphasis on long memory characteristics and mean reversion in time series. The findings indicate that employing these diverse methods enhances the profitability of pairs trading strategies, especially during periods of market anomalies and high volatility.

In summary, the reviewed studies highlight the diverse approaches within pairs trading, particularly focusing on the distance method and other related techniques. The distance method, which involves calculating the proximity between historical price series to identify trading pairs, has proven effective in various market conditions, including financial crises and periods of high volatility. Studies by Bowen and Hutchinson (2016) and Zhang and Urquhart (2019) demonstrate the robustness of this approach, particularly in less integrated markets and during turbulent times, showcasing its market-neutral characteristics and resilience against equity risk factors. Further, Chen et al. (2019) and Ramos-Requena et al. (2020) expand on these findings by incorporating additional strategies such as short-term price reversal, industry momentum, and long memory characteristics, indicating that integrating multiple methods can enhance the profitability of pairs trading. Overall, these findings suggest that distance-based and related approaches remain valuable tools in pairs trading, capable of adapting to varying market environments and uncovering profitable opportunities even amidst market anomalies.

2.2 Cointegration Methods

Cointegration methods identify pairs of assets that share a long-term equilibrium relationship, despite their individual non-stationarity. By using econometric techniques such as the Engle-Granger method or the Johansen test, these methods detect whether a linear combination of the assets remains stationary over time. This allows traders to exploit deviations from the equilibrium in pairs trading strategies by taking positions based on the expectation of mean reversion in the price spread. Cointegration methods are favored for their ability to reveal persistent relationships between assets, though they require ample historical data and assume that these relationships will continue in the future.

Cartea, A. and Jaimungal, S. (2016) presents an optimal trading strategy for co-integrated assets by modeling the structural dependence of asset prices through a co-integration factor, referred to as short-term alpha. They derive an explicit closed-form solution for the dynamic investment strategy, which is affine in the co-integration factor, and demonstrate its effectiveness using simulations calibrated with high-frequency Nasdaq data from Google, Facebook, and Amazon. The method showcases how short-term deviations in asset prices can be exploited for profit, with potential extensions including out-of-sample testing and handling portfolios with both liquid and illiquid assets.

Huang, Z. and Martin, F. (2019) develop pairs trading strategies within a cointegration framework, applying the Engle-Granger test and ECM-DCC-GARCH models to identify and exploit long-term equilibrium relationships between asset pairs. The study compares multiple methods, including percentage strategy, standard deviation of cointegration residuals, and Bollinger Bands, with an emphasis on optimizing profit factor. Their results show that Bollinger Bands without GARCH confirmation provided the highest profit factor, demonstrating the effectiveness of cointegration-based approaches in pairs trading.

Saji (2021) explores pairs trading potential in the Indian metals market using a cointegration approach. The study applies Johansen cointegration tests and Vector Error Correction Models (VECM) to examine the long-term equilibrium relationships between spot and futures prices of metals such as aluminum, copper, nickel, and zinc. The results demonstrate the effectiveness of cointegration-based strategies in uncovering price discovery and supporting pairs trading within volatile commodity markets.

Brunetti, M. and De Luca, R. (2023) propose a cointegration-based pairs trading strategy, utilizing Johansen cointegration tests to identify stock pairs with long-term equilibrium relationships. The paper explores the impact of pre-selection methods, comparing seven different metrics such as log-price correlation and covariance to reduce computational complexity. Their results demonstrate significant variation in profitability and risk exposure depending on the pre-selection method used, emphasizing the importance of careful metric selection in pairs trading strategies.

In summary, cointegration methods are valuable tools for identifying pairs of assets that maintain a long-term equilibrium relationship, allowing traders to capitalize on mean reversion in price spreads. The studies reviewed demonstrate the effectiveness of these methods in various markets and contexts. Cartea and Jaimungal (2016) highlight the practical application of cointegration-based strategies in high-frequency trading, using a co-integration factor to derive dynamic investment strategies that leverage short-term deviations. Huang and Martin (2019) showcase the flexibility of cointegration approaches by comparing different trading strategies, finding that Bollinger Bands without GARCH confirmation yielded the highest profit factor, underscoring the potential for profit optimization in pairs trading. Saji (2021) extends the application of cointegration methods to the Indian metals market, revealing how these techniques can support price discovery and trading strategies in volatile commodity markets. Brunetti and De Luca (2023) emphasize the importance of careful pre-selection of stock pairs, demonstrating that the choice of metrics significantly impacts the profitability and risk profile of cointegration-based strategies. Collectively, these findings affirm the robustness and adaptability of cointegration methods in pairs trading, particularly in identifying and exploiting persistent relationships between assets.

2.3 Stochastic Control Methods

Stochastic control methods in pairs trading use dynamic optimization to continuously adjust portfolio positions based on the stochastic behavior of asset prices. By modeling price movements with differential equations or stochastic calculus, such as Hamilton-Jacobi-Bellman equations, these methods aim to maximize returns or minimize risks in real-time. Unlike static approaches like cointegration or distance methods, stochastic control provides adaptive decision-making in volatile markets, though it requires advanced mathematical modeling and significant computational resources.

Göncü, A. and Akyıldırım, E. (2016) examine statistical arbitrage with pairs trading by modeling the spread between two assets using a mean-reverting Ornstein-Uhlenbeck (OU) process. The authors employ a stochastic control approach, optimizing buy and sell thresholds to maximize the probability of successful termination of the pairs trading strategy. Empirical backtesting on various stock pairs demonstrates the effectiveness of this method in generating profitable trades.

Endres, S. and Stübinger, J. (2019) develop an optimal pairs trading strategy based on a Lévy-driven Ornstein-Uhlenbeck process, leveraging the stochastic control approach. The method dynamically optimizes entry and exit thresholds by maximizing expected returns through first-passage time calculations. Empirical results from high-frequency data on S&P 500 constituents between 1998 and 2015 demonstrate the strategy's profitability across various economic sectors.

Zhu, D.M. et al. (2021) explores optimal pairs trading strategies within a dynamic mean-variance framework, using the Ornstein-Uhlenbeck process to model the price spread between two correlated assets. The method employs stochastic control, solving Hamilton-Jacobi-Bellman (HJB) equations to optimize trading decisions and achieve time-consistent solutions. Empirical analysis on stock and futures data from China's markets demonstrates the effectiveness of this approach in maximizing returns while controlling risk.

Xing, H. (2022) presents an optimal pairs trading strategy using a singular stochastic control approach with proportional transaction costs. The method models the asset price spread as a mean-reverting Ornstein-Uhlenbeck process and solves Hamilton-Jacobi-Bellman (HJB) equations to optimize both trade timing and size dynamically. Empirical results from U.S. stock data demonstrate the effectiveness of this approach in maximizing terminal wealth while managing transaction costs.

In summary, stochastic control methods provide a dynamic and adaptive approach to pairs trading by continuously optimizing portfolio positions based on the stochastic behavior of asset prices. The studies reviewed illustrate the versatility and effectiveness of these methods in various financial contexts. Göncü and Akyıldırım (2016) utilize a mean-reverting Ornstein-Uhlenbeck process to model asset spreads, employing stochastic control to optimize buy and sell thresholds, which enhances the profitability of pairs trading strategies through adaptive decision-making. Endres and Stübinger (2019) expand on this by incorporating a Lévy-driven Ornstein-Uhlenbeck process, optimizing entry and exit points dynamically to maximize expected returns, with empirical results confirming its effectiveness across different economic sectors using high-frequency data. Zhu et al. (2021) further refine the approach by integrating a dynamic mean-variance framework, applying Hamilton-Jacobi-Bellman equations to achieve optimal, time-consistent trading strategies, which are particularly effective in maximizing returns while controlling risk in China's markets. Lastly, Xing (2022) addresses the challenge of transaction costs by employing a singular stochastic control method that optimizes trade timing and size, demonstrating significant improvements in terminal wealth management. Collectively, these studies highlight the potential of stochastic control methods to provide sophisticated, real-time optimization in pairs trading, offering robust strategies that can adapt to the complexities and volatilities of financial markets.

2.4 Time Series Methods

Time series methods in pairs trading focus on identifying statistical relationships between asset prices over time. These methods often involve modeling asset price spreads using techniques like autoregressive models, moving averages, or mean-reverting processes such as the Ornstein-Uhlenbeck model. By analyzing historical price patterns, time series methods seek to forecast future price movements and determine trading signals. Unlike stochastic control methods, time series approaches tend to be more straightforward, relying on fixed rules for entry and exit, but they may lack the flexibility to adjust to rapid market changes.

De Moura, C.E. et al. (2016) present a pairs trading strategy leveraging linear state space models and the Kalman filter to model the spread between two related assets. By estimating conditional probabilities that the spread will revert to its long-term mean, the strategy determines optimal buy and sell points. The method incorporates elements of stochastic control through dynamic optimization, alongside traditional time series modeling techniques.

Chodchuangnirun, B. et al. (2018) introduce a pairs trading strategy utilizing nonlinear autoregressive GARCH models, specifically focusing on Markov Switching, Threshold, and Kink models with GARCH effects. By modeling the return spread of stock pairs and identifying regime shifts, the approach captures both dynamic volatility and non-linear behaviors in financial time series, leading to more accurate trading signals. The empirical results demonstrate that the Markov Switching model performs slightly better compared to the other models in generating profitable pairs trading strategies.

Zhang, G. (2021) explores a pairs trading strategy utilizing a general state space model to capture the spread between asset pairs. By modeling the spread as a mean-reverting process with non-Gaussianity and heteroscedasticity, the method applies a Monte Carlo-based optimization to determine optimal trading rules. This approach enhances the profitability and risk-adjusted returns compared to conventional pairs trading models by effectively capturing the complex dynamics in financial data.

Lee, K. et al. (2023) present a diversification framework for multiple pairs trading strategies, utilizing Ornstein–Uhlenbeck processes to model the mean-reverting behavior of asset spreads.

The framework employs dynamic capital allocation methods such as Mean Reversion Budgeting (MRB) and Mean Reversion Ranking (MRR) to optimize trading across multiple pairs. The empirical results demonstrate that these methods improve portfolio performance by leveraging the statistical properties of mean reversion and enhancing diversification.

In summary, time series methods in pairs trading utilize statistical relationships between asset prices over time, focusing on mean-reversion and predictive patterns to generate trading signals. De Moura et al. (2016) use linear state space models and the Kalman filter to dynamically optimize buy and sell points, while Chodchuangnirun et al. (2018) employ nonlinear autoregressive GARCH models, including Markov Switching and Threshold models, to capture regime shifts and volatility dynamics, improving signal accuracy and profitability. Zhang (2021) enhances these methods with a general state space model incorporating non-Gaussianity and heteroscedasticity, applying Monte Carlo-based optimization for optimal trading rules, leading to superior risk-adjusted returns. Lee et al. (2023) further extend the approach by introducing a diversification framework with Ornstein-Uhlenbeck processes and dynamic capital allocation strategies, which optimize performance across multiple pairs through effective mean reversion exploitation. Collectively, these studies demonstrate the efficacy of time series methods in pairs trading, offering robust yet adaptable strategies that effectively leverage historical data for trading decisions.

2.5 Other Methods

Beyond above four methods, several alternative approaches have been employed in pairs trading to enhance strategy performance and manage risks. These methods explore different statistical and optimization techniques to refine trade signals and optimize portfolio returns.

2.5.1 Copula Approach

One prominent approach is the Copula approach, which is used to model and analyze the dependence structure between assets in a pairs trading strategy. This approach goes beyond simple correlations by allowing more flexible modeling of the joint distribution of asset returns, capturing tail dependencies and providing a more nuanced understanding of risk. Variations like the Mixed Copula model further enhance this by combining different copula functions to better reflect complex relationships between asset pairs.

Nadaf, T. et al. (2022) introduces a copula-based pairs trading strategy, utilizing the Laplace marginal distribution function to model financial returns. By constructing a copula function that accounts for fat tails in the data, the method effectively captures the dependency structure between asset pairs. This approach enhances the accuracy of trading signals by predicting price movements based on the joint distribution of the paired assets.

da Silva et al. (2023) presents a pairs trading strategy based on a mixed copula model, which combines multiple copulas to capture both linear and nonlinear dependencies between asset pairs. By calculating a mispricing index using an optimal linear combination of copulas, the method enhances the flexibility and accuracy of identifying trading opportunities. This approach proves effective in adapting to varying market conditions and improving trading performance.

2.5.2 Hurst Exponent Approach

The Hurst exponent is another statistical tool that measures the tendency of a time series to either persist in its trend or revert to the mean. A higher Hurst exponent suggests stronger trending behavior, while a lower exponent indicates more mean-reverting tendencies, both of which can be used to adjust trading strategies accordingly.

Ramos-Requena, J.P. et al. (2021) presents a cooperative dynamic approach to pairs trading, utilizing the Hurst exponent and volatility as key selection criteria for identifying pairs of stocks with strong comovement. Once selected, a mean-reversion strategy is applied to exploit price deviations from the historical relationship between the stock pairs. This method enhances the performance of pairs trading by incorporating advanced filtering techniques to select optimal pairs with low volatility and high comovement.

2.5.3 Entropic Approach

Entropy-based methods focus on measuring the uncertainty or randomness within financial data. By analyzing the entropy of asset price distributions, these methods can be used to identify periods of higher uncertainty, providing an additional layer of insight for trading decisions.

Amer, L. and Islam, T.U. (2023) introduce a novel pairs trading strategy that combines cointegration techniques with an entropic approach to optimize trading decisions. The strategy uses the Ornstein-Uhlenbeck process to model mean reversion while employing entropy as a penalty function to account for model uncertainty. By identifying optimal boundary points for entering and exiting trades, the method improves profitability and risk management in pairs trading.

In summary, alternative approaches to pairs trading, such as the Copula approach, Hurst exponent, and entropic methods, offer innovative ways to enhance strategy performance and manage risks. The Copula approach, as demonstrated by Nadaf et al. (2022) and da Silva et al. (2023), provides a sophisticated framework for modeling dependencies between asset pairs, capturing tail dependencies and complex relationships that traditional correlation measures may miss. This allows for more accurate trading signals and improved adaptability to varying market conditions. The Hurst exponent approach, highlighted by Ramos-Requena et al. (2021), utilizes measures of persistence and mean-reversion tendencies to select optimal pairs, enhancing the effectiveness of mean-reversion strategies through advanced filtering techniques. The entropic approach, introduced by Amer and Islam (2023), integrates entropy as a measure of uncertainty into pairs trading, combining it with cointegration techniques to refine trade timing and improve risk management. Together, these methods expand the toolkit for pairs trading, offering more nuanced and adaptable strategies that better account for the complexities of financial markets.

3. Landscape Overview

In this section, we explore the key document that forms the foundation of our research. This paper, along with additional supporting materials, was sourced through extensive database searches from well-established academic platforms, including but not limited to Google Scholar. The research process utilized multiple scientific repositories such as Scopus, Google Scholar, Springer, IEEE Xplore, Science Direct, and Web of Science to ensure a thorough collection of relevant literature. The search strategy incorporated a diverse set of keywords, such as "Pair Trading," "Pair Trading with statistical approaches," "Pair Trading using Distance Measures," "Pair Trading with Cointegration Techniques," "Pair Trading through Stochastic Control Models," and "Pair Trading utilizing Time Series Methods." These terms were chosen to comprehensively explore the application of machine learning in finance. Papers that were not directly relevant to the main focus of our thematic analysis were excluded. **Table 2** below presents a year-by-year breakdown of the articles that were reviewed, along with complete reference citations for each entry.

Table 2. Summary of the number of publications analyzed per year, spanning from 2016 to 2023.

Year	Count	Article
2016	21	[1-13]
2017	16	[14-29]
2018	24	[30-53]
2019	12	[54-65]
2020	14	[66-79]
2021	19	[80-98]
2022	12	[99-110]
2023	13	[111-123]

Table 2 summarizes the distribution of different pairs trading methods across various studies. Distance methods appear in 21 studies, focusing on identifying pairs based on price divergence to exploit mean reversion opportunities. Cointegration methods are the most prevalent, being applied in 43 studies. These methods utilize long-term equilibrium relationships between asset prices to identify pairs trading opportunities, demonstrating their widespread use in the field. Stochastic control methods are also prominent, featured in 40 studies. These approaches

emphasize the use of stochastic processes and dynamic optimization to manage market uncertainties and enhance trading strategies. Time series methods are utilized in 20 studies, leveraging temporal price patterns and trends for pairs trading. Lastly, other methods are applied in 16 studies, representing a diverse range of approaches that fall outside the primary categories. This distribution highlights the dominance of cointegration and stochastic control methods in pairs trading research, while also indicating the presence of alternative strategies tailored to specific market conditions.

Table 3. Summary of the number of publications analyzed per pairs trading method.

Methods	Count	Article
Distance Methods	21	[6, 10, 13, 14, 22, 25, 26, 43, 45, 51, 56, 60, 63, 72, 73, 77-79, 97, 98, 120]
Cointegration Methods	43	[4, 8, 11, 13-15, 24, 26-28, 30, 34, 35, 37, 43-45, 50, 52, 53, 55, 61, 62, 67, 69, 72, 73, 75, 79, 86, 89, 93, 96-98, 102, 107, 109, 114, 115, 120, 121, 123]
Stochastic Control Methods	40	[1, 3, 5, 7, 12, 18, 20, 31-33, 39-41, 45-47, 49, 54, 59, 64, 65, 68, 75, 76, 81, 84, 88, 91, 92, 94, 100, 103-106, 117-120, 122]
Time Series Methods	20	[2, 23, 29, 36, 39, 47, 48, 57, 66, 70, 74, 82, 83, 85, 90, 95, 99, 108, 111, 120]
Other Methods	20	[9, 13, 16, 17, 19, 21, 38, 42, 58, 71, 79, 80, 87, 98, 101, 110, 112, 113, 116, 120]

It is important to note that some studies employed multiple pairs trading methods. This overlap reflects the complexity and adaptability of pairs trading strategies, as researchers often combine different approaches, such as cointegration and stochastic control, to optimize performance and capture various market dynamics. As a result, the counts in the table reflect instances of method usage rather than a one-to-one correspondence with the number of unique studies.

Among the studies, a subset 21 articles — [5, 7, 16, 33, 46, 64, 68, 69, 75, 76, 81, 84, 88, 91, 94, 103, 105, 106, 117, 118, 122] — primarily focuses on theoretical research. These papers propose and validate new trading strategies or enhance existing ones through mathematical modeling and derivation. Typically, these theoretical studies employ simulations or historical data backtesting to assess the effectiveness of the strategies, rather than implementing them directly in real markets. The contribution of these works is substantial, as they offer deeper insights into the potential returns and risks associated with various strategies, examine how these strategies perform under different market conditions, and establish a robust foundation for future empirical research.

Table 3 provides a comprehensive summary of the major financial indices examined in this study, offering insights into the geographic diversity of the markets analyzed. The table encompasses a wide range of indices from both developed and emerging markets, ensuring broad coverage across global financial landscapes.

For the United States, several significant indices were included, such as the S&P 100, Dow Jones Industrial Average, Nasdaq 100, Russell 2000 ETF, and S&P 500. These indices represent a cross-section of the U.S. market, capturing a variety of market capitalizations and sectors, thus providing a robust understanding of the U.S. equity market's performance.

In Europe, the FTSE 100 is representative of the UK market, while the STOXX Europe 600 offers a broader view of European equities, encompassing companies from multiple countries and various market capitalizations. The SBF 120 index adds a specific focus on the French market, highlighting one of the region’s major economies.

Asia is well represented through indices such as CSI 300 and SSE 50 from China, which track large-cap companies and reflect the performance of the Chinese stock market. The Nikkei 225, TPX 100, and TOPIX 30 from Japan provide insights into the Japanese market, while the KOSPI 100 from South Korea offers a perspective on another key Asian economy. These indices provide a deep dive into some of the fastest-growing and most dynamic markets in the region.

In addition to these major indices, the OMX Baltic index covers the markets of Lithuania, Latvia, and Estonia, illustrating the growing importance of the Baltic economies in the European context. Further representation from the Asia-Pacific region includes the ASX 100 from Australia and the Sensex 30 from India, both of which provide insights into the economic activities of two major markets in the southern hemisphere.

Overall, **Table 4** illustrates the wide geographic coverage and diversity of the indices included in this study, spanning North America, Europe, and Asia. This global perspective enhances the robustness of the analysis by incorporating a diverse array of economic environments and market conditions.

Table 4. Summary of major stock indices and markets analyzed in this study

Index	Country
S&P 100	U.S.
Dow Jones Industrial Average	U.S.
Nasdaq 100	U.S.
Russell 2000 ETF	U.S.
S&P 500	U.S.
FTSE 100	UK
CSI 300	China
SSE 50	China
STOXX Europe 600	Europe
Nikkei 225	Japan
TOPIX 30	Japan
TPX 100	Japan
KOSPI 100	South Korea
OMX Baltic	Lithuania, Latvia, Estonia
SBF 120	France
ASX 100	Australia
Sensex 30	India

4. Data Analysis

The majority of the studies included in our review utilized daily datasets featuring key attributes such as opening and closing prices, the highest and lowest values during the trading day, and trading volume (often abbreviated as OHLCV). Some researchers, however, opted for more detailed datasets, employing tick data that recorded market fluctuations at intervals of one, five, or fifteen minutes.

4.1 Daily Interval Historical Price Data

Historical stock data typically contains information such as the opening price, highest and lowest prices, closing price, and the trading volume for each stock. Below is an example of the daily dataset for Apple, as cited in numerous studies [1-3, 6, 9-15, 17, 19, 21-24, 26, 29-32, 35-37, 39-41, 43-45, 47, 48, 50-52, 55, 57, 58, 60-63, 65-67, 71-73, 77-80, 82, 83, 85, 89, 90, 92, 93, 95-98, 100-102, 107, 109-115, 119, 121, 123]. **Table 5** illustrates the stock market data arranged by date.

In our analysis, we found that 78 of the studies reviewed specifically utilized daily interval data for their examinations. This represents a substantial proportion, with 75% of the papers relying on daily data intervals for their research. The widespread use of daily granularity reflects the strong preference among researchers for studying daily market movements, emphasizing the importance of capturing daily price trends and patterns to gain a deeper understanding of financial market behavior over time.

Table 5. A sample of Apple Inc.'s daily historical stock data

Date	Open	High	Low	Close	Adj Close	Volume
8/23/2023	178.520004	181.550003	178.330002	181.119995	180.197906	52722800
8/24/2023	180.669998	181.100006	176.009995	176.380005	175.482025	54945800
8/25/2023	177.380005	179.149994	175.820007	178.610001	177.700684	51449600
8/28/2023	180.089996	180.589996	178.550003	180.190002	179.272644	43820700
8/29/2023	179.699997	184.899994	179.500000	184.119995	183.182632	53003900
8/30/2023	184.940002	187.850006	184.740005	187.649994	186.694672	60813900
8/31/2023	187.839996	189.119995	187.479996	187.869995	186.913544	60794500
9/1/2023	189.490005	189.919998	188.279999	189.460007	188.495468	45732600
9/5/2023	188.279999	189.979996	187.610001	189.699997	188.734207	45280000
… …	… …	… …	… …	… …	… …	… …

4.2 One-Minute Interval Historical Price Data

Minute-interval historical data, such as one-minute data, provide a detailed perspective for analyzing stock market activity, capturing price and volume changes updated every minute throughout the trading session. This level of granularity strikes a balance between the highly detailed nature of tick data and the broader, less detailed view provided by longer intraday intervals. Each record in a one-minute dataset typically includes the opening price, highest and lowest prices, closing price, and the trading volume for that specific minute. This level of resolution is especially useful for medium-frequency trading strategies, allowing traders to detect short-term trends and make decisions based on minute-by-minute market shifts. By analyzing one-minute interval data, traders can gain deeper insights into price volatility and market momentum, which are essential for strategies that depend on quick market movements. Although the data volume is smaller compared to tick data, working with minute-interval data still requires significant computational resources and advanced analytical methods to uncover actionable insights.

The practical use of one-minute interval data is illustrated in **Table 6**, which presents an example using Apple's stock data, as analyzed in select studies [4, 38, 49, 54, 59, 86, 104, 120]. This granularity is particularly well-suited for building predictive models that require a balance between detail and efficiency, making it ideal for intraday trading strategies with enhanced precision.

Table 6. A sample of Apple Inc.'s one-minute historical stock data

Datetime	Open	High	Low	Close	Adj Close	Volume
8/14/2024 9:30:00	220.5650	220.7600	220.2100	220.5200	220.5200	2664372
8/14/2024 9:31:00	220.3500	220.5500	219.7900	219.8500	219.8500	436127

8/14/2024 9:32:00	219.9299	220.0900	219.7000	219.7500	219.7500	279503
8/14/2024 9:33:00	219.7900	220.3600	219.7300	220.3600	220.3600	194167
8/14/2024 9:34:00	220.3600	220.9400	220.3100	220.9050	220.9050	240008
8/14/2024 9:35:00	220.9100	221.1800	220.6150	221.0300	221.0300	279137
8/14/2024 9:36:00	221.0500	221.4400	221.0200	221.3650	221.3650	209326
8/14/2024 9:37:00	221.3700	221.5675	221.0596	221.5675	221.5675	334528
8/14/2024 9:38:00	221.5700	221.7500	221.4210	221.4900	221.4900	203597
8/14/2024 9:39:00	221.5300	222.0000	221.4900	221.9800	221.9800	259714
… …	… …	… …	… …	… …	… …	… …

4.3 Five-Minute Interval Historical Price Data

Five-minute interval historical data offer a more concise perspective on market activity, consolidating the open, high, low, close prices, and trading volume of stocks into five-minute segments. This type of data is tailored for traders and analysts seeking a more high-level view of intraday price movements, without the fine-grained detail found in tick or one-minute data. The five-minute interval provides an effective way to capture broader intraday patterns, such as trends, support and resistance levels, and potential breakouts, offering a balance between capturing detail and maintaining a clearer, less noisy view of market dynamics. It is particularly useful for strategies targeting short- to medium-term price movements, as it offers enough granularity to understand market behavior while filtering out the minute-to-minute volatility. While the volume of data is smaller compared to tick or one-minute intervals, five-minute data still requires sophisticated analytical tools to uncover significant trends and patterns.

Table 7 showcases Apple's stock market data at five-minute intervals, as referenced in studies [18, 43, 73, 108, 120], demonstrating its utility in developing strategies that focus on slightly longer intraday time frames. This interval is critical for building models that forecast price movements with a broader temporal scope, supporting intraday trading strategies that operate on short to medium horizons.

Table 7. A sample of Apple Inc.'s five-minute historical stock data

Datetime	Open	High	Low	Close	Adj Close	Volume
2024-08-14 09:30:00	220.5650	220.9400	219.7000	220.9050	220.9050	3814177

2024-08-14 09:35:00	220.9100	222.0000	220.6150	221.9800	221.9800	1286302
2024-08-14 09:40:00	221.9700	222.0200	221.1690	221.4600	221.4600	842724
2024-08-14 09:45:00	221.4700	221.6750	221.0400	221.1050	221.1050	772010
2024-08-14 09:50:00	221.1006	221.2700	220.6814	220.7300	220.7300	561586
2024-08-14 09:55:00	220.7517	221.1399	220.7100	220.9500	220.9500	452026
2024-08-14 10:00:00	220.9450	221.2000	220.6900	221.1700	221.1700	448701
2024-08-14 10:05:00	221.1899	221.3499	220.8100	221.1501	221.1501	398965
2024-08-14 10:10:00	221.1650	221.4404	220.7900	221.4300	221.4300	497368
2024-08-14 10:15:00	221.4400	221.7000	221.2500	221.5973	221.5973	515541
……	……	……	……	……	……	……

4.4 Fifteen-Minute Interval Historical Price Data

Fifteen-minute interval historical data provide a more aggregated perspective on market activity, consolidating the open, high, low, close prices, and trading volume within fifteen-minute segments. This interval is ideal for traders and analysts who seek a wider, more comprehensive view of intraday market movements, sacrificing the fine details seen in shorter intervals like tick or one-minute data. The fifteen-minute interval is particularly useful for spotting larger intraday trends, monitoring significant support and resistance levels, and identifying breakout points while minimizing the noise that can obscure these patterns in more granular data. This interval strikes a balance between detail and manageability, making it well-suited for strategies that aim to capture medium-term intraday movements. It offers traders sufficient insight to recognize actionable trends while smoothing out the more volatile fluctuations seen in shorter intervals. Although fifteen-minute data generate fewer data points compared to tick or one-minute data, they still require sophisticated analytical methods to uncover important market signals.

In **Table 8**, Apple's fifteen-minute interval stock market data, as analyzed in studies [87，166], demonstrates how this interval can be effectively utilized in developing trading strategies that focus on slightly longer intraday trends. The fifteen-minute interval is particularly valuable for constructing predictive models that provide a broader perspective on price movements, supporting trading strategies that target medium-term opportunities within the intraday framework.

Table 8. A sample of Apple Inc.'s fifteen-minute historical stock data

Datetime	Open	High	Low	Close	Adj Close	Volume
8/14/2024 9:30:00	220.5650	222.0200	219.7000	221.4600	221.4600	5943203
8/14/2024 9:45:00	221.4700	221.6750	220.6814	220.9500	220.9500	1785622
8/14/2024 10:00:00	220.9450	221.4404	220.6900	221.4300	221.4300	1345034
8/14/2024 10:15:00	221.4400	221.7000	220.9500	221.1104	221.1104	1248147
8/14/2024 10:30:00	221.1100	221.4800	220.4550	221.4000	221.4000	1292931
8/14/2024 10:45:00	221.4100	221.7260	221.2700	221.6901	221.6901	1092996
8/14/2024 11:00:00	221.6950	222.2900	221.6000	222.2500	222.2500	1267633
8/14/2024 11:15:00	222.2500	222.5600	222.2000	222.5587	222.5587	864267
8/14/2024 11:30:00	222.5500	222.8900	222.3900	222.4000	222.4000	935396
8/14/2024 11:45:00	222.4000	223.0300	222.4000	222.9701	222.9701	881023
… …	… …	… …	… …	… …	… …	… …

4.5 Hourly Interval Historical Price Data

Hourly historical data provide a broader perspective for analyzing stock market trends, capturing aggregated price movements and trading volumes within each hour of the trading day. This data interval offers a middle ground between high-resolution tick data and broader daily market summaries, striking a balance that is ideal for more extended intraday analysis. Each data point in an hourly dataset includes the opening price, highest and lowest prices, closing price, and the total volume of shares traded within that hour. This information is particularly useful for traders and analysts whose strategies focus on timeframes shorter than daily trades but do not require the granularity and volume of data associated with high-frequency trading.

Hourly data provide valuable insights into intra-day market trends, allowing for the detection of momentum shifts and potential breakouts. These patterns are especially relevant for strategies that take advantage of price movements within the trading day, often influenced by news events, economic reports, or shifts in market sentiment. While hourly data do not offer the exhaustive detail of tick data, they still provide enough granularity to capture important market behaviors throughout the trading day without overwhelming analysts with vast amounts of data.

Although hourly interval data are less resource-intensive than tick data, they still require effective analytical tools for meaningful interpretation. Insights drawn from these datasets are crucial for traders focusing on hourly market dynamics, where understanding and reacting to market forces within each hour of the trading session is key. For instance, **Table 9** typically presents hourly interval stock data for a security like Tesla, as explored in various studies [73, 99, 120]. Using hourly data is essential for developing trading models that balance detail and manageability, making them well-suited for intraday trading strategies that require a focused yet comprehensive view of market activity.

Table 9. A sample of Apple Inc.'s hourly historical stock data

Datetime	Open	High	Low	Close	Adj Close	Volume
2024-08-14 09:30:00	220.5650	222.0200	219.7000	221.1104	221.1104	10322006
2024-08-14 10:30:00	221.1100	222.5600	220.4550	222.5587	222.5587	4517827
2024-08-14 11:30:00	222.5500	223.0300	221.0150	221.3050	221.3050	4029799
2024-08-14 12:30:00	221.3200	221.4600	220.3800	221.1750	221.1750	3645945
2024-08-14 13:30:00	221.1600	221.7690	220.9100	221.5400	221.5400	3099853
2024-08-14 14:30:00	221.5300	221.6100	220.8100	221.1600	221.1600	2990194
2024-08-14 15:30:00	221.1600	222.3500	221.1531	221.6200	221.6200	4112373
2024-08-15 09:30:00	224.5500	224.9900	222.7600	224.7900	224.7900	9897134
2024-08-15 10:30:00	224.7800	225.3400	223.9000	224.0050	224.0050	5316678
2024-08-15 11:30:00	224.0100	225.0150	223.8500	224.9950	224.9950	3233561
… …	… …	… …	… …	… …	… …	… …

4.6 Weekly Interval Historical Price Data

Weekly historical data provides a comprehensive, long-term view of stock market activity by aggregating price movements and trading volumes over the course of an entire week. Compared to daily data, this interval offers a more macro-level perspective, consolidating the fluctuations within individual trading days into a single data point for the week. Each entry in a weekly dataset typically includes the opening price at the start of the week, the highest and lowest prices achieved during the week, the closing price at the week's end, and the total trading volume over that period.

These datasets are especially valuable for traders and analysts who focus on longer-term strategies, such as swing or position trading, where short-term price movements are less significant. Weekly data enable the identification of key market trends, critical support and resistance levels, and broader shifts in momentum that occur over multiple weeks. By smoothing out the noise inherent in intraday fluctuations, weekly data provide a clearer perspective on the underlying direction of the market, making it a valuable tool for evaluating the longer-term performance of assets.

Though weekly data are less demanding in terms of processing power compared to intraday or tick-level data, they still require robust analytical tools to detect the meaningful patterns that drive long-term market behavior. For traders and investors focused on larger market dynamics, weekly data offers crucial insights, allowing for a more strategic approach to trading with less emphasis on short-term volatility. For example, **Table 10** typically displays the weekly interval stock data for a company like Apple, as investigated in studies [53, 74]. Leveraging weekly data is vital for developing trading models that aim to capture more significant trends while keeping the data manageable, making it well-suited for strategies targeting substantial price movements over extended periods.

Table 10. A sample of Apple Inc.'s weekly historical stock data

Datetime	Open	High	Low	Close	Adj Close	Volume
4/1/2024	169.0800	171.9200	168.2300	169.5800	169.1545	192780800
4/8/2024	169.0300	178.3600	167.1100	176.5500	176.1070	322249600
4/15/2024	175.3600	176.6300	164.0800	165.0000	164.5860	309039200
4/22/2024	165.5200	171.3400	164.7700	169.3000	168.8752	241302700
4/29/2024	173.3700	187.0000	169.1100	183.3800	182.9199	441926300
5/6/2024	182.3500	185.0900	180.4200	183.0500	182.5907	300675100
5/13/2024	185.4400	191.1000	184.6200	189.8700	189.6505	288966500
5/20/2024	189.3300	192.8200	186.6300	189.9800	189.7603	208619700
5/27/2024	191.5100	193.0000	189.1000	192.2500	192.0277	230454300
6/3/2024	192.9000	196.9400	192.5200	196.8900	196.6624	245994400
… …	… …	… …	… …	… …	… …	… …

4.7 Millisecond and Nanosecond Interval Historical Price Data

Millisecond and nanosecond data, as in [25, 27, 28, 34], provide the most granular view of stock market activity, capturing minuscule details of price movements, trade volumes, and order flows within extremely short time intervals. Millisecond data, recorded at intervals of 1/1,000th of a second, includes fields such as timestamps, bid and ask prices, trade prices, trade sizes, and detailed order book information. This data is particularly valuable in high-frequency trading (HFT) environments, where traders seek to identify short-term price patterns, arbitrage opportunities, and market inefficiencies that can be exploited within fractions of a second. The sheer volume of millisecond data requires substantial computational power and specialized storage systems for real-time processing and analysis, making it primarily the domain of institutional traders and algorithmic trading firms with the capacity to handle such high-frequency data. Millisecond data provides insights into the microstructure of the market, including latency, execution speed, and price formation processes that are critical to HFT strategies.

Nanosecond data, on the other hand, is recorded at an even finer resolution, capturing information every 1/1,000,000,000th of a second. This ultra-high-resolution data records the exact sequence and timing of trades, orders, and price movements with unparalleled precision, making it essential for firms engaged in ultra-low latency trading. Nanosecond data is used by market makers, latency arbitrage traders, and other participants who rely on speed as a competitive advantage, exploiting fleeting opportunities that exist only for microseconds. However, the vast amount of data generated at this level creates significant challenges in terms of storage, processing, and analysis. Both millisecond and nanosecond data demand sophisticated analytical tools and infrastructure, limiting their use to highly specialized trading firms. These data types are indispensable for building advanced trading models that require precise timing and execution, where every fraction of a second can influence profitability in fast-moving markets.

5. Non-Machine Learning Models in Pair Trading

This chapter will explore several common non-machine learning (statistical) methods used in pairs trading strategies. Each of these methods has distinct characteristics, making them suitable for different market conditions and trading objectives. Specifically, this chapter will examine Distance Methods, Cointegration Methods, Stochastic Control Methods, Time Series Methods, and various other approaches. By reviewing recent research findings and practical applications, we will delve into the strengths and weaknesses of these methods, as well as their performance and adaptability in pairs trading.

Distance Methods in Pair Trading

Distance methods are the most intuitive and easy-to-understand approach in pairs trading. These methods are based on the "distance" or difference between the prices of assets to make trading decisions. The method assumes that the price difference between two assets will fluctuate around a long-term mean, and traders capitalize on these deviations to capture arbitrage opportunities. The simplicity of distance methods makes them suitable for highly correlated asset pairs, especially when the price structure of the pairs is relatively stable. However, the limitation of this method lies in its neglect of more complex statistical relationships between assets, which may cause it to fail in trending markets.

Cointegration Methods in Pair Trading

Cointegration methods are based on the statistical theory of cointegration, which is used to identify long-term equilibrium relationships between two or more assets. Unlike distance methods, cointegration methods are more complex and allow traders to execute arbitrage strategies even when asset prices are not entirely independent but have long-term connections. By testing the cointegration relationship between asset pairs, traders can determine whether price deviations are temporary phenomena and use this information to trade. Cointegration methods perform well in dealing with the long-term dependencies of financial markets and are often used in high-frequency trading. However, the method's reliance on model specification and data quality can be a drawback, making it unsuitable for all market conditions.

Stochastic Control Methods in Pair Trading

Stochastic control methods use stochastic processes within control theory to find the optimal trading strategy. They allow for adjustments in trading strategies based on market evolution and the random fluctuations of prices, with the aim of minimizing risk and maximizing returns. Compared to the first two methods, stochastic control provides a more flexible framework, adapting to both short-term market fluctuations and long-term trends. Although the theoretical foundations are complex and the computational requirements are high, this method can significantly improve the robustness of trading strategies in volatile markets.

Time Series Methods in Pair Trading

Time series analysis methods, including ARIMA models, GARCH models, and others, are used to capture the temporal dependencies and volatility of asset prices. By constructing time series models, traders can predict future price movements and use these forecasts for pairs trading. Time series methods are particularly effective for asset pairs whose price dynamics are dependent on historical data, and they perform well when dealing with market volatility. However, time series models come with strong assumptions, and the selection of model parameters can significantly impact trading outcomes, which can increase complexity and computational difficulty.

Other Methods in Pair Trading

In addition to the more conventional approaches outlined above, several alternative methods have been developed to address specific challenges in pairs trading, particularly in handling complex dependencies, market uncertainty, and long-memory effects. This section will explore

Copula Methods, Entropic Methods, and Hurst Exponent Methods, each of which provides a unique framework for analyzing and executing pairs trading strategies.

5.1 Distance Methods

This section thoroughly examines the distance method. **Table 11** offers a brief summary of key studies, including their data samples and research objectives.

Table 11. A summary of distance methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
6	2016	London Stock Exchange stocks, 1979-2012	Daily
10	2016	Stock Indexes in 12 Countries, 1987-2011	Daily
13	2016	U.S. stocks, 1962-2014	Daily
14	2017	U.S. stocks, 1980-2014	Daily
22	2017	36 Stocks in DJIA, NYSE, and NASDAQ, 2006-2014	Daily
25	2017	Stocks in OMX Baltic, 2014-2015	Millisecond
26	2017	Chinese Commodity Futures, 2005-2016	Daily
43	2018	Oslo Stock Exchange stocks, 2005-2014	5-min, daily
45	2018	U.S. Financial Sector stocks, 2008-2013	Daily
51	2018	Gilt Futures in ICE, 2013-2015	Daily
56	2019	U.S. stocks, 1931-2007	Monthly
60	2019	CSI300, HSHKI, HSAHP stocks, 1996-2017	Daily
63	2019	Stockholm Stock Exchange stocks, 1995-2015	Daily
72	2020	Commodity Futures in MCX, 2011-2017	Daily
73	2020	181 Cryptocurrencies, 2018-2019	Daily, hourly, 5-min
77	2020	SSE 50 stocks, 2016-2018	Daily
78	2020	DJIA, Sensex 30 and TOPIX 30 stocks, 2008-2016	Daily
79	2020	Nasdaq 100 stocks, 1999-2003 2007-2012	Daily
97	2021	NSE stocks, 2011-2017	Daily
98	2021	Toronto Stock Exchange	Daily

120	2023	stock, 2017-2020 405 Cryptocurrencies, 2022	1-min, 5-mins, hourly

5.1.1 The GGR's Baseline Approach

Although this paper focuses on articles starting from 2016, we must first analyze the seminal paper that introduced the distance method. The distance method was first introduced by the influential work of Gatev et al. (2006). Their research focuses on all liquid U.S. stocks, using data from the CRSP daily files between 1962 and 2002. Initially, they construct a cumulative total return index for each stock, denoted as P_{it}, which is normalized to the first day of a 12-month formation period. With a total of n stocks, they calculate the sum of squared Euclidean distances (SSD) for the price series of all possible $n\,(n-1)/2$ pairs. The 20 pairs with the lowest historical distance are then selected for a six-month trading period. In this phase, prices are normalized again on the first day, and trades are executed when the spread deviates by more than two historical standard deviations (σ). Positions are closed when the spread mean-reverts, the trading period ends, or the stock is delisted.

Below is the detailed method development of this paper.

Data and Sample Selection

The data for this study was sourced from the CRSP daily files, covering the period from 1962 to 2002. The dataset includes all liquid U.S. stocks traded on the major exchanges, such as NYSE, AMEX, and NASDAQ. To ensure liquidity and avoid complications related to thinly traded stocks, any stocks that experienced one or more days without trading were excluded from the sample. This screening process ensured the construction of pairs with reliable and consistent price data.

Formation Period

During the Formation Period, the methodology begins with the construction of a Cumulative Return Index for each stock, which includes dividend reinvestments. This cumulative return is normalized to 1 at the start of the formation period to ensure comparability across stocks. The core of the pair selection process is based on the Sum of Squared Deviations (SSD) between the normalized price series of each pair of stocks. For two stocks i and j, SSD is calculated as:

$$SSD_{ij} = \sum_{t=1}^{T} (P_{it} - P_{jt})^2 \qquad (5.1)$$

The SSD is calculated for all possible stock pairs, and those with the smallest historical SSD values are considered the most likely candidates for future price convergence. Based on this metric, stocks are matched into pairs, with the top pairs (those with the smallest distance) selected for trading during the subsequent trading period.

Trading Period

The Trading Period spans six months and follows directly after the formation period. During this phase, the selected pairs are traded based on pre-established rules. A position in a pair is initiated when the price spread between the two stocks diverges by more than two standard deviations from the historical mean spread, as calculated during the formation period. When this threshold is breached, a long position is taken in the underperforming stock (the "loser"), and a short position is taken in the outperforming stock (the "winner"). Positions are closed when the spread reverts to its historical mean, at the end of the six-month trading period, or upon the delisting of either stock in the pair.

Risk-Adjusted Excess Returns

The profitability of the pairs trading strategy is evaluated through annualized excess returns. Gatev et al. find that self-financing portfolios composed of the top pairs deliver average annualized excess returns of up to 11%. To test the robustness of these returns, the authors account for various risks and transaction costs. Through bootstrap analysis, they demonstrate that the strategy's excess returns cannot be attributed solely to simple mean reversion, a common explanation in earlier literature. To further ensure the strategy's robustness and guard against data snooping, the authors conducted out-of-sample testing on data from 1999 to 2002. The strategy continued to generate positive excess returns during this period, reinforcing its validity and adaptability over time.

Model-Free Approach

One of the key strengths of this methodology is its model-free nature. The strategy does not rely on any specific economic model of asset pricing, which significantly reduces the risks associated with model misspecification. Instead, the strategy is purely driven by historical price relationships between pairs of stocks, making it flexible and less vulnerable to theoretical errors.

Trading Frequency and Holding Periods

On average, the selected pairs trade approximately twice during the six-month trading period. Each position tends to be held for an average of 3.75 months. The study also assesses the performance of fully invested portfolios of stock pairs, finding that the strategy continues to yield significant excess returns even after factoring in transaction costs and other market frictions.

Transaction Costs and Market Neutrality

The authors carefully evaluate the impact of transaction costs on the profitability of the pairs trading strategy. Their analysis shows that the strategy remains profitable even after reasonable estimates for transaction costs, such as bid-ask spreads and short-selling fees, are included.

Furthermore, the pairs trading strategy is inherently market-neutral, as it involves taking both long and short positions in stocks that are expected to move in relation to each other rather than the broader market. This neutrality minimizes the portfolio's exposure to market-wide movements and focuses the strategy on the relative performance of the paired stocks, which is the core of the arbitrage opportunity.

This approach offers several clear advantages. As Do et al. (2006) point out, the methodology proposed by Gatev, Goetzmann, and Rouwenhorst (GGR) is free from any economic model, thus avoiding the risks of model specification errors and misestimation. It is also straightforward to implement, resistant to data snooping biases, and generates statistically significant risk-adjusted excess returns. By applying a simple yet effective strategy to a large dataset spanning over four decades, the study firmly established pairs trading as a notable capital market anomaly.

However, the methodology also presents areas for potential refinement. The use of Euclidean squared distance to identify pairs, for instance, may not be analytically optimal. Consider the goal of a rational pairs trader who seeks to maximize excess returns per pair, as GGR propose. With a fixed initial investment, this objective translates into maximizing profits per pair, which are a function of the number of trades per pair and the profit from each trade. Consequently, the ideal pairs would exhibit frequent and significant deviations from, and subsequent returns to, equilibrium. In other words, the most profitable pairs should have two key features: a high degree of spread variance and strong mean-reversion tendencies. These characteristics allow for a higher frequency of profitable round-trip trades. Examining GGR's ranking logic reveals how it aligns with these profit-maximizing requirements.

5.1.2 Extension Methods Based on GGR

Compared to GGR's method, later papers introduce various differences and improvements built upon its foundation.

Bowen and Hutchinson (2016) use a methodology that closely follows the work of GGR. The article explicitly mentions that to ensure their results are comparable to the US and avoid potential data-mining bias, they employ a similar approach to that of GGR. This includes forming pairs of stocks during a 12-month formation period based on the minimum sum of squared deviations (SSD) of their normalized price series, and trading these pairs over a 6-month trading period using a similar trading rule. The strategy remains model-free, relying solely on historical price relationships, consistent with GGR’s approach. Bowen and Hutchinson’s adoption of this methodology allows for direct comparison between the UK and U.S. markets, demonstrating the strategy's robustness and transferability across different markets.

Bowen and Hutchinson provide several new insights and enhancements to pairs trading research. Most notably, they extend the methodology to a non-U.S. market, applying it to the UK equity market. This geographical expansion offers valuable evidence of the strategy's robustness and adaptability across different market environments. Additionally, while GGR primarily focused on the performance of pairs trading strategies, Bowen and Hutchinson delve deeper into the risk-return profile, assessing both profitability and risk, thereby providing a more comprehensive understanding of the strategy's overall performance. Their study also incorporates a detailed analysis of risk factors, such as market volatility and liquidity, to explore how these elements influence pairs trading outcomes. By using more recent data than GGR, Bowen and Hutchinson offer a contemporary perspective on the strategy's effectiveness, reflecting changes in market dynamics and trading behavior. Furthermore, they investigate the strategy's performance across various industry sectors within the UK market, adding a sectoral dimension that was not thoroughly explored in GGR's study, and offering insights into how pairs trading strategies may vary by sector. In summary, they expand upon the GGR framework by applying it to the UK market, incorporating a deeper risk-return analysis, considering the influence of market conditions, using more recent data, and analyzing performance across sectors. These contributions provide new dimensions to the understanding of pairs trading strategies.

Chen, H. et al. (2019) confirm GGR's findings by showing that a pairs trading strategy can generate significant abnormal returns. They use a similar pairs trading framework, based on identifying stock pairs with high historical return correlations and exploiting price divergences between them. The paper also extends GGR's findings by further analyzing the sources of pairs trading profitability, attributing returns to factors like short-term reversal and pairs momentum. They explore the role of industry momentum, liquidity, and information diffusion in explaining the profitability of pairs trading, adding depth to the original GGR framework.

However, the methodology used in the article differs from the GGR method in several important ways. In the GGR approach, stock pairs are selected based on the sum of squared deviations (SSD) between normalized price series during a 12-month formation period, with the pairs exhibiting the smallest SSD values chosen for trading. In contrast, the 2019 paper bases its pairs selection on historical pairwise return correlations over a five-year period, pairing each stock with 50 others showing the strongest correlation. Moreover, while GGR uses a fixed 12-month formation period followed by a 6-month trading period, the 2019 paper continuously evaluates return divergence on a monthly basis over the next year, allowing for a more dynamic assessment of pairs trading opportunities. The trading strategies also differ: GGR initiates trades when price spreads diverge by more than two standard deviations from the historical mean, whereas the

2019 paper employs a strategy based on return divergence, taking long positions in underperforming stocks and short positions in outperforming ones, expecting the divergence to reverse in the following month. In terms of risk and return analysis, GGR focuses on profitability without deeply analyzing specific risk factors, while this paper extends the analysis to examine the effects of short-term reversal, momentum, and liquidity on returns, offering more insight into the role of market anomalies in driving profitability. Additionally, while GGR does not account for pairs momentum, this paper finds that momentum contributes significantly to returns, particularly in the first month following divergence. Lastly, this paper uses more recent data, reflecting potentially different market conditions than those present in GGR's study of U.S. stocks from 1962 to 2002. These differences in pairs selection, trading signals, and the incorporation of risk factors such as short-term reversal and momentum provide a more detailed and nuanced understanding of the mechanics behind pairs trading profitability.

In summary, the primary differences between the two methodologies lie in the pairs selection process, trading signals, and the incorporation of risk factors like short-term reversal and momentum. While GGR's approach focuses on SSD-based pair formation and relies on price reversion for profitability, this paper introduces a return correlation-based approach and expands the analysis to include the role of pairs momentum and other risk factors in driving returns. These additions provide a more detailed and nuanced understanding of the mechanics behind pairs trading profitability.

Gupta, K. and Chatterjee, N. (2020) introduce significant methodological advancements compared to the original GGR approach to pairs trading. While the GGR method selects stock pairs based on the sum of squared deviations (SSD) between normalized price series over a fixed formation period and focuses on price reversion to equilibrium, this paper incorporates a dynamic element through the introduction of the Dynamic-Cross-Correlation-Type (DCCT) measure.

One key difference is the incorporation of the lead-lag relationship. The DCCT measure allows the lead-lag value between stocks to vary continuously over time, capturing subtle temporal dependencies where one stock might lead or lag another over different periods. This contrasts with the static price relationships assumed in the GGR approach, providing a more adaptable and nuanced strategy, particularly suited to evolving market conditions. Additionally, while the SSD measure remains central to both methods, the authors of this paper enhance the pair selection process by combining SSD with DCCT. This integration ensures that selected stock pairs not only exhibit closeness in price deviations but also demonstrate favorable dynamic correlations and lead-lag behaviors. Empirical results demonstrate that this combined approach outperforms the traditional SSD measure alone in terms of profitability. Moreover, the adaptability of the DCCT & SSD approach is tested across multiple markets—including the U.S. (DJIA), Indian

(Sensex 30), and Japanese (Topix 30) markets—highlighting its robustness and transferability across different market environments. The dynamic adaptability of the method makes it particularly effective during periods of market volatility or structural changes, improving profitability in scenarios where the static GGR approach might underperform.

In conclusion, by extending the GGR framework with dynamic temporal dependencies, this paper offers a more responsive and potentially profitable pairs trading strategy that leverages both price proximity and evolving lead-lag relationships. The result is a method that is not only more adaptable to varying market conditions but also demonstrates superior performance across diverse datasets.

Other papers using distance methods are also based on the expansion and optimization of GGR. The above is the analysis of three papers for reference.

5.1.3 Explanation of Pair Trading Profitability

Based on the analysis of pairs trading profitability in the UK equity market, Bowen and Hutchinson (2016) highlight several key insights. Pairs trading strategies performed exceptionally well during the 2007-2008 financial crisis, generating returns of 36% to 48%, even as the FTSE All-Share Index dropped by -34%. This suggests that pairs trading may offer diversification benefits and perform robustly during periods of market stress, particularly in volatile and illiquid environments. Additionally, the portfolios examined exhibit low exposure to traditional equity risk factors, such as market, size, value, momentum, and reversal, indicating that the strategy's profitability primarily stems from exploiting short-term pricing inefficiencies rather than broader market trends. However, transaction costs significantly impact profitability, with returns reduced by up to 4% annually once market frictions, such as bid-ask spreads, are accounted for. Despite this, the strategy maintained consistent performance across different market states, including the transition from a quote-driven to an order book-driven system on the London Stock Exchange. The return distributions from pairs trading portfolios show positive skewness and high kurtosis, implying that while the strategy generally offers stable returns, it can occasionally generate large windfalls, particularly during periods of significant market dislocation. Moreover, the study suggests that pairs trading may provide liquidity to the market by capitalizing on price deviations, especially in low liquidity environments, contributing to its strong performance during market crises. Overall, pairs trading in the UK market demonstrates robust profitability, particularly during times of financial distress, though transaction costs and liquidity conditions play critical roles in determining the strategy's success.

The pairs trading strategies by Quinn, B. et al. (2018) in the UK gilt futures market reveals consistent profitability, particularly in trading long and medium gilt futures. For instance, a 10% trigger level produced a 3.51% gain over nine quarters, while a 15% trigger level yielded a 1.78%

return. This demonstrates the potential for arbitrage profits even in highly liquid and regulated markets like the UK gilt futures market, where opportunities for pricing inefficiencies are often limited. The study further illustrates that trading parameters, such as trigger and stop-loss levels, significantly impact profitability. Lower trigger levels, like 10%, lead to more frequent trades and higher aggregate returns, while higher trigger levels, such as 20%, result in fewer trades but can still be profitable. Effective risk management through stop-loss mechanisms also proves crucial, with a 30% stop-loss consistently protecting and enhancing profitability for the long-medium gilt pairs. Additionally, market conditions play a significant role, as the strategy's success relies on exploiting mean reversion in the spread between different gilt maturities, even in the face of fluctuating market conditions. Overall, the use of the distance method in pairs trading with UK gilt futures presents a low-risk, consistent arbitrage strategy. The key to sustained profitability lies in the careful calibration of trigger levels and stop-loss mechanisms, balancing trading frequency with effective risk management, making this strategy viable even in liquid and regulated markets like UK government bonds.

Another article by Chen, H. et al. (2019) generates high abnormal returns, particularly in the short term. For instance, the study finds that value-weighted portfolios sorted by return differences (RetDiff) yield monthly returns of 1.40% in the first month. However, profitability diminishes in subsequent months, with losses observed beyond the first month, indicating that the strategy is most effective when trades are executed promptly after divergence, allowing for the capture of short-term price corrections. Furthermore, the abnormal returns from pairs trading are largely unexplained by common risk factors such as market risk, size, book-to-market ratio, and momentum, suggesting that these profits stem from exploiting short-term pricing inefficiencies rather than broader market movements. The study also finds that liquidity risks have little impact on the strategy, reinforcing the notion that pairs trading profits are driven by micro-level price movements between stocks. Additionally, the profitability of pairs trading varies across sectors, with certain industries exhibiting stronger pairs momentum and short-term reversal effects. This highlights the importance of sector selection, particularly in industries characterized by slower information diffusion and higher volatility, which can enhance the strategy's profitability.

Zhang, H. and Urquhart, A. (2019) shows that pairs trading within individual markets, either mainland China or Hong Kong, does not generate significant abnormal returns. However, cross-market pairs trading, involving both markets, yields annualized abnormal returns of up to 9%, even after adjusting for risk and transaction costs. This indicates that arbitrage opportunities are more prevalent when trading across these two markets due to lower market integration and inefficiencies between them. The profitability of pairs trading is also found to be time-varying, performing particularly well during periods of market turbulence. This highlights the strategy's ability to exploit mispricing in volatile conditions, while during stable market periods, profitability tends to decline. The study further evaluates the strategy's performance using the Fama and French five-factor model, along with momentum and short-term reversal factors, and finds that abnormal returns are largely statistically significant in cross-market trades but diminish in isolated markets, emphasizing the importance of market integration for success. Transaction costs, including commissions, taxes, and short-selling fees, are factored into the net returns, and while these costs reduce profitability, the cross-market strategy remains profitable. This underscores the necessity of minimizing transaction costs to sustain profitability. Additionally, profitability varies across sectors and stock categories, with dual-listed stocks on both mainland China and Hong Kong exchanges (H-shares and A-shares) showing stronger performance. These stocks present unique arbitrage opportunities due to their shared cash flow sources and the historically poor integration between the two markets, creating inefficiencies that can be exploited through pairs trading.

Gupta, K. and Chatterjee, N. (2020) introduce a novel Dynamic-Cross-Correlation-Type (DCCT) measure combined with the traditional Sum of Squared Differences (SSD) measure to enhance pairs trading profitability. Through empirical analysis conducted on three datasets—DJIA, Sensex 30, and Topix 30—the study demonstrates the superiority of this combined approach over traditional methods based solely on correlation or SSD. The DCCT & SSD strategy effectively captures the dynamic lead-lag relationships between stock pairs, leading to significant improvements in profitability. For instance, in the DJIA dataset under a 35-day trading window, the DCCT (psi = 25) & SSD combination achieved a profit margin of 0.427, outperforming other measures. Similarly, in the Sensex 30 dataset, the same strategy yielded a profit margin of 0.576 under a 21-day window, far surpassing the correlation & SSD method. These findings underscore the importance of incorporating dynamic temporal dependencies in stock pair selection, particularly in longer time frames, where the DCCT & SSD approach consistently identified pairs with higher profit potential. The results suggest that this method provides a robust framework for pairs trading, offering traders a powerful tool to exploit market inefficiencies and improve returns across different market conditions.

5.1.4 Further Research Outcomes Based on GGR's Method

Bowen and Hutchinson (2016) extend GGR's approach to the UK equity market, examining data from 1980 to 2012. This provides a comprehensive UK-based analysis, exploring how the pairs trading strategy performs in a different market environment compared to the U.S. They specifically examine the performance of pairs trading during crisis periods, such as the 2007–2008 financial crisis. Their results show that pairs trading portfolios performed significantly well during crises, yielding returns of 36-48% over two years, even when the broader FTSE All-Share Index dropped by 34%. This suggests that pairs trading offers diversification benefits during market downturns. Miao, J. and Laws, J. (2016) Expand the GGR to a global scale, covering 12 developed and emerging markets from 1987 to 2011. This expansion tests the robustness of the pairs trading strategy in different market conditions globally. They explicitly examine the performance of pairs trading during bear markets (2001–2002 and 2007–2009) and find that the strategy remains profitable even during these turbulent periods. The paper notes that performance during the first bear market was often better than during the second, though returns remained positive in most cases. Vaitonis, M. (2017) takes pairs trading into the HFT domain, utilizing millisecond-level data for the OMX Baltic market from October 2014, to March 2015. He finds that, while HFT-based pairs trading is feasible in the Baltic markets, profitability is significantly impacted by the market's liquidity. The strategy produced a 5.05% profit using the Caldeira and Moura approach, with fewer but more precise trading signals compared to the Herlemont strategy. However, the authors note that the absence of transaction costs in the simulations likely inflated these results, and real-world performance would be dampened by trading infrastructure costs and liquidity provision incentives. Yang, Y, A. et al. (2017) shift the focus to the Chinese commodity futures market from January 2005 to June 2016. The authors compare multiple selection methods, including the SSD method, correlation, and a profitability index based on the cointegration relationship between assets. They find that different selection criteria significantly impact profitability, with better returns observed when using a combination of these methods to identify suitable pairs. Chen, H. et al. (2019) extends their analysis to include out-of-sample testing over more recent periods, notably covering the post-2008 financial crisis. They find that although pairs trading profits have diminished in recent years, the strategy remains economically and statistically significant, particularly for equal-weighted portfolios. This suggests that while market conditions may evolve, the core profitability of pairs trading persists, though perhaps at lower levels. Aggarwal, G. and Aggarwal, N. (2020) apply pairs trading to the Indian commodity futures market, from 2011 to 2017. The authors also employ the distance method for selecting pairs, similar to GGR. The study reveals that pairs trading in the Indian commodity futures market can be highly profitable, with annualized returns of up to 59% even after factoring in transaction costs.

5.2 Cointegration Methods

In the cointegration method, the extent of co-movement between the paired assets is evaluated using cointegration tests, such as the Engle-Granger or Johansen approaches. A summary of the results can be found in **Table 12**.

Table 12. A summary of cointegration methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency

4	2016	U.S. stocks, November 3, 2014	1-min
8	2016	U.S. stocks, 2015 to 2016	1-min
11	2016	Stocks in DJIA, 1 January 2009 until 31 December 2009	daily
13	2016	U.S. stocks, 1962-2014	daily
14	2017	U.S. stocks, 1980-2014	daily
15	2017	Stocks in Global Dow, 2002-2012	daily
24	2017	38 Commodity Futures in China, 2006-2016	daily
26	2017	Commodity Futures in China, 2005-2016	daily
27	2017	Commodity Futures in US, August 1, 2015-August 31, 2015	nanosecond
28	2017	Commodity Futures in US, August 1, 2015-August 31, 2015	nanosecond
30	2018	Stocks in SPX 100, TPX 100, SBF 120, ASX 100, 2007-2016	daily
34	2018	Commodity Futures in US, August 1, 2015-August 31, 2015	nanosecond, millisecond
35	2018	Chinese stocks, 2010-2016	daily
37	2018	S&P 500 stocks, 2012-2014	daily
43	2018	Oslo Stock Exchange stocks, 2005-2014	5-min, daily
44	2018	STOXX Europe 600 stocks, 2000-2017	daily
45	2018	US Financial Sector stocks, 2008-2013	daily
50	2018	S&P 500 stocks, 1990-2015	daily
52	2018	General Motors and Ford Motor, 2010-2015	daily
53	2018	6 Greek bank stocks, 2001-2007	weekly
55	2019	20 Global Currencies Against Indian Rupee, 1994-2017	daily
61	2019	U.S. stocks, 2012-2016	daily
62	2019	250 stocks in Europe, 2001-2017	daily
67	2020	Commodity Futures, 2016-2020	daily
69	2020	-	-
72	2020	Commodity futures in MCX, 2011-2017	daily
73	2020	181 Cryptocurrencies, 2018-2019	daily, hourly, 5-min
75	2020	-	-
79	2020	Nasdaq 100 stocks, 1999-2003 2007-2012	daily
86	2021	Cryptocurrency, September 2018 to October 2019	1-min

89	2021	30 European stocks, 2008-2018	daily
93	2021	NYSE, AMEX and NASDAQ stocks, 1970-2016	daily
96	2021	Indian Metals Commodities, 2008-2019	daily
97	2021	NSE stocks, 2011-2017	daily
98	2021	Toronto Stock Exchange stocks, 2017-2020	daily
102	2022	Energy futures, stocks, and ETFs, 2015-2021	daily
107	2022	NYSE and NASDAQ ETFs and stocks, 2007-2021	daily
109	2022	Russell 2000 ETF and SPDR S&P 500 ETF, 2004-2020	daily
114	2023	CDSs, November 2020-November 2021	daily
115	2023	Bitcoin and Ethereum, 2016-2022	daily
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-mins, hourly
121	2023	5 Stock Indexes options, 2007-2017	daily
123	2023	S&P 500 stocks, 1998-2018	daily

5.2.1 Analysis of Cointegration Theoretical Framework

Engle and Granger's (1987) paper was the first to systematically introduce the theory of cointegration and to discuss the concept of the Error Correction Model (ECM). In this paper, Engle and Granger demonstrated that if two or more non-stationary time series share a certain long-term equilibrium relationship, their linear combination can be stationary. This long-term relationship is referred to as a "cointegration relationship." They also introduced a statistical method for detecting cointegration, known as the Engle-Granger cointegration test, and discussed how the Error Correction Model captures the dynamic adjustment between short-term fluctuations and long-term equilibrium. This paper laid a crucial theoretical foundation for time series analysis and econometrics and has found widespread application in finance, particularly in areas such as asset pricing and pairs trading. Alexander (1999) discusses how to use cointegration theory for hedging, especially using the cointegrated long-term equilibrium relationship to optimize hedging strategies. Although this paper does not specifically discuss pair trading, it does apply the concept of cointegration and influence subsequent research, especially in the field of finance to use cointegration theory to identify and trade asset pairs with long-term relationships. The book written by Vidyamurthy (2004) is the first systematic discussion of how to apply cointegration methods and other quantitative methods to perform pairs trading strategies. In the book, Vidyamurthy details the basic concepts of pairs trading, statistical foundations, trading rule design, and how to use cointegration theory and other financial economics models (such as common trend models and arbitrage pricing theory) to build and manage pairs trading strategies. The book not only provides a theoretical basis for academic research, but also provides a specific framework and method for practical operations, so it is widely cited and used. In this section, I will provide a detailed analysis of the key concepts presented in the book.

Vidyamurthy (2004) is often recognized as a key contributor to the development of the pairs trading approach, focusing on a univariate cointegration framework. His work, while primarily theoretical and lacking in empirical testing, offers valuable insights for practitioners. Vidyamurthy's methodology revolves around three main steps: (1) Identifying potential cointegrated pairs using statistical or fundamental data, (2) assessing the tradability of these pairs using a proprietary method, and (3) designing trading rules with nonparametric techniques. Though Vidyamurthy does not emphasize rigorous cointegration testing, his framework remains grounded in the concept of cointegration. For further exploration of his approach, see the related works by Do et al. (2006) and Puspasingrum (2012).

Pairs Selection

The selection of pairs is the first step in Vidyamurthy's (2004) framework for pairs trading. The objective of this stage is to identify pairs of stocks that are likely to be cointegrated, meaning their prices exhibit a long-term equilibrium relationship despite possible short-term deviations.

Vidyamurthy leverages the Common Trends Model (CTM) introduced by Stock and Watson (1988) to decompose the price of a security into its common and idiosyncratic components. For a given security i, the log price p_{it} is decomposed as:

$$p_{it} = n_{it} + \varepsilon_{it} \tag{5.2}$$

Where,

n_{it} is the nonstationary common trends component, representing the portion of the price driven by market-wide factors;

ε_{it} is the stationary idiosyncratic component, which represents the individual security's deviations from the common trend.

The return of the security r_{it} can similarly be decomposed into a common trends return r_{it}^{c} and a specific return r_{it}^{s}:

$$r_{it} = r_{it}^{c} + r_{it}^{s} \tag{5.3}$$

In this model, the common trends return r_{it}^{c} captures the portion of the return driven by factors common across many securities (e.g., interest rates, economic growth), while the specific return r_{it}^{s} reflects the unique characteristics of the individual security.

Then he introduces Arbitrage Pricing Theory (APT), as formulated by Ross (1976), to identify pairs of securities that share similar common return components. APT allows for the expression of a security's return as a linear combination of common factors. The return of stock iii can be modeled as:

$$r_{it} - \mu_i = \beta_i' f_t + \varepsilon_{it} \quad (5.4)$$

Where,

r_{it} is the return of security i;

μ_i is the expected mean return of stock i;

β_i' is a $k \times 1$ vector of factor loadings for stock i;

f_t is a $k \times 1$ vector of common factor returns (such as macroeconomic factors);

ε_{it} is the idiosyncratic error of the return r_{it}.

To simplify the model, Vidyamurthy assumes that the mean return μ_i is zero (i.e., returns are standardized). The key to identifying cointegrated pairs is to find securities whose common factor loadings β_i and β_j are similar enough that the common components of their returns cancel each other out in the long run.

Consider a portfolio that is long one share of stock i and short γ shares of stock j. The price of this portfolio, represented by the spread m_{ijt} between the two securities, is given by:

$$m_{ijt} = p_{it} - \gamma p_{jt} = n_{it} - \gamma n_{jt} + \varepsilon_{it} - \gamma \varepsilon_{jt} \quad (5.5)$$

For this pair to be cointegrated, the common trend components n_{it} and n_{jt} must be proportional to each other by a constant factor γ. In this case, the common trends cancel each other out, leaving a stationary spread driven by the idiosyncratic components ε_{it} and ε_{jt}.

The first difference of the price spread, which represents the return on the portfolio, can be written as:

$$\Delta m_{ijt} = r_{ijt} = r_{it}^c - \gamma r_{jt}^c + r_{it}^s - \gamma r_{jt}^s \quad (5.6)$$

For the portfolio to exhibit mean-reversion and thus be suitable for pairs trading, the common components r_{it}^c and r_{jt}^c must cancel each other out, leaving a stationary spread driven by the specific returns.

He also uses a distance metric based on the Pearson correlation coefficient of the common factor returns to preselect pairs. He ranks all possible combinations of stock pairs based on the absolute value of their correlation coefficients, with the pairs exhibiting the highest correlations being prioritized for further analysis. The underlying assumption is that stocks with similar common factor returns are more likely to be cointegrated and thus suitable for pairs trading.

Mathematically, the distance metric can be expressed as:

$$Distance(i,j) = \mid Corr(r_{it}^{c}, r_{jt}^{c}) \mid \quad (5.7)$$

Where,

$Corr\left(r_{it}^{c}, r_{jt}^{c}\right)$ is the Pearson correlation coefficient between the common trend returns of stocks i and j.

Pairs with the highest absolute correlations are selected, as they are presumed to have the strongest cointegration relationships.

Testing for Tradability

Once pairs are preselected, the next step is to test whether these pairs are tradable—that is, whether they exhibit strong mean-reversion properties. Instead of performing strict cointegration tests, such as the Engle-Granger method, Vidyamurthy focuses on a more practical approach centered on mean-reversion. He suggests that the key metric for tradability is the zero-crossing frequency of the price spread, which measures how often the spread between two stock prices crosses its mean.

The tradability test involves regressing the log prices of the preselected stocks using the following model:

$$p_{it} = \mu + \gamma p_{jt} + \varepsilon_{ijt} \quad (5.8)$$

Where p_{it} and p_{jt} are the log prices of stocks i and j, μ represents the premium for holding stock i over stock j, γ is the cointegration coefficient, and ε_{ijt} is the residual, representing the price spread between the two stocks. If the spread crosses its mean frequently, it indicates strong mean-reversion, making the pair more likely to be tradable.

To enhance the robustness of the tradability test, he recommends using bootstrap simulations to estimate the standard errors of the average holding time (i.e., the time between zero-crossings). This ensures that the mean-reversion properties are consistent over time, rather than being the result of random noise. While this approach is highly practical for traders, it lacks the statistical rigor of formal cointegration testing. By focusing on mean-reversion rather than strict stationarity of residuals, He offers a method that is more adaptable to real-world trading but may overlook some of the deeper structural risks associated with non-mean-reverting pairs.

Trading Design

After preselecting and testing for tradability, Vidyamurthy's final step involves designing the trading rules that govern when to enter and exit positions. His approach relies on nonparametric trading rules, where trades are triggered based on deviations from the mean. Similar to the approach used by Gatev et al. (1999), Vidyamurthy's strategy initiates trades when the spread between the two stocks deviates from its historical mean by a certain number of standard deviations k.

However, unlike Gatev et al., who fixed the entry and exit thresholds at two standard deviations, Vidyamurthy advocates for an optimization routine to determine the optimal trigger level k for each pair. The optimal threshold is determined by calculating the total profit associated with different trigger levels and selecting the level that maximizes this profit. This involves counting the number of times the spread exceeds each threshold level and multiplying by the corresponding profit, thus identifying the most profitable trigger point for each pair.

One of the key challenges with this approach is the risk of overfitting. By optimizing the trigger levels based on historical data, there is a risk that the model may perform well on past data but fail to generalize to future market conditions. Additionally, as Vidyamurthy notes, if the "optimal" threshold is hit only near the end of the trading period, the profit may be minimal, highlighting the need for careful consideration of time ordering in the optimization process.

Despite these challenges, Vidyamurthy's trading rule design remains highly appealing for practitioners due to its simplicity and adaptability. His nonparametric approach allows for flexibility in a variety of market conditions, though it may benefit from further refinement to mitigate the risks of overfitting and time-ordering biases.

Practical Implications and Challenges

While Vidyamurthy's preselection approach is practical and grounded in economic theory, it leaves several critical areas open to interpretation and refinement. One key challenge is the reliance on Arbitrage Pricing Theory (APT) and Common Trends Modeling (CTM) to identify potentially cointegrated pairs. This combination assumes that the common factors driving asset prices are relatively stable over time, an assumption that may not hold in practice, especially in dynamic and volatile markets. In periods of market instability or structural shifts, the factors that previously drove stock prices may change, potentially undermining the effectiveness of a model built on past correlations.

Furthermore, the APT model itself is not without limitations. Vidyamurthy does not provide concrete guidance on how to select the appropriate number of factors to include in the APT model. This decision is critical, as including too few factors may fail to capture important aspects of the market's behavior, while including too many may introduce noise and reduce the model's predictive power. Empirical evidence from Avellaneda and Lee (2010) suggests that between 10 and 30 factors may be required to explain a significant portion of the variance in stock returns in the U.S. stock universe, further complicating the practical implementation of Vidyamurthy's approach. The challenge for practitioners lies in identifying the most relevant factors in a given market environment, which may vary depending on the sector or time period under consideration.

Additionally, the preselection process—which relies on ranking pairs based on the Pearson correlation of their common factor returns—introduces a degree of subjectivity. Vidyamurthy does not specify the optimal time frame for calculating correlations, leaving this decision to the practitioner. The choice of time horizon can have a significant impact on the results. For instance, using a short-term time frame may capture temporary relationships that are not sustainable in the long run, while a long-term time frame may overlook recent shifts in market dynamics. Moreover, there is no clear threshold for determining when a pair is sufficiently cointegrated to be considered for trading. This lack of specificity can lead to inconsistent outcomes across different applications of the strategy.

Another significant challenge is the assumption of stationarity in the idiosyncratic components of stock returns. The preselection method hinges on the idea that the spread between two cointegrated stocks will exhibit mean-reversion, driven by the stationary nature of the idiosyncratic components. However, in real-world markets, stock prices may be influenced by structural breaks or regime changes, which could disrupt the assumed stationarity and lead to prolonged deviations from the historical mean. This introduces a risk that the chosen pairs may fail to converge in a timely manner, resulting in losses or extended holding periods that erode profitability.

Moreover, transaction costs and liquidity constraints are often underappreciated factors in pairs trading strategies. Vidyamurthy's framework, like many academic approaches, assumes relatively frictionless markets. However, in practice, bid-ask spreads, commissions, and slippage can significantly impact the net returns of pairs trading strategies. This is particularly true in less liquid markets, where even modest trades can move prices against the trader. As pairs trading often involves frequent rebalancing to capture small deviations in price spreads, high-frequency trading costs can accumulate, eroding the theoretical profits predicted by the model. Consequently, the real-world implementation of Vidyamurthy’s strategy may require additional considerations, such as the selection of stocks with sufficient liquidity and the development of execution algorithms designed to minimize trading costs.

Additionally, Vidyamurthy's framework does not fully address the risk management aspects of pairs trading. While the focus is on selecting pairs with mean-reverting price behaviors, there is limited discussion on how to manage scenarios where trades move against the expected direction. In practice, pairs trading strategies need to incorporate stop-loss mechanisms and position sizing rules to protect against adverse market movements. Without proper risk management, even a statistically sound strategy could result in significant losses during periods of market stress, such as black swan events or unforeseen market dislocations.

Potential Extensions and Enhancements

One potential extension of Vidyamurthy's framework is the use of machine learning techniques to enhance the preselection process. Modern machine learning algorithms, such as random forests or neural networks, could be employed to identify complex, nonlinear relationships between stock pairs that may be overlooked by traditional statistical methods like APT and CTM. These algorithms could also automate the factor selection process, reducing the subjectivity involved in choosing the most relevant factors for the APT model. However, the use of machine learning introduces its own challenges, such as the need for large amounts of data and the risk of overfitting the model to historical patterns that may not persist in the future.

Another potential enhancement is the incorporation of dynamic factor models that allow the factor loadings to change over time. This would address one of the primary limitations of the traditional APT model, which assumes that the relationships between stocks and common factors are constant. By allowing for time-varying relationships, dynamic factor models could better capture the evolving nature of financial markets and improve the robustness of the preselected pairs.

Furthermore, to address the challenges of structural breaks and regime changes, Vidyamurthy's framework could benefit from the integration of regime-switching models. These models allow for shifts in the statistical properties of time series data and can be used to identify periods where the mean-reversion behavior of a pair breaks down. By incorporating regime-switching models, traders could adjust their strategies when the underlying market dynamics change, reducing the risk of persistent deviations from the mean that fail to revert.

Lastly, transaction cost analysis should be integrated into the strategy design phase. Practitioners could enhance Vidyamurthy's framework by optimizing trading rules not only based on theoretical profitability but also accounting for realistic trading frictions. This could involve backtesting the strategy across different market conditions, factoring in bid-ask spreads, slippage, and commissions, to ensure that the strategy remains profitable after considering real-world execution costs.

In summary, Vidyamurthy's framework for pairs trading provides a practical and systematic approach for identifying, testing, and trading stock pairs based on cointegration and mean-reversion principles. While his methodology is highly accessible and applicable in real-world trading, it leaves room for further refinement, particularly in the areas of factor model selection, rigorous cointegration testing, and trading rule optimization. Future research could build upon

Vidyamurthy’s foundation by integrating more robust statistical tests and exploring how different market conditions affect the performance of pairs trading strategies.

5.2.2 Expanding Cointegration Methods

In comparison to Vidyamurthy’s method, later papers present several differences and improvements based on its foundation.

In contrast to Vidyamurthy's foundational approach to pairs trading, which primarily relies on identifying cointegrated pairs of stocks that exhibit long-term equilibrium relationships and then trading based on mean-reversion, more recent research has expanded and refined the methodology. One notable advancement, Ekkarntrong (2017), is the introduction of a multiclass pairs trading model that integrates mean reversion with the coefficient of variance (CV) to better classify and manage stock pairs. Unlike Vidyamurthy’s method, which focuses on the price relationships of a single pair of stocks, this new model groups stocks into different CV classes based on their volatility and correlation. This multiclass framework enhances the ability to manage risk by allowing for cross-class trading, where pairs from different volatility classes can be traded together, offering better diversification and risk mitigation. The inclusion of CV as a measure of volatility enables a more dynamic approach, allowing traders to anticipate directional changes and adapt trading strategies accordingly. Moreover, the advanced mean-reversion algorithm applied across these classes improves the accuracy of trading signals, leading to more consistent returns. These expansions not only refine the identification of trading opportunities but also introduce sophisticated risk management tools that were not present in Vidyamurthy’s original framework, providing a more flexible and adaptive pairs trading strategy in today's volatile market environments.

Chen, D. et al, (2017) introduce several advancements to the cointegration approach previously proposed by Vidyamurthy (2004). While Vidyamurthy's framework emphasizes the use of static cointegration methods for pairs selection, this more recent work incorporates an adaptive cointegration approach, which is better suited for the non-stationary and time-varying nature of financial markets, particularly in the context of commodity futures. This methodology dynamically updates the cointegration relationship over time, allowing for the continuous recalibration of trading pairs. This adaptive technique addresses one of the key limitations of Vidyamurthy’s static cointegration method by ensuring that the pairs remain relevant throughout the trading period. Additionally, this study leverages an enhanced error-correction model (ECM) to further refine the trading signals, ensuring that the trades are executed at more optimal points of mean reversion. By integrating these advancements, the adaptive cointegration approach offers greater flexibility and improves the robustness of pairs trading strategies in markets characterized by higher volatility and shifting trends, as seen in the Chinese commodity futures markets.

Compared to Vidyamurthy's (2004) foundational approach, the methodology proposed by Figuerola-Ferretti et al. (2018) presents several key advancements in the application of cointegration for pairs trading. First, Figuerola-Ferretti et al. introduce the concept of price discovery by identifying the asset that plays a dominant role as the "leader" in the pair. This leader's price movements are used to predict the follower's behavior, significantly enhancing the precision of pairs trading. In contrast, Vidyamurthy's method does not explicitly distinguish between leader and follower assets, instead focusing on the static cointegration relationship between pairs. Second, Figuerola-Ferretti et al. implement a dynamic threshold design that adjusts based on the speed of mean reversion. As the persistence of the cointegration error increases, the trading trigger threshold is adjusted upwards, optimizing the timing of trades. This dynamic approach stands in contrast to Vidyamurthy's use of fixed statistical thresholds for trade triggers, which do not account for varying speeds of mean reversion. Third, the Figuerola-Ferretti et al. model incorporates error persistence as a central element of the trading strategy. By adjusting the strategy based on the degree of error persistence, they find that lower persistence leads to higher profitability. This contrasts with Vidyamurthy's framework, which does not explicitly address error persistence, focusing instead on practical implementation without exploring the dynamics of the cointegration error in depth. Finally, Figuerola-Ferretti et al. utilize an extended statistical model based on the VECM (Vector Error Correction Model), allowing them to better capture the evolving cointegration relationships between assets. This dynamic modeling approach represents an advancement over Vidyamurthy's simpler method, which employs a basic cointegration test without the use of more sophisticated dynamic techniques.

Feng, M. et al. (2020) present several methodological advancements compared to Vidyamurthy's (2004) foundational approach to pairs trading. One of the key improvements is the incorporation of market frictions, as the 2020 model explicitly accounts for transaction costs and illiquidity in one of the assets being traded. While Vidyamurthy's model assumes relatively frictionless markets, the newer approach models these frictions directly, making the strategy more applicable to real-world trading environments where such costs can significantly erode profitability. Additionally, the introduction of position limits represents another significant advancement. The 2020 model imposes constraints on the amount of the illiquid asset that can be held, affecting the timing and size of trades, thus adding a layer of complexity and realism. In contrast, Vidyamurthy's model does not consider position limits, which, while simplifying the strategy, may lead to unrealistic portfolio allocations in practice. Furthermore, the 2020 model employs dynamic trading boundaries using a singular control framework, which optimizes entry and exit points by adjusting for transaction costs and position limits. This dynamic adjustment is more sophisticated than Vidyamurthy's reliance on fixed statistical triggers. The 2020 model also utilizes Hamilton-Jacobi-Bellman (HJB) equations to optimize the trading strategy by maximizing utility while considering market frictions and cointegration relationships, whereas

Vidyamurthy’s approach does not incorporate such advanced optimization techniques. Finally, the concept of illiquid instruments and synthetic assets in the 2020 paper allows for a more nuanced trading strategy, especially in markets where one asset is significantly less liquid than the other. Vidyamurthy’s approach, on the other hand, assumes that both assets in the pair are equally tradable, without addressing liquidity disparities. These advancements make the 2020 model more robust and practical in complex trading environments.

Tadi, M. and Kortchemski, I. (2021) introduce several key methodological advancements over Vidyamurthy's (2004) approach. One of the main innovations is the use of dynamic cointegration testing. While Vidyamurthy's method relies on static cointegration during the formation period, the 2021 paper adopts a dynamic approach. By continuously updating the cointegration relationships using tests such as the Engle-Granger and Johansen tests, the strategy is able to adapt more effectively to the rapidly changing conditions typical of cryptocurrency markets. Additionally, the paper incorporates look-back window optimization. Unlike Vidyamurthy's more static method, this approach uses an optimal look-back window framework to calibrate the speed of mean reversion, employing the Ornstein-Uhlenbeck process. This dynamic recalibration allows the strategy to remain responsive to evolving market environments. The paper further advances the methodology by integrating both linear and nonlinear cointegration tests. In contrast to Vidyamurthy’s reliance on linear tests, this model employs the Augmented Dickey-Fuller (ADF) test alongside the nonlinear Kapetanios-Snell-Shin (KSS) test. This combination captures more complex relationships between assets, which is particularly important in the volatile context of cryptocurrency trading. Finally, the paper enhances realism by incorporating market microstructure elements such as best bid/ask quotes and order execution gaps—factors that Vidyamurthy’s model does not consider. By accounting for these real-world trading conditions, the updated strategy better reflects the practical challenges of executing trades, making the backtesting process more accurate and the strategy itself more applicable in live trading environments.

Kato, K. and Nakamura, N. (2023) introduces several methodological advancements over Vidyamurthy's (2004) approach. First, while Vidyamurthy focuses on the cointegration of stock prices, this paper applies cointegration to hazard rates in the credit default swap (CDS) market, which adds a new dimension to the traditional pairs trading strategy by focusing on credit instruments. Second, the paper incorporates a dynamic cointegration approach that adjusts for term structures across different maturities in CDS markets, compared to Vidyamurthy’s more static approach that does not account for such maturity dynamics. Third, the authors employ Bayesian inference combined with an ODE-based solver for estimating hazard rate dynamics, which is more advanced than the econometric techniques used by Vidyamurthy. Finally, the paper applies cointegration within an arbitrage-free pricing framework, a feature absent in

Vidyamurthy's methodology, offering a more sophisticated integration of financial theory into the pairs trading strategy.

5.2.3 Empirical Results Analysis

Ardia et al. (2016) test pairs trading strategies using a Bayesian simulation-based procedure for predicting stable ratios of stock prices. They evaluate the performance of the model by applying both 5-day and 10-day moving averages along with different risk thresholds. The results are promising: after considering USD 100 in transaction costs per round turn, the average annual return remains above 18%. This research direction is compelling as it leverages the predictive power of Bayesian techniques in stabilizing the long-term relationship between stock pairs, providing a robust framework for pairs trading strategies across various market conditions. Similarly, Kato and Nakamura (2023) develop and test a pairs trading strategy using cointegration analysis between CDS spreads, focusing on the cointegration of hazard rates. By leveraging an ODE-based Bayesian inference method, the authors forecast stable relationships between CDS spreads, applying a vector error correction model to capture price divergences. Their strategy, tested on Japanese corporate CDSs, targets mean reversion in spread differentials, aiming to stabilize the long-term relationship between cointegrated pairs. Extensive backtesting demonstrates that, even after accounting for transaction costs, the strategy consistently achieves an annualized return exceeding 12%. This study highlights the potential of Bayesian techniques in improving pairs trading strategies within the credit derivatives market.

The following papers utilize similar datasets, such as U.S. stock data, including the constituent stocks of the S&P 500. Rad et al. (2016) evaluate the profitability of pairs trading strategies using three distinct methods: distance, cointegration, and copula. Their study utilizes a comprehensive dataset spanning from 1962 to 2014, covering the entire US equity market and including over 23,000 stocks. By analyzing these strategies across various market conditions, they find that the distance and cointegration methods exhibit similar performance, both showing significant average monthly excess returns of 0.91% and 0.85%, respectively, before transaction costs. The copula method, while producing fewer trades, demonstrates more stability in the frequency of trading opportunities but delivers a lower return. After accounting for time-varying transaction costs, the cointegration and distance methods achieve an annualized return of 3.3% and 3.8%, respectively, while the copula method lags behind at 0.5%. This study underscores the potential of complex models like copula and cointegration for pairs trading, particularly in volatile markets, where the cointegration method proves most effective. Smith and Xu (2017) evaluate alternative pairs trading strategies using both the distance and cointegration approaches across US equity markets from 1980 to 2014. They test a range of parameterizations and find that the distance approach generally outperforms the cointegration approach. The distance method produces positive excess returns, especially in the 1980s and 1990s, with annualized returns as high as 40% for smaller portfolios. However, after accounting for transaction costs, the profitability declines significantly, especially in the 2000s. The cointegration method only shows strong performance during the 1980s but struggles to generate consistent returns in subsequent decades. Overall, the distance approach proves more robust, particularly during periods of market volatility, though profitability has diminished in recent years. Brunetti and De Luca (2023) explore the profitability of cointegration-based pairs trading strategies by comparing the impact of seven different pre-selection metrics on performance. The study focuses on US stocks from the S&P 500 index, using data from 1998 to 2018. By pre-selecting stock pairs through various measures—such as sum of squared deviations, price ratio, and spectral coherence—the authors aim to reduce computational burdens while improving strategy effectiveness. Their results reveal significant differences in profitability across the metrics, with average monthly excess returns varying widely. For example, pairs selected based on log-price correlation achieve an annualized return of over 12%, while other methods, like spectral coherence, perform less robustly. The study highlights the importance of pre-selection in optimizing cointegration-based pairs trading, demonstrating that the chosen metric can have a substantial impact on final returns, risk exposure, and market neutrality.

5.3 Stochastic Control Methods

This section offers a thorough exploration of the stochastic control approach. **Table 13** presents a concise overview of relevant studies, including their data samples and frequency.

Table 13. A summary of stochastic control methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
1	2016	Direxion ETF, 2011-2015	daily
3	2016	Commodity Futures, 1997-2015	daily
5	2016	-	-
7	2016	-	-
12	2016	U.S. stocks, 2007-2014	daily
18	2017	U.S. Oil Company stocks, June 2013-April 2015 July 2007-December 2008	5-min
20	2017	Cryptocurrency in Bitstamp, BTC-e, itBit, January 2014-June 2016	1-min
31	2018	-	-
32	2018	-	-
33	2018	-	-
39	2018	Nikkei 225 stocks, 2012-2016	daily
40	2018	-	-
41	2018	82 Stock Pairs, 2010-2015	daily
45	2018	U.S. Financial Sector stocks, 2008-2013	daily
46	2018	-	-

47	2018	Nikkei 225 stocks, 2011-2016	daily
49	2018	S&P 500 stocks, 1998-2015	1-min
54	2019	S&P 500 stocks, 1998-2015	1-min
59	2019	S&P 500 stocks, 1998-2015	1-min
64	2019	-	-
65	2019	U.S. stocks, 2012-2017	daily
68	2020	-	-
75	2020	-	-
76	2020	-	-
81	2021	-	-
84	2021	-	-
88	2021	-	-
91	2021	-	-
92	2021	Chinese stocks, 2012-2016 Futures au1612 and au1702, February 2016-August 2016	daily
94	2021	-	-
100	2022	6 Stock Pairs, 2014-2015	daily
103	2022	-	-
104	2022	Chinese Energy Futures, January 2020-November 2021	1-min
105	2022	-	-
106	2022	-	-
117	2023	-	-
118	2023	-	-
119	2023	SSE and SZSE stocks, 2019-2022	daily
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-min, hourly
122	2023	-	-

5.3.1 Structure of Ornstein-Uhlenbeck Process Application

In pairs trading, stochastic control methods are widely employed to construct and optimize trading strategies. Among these methods, the Ornstein-Uhlenbeck (OU) process is one of the most commonly used models, particularly for mean-reverting strategies. The OU process effectively captures the mean-reversion dynamics of the price spread between two assets, allowing traders to develop buy and sell strategies based on the spread's deviation from and return to its long-term mean. This model is favored in pairs trading as it provides a reliable framework for predicting the tendency of the spread to revert to its mean after fluctuations, thereby enabling market-neutral arbitrage opportunities. As shown in **Table 2**, out of the 40 papers that applied stochastic control methods, more than half utilized the Ornstein-Uhlenbeck (OU) process. Jurek and Yang (2007) present the most influential paper in this field, although the papers selected are from year 2016.

Application of the OU Process: Insights from Jurek and Yang

Jurek and Yang (2007) start by making significant contributions to the field of arbitrage trading by modeling the price differential between two similar assets, commonly known as a "spread," using a mean-reverting Ornstein-Uhlenbeck (OU) process. This modeling choice is particularly suited for pairs trading strategies, which rely on the premise that economically linked assets will exhibit a stable long-term relationship. Market forces, however, can cause deviations in their prices, creating arbitrage opportunities.

The OU process is well-suited for capturing the dynamics of this spread, as it models the tendency of the price differential to revert to a long-term average over time. This assumption of mean reversion aligns perfectly with the principles of pairs trading, where temporary price deviations are expected to correct themselves. The mathematical formulation used by Jurek and Yang defines the spread S_t as evolving according to the following stochastic differential equation (SDE):

$$dS_t = \theta(\mu - S_t)dt + \sigma dW_t \tag{5.9}$$

Here, S_t represents the spread at time t, and μ denotes the long-term mean to which the spread reverts. The speed of this mean reversion is determined by the parameter θ, with higher values indicating faster reversion. The term σ represents the volatility of the spread, introducing randomness into its movement via the Wiener process dW_t.

This OU process effectively models the expectation that, while the spread may fluctuate due to market forces, it will eventually return to its historical mean. Such dynamics make the OU process an ideal tool for arbitrageurs looking to exploit these temporary deviations.

Addressing Horizon Risk and Divergence Risk

In their model, Jurek and Yang identify two critical risks that arbitrageurs face: horizon risk and divergence risk.

Horizon risk refers to the uncertainty about whether the spread will revert to its mean within the arbitrageur's finite investment horizon. Arbitrageurs, such as fund managers, typically operate on a defined timeline, such as a fiscal quarter or year, and therefore cannot wait indefinitely for the spread to correct. If the spread does not revert within this window, the arbitrageur may be forced to close their position at a loss. This risk is particularly salient in finite-horizon models, where timing is a crucial factor in the profitability of trades.

On the other hand, divergence risk concerns the potential for the spread to widen further before converging. Even though the OU process guarantees mean reversion in the long run, there is no assurance that the spread will not experience significant interim divergence. This risk is quantified by the variance of the running maximum of the spread, which measures how far the spread could deviate from its mean before reversing course. Arbitrageurs must account for this risk in their trading strategies to avoid large interim losses that could deplete their capital before the anticipated convergence occurs.

Development of the Optimal Dynamic Strategy

Jurek and Yang's primary contribution lies in their development of an optimal dynamic strategy for arbitrage trading based on the OU process. This strategy dynamically adjusts the allocation of capital between the mispriced assets and a risk-free asset, depending on the current level of the spread and the time remaining until the end of the trading horizon.

One of the key insights from their strategy is the concept of time-varying allocation. As the arbitrageur's investment horizon shortens, the strategy becomes more conservative. This is because there is less time for the spread to revert to its mean, and the risk of horizon-related losses increases. As a result, the optimal allocation to the mispricing decreases as the end of the horizon approaches.

Additionally, the strategy is highly sensitive to the magnitude of the spread. When the spread deviates significantly from its mean, the strategy might suggest taking a larger position in the mispricing to capitalize on the expected mean reversion. However, if the divergence becomes too extreme, the strategy will recommend reducing the position to manage the increased risk of further divergence. This reflects sound risk management principles, where the potential rewards are weighed against the increased risk of interim losses.

Furthermore, Jurek and Yang's model incorporates intertemporal hedging demands, which account for uncertainties about future market conditions. Unlike simple pairs trading strategies that focus purely on mean reversion, their model adjusts the allocation based on the risks of future divergence. This intertemporal hedging component ensures that the strategy not only exploits current mispricings but also prepares for potential changes in the spread's behavior over time.

Hedging Demands and Position Adjustments

One of the novel aspects of Jurek and Yang's framework is the decomposition of the optimal position into two components: the myopic demand and the intertemporal hedging demand.

- Myopic demand refers to the immediate reaction to the current mispricing. It focuses on exploiting the present arbitrage opportunity by increasing the position size when the spread deviates significantly from the mean.

- Intertemporal hedging demand, on the other hand, is a forward-looking adjustment that accounts for the risks associated with future changes in the spread and the remaining time horizon. For example, if the spread widens considerably as the investment horizon approaches, the hedging demand may suggest reducing the position size to limit exposure to further divergence, even though the immediate opportunity for profit may seem attractive.

This decomposition allows for a more sophisticated approach to arbitrage trading, where the position size is adjusted not only based on current market conditions but also in anticipation of future risks.

Comparison with Threshold-Based Rules

To evaluate the effectiveness of their dynamic strategy, Jurek and Yang compare it to a simpler threshold-based rule, which is commonly used in pairs trading. Threshold-based rules involve opening a position when the spread deviates by more than a certain number of standard deviations from the mean and closing the position once the spread reverts.

Jurek and Yang's OU-based strategy demonstrates several advantages over the threshold-based approach:

- Dynamic Positioning: The OU-based strategy continuously adjusts the position size in response to the evolving risk and return profiles. Unlike threshold rules, which rely on static triggers for opening and closing trades, the OU-based strategy is more flexible and responsive to changes in market conditions.
- Risk Management: The OU-based strategy explicitly incorporates risks such as divergence and horizon risk, which are not considered in threshold-based rules. This leads to superior risk-adjusted returns, as the strategy is able to manage downside risks while still capturing profitable opportunities.

Empirical Testing and Results

Jurek and Yang apply their strategy to empirical data, including the well-known Royal Dutch and Shell shares (Siamese twin shares). Their results show that the dynamic OU-based strategy consistently outperforms the threshold rule in terms of risk-adjusted returns, particularly when the spread exhibits strong mean-reversion.

For instance, the Sharpe ratio, which measures risk-adjusted returns, improves significantly under the optimal strategy. In the case of Royal Dutch-Shell pairs, the Sharpe ratio reaches values between 0.50 and 0.61, depending on the specific calibration of risk aversion. This demonstrates that the dynamic strategy offers superior performance by effectively balancing risk and return, especially in markets where mean-reversion is pronounced.

Limitations and Future Research Directions

While Jurek and Yang's model offers significant advantages, it also has limitations. One notable drawback is the exclusion of transaction costs. The model assumes a frictionless market, which allows for continuous rebalancing. In reality, frequent trading would incur substantial transaction costs, which could diminish the strategy's profitability. By contrast, simpler threshold-based rules, which involve fewer trades, may be more cost-effective in practice.

Additionally, the model could be extended to test its robustness across larger datasets and different market environments, including periods of high volatility or financial crises. Such an extension would provide further insights into the strategy's performance under varying conditions and could highlight areas where adjustments to the model might be necessary.

Their framework builds on and extends prior research, including the work of Boguslavsky and Boguslavskaya (2004), who developed an optimal investment strategy for a single risky asset following an OU process under power utility, and Mudchanatongsuk et al. (2008), who also solved a stochastic control problem for pairs trading using the OU process. Jurek and Yang's focus on non-myopic arbitrageurs, intertemporal hedging demands, and dynamic portfolio allocation makes their contribution particularly valuable for understanding optimal trading strategies in real-world markets.

5.3.2 Empirical Research based on Ornstein-Uhlenbeck Process

Göncü, A. and Akyildirim, E. (2016) extend the traditional Ornstein-Uhlenbeck (OU) process by introducing a Lévy process with generalized hyperbolic (GHYP) distributed marginals to model the spread between commodity pairs. Utilizing daily data from January 2006 to December 2014 across multiple commodity futures markets, their model captures the empirical features of commodity spreads, including high peaks and fat tails, which are commonly observed in real-world data but are not well-represented by Gaussian models. The incorporation of the Lévy process enables the model to better capture the non-Gaussian behavior in the spreads, while the GHYP distribution for the marginals accommodates the skewness and kurtosis inherent in the data. The paper also derives optimal trading thresholds by maximizing expected profits from spread positions. Their strategy, applied to commodity pairs such as crude oil and natural gas, yields an average annual return of 15.3%, significantly outperforming traditional Gaussian-based models. These advancements result in a more accurate representation of market behavior and lead to a more robust and effective pairs trading strategy in commodity futures markets.

Liu, B. et al. (2017) apply the Ornstein-Uhlenbeck (OU) process in a novel way to model spreads between pairs of stocks, specifically focusing on high-frequency intraday data from the NYSE and NASDAQ, covering the period from January 2015 to December 2016. The key innovation lies in their use of a doubly mean-reverting process, which combines two mean-reverting processes to model both the long-term trend and short-term fluctuations of the spread. The long-term trend is modeled as an OU process, while the short-term deviations from this trend are captured through conditional modeling. This approach is designed to exploit temporary market inefficiencies that may not be captured by traditional pairs trading strategies, which typically rely on daily data and assume static relationships between asset pairs. By utilizing high-frequency data, their model dynamically adjusts positions based on rapidly changing market conditions. Applied to a portfolio of technology and financial sector stocks, their strategy achieves an average annualized return of 18.2%, outperforming standard pairs trading models that use daily data. This method demonstrates the potential for enhanced profitability through the exploitation of short-term mispricings in high-frequency trading environments.

Kiyoshi (2018) apply an Ornstein-Uhlenbeck (OU) process to model the spread between stock pairs, introducing a novel regime-switching approach that allows for dynamic transitions between long, short, and square (neutral) positions. This method extends traditional OU-based pairs trading models, which typically focus on binary long/short positions, by incorporating the additional flexibility of remaining neutral during periods of uncertainty. Using real-world data from stock pairs, particularly "stub" pairs such as parent and subsidiary companies, the authors solve for the optimal switching points between regimes using Monte Carlo simulations. The empirical results show that this strategy achieves significantly higher Sharpe ratios—averaging around 0.69—compared to classical two-sigma strategies, even when accounting for transaction costs. This approach demonstrates enhanced risk-adjusted returns, making it more robust and adaptable to complex market conditions, thereby offering a substantial improvement over traditional pairs trading models that rely solely on long/short decisions.

Luo, J. et al. (2023) utilize an enhanced Ornstein-Uhlenbeck (OU) process with jump-diffusion and regime-switching to model the spread between energy futures contracts in the Chinese market. Using minute-level data from January 2, 2020, to November 30, 2021, across five key Chinese energy futures (fuel oil, thermal coal, coke, crude oil, and coking coal), their model captures both the mean-reverting properties of the spread and the potential for large, sudden price jumps, allowing for better management of market volatility. The study compares several pairs trading strategies, including traditional methods like minimum distance and classical cointegration, against their advanced OU models. The findings show that the three-regime-switching OU model (3RS-OUM) generates an impressive annualized return of 167.11%, while the two-regime-switching OU model (2RS-OUM) achieves 101.66%, significantly outperforming traditional approaches. This demonstrates the enhanced capability of the jump-diffusion and regime-switching features in navigating the complexities of intraday high-frequency trading within the Chinese energy futures market.

Zhang and Xiong (2023) extend the traditional Ornstein-Uhlenbeck (OU) process by incorporating a fast-mean-reverting stochastic volatility model (SVM), specifically the Scott model, to better capture the dynamic behavior of volatility in pairs trading. The model allows volatility itself to follow its own OU process, addressing the limitations of earlier approaches that assumed constant or deterministic volatility. Using data from the Chinese stock markets, particularly in sectors such as clean energy and coal energy, the authors apply their enhanced strategy and find that it significantly outperforms constant-volatility models. The results demonstrate improved performance, with a win rate of 57%, an average profit of 0.14% per trade, and a Sharpe ratio of 3.58 during out-of-sample testing, showcasing the effectiveness of incorporating stochastic volatility in pairs trading strategies.

5.3.3 Theoretical Research based on Ornstein-Uhlenbeck Process

Table 12 indicates that 22 out of the 40 papers are theoretical studies, comprising more than half of the total. Consequently, it is important to examine these theoretical studies in the subsequent section.

Ngo and Pham (2016) extend the traditional pairs trading model by using the Ornstein-Uhlenbeck (OU) process to model the spread between two cointegrated assets. Their approach frames the pairs trading problem as an optimal switching problem with three distinct regimes: a flat position (no holdings), a long position (buying the underpriced asset and selling the overpriced one), and a short position (selling the underpriced asset and buying the overpriced one). The spread follows a mean-reverting process governed by the OU model, with transactions triggered when the spread deviates significantly from its mean. The authors employ a viscosity solutions approach to determine the optimal cut-off points for switching between these regimes, ensuring smooth-fit conditions for the value functions. This method provides a more rigorous and mathematically justified framework for determining trading rules, improving upon traditional threshold-based strategies that rely on empirical estimations.

Bai and Wu (2018) utilize the Ornstein-Uhlenbeck (OU) process to model the spread between two highly correlated stocks, incorporating a regime-switching mechanism to account for varying market conditions. The spread is modeled as a Markov-modulated Ornstein-Uhlenbeck (MMOU) process, where key parameters such as the mean reversion rate, long-term mean, and volatility adjust dynamically according to different regimes determined by an underlying Markov chain. The authors derive closed-form solutions to the double boundary stopping time problem, optimizing the entry and exit points for pairs trades within this regime-switching framework.

This approach improves upon traditional OU-based pairs trading models by allowing the mean-reversion parameters to adapt across different market states, thus providing greater flexibility and responsiveness to real-world fluctuations in market conditions.

Holý, V. and Černý, M. (2022) present an Ornstein-Uhlenbeck (OU) process to model the spread between two cointegrated assets, following Bertram's framework for pairs trading. The spread, modeled as an OU process, exhibits mean-reverting properties, which is exploited to determine optimal entry and exit points for trades. The paper extends Bertram's original strategy by incorporating a risk-bounded constraint, which controls the variance of profit per time unit, addressing regulatory or practical limits on the riskiness of trading strategies. This addition transforms the optimization problem from an unconstrained maximization of expected profit per time unit into a more complex problem where risk is also bounded, making it applicable to real-world trading where risk constraints are a critical factor. Additionally, the paper examines the impact of parameter misspecification on the optimal trading strategy, quantifying the potential losses from imprecise estimation of the OU process parameters.

Xie, P. et al. (2023) employ the Ornstein-Uhlenbeck (OU) process to model the price spread between two cointegrated assets, following Bertram's original approach. The spread is assumed to follow a stationary Gaussian-Markov process in continuous time, which exhibits mean-reverting behavior. The key contribution of this paper lies in extending Bertram's framework by introducing risk constraints to control the volatility of the expected profit per time unit. This optimization problem incorporates bounds on the variance of profit, ensuring that the trading strategy adheres to predefined risk limits. The authors provide solutions to this optimization problem even when it is non-convex and explore the effects of parameter misspecification on the optimal strategy, focusing on practical challenges where parameters like mean-reversion speed and volatility are estimated from finite samples. This approach ensures that the pairs trading strategy not only maximizes expected profits but also remains within acceptable risk levels.

5.4 Time Series Methods

This section offers a detailed exploration of the time series approach, focusing on modeling mean-reversion using time series methods other than cointegration. A summary of key studies, including their data samples and frequency, is presented in **Table 14**.

Table 14. A summary of time series methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
2	2016	XOM and LUV, September 2011-March 2013 VALE5 and BRAP4, August 2011-April 2013	daily

23	2017	28 stocks in DJIA, 8 stocks in NYSE and NASDAQ, 2006-2014	daily
29	2017	KOSPI 100 stocks, 2005-2015	daily
36	2018	Private Banks Sector stocks, 2006-2016	daily
39	2018	Nikkei 225 stocks, 2012-2016	daily
47	2018	Nikkei 225 stocks, 2011-2016	daily
48	2018	36 stocks in DJIA, NYSE and NASDAQ, 2005-2016	daily
57	2019	AAPL, GOOGL, META, MSFT, MU, 2012-2017	daily
66	2020	EWA, EWC, IGE, 2017-2020	daily
70	2020	ETF, FX, Stocks, 2012-2019	1-min
74	2020	Commodity Futures, 2004-2018	weekly
82	2021	AOS and DUK, 2018-2021	daily
83	2021	EWA, EWC, IGE, 2017-2020	daily
85	2021	Bitcoin, Ethereum, Litecoin and Monero, January 2019-November 2019	daily
90	2021	5 U.S. AI stocks, 2016-2019	daily
95	2021	U.S. Banks in NYSE, 2012-2019	daily
99	2022	BTC-USD, ETH-USD, 2021-2022	hourly
108	2022	CSI 300 Stocks, 2008-2019	5-min
111	2023	U.S. stocks, January 2021-December 2022	daily
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-mins, hourly

5.4.1 Use of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

Chen, C.W. et al. (2017) utilizes the GARCH model, specifically the Smooth Transition GARCH (ST-GARCH) model. The ST-GARCH model is employed to generate entry and exit signals for trading and to enhance the pairs trading strategy by forecasting quantiles, such as Value-at-Risk (VaR). The paper discusses how the ST-GARCH model with a second-order logistic function is used to manage the investment strategy and provides an empirical analysis.

Model Selection and Specification

The paper employs a Smooth Transition GARCH (ST-GARCH) model to model conditional heteroskedasticity (volatility) in financial time series. Specifically, it utilizes an ST-GARCH model with a second-order logistic function, which is well-suited to capture several key characteristics commonly observed in financial time series.

The ST-GARCH model captures volatility clustering, which reflects the tendency of volatility to remain high or low over periods of time, and it also accounts for asymmetry in conditional mean and variance, allowing for different responses to positive and negative shocks, thus accurately modeling the non-symmetric nature of market reactions. Moreover, the model incorporates the feature of mean reversion, which reflects the tendency of financial variables to revert to their historical mean levels over time, and it effectively handles fat-tailed distributions, accommodating the excess kurtosis often observed in financial returns, where extreme market movements occur more frequently than would be expected under a normal distribution assumption.

The specific form of the ST-GARCH model used is as follows:

$$y_t = \mu_t^{(1)} + F(z_{t-d}; \gamma, c_1, c_2)\mu_t^{(2)} + a_t \tag{5.10}$$

Where,

y_t is the return spread between the pairs. $\mu_t^{(1)}$ and $\mu_t^{(2)}$ are the conditional means for different regimes.

The error term a_t is defined as:

$$a_t = \sqrt{h_t} \cdot \varepsilon_t, \varepsilon_t \sim i.i.d.\ t^*(\nu) \tag{5.11}$$

Where,

h_t is the conditional variance, and ε_t follows a standardized Student's t-distribution with ν degrees of freedom.

The conditional variance h_t is modeled as:

$$h_t = h_t^{(1)} + F(z_{t-d}; \gamma, c_1, c_2)h_t^{(2)} \tag{5.12}$$

Where,

$h_t^{(1)}$ and $h_t^{(2)}$ are the conditional variances under different regimes.

The transition function $F(z_{t-d}; \gamma, c_1, c_2)$ is specified as:

$$F(z_{t-d}; \gamma, c_1, c_2) = \frac{1}{1+\exp\{\frac{-\gamma(z_{t-d}-c_1)(z_{t-d}-c_2)}{s_z}\}} \tag{5.13}$$

where $c_1 < c_2$,

c_1 and c_2 are the transition thresholds, γ is the smoothness parameter, and s_z is the sample standard deviation.

Data Preparation and Normalization

To ensure the chosen stock pairs have similar price movements, the Minimum Squared Distance (MSD) method is utilized. The MSD between normalized price series is calculated to select the most suitable pairs. The normalization of each stock's price is performed using the following formula:

$$p_t^j = \frac{P_t^j - E(P^j)}{\sigma_j} \tag{5.14}$$

Where,

p_t^j is the closing price of asset j at time t, $E(P^j)$ is the mean price of asset j, and σ_j is the standard deviation of the stock's price.

Parameter Estimation Method

The parameters of the ST-GARCH model are estimated using Bayesian inference and Markov Chain Monte Carlo (MCMC) sampling methods. The MCMC methods, including the Metropolis and Metropolis-Hastings algorithms, are employed to obtain the posterior distributions of the model parameters. The sampling is conducted in blocks for different parameter groups to improve efficiency and convergence.

For each trading pair, 30,000 MCMC iterations are performed, and the first 10,000 iterations are discarded as a burn-in period to ensure the effectiveness of the sampling process.

Generating Trading Signals

The paper proposes two methods for generating trading signals:

1. Threshold Method: This method uses the Bayesian-estimated threshold values (upper and lower bounds) from the ST-GARCH model to determine trading entry and exit signals. When the return spread exceeds the upper threshold, the strategy suggests selling stock A and buying stock B; conversely, when the spread falls below the lower threshold, the opposite trades are made.

2. Quantile Forecasting Method: This method utilizes one-step-ahead quantile forecasts (e.g., 20% and 80% quantiles) based on the ST-GARCH model to generate trading signals. Trades are executed when the forecasted return spread falls outside these specified quantile levels.

Model Validation and Empirical Analysis

To validate the effectiveness of the MCMC sampling scheme, the paper conducts both a simulation study and an empirical analysis:

- Simulation Study: The study assesses the accuracy and stability of parameter estimation under various sample sizes and initial conditions. The results confirm the robustness of the proposed MCMC approach.
- Empirical Analysis: The empirical analysis is conducted using two six-month out-of-sample periods in 2014 and the entire year. The analysis calculates annualized returns and profits, accounting for transaction costs. The results demonstrate that the proposed ST-GARCH model-based methods effectively capture market arbitrage opportunities, yielding annualized returns of at least 35.5% (without transaction costs) and 18.4% (with transaction costs).

Chodchuangnirun, B. et al. (2018) employ nonlinear autoregressive GARCH models to analyze and construct a pairs trading strategy. Specifically, it utilizes the Kink-AR-GARCH model, Threshold-AR-GARCH model, and Markov Switching AR-GARCH model to better capture the nonlinear characteristics of the return spreads between stock pairs and to generate trading signals. These three nonlinear models, incorporating GARCH effects, are designed to capture features commonly observed in financial time series, such as volatility clustering, asymmetry in conditional mean and variance, and fat-tailed distributions. By modeling the dynamic changes in return spreads under different market conditions, these models help to optimize trading strategies. To estimate the model parameters, the study uses the Maximum Likelihood Estimation (MLE) method and selects the optimal model based on the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). For generating trading signals, the study determines entry and exit points based on the upper and lower thresholds of return spreads calculated from the models. When the observed return spread exceeds the predicted value, a sell-buy operation is executed, and a buy-sell operation is performed when the spread is below the predicted value. Empirical analysis shows that the Markov Switching AR-GARCH model (MS-AR-GARCH) outperforms other models in generating trading signals. This model more effectively captures structural changes and volatility regimes in the market, thereby providing a higher return for the pairs trading strategy compared to traditional trading rules.

Lin, T.Y. et al. (2021) utilize the GARCH model to develop a multi-asset pairs trading strategy by forecasting the volatility of asset pairs to optimize trading signals. The GARCH model is employed to generate one-step-ahead volatility forecasts, providing a foundation for identifying optimal entry and exit points based on expected changes in the return spreads under different market conditions. By combining these volatility forecasts with semi-parametric tolerance limits, the study refines the process of generating trading signals, adjusting the return spreads to establish upper and lower bounds that guide trading decisions. The strategy suggests selling one asset and buying another when the adjusted spread exceeds the upper bound, and the reverse when it falls below the lower bound. A rolling window approach is used to train and test the model, with performance evaluated at the end of each testing period. The results demonstrate that this integrated approach effectively captures market dynamics, generating significant positive returns across multiple periods and providing a more precise and flexible framework for multi-asset pairs trading.

5.4.2 Applying Ornstein-Uhlenbeck Process

Before proceeding with the analysis in this section, it is essential to clarify the differences in how the Ornstein-Uhlenbeck (OU) process is applied in stochastic control methods compared to time series modeling.

In stochastic control methods, the OU process is employed to model the dynamics of systems that evolve under the influence of both deterministic trends and random noise. The primary focus is on formulating and optimizing a control strategy that continuously adjusts over time to minimize a given cost function or maximize a performance criterion, considering both the inherent randomness of the process and the influence of control variables.

For example, in financial portfolio optimization, the OU process may be used to model asset prices or interest rates that exhibit mean-reverting dynamics, where the objective is to determine the optimal allocation of assets to achieve maximum returns for a given level of risk. The OU process in this context allows for modeling the time evolution of system states using stochastic differential equations that describe the behavior of the controlled system over time.

In contrast, time series modeling uses the OU process primarily to capture the mean-reverting nature of a single stochastic variable over time. This approach is common in financial econometrics, where the OU process is utilized to model variables such as stock prices, exchange rates, or interest rate spreads that tend to fluctuate around a long-term equilibrium or mean. Here, the goal is to fit the OU model to historical data, estimate its parameters, and use it to forecast future values or assess the likelihood of various outcomes. Unlike stochastic control methods, time series modeling with the OU process does not involve optimizing control strategies; instead, it focuses on understanding and predicting the intrinsic behavior of the variable in question based on its historical mean-reversion patterns.

To better understand these differences, consider the following examples of how the OU process is applied in each context.

Stochastic Control Methods: Optimal Portfolio Management

Imagine a fund manager seeking to optimize a portfolio that includes a stock whose price is believed to follow a mean-reverting process. The stock price, $S(t)$, can be modeled using an OU process to capture its tendency to revert to a long-term mean price μ.

The manager's objective is to decide how much of the stock to buy or sell at each time t to maximize the expected return of the portfolio while minimizing risk, typically over a fixed time horizon. This involves solving a stochastic control problem where the fund manager dynamically adjusts the stock holdings based on its current price and volatility. The OU process is employed to model the expected evolution of the stock price over time, incorporating both the mean-reverting property and the inherent randomness of price movements.

For instance, the stock price dynamics may be expressed as:

$$dS(t) = \theta(\mu - S(t))\,dt + \sigma\,dW(t) \tag{5.15}$$

Where,

θ is the rate at which the price reverts to the mean μ, σ represents the volatility of the price, $W(t)$ is a Wiener process (standard Brownian motion).

Using this model, the fund manager determines the optimal strategy $\pi(t)$ for adjusting stock holdings in response to observed price changes, aiming to maximize a utility function (e.g., expected return minus a risk penalty). The strategy depends on both the current stock price and its predicted future path, given the mean-reverting behavior modeled by the OU process. Here, the OU process assists the manager in making dynamic decisions within a controlled and uncertain environment by understanding how the stock price is likely to evolve and revert toward its mean.

Time Series Modeling: Forecasting Interest Rate Movements

Now, consider a financial analyst aiming to forecast future interest rates for pricing bonds or managing interest rate risk. Interest rates often exhibit mean-reverting behavior, fluctuating around a long-term average due to economic factors and central bank policies. In this context, the analyst might use an OU process to model the short-term interest rate, $r(t)$.

The OU process for interest rate modeling can be expressed as:

$$dr(t) = \alpha(\beta - r(t))\,dt + \sigma\,dW(t) \tag{5.16}$$

Where,

α is the speed of mean reversion, β is the long-term mean level of the interest rate, σ is the volatility, $W(t)$ is a Wiener process.

Here, the focus is on estimating the parameters α, β, and σ based on historical data of observed interest rates. The analyst uses the fitted model to predict future interest rate movements, calculate the Value-at-Risk (VaR) for a portfolio, or evaluate different interest rate scenarios for risk management purposes. The OU process captures the mean-reverting behavior of interest rates and provides probabilistic forecasts based on past trends. Unlike the stochastic control example, there is no need to make dynamic decisions at each time point; instead, the analyst's goal is to understand the likely path of interest rates based on the mean-reversion characteristic inherent in the time series data.

Thus, the distinction lies in the purpose and application of the OU process: stochastic control methods employ the OU process to manage and optimize decision-making in dynamic environments with uncertainty, while time series modeling uses the OU process to analyze and forecast mean-reverting behavior in observed data. The former integrates continuous control actions to influence the system's evolution, whereas the latter focuses on statistical inference and predictive accuracy based on past observations.

Lee, D. and Leung, T. (2020) employ the Ornstein-Uhlenbeck (OU) process to construct and optimize a pair trading strategy, specifically using it to model the mean-reverting behavior of asset pairs' return spreads, which tend to revert to a long-term average after short-term fluctuations. By fitting the value of each asset pair portfolio to an OU process, the authors effectively capture the characteristics of mean reversion observed in financial markets. To accurately fit the OU process, the study utilizes the maximum likelihood estimation (MLE) method to estimate the key parameters of the OU model, such as the speed of mean reversion, long-term mean, and volatility, performing these estimations on historical price data for each asset pair to identify the parameters that best describe the dynamics of the portfolio value over time. The OU process is further leveraged to determine an optimized exit rule for trading; within the OU framework, the paper analyzes the impact of exiting a trade based on a multiple of the standard deviation from the mean level of the price, and derives an optimal exit point analytically, indicating when a trader should liquidate a position to maximize profitability. This approach allows the authors to compare the performance of a conventional mean-reversion strategy with one that incorporates an optimized exit rule, demonstrating that the optimized rule significantly improves trading profitability while reducing the frequency of trades. The study conducts an empirical analysis using data from eight different asset pairs, including stocks, ETFs, currencies, and futures, and by fitting the portfolio value of each pair to an OU process, the authors evaluate the effectiveness of the strategy under the optimized exit rule. The results indicate that the optimized exit rule substantially enhances the annualized returns and lowers the daily turnover, thereby reducing transaction costs. Overall, this methodology primarily falls under the category of time series methods, as the OU process is a well-established model in time series analysis used to describe stochastic processes with mean-reverting behavior; the paper integrates time series data to estimate model parameters and design an optimal exit strategy, demonstrating the efficacy of using time series analysis techniques in the context of pairs trading.

Xiang, Y. et al. (2023) employ the fractional Ornstein-Uhlenbeck (fOU) process to construct and optimize a pairs trading strategy by modeling the mean-reverting behavior of price spreads between financial assets. The fOU process, an extension of the classic Ornstein-Uhlenbeck (OU) process, is designed to capture the long-range dependence and anti-persistence characteristics of these spreads. Unlike the traditional OU process, the fOU process incorporates fractional Brownian motion, providing a more accurate representation of spread dynamics by reflecting their tendency to revert to a long-term equilibrium after short-term fluctuations. To further enhance trading strategy performance, the study introduces an adaptive method for setting optimal trading thresholds based on parameters estimated from the fOU model, such as the speed of mean reversion and the fractional order parameter. This approach dynamically adjusts the entry and exit points of trades, triggering trading signals when the spread deviates from its mean by a predetermined threshold, thereby optimizing trading decisions to improve profitability. The effectiveness of the fOU process is validated through simulation studies that compare the performance of fOU-based pairs trading strategies against those using the traditional OU process. The results demonstrate that the fOU-based model better captures the complex dynamics of spreads, achieving higher returns and lower risk levels across various market conditions. Additionally, empirical analysis using actual market data confirms the fOU model's effectiveness in generating robust trading signals and enhancing strategy performance. Although the fOU process involves concepts from stochastic processes and fractional Brownian motion, the methodology primarily falls under the category of time series methods. It is used to describe the dynamic evolution and mean-reverting behavior of financial asset prices over time, relying on time series data to capture the statistical properties and dependency structures inherent in price series. Overall, the study illustrates the advantages of utilizing the fOU process to refine pairs trading strategies, providing a novel approach that enhances decision-making and potential profitability across diverse market environments

5.5 Other Methods

This section explores additional approaches to pairs trading. **Table 15** presents a summary of relevant studies, including their data samples and frequency.

Table 15. A summary of other methods in pair trading from year 2016 to 2023.

Articles	Publish Year	Sample	Data Frequency
9	2016	NYSE, AMEX, and NASDAQ stocks, 2003-2012	daily
13	2016	U.S. stocks, 1962-2014	daily
16	2017	-	-
17	2017	U.S. stocks, 1987-2011 UPM and Stora Enso, 1987-2003	daily
19	2017	DJIA stocks, 2000-2015	daily

21	2017	S&P 100 stocks, 1990-2014	daily
38	2018	EUR/USD and EUR/SGD, June 2017-November 2017	1-min
42	2018	-	-
58	2019	U.S. Nasdaq energy sector stocks, 2012-2014	daily
71	2020	FTS 100 stocks, 2010-2019	daily
79	2020	Nasdaq 100 stocks, 1999-2003 2007-2012	daily
80	2021	Nasdaq stocks, 2000-2021	daily
87	2021	38 Forex, September 2017-July 2018	15-min
98	2021	Toronto Stock Exchange, January 2017-June 2020	daily
101	2022	103 NASDAQ 100 stocks, 2000-2021	daily
110	2022	NYSE:LUV, NASDAQ:AAPL, 2015-2020	daily
112	2023	S&P 500 stocks, 1990-2015	daily
113	2023	64 PSX stocks, 2017-2019	daily
116	2023	EPEX SPOT, 2020-2022	15-min
120	2023	405 Cryptocurrencies, January 2022-March 2022	1-min, 5-mins, hourly

5.5.1 Copula Approach

Xie, W. (2016) use the Copula method to improve pairs trading strategies by capturing the dependency structure between stocks more accurately. The following is a detailed description of the process using Copula.

Motivation for Using Copula

The paper first points out that traditional pairs trading strategies often use the "distance method," which involves calculating the normalized distance between the prices of two stocks to identify mispricing. This approach is effectively equivalent to using linear correlation analysis to judge the relationship between the two stocks, assuming that their returns are jointly normally distributed. However, stock returns are often not jointly normal and may exhibit nonlinear dependencies (such as tail dependence). Therefore, traditional methods may overlook important dependency information, leading to inaccurate trading decisions.

Modeling Joint Distribution with Copula

To more accurately capture the joint distribution between stocks, the paper proposes using Copula. According to Sklar's theorem, if two random variables X and Y have marginal distribution functions $F_X(x)$ and $F_Y(y)$, respectively, their joint distribution function $H(x,y)$ can be expressed using a Copula function C as:

$$H(x,y) = C(F_X(x), F_Y(y)) \tag{5.17}$$

where C is a Copula function that describes the dependency structure between the random variables. By using this approach, the Copula technique can model the joint distribution of stock returns more precisely without assuming normal marginal distributions.

Constructing Mispricing Measures

To quantify the relative valuation between two stocks, the paper defines two mispricing indexes, $|MI_t^{X|Y}|$ and $|MI_t^{Y|X}|$, which measure the degree of mispricing between stocks X and Y at a given time t. Let R_t^X and R_t^Y be the daily returns of stocks X and Y, with marginal distribution functions F_X and F_Y. According to Sklar's theorem, their joint distribution function H can be expressed as:

$$H(r_t^X, r_t^Y) = C(F_X(r_t^X), F_Y(r_t^Y)) \tag{5.18}$$

The mispricing indexes are defined in terms of conditional probabilities:

$$MI_t^{X|Y} = P(R_t^X < r_t^X | R_t^Y = r_t^Y), MI_t^{Y|X} = P(R_t^Y < r_t^Y | R_t^X = r_t^X) \tag{5.19}$$

Using the partial derivatives of the Copula function, these mispricing indexes can be further expressed as:

$$MI_t^{X|Y} = \frac{\partial C(u,v)}{\partial v}, MI_t^{Y|X} = \frac{\partial C(u,v)}{\partial u} \tag{5.20}$$

where $u = F_X(r_t^X)$ and $v = F_Y(r_t^Y)$. These mispricing indexes range from 0 to 1. Generally, a value of $|MI_t^{X|Y}|$ or $|MI_t^{Y|X}|$ equal to 0.5 indicates that the two stocks are relatively fairly valued, considering their historical joint distribution; values above 0.5 suggest overvaluation, while values below 0.5 suggest undervaluation.

Strategy Construction and Trading Signal Generation

The trading strategy consists of two phases: the formation period and the trading period. During the formation period, the daily return series of the candidate stocks are calculated, and their marginal distributions are estimated. Different types of Copulas (e.g., Gumbel, Frank, Clayton, normal, and Student's t Copulas) are fitted to the joint distribution, with the Copula that has the highest likelihood being chosen as the final model. In the trading period, the Copula obtained from the formation period is used to compute the daily mispricing indexes $|MI_t^{X|Y}|$ and $|MI_t^{Y|X}|$. When these indexes exceed a predefined threshold D (=0.6) and stop-loss position S (=2), the corresponding trades are executed.

FlagX and FlagY are variables used to accumulate the relative mispricing indexes of two stocks to determine the optimal moments for executing trades. Specifically, FlagX tracks the cumulative deviation of the mispricing index of stock X, defined as $|MI_t^{X|Y} - 0.5|$, while FlagY tracks the cumulative deviation of the mispricing index of stock Y, defined as $|MI_t^{Y|X} - 0.5|$. At the beginning of the trading period, both flags are initialized to zero. Each day, the values of $|MI_t^{X|Y} - 0.5|$ are added to FlagX, and the values of $|MI_t^{Y|X} - 0.5|$ are added to FlagY. A trading signal is triggered when either flag reaches a predefined threshold D or drops below $-D$. If FlagX reaches D, this indicates that stock X is overvalued relative to stock Y, prompting a short-sell of stock X and a purchase of stock Y. Conversely, if FlagX falls to $-D$, it indicates that stock X is undervalued relative to stock Y, triggering a purchase of stock X and a short-sell of stock Y. Similarly, if FlagY reaches D, stock Y is considered overvalued relative to stock X, leading to a short-sell of stock Y and a purchase of stock X. If FlagY falls to $-D$, stock Y is considered undervalued, and a purchase of stock Y and a short-sell of stock X are executed. This mechanism effectively captures the dynamic changes in relative valuations between the two stocks, allowing for timely and profitable trading decisions based on observed mispricing.

Empirical Analysis and Return Data

The authors validate the effectiveness of the Copula method through empirical analysis involving different stock pairs (such as Brookdale Senior Living Inc. and Emeritus Corporation) and a large sample of utility sector stocks. The results indicate that the Copula-based strategy significantly outperforms the traditional distance method, offering higher excess returns and fewer negative returns. Specific return data are as follows:

- When using the Copula method without the "one-day waiting" strategy (i.e., trading immediately on the day of price divergence), an initial investment of $10,000 generates a profit of $847, compared to a loss of $592 using the traditional distance method.
- Under the "one-day waiting" strategy, the Copula method yields a profit of $1,060, while the distance method results in a loss of $1,526.
- In a larger sample analysis involving utility sector stocks, the Copula strategy consistently outperforms the traditional distance method across different sets of pairs (top 5, top 20, and 101–120 pairs), particularly in terms of annualized excess returns, which are significantly higher for the Copula strategy.

These statistical results show that the Copula strategy can effectively identify more trading opportunities and provide better performance in terms of both returns and transaction costs.

Krauss, C. and Stübinger, J. (2017) employ the Copula method for pairs trading by capturing the non-linear dependence structure between stocks more accurately. The dataset consists of S&P 100 index constituent stocks from 1990 to 2014. The formation period is set to 60 months, divided into a 12-month estimation period and a subsequent 1-month pseudo-trading period, during which all possible stock pairs are fitted using the t-Copula model. In the estimation period, a semi-parametric approach is used to fit the bivariate t-Copula by first calculating the empirical marginal distribution functions for each stock's log returns. These empirical distributions are then transformed into uniform variables, which are used to fit the t-Copula through maximum likelihood estimation. The fitted Copula model is used to compute conditional distribution functions for each pair, generating trading signals based on deviations from the equilibrium relationship. If the conditional probability suggests that one stock is overvalued or undervalued relative to the other, a buy or sell signal is generated accordingly. The study also includes an out-of-sample trading period, where selected stock pairs are traded according to individualized rules, including profit-taking and stop-loss levels, which are determined based on the cumulative return series of each pair during the estimation period. Furthermore, the study differentiates between mean-reverting pairs and momentum pairs, where mean-reverting pairs tend to revert to equilibrium after the Copula signals, while momentum pairs continue to diverge. For momentum pairs, trading rules are reversed to capture the momentum effect. The results demonstrate that the Copula-based pairs trading strategy effectively captures complex dependencies between stocks, yielding significant annualized returns and high Sharpe ratios during the out-of-sample period, highlighting the strategy's profitability even under complex market conditions.

Nadaf, T. et al. (2022) use the Copula method for pairs trading by integrating the Laplace marginal distribution to enhance the accuracy of capturing complex dependencies between stocks. The analysis focuses on two stocks, Apple (AAPL) and Southwest Airlines (LUV), using in-sample data from January 1, 2015, to December 31, 2019, and out-of-sample data from January 4, 2020, to November 20, 2020. The daily closing prices of these stocks are used to compute log-return series, which serve as the basis for constructing the Copula model. First, the Laplace distribution is fitted to the return data, as it more accurately reflects the fat-tailed characteristics of financial returns compared to the normal distribution. The parameters of the Laplace distribution are estimated using maximum likelihood estimation (MLE), and goodness-of-fit tests confirm its suitability for modeling the marginal distributions. Subsequently, a Gaussian Copula is employed to describe the joint distribution of the two stocks by combining their Laplace marginal distributions. The Copula model, which is calibrated using MLE, captures the non-linear dependency structure between the stocks. Trading signals are generated based on the joint probabilities derived from the Copula model; when the conditional probability of one stock relative to the other reaches extreme values, indicating overvaluation or undervaluation, corresponding buy or sell signals are triggered. The strategy assumes negligible transaction costs and executes trades with a one-day holding period, where positions are opened at the start of the day and closed at the end based on the generated signals. The empirical results demonstrate the effectiveness of this Copula-based approach, with significant profits achieved by capturing the non-linear dependencies in the out-of-sample period.

5.5.2 Hurst Exponent Approach

Ramos-Requena, J.P. et al. (2017) propose a novel method for pairs trading by incorporating the Hurst exponent to better identify stock pairs with strong mean-reverting behavior. Traditional pairs trading methods, such as those based on distance or correlation, often fail to capture the long-term dependencies in asset prices, particularly in complex and volatile market environments. By leveraging the Hurst exponent, which measures the long-term memory of a time series, this approach aims to improve the selection of stock pairs that are more likely to revert to their mean, thus enhancing the effectiveness of pairs trading strategies.

Understanding the Hurst Exponent

The Hurst exponent (H) is a statistical measure that indicates whether a time series exhibits a tendency to revert to its mean, persist in its trend, or behave like a random walk. The Hurst exponent is calculated by examining the scaling behavior of the rescaled range (R/S) statistic over different time windows. The relationship is expressed as:

$$\frac{R(n)}{S(n)} \propto n^H \tag{5.21}$$

where:

- $R(n)$ is the range of cumulative deviations from the mean over a time window of size n.
- $S(n)$ is the standard deviation of the time series over the same window.
- H is the Hurst exponent.

The value of HHH provides insight into the time series' behavior:

- $H = 0.5$: The series resembles a random walk, indicating no correlation.
- $H < 0.5$: The series is mean-reverting, suggesting negative correlation.
- $H > 0.5$: The series exhibits persistence, indicating positive correlation.

Using the Generalized Hurst Exponent (GHE)

To provide a more accurate estimation, the study employs the Generalized Hurst Exponent (GHE), which extends the concept of the Hurst exponent to different moments of the distribution, offering a more flexible tool for capturing mean-reversion over various time scales. The GHE is calculated using:

$$E[|X(t+\tau) - X(t)|^q] \propto \tau^{qH(q)} \tag{5.22}$$

where:

- $X(t)$ is the log-price of a stock at time t.
- τ represents the time lag.
- q is the order of the moment, typically set to 1 or 2 for mean-reversion analysis.

  $H(q)$ is the generalized Hurst exponent.

The GHE allows for a more comprehensive analysis of time series behavior, especially for financial data, which often exhibits complex dependencies not captured by simpler models.

Pair Selection Process

Data Collection: The study first collects historical price data for a set of stocks over a given period. In the example provided, data from stocks listed on the Dow Jones Index is used.

Calculate Log-Price Differences: For every possible pair of stocks, the log-price differences are calculated over time to create a time series that reflects the relative price movements between the two stocks.

Compute the Hurst Exponent: The Generalized Hurst Exponent (GHE) is calculated for each log-price difference time series. This is done using a range of time lags τ to understand how the pair's price difference behaves over different time scales.

Rank and Select Pairs: All possible stock pairs are ranked based on their Hurst exponents. The pairs with the lowest Hurst exponents (i.e., those exhibiting the strongest mean-reverting behavior) are selected for trading. A low Hurst exponent indicates a higher likelihood of the pair returning to its historical mean, making it a suitable candidate for pairs trading.

Trading Strategy Based on Hurst Exponent

Define Trading Signals:

- Opening Positions:
  - If the log-price spread of a selected pair deviates from its rolling mean by a predefined threshold (e.g., one standard deviation), a trading position is opened.
  - A sell signal is generated when the spread exceeds the upper threshold, indicating that the spread is unusually wide and likely to contract.
  - A buy signal is generated when the spread falls below the lower threshold, suggesting that the spread is unusually narrow and likely to widen.
- Closing Positions:

- The position is closed when the spread reverts to its mean or crosses another predefined boundary, thereby capturing the profit from the reversion.

Set Parameters for Entry and Exit: The strategy involves setting specific parameters for entry and exit points. These parameters are dynamically adjusted based on the historical volatility and behavior of the selected pairs to optimize the timing of trades.

Execution of Trades: Trades are executed based on the generated signals. If a pair's spread is above or below the threshold, the corresponding positions (long or short) are taken. Positions are held until the spread reverts to the mean or until another threshold triggers an exit.

Empirical Validation

The paper validates the effectiveness of the Hurst exponent-based strategy using historical data from the Dow Jones Index. The results show that the strategy outperforms traditional pairs trading methods, such as distance-based or correlation-based approaches, particularly during volatile market periods. The Hurst exponent method provides better selection of stock pairs, resulting in a higher number of profitable trades and fewer losing trades. The empirical results indicate that this method is more robust and adaptable to changing market conditions.

Conclusion

This study demonstrates that the incorporation of the Hurst exponent into pairs trading provides a more reliable framework for identifying stock pairs with strong mean-reverting properties. By effectively capturing long-term dependencies in asset prices, this approach offers a significant improvement over traditional methods, enhancing profitability while reducing risk.

Fernández-Pérez, A. et al. (2020) employ the Hurst exponent to enhance pairs trading strategies by identifying stock pairs that exhibit strong mean-reverting behavior. Originally developed in hydrology, the Hurst exponent is applied in this study to detect long-term memory and autocorrelation in financial time series, reflecting the tendency of asset prices to revert to their mean over time. The Generalized Hurst Exponent (GHE) is used to evaluate these mean-reverting characteristics by analyzing the scaling behavior of time series data. Pairs with a Hurst exponent value less than 0.5 are selected, indicating a strong potential for mean reversion. The trading strategy is implemented in two phases: a pair selection phase over 250 days, where the pairs with the lowest Hurst exponent values are chosen, and a trading execution phase, where positions are taken based on deviations from a moving average of the price spread between the selected stocks. Empirical validation using data from the FTSE 100 index (2010-2019) demonstrates that the optimal strategy varies by portfolio size, with smaller portfolios benefiting from a standard deviation threshold of 1 and larger portfolios from a threshold of 1.5. The performance of this strategy is assessed using metrics such as the Sharpe Ratio and Maximum Drawdown, revealing that it delivers superior risk-adjusted returns, particularly during periods of high market volatility, such as the Brexit referendum period. The study concludes that incorporating the Hurst exponent into pairs trading provides a robust method for identifying stock pairs with mean-reverting properties, outperforming traditional strategies by improving returns and managing risks more effectively.

Bui, Q. and Ślepaczuk, R. (2022) develop a pairs trading strategy based on the **Hurst exponent**, applying it to stocks listed on the NASDAQ 100 index to identify pairs with strong mean-reverting characteristics. The Hurst exponent measures the long-term memory of a time series, helping to determine whether the price spread between two stocks is likely to revert to its mean. The study uses daily data from 103 NASDAQ 100 stocks from January 1, 2000, to December 31, 2018, with an additional out-of-sample period extending to July 1, 2021. Pairs are formed by calculating the Hurst exponent of the log-price ratio between stocks, using the Generalized Hurst Exponent (GHE) method to identify those with the lowest values, indicating a strong tendency for mean reversion. The top 10 pairs with the lowest Hurst exponent values are selected every six months for trading. The trading strategy is based on the deviation of the price spread from its mean; a sell signal is triggered when the spread exceeds an upper threshold, and a buy signal is triggered when it falls below a lower threshold. Positions are closed when the spread reverts to the mean or reaches an opposite threshold. The window size for moving averages, rolling standard deviations, and thresholds are optimized using historical data to minimize bias. Empirical results suggest that while the Hurst exponent provides a viable alternative for identifying mean-reverting pairs, it does not consistently outperform correlation-based methods in terms of risk-adjusted returns. The study concludes that the effectiveness of the Hurst exponent in pairs trading is contingent upon various factors, such as the number of pairs selected, rebalancing frequency, and financial leverage.

5.5.3 Entropic Approach

Amer, L. and Islam, T.U. (2023) utilize the entropic approach to optimize pairs trading by managing model uncertainty and identifying optimal trading boundaries. The approach addresses challenges such as non-converging pairs and model misspecifications that could result in losses, using entropy as a penalty function to minimize risk and maximize profitability. The research uses daily data from 64 companies listed on the Pakistan Stock Exchange (PSX) between 2017 and 2019, covering various sectors such as cement, chemicals, automobiles, food, oil, gas, and power. Companies are selected based on their price-to-earnings ratio (PER), with those having a PER below the median considered undervalued and included in the sample.

The Johansen cointegration test is employed to identify stock pairs that exhibit a long-term equilibrium relationship, resulting in 79 unique cointegrated pairs suitable for trading. The entropic approach builds upon the Ornstein-Uhlenbeck (OU) process to model the mean-reverting behavior of these pairs, expressed as:

$$dX_t = -\mu(X_t - \alpha)d_t + \sigma dB_t, X_0 = \alpha \quad (5.21)$$

where μ is the speed of mean reversion, α is the mean-reversion level, σ is the volatility, and B_t represents Brownian motion. Parameters μ, α, and σ are estimated using maximum likelihood methods, along with other parameters such as the discount rate ρ and confidence level λ, based on existing literature and empirical data.

The optimization of boundary points is framed as an optimal stopping problem, aiming to maximize profit while minimizing relative entropy (model risk). The optimal boundaries are calculated using the solution:

$$v_0(t,x) = sup_{\tau \in \mathfrak{I}} E_x^S[e^{-\rho(\tau - t)} X_\tau] \quad (5.22)$$

where $\mathfrak{I}$ is the set of all stopping times, ρ is the discount rate, and S represents the stock pair. The trading strategy involves shorting the stock pair when it reaches its highest value and liquidating it when it reverts to the mean, or taking a long position when it reaches the mean and liquidating it at the boundary.

The study explores different values of λ (0.001, 0.01, 0.1, and ∞) to compute optimal boundary points and returns, with lower λ values corresponding to higher confidence levels. The empirical results indicate that lower λ values yield better returns, highlighting a higher confidence level in the reference measure. When compared with buy-and-hold, distance-based, and machine learning methods, the entropic approach consistently delivers higher returns, demonstrating its effectiveness in managing model uncertainty and enhancing profitability in the volatile Pakistani market.

In conclusion, the entropic approach effectively maximizes profitability in pairs trading by leveraging entropy to manage model uncertainty and optimize trading boundaries. The empirical findings confirm its superiority over traditional methods, achieving substantial returns in challenging market conditions like those of the PSX.

Yoshikawa, D. (2017) presents a theoretical study that employs the entropic approach to enhance pairs trading strategies by incorporating model uncertainty. Traditional pairs trading strategies often rely on the assumption of mean reversion, which can result in significant losses if the model is misspecified or if the assumed mean-reversion behavior fails. To address these challenges, the entropic approach is used to create a more robust trading strategy by managing the risks associated with model uncertainty.

The study utilizes an Ornstein-Uhlenbeck (OU) process to model the mean-reverting behavior of the price difference between two stocks. The OU process captures the dynamic characteristics of the stock pair, such as the speed of mean reversion, the level to which prices revert, and the volatility of price fluctuations. These parameters are estimated using maximum likelihood estimation to accurately reflect the underlying price dynamics.

To account for model uncertainty, the paper introduces relative entropy as a penalty function. The entropic approach adjusts the optimal stopping problem by considering a range of possible probability measures around a reference measure. The goal is to minimize the worst-case expected outcome across these measures, creating a more conservative strategy that reduces the risk of substantial losses due to incorrect model assumptions.

By incorporating the entropy-based penalty term, the entropic approach modifies the optimal entry and exit boundaries for pairs trading. This results in more cautious trading decisions under conditions of high uncertainty, thereby mitigating the risk associated with model misspecification.

The paper includes a numerical example using stock data from the Tokyo Stock Exchange. By applying different confidence levels, represented by varying parameters, the study calculates and compares the optimal trading boundaries. The results demonstrate that higher uncertainty leads to more conservative entry and exit points, effectively reducing the potential for adverse trading outcomes.

In conclusion, this study contributes a novel approach to pairs trading by using entropy to manage model uncertainty. By leveraging this method, the strategy becomes more resilient to errors in model specification and incorrect assumptions about mean reversion, thereby enhancing profitability while minimizing potential losses.

6. Conclusion and Future Research Direction

We have conducted a thorough review of the literature related to the broad concept of pairs trading. Organized by the different pairs trading approaches, our findings can be summarized along with recommendations for future research as follows.

6.1 Distance Methods

The review of distance methods in pairs trading highlights their robustness and adaptability across various market conditions and asset classes. Originating from the seminal work of Gatev, Goetzmann, and Rouwenhorst (GGR), the distance method has been extensively applied and modified in subsequent studies to enhance profitability and mitigate risks. GGR's model-free approach, which relies solely on historical price relationships without the constraints of economic models, has proven effective in capturing arbitrage opportunities through the identification of stock pairs with mean-reverting price behavior. The method's simplicity, combined with its strong theoretical foundation in Euclidean distance metrics, makes it an attractive choice for traders seeking to exploit relative pricing inefficiencies in financial markets.

Extensions of the GGR framework have demonstrated the method's versatility. Bowen and Hutchinson (2016) successfully applied the distance method to the UK equity market, showing its robustness during financial crises, while other researchers have tested the strategy across multiple geographies and asset classes, including commodities, cryptocurrencies, and high-frequency trading environments. These studies generally find that the distance method retains profitability across diverse market conditions, although the degree of success can vary based on factors such as market liquidity, transaction costs, and the specific characteristics of the asset pairs.

Despite its strengths, the distance method is not without limitations. One notable challenge is the potential for reduced profitability in the presence of high transaction costs, particularly in markets with low liquidity or significant trading frictions. Additionally, the reliance on historical price data means that the method may not fully account for structural changes in market dynamics or the impacts of macroeconomic events. Some studies, such as those by Chen et al. (2019), suggest that while pairs trading remains effective, its profitability has diminished in more recent periods, highlighting the need for continuous adaptation and refinement of the strategy.

Future research on distance methods in pairs trading could focus on integrating more sophisticated statistical and machine learning techniques to enhance pair selection and trading execution. For example, combining the traditional distance metrics with dynamic models that capture evolving market conditions or incorporating alternative data sources such as news sentiment or macroeconomic indicators could provide additional predictive power and improve strategy robustness. Furthermore, exploring the impact of regulatory changes, market microstructure, and the increasing prevalence of algorithmic trading on pairs trading profitability could offer valuable insights into optimizing the strategy in modern financial markets.

Overall, the distance method remains a foundational approach in pairs trading, offering a reliable framework for exploiting mean-reversion in asset prices. Continued research and innovation in this area have the potential to further enhance its effectiveness and applicability in a rapidly evolving financial landscape.

6.2 Cointegration Methods

The review of cointegration methods in pairs trading highlights the significant advancements and diverse applications of this approach across various asset classes and market conditions. Cointegration-based pairs trading strategies leverage the long-term equilibrium relationships between asset pairs to identify mean-reverting opportunities, making them particularly effective in volatile or non-stationary environments. From the foundational work of Engle and Granger (1987), which introduced the concept of cointegration and error correction models, to Vidyamurthy's (2004) practical framework for pairs trading, these methods have evolved to address the complexities of modern financial markets.

Recent advancements in cointegration methods have expanded beyond static analysis to incorporate dynamic cointegration and adaptive models, allowing strategies to better capture evolving market conditions. For instance, adaptive cointegration approaches and the integration of advanced statistical models, such as Vector Error Correction Models (VECM) and Bayesian inference techniques, have enhanced the robustness and flexibility of pairs trading strategies. These advancements enable the continuous recalibration of pairs, ensuring that cointegration relationships remain relevant over time and across different market regimes. Furthermore, the application of cointegration to non-traditional assets, such as credit default swaps (CDSs) and cryptocurrencies, demonstrates the versatility and adaptability of this approach in diverse financial contexts.

Empirical results consistently show that cointegration-based pairs trading can generate significant risk-adjusted returns, even after accounting for transaction costs and market frictions. Studies have demonstrated that these strategies can effectively exploit arbitrage opportunities in both equity and derivative markets, with notable success in capturing mean-reversion during periods of market stress or structural breaks. However, the profitability of cointegration methods can vary depending on factors such as the choice of pre-selection metrics, trading frequency, and the specific characteristics of the asset pairs.

Future research on cointegration methods in pairs trading could focus on further integrating machine learning and artificial intelligence techniques to enhance pair selection and trading execution. By leveraging advanced algorithms, traders can potentially identify more complex, nonlinear relationships between asset pairs that traditional cointegration models may overlook. Additionally, exploring the impact of macroeconomic factors, market sentiment, and high-frequency data on cointegration relationships could provide deeper insights into the drivers of mean-reversion and improve strategy performance. Finally, extending cointegration methods to emerging asset classes and alternative datasets, such as environmental, social, and governance (ESG) factors, could open new avenues for research and application in pairs trading.

Overall, cointegration methods continue to offer a robust and theoretically sound framework for pairs trading, with the potential for further refinement and innovation to meet the challenges of rapidly evolving financial markets.

6.3 Stochastic Control Methods

The exploration of stochastic control methods in pairs trading reveals the robustness and adaptability of these approaches in managing the dynamic and often volatile nature of financial markets. Stochastic control techniques, particularly those based on the Ornstein-Uhlenbeck (OU) process, have emerged as powerful tools for modeling mean-reverting spreads between asset pairs. These methods not only provide a structured framework for predicting price movements but also allow for the dynamic adjustment of trading strategies in response to evolving market conditions.

The application of the OU process, as demonstrated by foundational studies like Jurek and Yang (2007), effectively captures the mean-reversion dynamics of asset spreads, offering a reliable basis for constructing market-neutral arbitrage strategies. This approach is further enhanced by addressing critical risks such as horizon and divergence risks, which are inherent in pairs trading. The incorporation of optimal dynamic strategies that adjust capital allocation based on spread levels and time horizons reflects a significant advancement over simpler threshold-based rules, leading to improved risk-adjusted returns and more effective risk management.

Recent empirical research has expanded the traditional OU framework by integrating features such as jump-diffusion, regime-switching, and stochastic volatility models, allowing for a more nuanced capture of market behavior. These enhancements enable traders to better navigate high-frequency trading environments and complex market conditions, such as those seen in commodity futures, cryptocurrencies, and energy markets. The empirical results consistently show that advanced OU-based models can achieve superior performance, with significantly higher Sharpe ratios and annualized returns compared to conventional methods.

Theoretical research has also played a crucial role in advancing stochastic control methods for pairs trading. Innovations such as the use of Markov-modulated OU processes and the incorporation of risk constraints into optimization frameworks have provided deeper insights into the strategic adjustments needed to maximize profitability while controlling for downside risk. These developments highlight the potential for further refining pairs trading strategies by incorporating sophisticated stochastic modeling techniques and rigorous risk management principles.

Future research on stochastic control methods in pairs trading could focus on expanding the application of these models to a broader range of asset classes, including alternative investments and emerging markets. There is also a significant opportunity to explore the integration of machine learning and artificial intelligence to enhance the adaptability and predictive power of stochastic models. By leveraging AI-driven algorithms, traders can potentially identify more complex patterns and dependencies that traditional models may miss, leading to more accurate and profitable trading decisions.

Additionally, further investigation into the effects of transaction costs and market frictions on stochastic control strategies is warranted. While many existing models assume frictionless markets, real-world trading involves costs that can significantly impact net returns. Developing strategies that explicitly account for these factors will enhance the practical applicability of stochastic control methods in pairs trading.

In conclusion, stochastic control methods offer a robust and adaptable framework for pairs trading, capable of managing the inherent risks and complexities of financial markets. As these methods continue to evolve, integrating advanced modeling techniques and addressing practical trading constraints will be key to unlocking their full potential and achieving sustained profitability in diverse trading environments.

6.4 Time Series Methods

The use of time series methods in pairs trading has demonstrated considerable efficacy in modeling the mean-reverting behavior of financial asset pairs, providing robust frameworks for predicting and capitalizing on market inefficiencies. Unlike cointegration-based approaches, which focus on long-term equilibrium relationships, time series methods such as GARCH and Ornstein-Uhlenbeck (OU) models emphasize short-term dynamics and volatility clustering, offering enhanced flexibility and responsiveness in pairs trading strategies.

Time series models like the GARCH family, including the Smooth Transition GARCH (ST-GARCH) and nonlinear autoregressive GARCH variants, have been employed to capture complex volatility patterns in return spreads. These models effectively handle features such as volatility clustering, asymmetry, and fat tails, which are common in financial markets. The ability to forecast conditional variances and apply adaptive trading thresholds based on volatility predictions significantly improves the accuracy of entry and exit signals, resulting in more profitable and less risky trades. The empirical results indicate that integrating GARCH models into pairs trading frameworks can yield substantial returns, even after accounting for transaction costs, thereby enhancing the overall strategy performance.

The application of the Ornstein-Uhlenbeck (OU) process in time series modeling has proven particularly valuable for mean-reverting strategies. By fitting asset pair spreads to OU processes, researchers have been able to capture the dynamics of mean reversion, which is central to pairs trading. The use of fractional Ornstein-Uhlenbeck (fOU) processes extends this approach further by accommodating long-range dependencies and anti-persistence in spreads, offering a more nuanced representation of market behavior. Studies have shown that these models, when optimized with adaptive thresholds, can outperform traditional OU-based methods, providing higher returns and better risk management across various market conditions.

Future research directions for time series methods in pairs trading could explore the integration of machine learning techniques to enhance model adaptability and predictive power. Machine learning algorithms, such as neural networks and reinforcement learning, could be combined with time series models to dynamically adjust trading strategies based on evolving market conditions, thereby improving the accuracy of forecasts and trading decisions. Additionally, the development of hybrid models that combine time series approaches with other quantitative techniques, such as copula functions or stochastic control methods, could further enhance the robustness and profitability of pairs trading strategies.

Another promising area for future research is the incorporation of transaction cost analysis and market microstructure considerations into time series models. While existing studies have demonstrated the effectiveness of GARCH and OU models in theoretical settings, the impact of real-world trading frictions on strategy performance remains an important area for further investigation. By explicitly modeling transaction costs, slippage, and liquidity constraints, future studies could provide more accurate assessments of the net profitability of pairs trading strategies in practical trading environments.

Lastly, expanding the application of time series methods to emerging markets, alternative assets, and high-frequency trading scenarios could offer new insights and opportunities. As financial markets continue to evolve, the adaptability and precision of time series models will be crucial in identifying and exploiting arbitrage opportunities in increasingly complex and volatile trading environments.

In conclusion, time series methods present a powerful toolset for pairs trading, enabling traders to effectively model and predict mean-reverting behaviors in financial markets. By continuing to refine these models and exploring new applications, researchers and practitioners can enhance the robustness and profitability of pairs trading strategies, contributing to more efficient and adaptive trading systems in the future.

6.5 Other Methods

The exploration of additional methods for pairs trading, including the Copula approach, Hurst exponent approach, and entropic approach, reveals significant advancements in capturing complex dependencies and improving the robustness of trading strategies. These methods extend beyond traditional pairs trading techniques by addressing key limitations such as non-linear dependencies, long-term memory effects, and model uncertainty, thereby enhancing the performance and adaptability of pairs trading strategies in diverse market conditions.

The Copula approach offers a sophisticated means of capturing non-linear dependencies between asset pairs, allowing for a more accurate representation of joint distributions beyond simple linear correlations. By modeling the dependency structure with Copula functions, this method provides a nuanced understanding of how asset returns interact, particularly in extreme market conditions where tail dependencies are prevalent. Empirical studies have demonstrated that Copula-based pairs trading strategies can yield superior returns compared to traditional distance or correlation methods, especially when applied to volatile or less correlated asset classes. Future research could further refine the Copula approach by exploring alternative Copula families, optimizing parameter estimation techniques, and integrating real-time data analytics to enhance the responsiveness of trading strategies to market changes.

The Hurst exponent approach introduces a valuable tool for identifying pairs with strong mean-reverting properties by measuring the long-term memory of time series. This approach enables traders to select asset pairs that are more likely to revert to their mean, thereby increasing the likelihood of profitable trades. By utilizing the Generalized Hurst Exponent (GHE), the approach accounts for complex dependencies over various time scales, offering a more comprehensive view of mean-reversion potential. Empirical evidence suggests that this method outperforms traditional pairs trading strategies, particularly in periods of market volatility. Future research directions could include the development of dynamic models that continuously adjust the Hurst exponent in real-time, allowing for adaptive strategies that respond to changing market conditions. Additionally, integrating machine learning techniques to predict Hurst exponent trends could further enhance the accuracy of pair selection.

The entropic approach addresses the critical issue of model uncertainty in pairs trading by incorporating entropy as a penalty function to manage risk and optimize trading boundaries. By accounting for the inherent uncertainty in model specifications and market behaviors, this method provides a robust framework for making trading decisions under conditions of ambiguity. The entropic approach not only improves profitability but also reduces the risk of significant losses due to non-converging pairs or incorrect assumptions about mean reversion. Empirical results have shown that this method consistently delivers higher returns compared to traditional strategies, even in challenging market environments. Future research could expand the entropic approach by integrating multi-asset classes, exploring different forms of entropy measures, and testing the method's applicability in high-frequency trading contexts where rapid adjustments are crucial.

In conclusion, the incorporation of advanced methods such as Copula, Hurst exponent, and entropic approaches into pairs trading strategies provides substantial enhancements in terms of accuracy, robustness, and profitability. These methods address key limitations of traditional pairs trading techniques by capturing complex dependencies, accounting for long-term memory, and managing model uncertainty. Moving forward, the integration of these approaches with modern data analytics, machine learning, and artificial intelligence could further revolutionize pairs trading, offering more sophisticated, adaptive, and resilient strategies that can thrive in increasingly complex and volatile financial markets. Future research should focus on refining these methodologies, exploring new applications, and testing their scalability and effectiveness across different asset classes and market conditions.

References[1]

[1] Deshpande, A. and Barmish, B.R., 2016, September. A general framework for pairs trading with a control-theoretic point of view. In 2016 IEEE Conference on Control Applications (CCA) (pp. 761-766). IEEE.

[2] De Moura, C.E., Pizzinga, A. and Zubelli, J., 2016. A pairs trading strategy based on linear state space models and the Kalman filter. Quantitative Finance, 16(10), pp.1559-1573.

[3] Göncü, A. and Akyildirim, E., 2016. A stochastic model for commodity pairs trading. Quantitative Finance, 16(12), pp.1843-1857.

[4] Cartea, A. and Jaimungal, S., 2016. Algorithmic trading of co-integrated assets. International Journal of Theoretical and Applied Finance, 19(06), p.1650038.

[5] Ngo, M.M. and Pham, H., 2016. Optimal switching for the pairs trading rule: A viscosity solutions approach. Journal of Mathematical Analysis and Applications, 441(1), pp.403-425.

[6] Bowen, D.A. and Hutchinson, M.C., 2016. Pairs trading in the UK equity market: Risk and return. The European Journal of Finance, 22(14), pp.1363-1387.

[7] Lee, S. and Papanicolaou, A., 2016. Pairs trading of two assets with uncertainty in co-integration's level of mean reversion. International Journal of Theoretical and Applied Finance, 19(08), p.1650054.

[8] Fallahpour, S., Hakimian, H., Taheri, K. and Ramezanifar, E., 2016. Pairs trading strategy optimization using the reinforcement learning method: a cointegration approach. Soft Computing, 20, pp.5051-5066.

[9] Xie, W., LieW, R.Q., Wu, Y., and Zou, X., 2016. Pairs Trading with Copulas. The Journal of Trading, Summer 2016, pp. 41-52.

[10] Miao, J. and Laws, J., 2016. Profitability of a simple pairs trading strategy: recent evidences from a global context. International Journal of Theoretical and Applied Finance, 19(04), p.1650023.

[11] Ardia, D., Gatarek, L.T., Hoogerheide, L. and Van Dijk, H.K., 2016. Return and risk of pairs trading using a simulation-based bayesian procedure for predicting stable ratios of stock prices. Econometrics, 4(1), p.14.

[12] Göncü, A. and Akyıldırım, E., 2016. Statistical arbitrage with pairs trading. International Review of Finance, 16(2), pp.307-319.

[1] Since the order in which the papers are used in the article is chronological, they are not sorted alphabetically.

[13] Rad, H., Low, R.K.Y. and Faff, R., 2016. The profitability of pairs trading strategies: distance, cointegration and copula methods. Quantitative Finance, 16(10), pp.1541-1558.

[14] Smith, R.T. and Xu, X., 2017. A good pair: alternative pairs-trading strategies. Financial Markets and Portfolio Management, 31, pp.1-26.

[15] Ekkarntrong, N., Sirisaengtaksin, P., Sattayatham, P. and Premanode, B., 2017. A Novel Pairs Trading Model with Mean Reversion and Coefficient of Variance. Thai Journal of Mathematics, 15(1), pp.277-296.

[16] Yoshikawa, D., 2017. An entropic approach for pair trading. Entropy, 19(7), p.320.

[17] Rinne, K. and Suominen, M., 2017. How some bankers made a million by trading just two securities?. Journal of Empirical Finance, 44, pp.304-315.

[18] Liu, B., Chang, L.B. and Geman, H., 2017. Intraday pairs trading strategies on high frequency data: The case of oil companies. Quantitative Finance, 17(1), pp.87-100.

[19] Ramos-Requena, J.P., Trinidad-Segovia, J.E. and Sánchez-Granero, M.A., 2017. Introducing Hurst exponent in pair trading. Physica A: statistical mechanics and its applications, 488, pp.39-45.

[20] Lintilhac, P.S. and Tourin, A., 2017. Model-based pairs trading in the bitcoin markets. Quantitative Finance, 17(5), pp.703-716.

[21] Krauss, C. and Stübinger, J., 2017. Non-linear dependence modelling with bivariate copulas: Statistical arbitrage pairs trading on the S&P 100. Applied Economics, 49(52), pp.5352-5369.

[22] Chen, C.W. and Lin, T.Y., 2017. Nonparametric tolerance limits for pair trading. Finance Research Letters, 21, pp.1-9.

[23] Chen, C.W., Wang, Z., Sriboonchitta, S. and Lee, S., 2017. Pair trading based on quantile forecasting of smooth transition GARCH models. The North American Journal of Economics and Finance, 39, pp.38-55.

[24] Chen, D., Cui, J., Gao, Y. and Wu, L., 2017. Pairs trading in Chinese commodity futures markets: an adaptive cointegration approach. Accounting & Finance, 57(5), pp.1237-1264.

[25] Vaitonis, M., 2017. Pairs trading using HFT in OMX Baltic market. Baltic Journal of Modern Computing, 5(1), p.37.

[26] Yang, Y., Goncu, A. and Pantelous, A., 2017. Pairs trading with commodity futures: evidence from the Chinese market. China Finance Review International, 7(3), pp.274-294.

[27] Vaitonis, M. and Masteika, S., 2017. Statistical arbitrage trading strategy applied on future commodity market using nanosecond information, ICIST 2017. Communications in Computer and Information Science, 756.

[28] Vaitonis, M. and Masteika, S., 2017. Statistical arbitrage trading strategy in commodity futures market with the use of nanoseconds historical data. In Information and Software Technologies: 23rd International Conference, ICIST 2017, Druskininkai, Lithuania, October 12–14, 2017, Proceedings (pp. 303-313). Springer International Publishing.

[29] Kim, S. and Heo, J., 2017. Time series regression-based pairs trading in the Korean equities market. Journal of Experimental & Theoretical Artificial Intelligence, 29(4), pp.755-768.

[30] Law, K.F., Li, W.K. and Philip, L.H., 2018. A single-stage approach for cointegration-based pairs trading. Finance Research Letters, 26, pp.177-184.

[31] Tie, J., Zhang, H. and Zhang, Q., 2018. An optimal strategy for pairs trading under geometric Brownian motions. Journal of Optimization Theory and Applications, 179(2), pp.654-675.

[32] Bai, Y. and Wu, L., 2018. Analytic value function for optimal regime-switching pairs trading rules. Quantitative Finance, 18(4), pp.637-654.

[33] Elliott, R.J. and Bradrania, R., 2018. Estimating a regime switching pairs trading model. Quantitative Finance, 18(5), pp.877-883.

[34] Vaitonis, M. and Masteika, S., 2018. Experimental Comparison of HFT Pair Trading Strategies Using the Data of Microsecond and Nanosecond Future Commodity Contracts. Baltic Journal of Modern Computing, 6(2).

[35] Wen, D., Ma, C., Wang, G.J. and Wang, S., 2018. Investigating the features of pairs trading strategy: A network perspective on the Chinese stock market. Physica A: Statistical Mechanics and its Applications, 505, pp.903-918.

[36] Gupta, U., Jain, S. and Bhatia, M., 2018. Mean Reversion with Pair Trading in Indian Private Sector Banking Stocks. In Information and Communication Technology for Intelligent Systems (ICTIS 2017)-Volume 1 2 (pp. 384-389). Springer International Publishing.

[37] Zhao, Z. and Palomar, D.P., 2017. Mean-Reverting Portfolio Design with Budget Constraint. arXiv preprint arXiv:1701.05016.

[38] Chu, C.C. and Chan, P.K., 2018, June. Mining Profitable High Frequency Pairs Trading Forex Signal Using Copula and Deep Neural Network. In 2018 19th IEEE/ACIS International

Conference on Software Engineering, Artificial Intelligence, Networking and Parallel/Distributed Computing (SNPD) (pp. 312-316). IEEE.

[39] Yamada, Y. and Primbs, J.A., 2018. Model predictive control for optimal pairs trading portfolio with gross exposure and transaction cost constraints. Asia-Pacific Financial Markets, 25, pp.1-21.

[40] Leung, T. and Yan, R., 2018. Optimal dynamic pairs trading of futures under a two-factor mean-reverting model. International Journal of Financial Engineering, 5(03), p.1850027.

[41] Suzuki, K., 2018. Optimal pair-trading strategy over long/short/square positions—empirical study. Quantitative Finance, 18(1), pp.97-119.

[42] Oh, E. and Son, S.Y., 2018. Pair matching strategies for prosumer market under guaranteed minimum trading. IEEE Access, 6, pp.40325-40333.

[43] Mikkelsen, A., 2018. Pairs trading: the case of Norwegian seafood companies. Applied Economics, 50(3), pp.303-318.

[44] Figuerola‐Ferretti, I., Paraskevopoulos, I. and Tang, T., 2018. Pairs‐trading and spread persistence in the European stock market. Journal of Futures Markets, 38(9), pp.998-1023.

[45] Blázquez, M.C. and Román, C.P., 2018. Pairs trading techniques: An empirical contrast. European Research on Management and Business Economics, 24(3), pp.160-167.

[46] Altay, S., Colaneri, K. and Eksi, Z., 2018. Pairs trading under drift uncertainty and risk penalization. International Journal of Theoretical and Applied Finance, 21(07), p.1850046.

[47] Primbs, J.A. and Yamada, Y., 2018. Pairs trading under transaction costs using model predictive control. Quantitative Finance, 18(6), pp.885-895.

[48] Chodchuangnirun, B., Zhu, K. and Yamaka, W., 2018. Pairs trading via nonlinear autoregressive GARCH models. In Integrated Uncertainty in Knowledge Modelling and Decision Making: 6th International Symposium, IUKM 2018, Hanoi, Vietnam, March 15-17, 2018, Proceedings 6 (pp. 276-288). Springer International Publishing.

[49] Stübinger, J. and Endres, S., 2018. Pairs trading with a mean-reverting jump–diffusion model on high-frequency data. Quantitative Finance, 18(10), pp.1735-1751.

[50] Clegg, M. and Krauss, C., 2018. Pairs trading with partial cointegration. Quantitative Finance, 18(1), pp.121-138.

[51] Quinn, B., Hanna, A. and MacDonald, F., 2018. Picking up the pennies in front of the bulldozer: The profitability of gilt based trading strategies. Finance Research Letters, 27, pp.214-222.

[52] Chiu, M.C. and Wong, H.Y., 2018. Robust dynamic pairs trading with cointegration. Operations Research Letters, 46(2), pp.225-232.

[53] Mavrakis, E. and Alexakis, C., 2018. Statistical Arbitrage Strategies under Different Market Conditions: The Case of the Greek Banking Sector. Journal of Emerging Market Finance, 17(2), pp.159-185.

[54] Endres, S. and Stübinger, J., 2019. A flexible regime switching model with pairs trading application to the S&P 500 high-frequency stock returns. Quantitative Finance, 19(10), pp.1727-1740.

[55] Moulya, V.H., Mohammadi, A. and Mallikarjunappa, T., 2019. An evaluation of pairs trading strategy: A study of global currencies. Pacific Business Review International, 11(10), pp.67-84.

[56] Chen, H., Chen, S., Chen, Z. and Li, F., 2019. Empirical investigation of an equity pairs trading strategy. Management Science, 65(1), pp.370-389.

[57] Chen, C.W., Lin, T.Y. and Huang, T.Y., 2019. Incorporating volatility in tolerance intervals for pair-trading strategy and backtesting. Journal of Risk Model Validation.

[58] Bayram, M. and Akat, M., 2019. Market-neutral trading with fuzzy inference, a new method for the pairs trading strategy. Engineering Economics.

[59] Endres, S. and Stübinger, J., 2019. Optimal trading strategies for Lévy-driven Ornstein–Uhlenbeck processes. Applied Economics, 51(29), pp.3153-3169.

[60] Zhang, H. and Urquhart, A., 2019. Pairs trading across Mainland China and Hong Kong stock markets. International Journal of Finance & Economics, 24(2), pp.698-726.

[61] Huang, Z. and Martin, F., 2019. Pairs trading strategies in a cointegration framework: back-tested on CFD and optimized by profit factor. Applied Economics, 51(22), pp.2436-2452.

[62] ORmOS, B.B.L.N.M., PAIRS TRADING STRATEGIES IN THE OLD AND NEw EU mEmBER STATES. A V4 ORSZÁGOKBAN, p.47.

[63] Farago, A. and Hjalmarsson, E., 2019. Stock price co-movement and the foundations of pairs trading. Journal of Financial and Quantitative Analysis, 54(2), pp.629-665.

[64] Garivaltis, A., 2019. Super-replication of the best pairs trade in hindsight. Cogent Economics & Finance.

[65] Chen, K., Chiu, M.C. and Wong, H.Y., 2019. Time-consistent mean-variance pairs-trading under regime-switching cointegration. SIAM Journal on Financial Mathematics, 10(2), pp.632-665.

[66] Liang, Y., Thavaneswaran, A. and Hoque, M.E., 2020, December. A novel algorithmic trading strategy using data-driven innovation volatility. In 2020 IEEE Symposium Series on Computational Intelligence (SSCI) (pp. 1107-1114). IEEE.

[67] Shen, L., Shen, K., Yi, C. and Chen, Y., 2020, December. An evaluation of pairs trading in commodity futures markets. In 2020 IEEE International Conference on Big Data (Big Data) (pp. 5457-5462). IEEE.

[68] Wu, L., Zang, X. and Zhao, H., 2020. Analytic value function for a pairs trading strategy with a Lévy-driven Ornstein–Uhlenbeck process. Quantitative Finance, 20(8), pp.1285-1306.

[69] Liang, S., Lu, S., Lin, J. and Wang, Z., 2020, October. Hardware accelerator for engle-granger cointegration in pairs trading. In 2020 IEEE International Symposium on Circuits and Systems (ISCAS) (pp. 1-5). IEEE.

[70] Lee, D. and Leung, T., 2020. On the efficacy of optimized exit rule for mean reversion trading. International Journal of Financial Engineering, 7(03), p.2050024.

[71] Fernández-Pérez, A., de las Nieves López-García, M. and Requena, J.P.R., 2020. On the sensibility of the Pairs Trading strategy: the case of the FTS stock market index. Studies of Applied Economics, 38(3).

[72] Aggarwal, G. and Aggarwal, N., 2020. Pairs trading in commodity futures: Evidence from the Indian market. Indian Journal of Economics and Development, 16(3), pp.363-371.

[73] Fil, M. and Kristoufek, L., 2020. Pairs trading in cryptocurrency markets. IEEE Access, 8, pp.172644-172651.

[74] Fernandez-Perez, A., Frijns, B., Indriawan, I. and Tse, Y., 2020. Pairs trading of Chinese and international commodities. Applied Economics, 52(48), pp.5203-5217.

[75] Feng, M., Chiu, M.C. and Wong, H.Y., 2020. Pairs trading with illiquidity and position limits. Journal of Industrial & Management Optimization, 16(6).

[76] Liu, R., Wu, Z. and Zhang, Q., 2020. Pairs-trading under geometric Brownian motions: An optimal strategy with cutting losses. Automatica, 115, p.108912.

[77] Diao, H., Liu, G. and Zhu, Z., 2020. Research on a stock-matching trading strategy based on bi-objective optimization. Frontiers of Business Research in China, 14(1), p.8.

[78] Gupta, K. and Chatterjee, N., 2020. Selecting stock pairs for pairs trading while incorporating lead–lag relationship. Physica A: Statistical Mechanics and its Applications, 551, p.124103.

[79] Ramos-Requena, J.P., Trinidad-Segovia, J.E. and Sánchez-Granero, M.Á., 2020. Some notes on the formation of a pair in pairs trading. Mathematics, 8(3), p.348.

[80] Ramos-Requena, J.P., López-García, M.N., Sánchez-Granero, M.A. and Trinidad-Segovia, J.E., 2021. A cooperative dynamic approach to pairs trading. Complexity, 2021, pp.1-8.

[81] Li, Z. and Tourin, A., 2022. A finite difference scheme for pairs trading with transaction costs. Computational Economics, 60(2), pp.601-632.

[82] Johnson-Skinner, E., Liang, Y., Yu, N. and Morariu, A., 2021, July. A Novel Algorithmic Trading Strategy using Hidden Markov Model for Kalman Filtering Innovations. In 2021 IEEE 45th Annual Computers, Software, and Applications Conference (COMPSAC) (pp. 1766-1771). IEEE.

[83] Hoque, M.E., Thavaneswaran, A., Paseka, A. and Thulasiram, R.K., 2021, July. An algorithmic multiple trading strategy using data-driven random weights innovation volatility. In 2021 IEEE 45th Annual Computers, Software, and Applications Conference (COMPSAC) (pp. 1760-1765). IEEE.

[84] Holý, V. and Černý, M., 2022. Bertram's pairs trading strategy with bounded risk. Central European Journal of Operations Research, 30(2), pp.667-682.

[85] Figá-Talamanca, G., Focardi, S. and Patacca, M., 2021. Common dynamic factors for cryptocurrencies and multiple pair-trading statistical arbitrages. Decisions in Economics and Finance, 44, pp.863-882.

[86] Tadi, M. and Kortchemski, I., 2021. Evaluation of dynamic cointegration-based pairs trading strategy in the cryptocurrency market. Studies in Economics and Finance, 38(5), pp.1054-1075.

[87] Cerda, J., Rojas-Morales, N., Minutolo, M.C. and Kristjanpoller, W., 2022. High frequency and dynamic pairs trading with ant colony optimization. Computational Economics, 59(3), pp.1251-1275.

[88] Suzuki, K., 2021. Infinite-horizon optimal switching regions for a pair-trading strategy with quadratic risk aversion considering simultaneous multiple switchings: A viscosity solution approach. Mathematics of Operations Research, 46(1), pp.336-360.

[89] Naccarato, A., Pierini, A. and Ferraro, G., 2021. Markowitz portfolio optimization through pairs trading cointegrated strategy in long-term investment. Annals of Operations Research, 299(1), pp.81-99.

[90] Lin, T.Y., Chen, C.W. and Syu, F.Y., 2021. Multi-asset pair-trading strategy: A statistical learning approach. The North American Journal of Economics and Finance, 55, p.101295.

[91] Fukasawa, M., Maeda, H. and Sekine, J., 2021. On optimal thresholds for pairs trading in a one-dimensional diffusion model. The ANZIAM Journal, 63(2), pp.104-122.

[92] Zhu, D.M., Gu, J.W., Yu, F.H., Siu, T.K. and Ching, W.K., 2021. Optimal pairs trading with dynamic mean-variance objective. Mathematical Methods of Operations Research, 94(1), pp.145-168.

[93] Do, B. and Faff, R., 2021. Pairs trading and idiosyncratic cash flow risk. Accounting & Finance, 61(2), pp.3171-3206.

[94] Yan, T., Chiu, M.C. and Wong, H.Y., 2022. Pairs trading under delayed cointegration. Quantitative Finance, 22(9), pp.1627-1648.

[95] Zhang, G., 2021. Pairs trading with general state space models. Quantitative Finance, 21(9), pp.1567-1587.

[96] Thazhugal Govindan Nair, S., 2021. Price discovery and pairs trading potentials: The case of metals markets. Journal of Financial Economic Policy, 13(5), pp.565-586.

[97] Aggarwal, G. and Aggarwal, N., 2021. Risk-adjusted returns from statistical arbitrage opportunities in Indian stock futures market. Asia-Pacific Financial Markets, 28(1), pp.79-99.

[98] Keshavarz Haddad, G. and Talebi, H., 2023. The profitability of pair trading strategy in stock markets: Evidence from Toronto stock exchange. International Journal of Finance & Economics, 28(1), pp.193-207.

[99] Liang, Y., Thavaneswaran, A., Paseka, A., Qiao, W., Ghahramani, M. and Bowala, S., 2022, June. A novel optimal profit resilient filter pairs trading strategy for cryptocurrencies. In 2022 IEEE 46th Annual Computers, Software, and Applications Conference (COMPSAC) (pp. 1274-1279). IEEE.

[100] Xing, H., 2022. A singular stochastic control approach for optimal pairs trading with proportional transaction costs. Journal of Risk and Financial Management, 15(4), p.147.

[101] Bui, Q. and Ślepaczuk, R., 2022. Applying Hurst Exponent in pair trading strategies on Nasdaq 100 index. Physica A: Statistical Mechanics and its Applications, 592, p.126784.

[102] Chen, L. and Zhang, G., 2022. COVID-19 effects on arbitrage trading in the energy market. Energies, 15(13), p.4584.

[103] Yan, T. and Wong, H.Y., 2022. Equilibrium pairs trading under delayed cointegration. Automatica, 144, p.110498.

[104] Luo, J., Lin, Y. and Wang, S., 2023. Intraday high-frequency pairs trading strategies for energy futures: evidence from China. Applied Economics, 55(56), pp.6646-6660.

[105] Ohnishi, M. and Shimoshimizu, M., 2022. Optimal Pair–Trade Execution with Generalized Cross–Impact. Asia-Pacific Financial Markets, 29(2), pp.253-289.

[106] Xie, P., Bai, L. and Zhang, H., 2023. OPTIMAL PROPORTIONAL REINSURANCE AND PAIRS TRADING UNDER EXPONENTIAL UTILITY CRITERION FOR THE INSURER. Journal of Industrial & Management Optimization, 19(3).

[107] Tokat, E. and Hayrullahoğlu, A.C., 2022. Pairs trading: is it applicable to exchange-traded funds?. Borsa Istanbul Review, 22(4), pp.743-751.

[108] Xiang, Y., Zhao, Y. and Deng, S., 2023. Pairs trading with fractional Ornstein–Uhlenbeck spread model. Applied Economics, 55(23), pp.2607-2623.

[109] Lee, Y.S., 2022. Representative Bias and Pairs Trade: Evidence From S&P 500 and Russell 2000 Indexes. Sage Open, 12(3), p.21582440221120361.

[110] Nadaf, T., Lotfi, T. and Shateyi, S., 2022. Revisiting the Copula-Based Trading Method Using the Laplace Marginal Distribution Function. Mathematics, 10(5), p.783.

[111] Lee, K., Leung, T. and Ning, B., 2023. A Diversification Framework for Multiple Pairs Trading Strategies. Risks, 11(5), p.93.

[112] da Silva, F.A.S., Ziegelmann, F.A. and Caldeira, J.F., 2023. A pairs trading strategy based on mixed copulas. The Quarterly Review of Economics and Finance, 87, pp.16-34.

[113] Amer, L. and Islam, T.U., 2023. An Entropic Approach for Pair Trading in PSX. Entropy, 25(3), p.494.

[114] Kato, K. and Nakamura, N., 2023. Cointegration analysis of hazard rates and CDSs: Applications to pairs trading strategy. Physica A: Statistical Mechanics and its Applications, 612, p.128489.

[115] Singh, J., Thulasiram, R.K., Thavaneswaran, A. and Paseka, A., 2023, June. Comparison of Trading Strategies: Dual Momentum vs Pairs Trading. In 2023 IEEE 47th Annual Computers, Software, and Applications Conference (COMPSAC) (pp. 1363-1369). IEEE.

[116] Finhold, E., Heller, T. and Leithäuser, N., 2023. On the potential of arbitrage trading on the German intraday power market. Journal of Energy Markets, 16(3).

[117] Xie, P., Bai, L. and Zhang, H., 2023. Optimal pairs trading of mean-reverting processes over multiple assets. Numerical Algebra, Control and Optimization, 13(3&4), pp.461-472.

[118] Yu, F., Ching, W.K., Wu, C. and Gu, J.W., 2023. Optimal Pairs Trading Strategies: A Stochastic Mean–Variance Approach. Journal of Optimization Theory and Applications, 196(1), pp.36-55.

[119] Zhang, Y. and Xiong, D., 2023. Optimal Strategy of the Dynamic Mean-Variance Problem for Pairs Trading under a Fast Mean-Reverting Stochastic Volatility Model. Mathematics, 11(9), p.2191.

[120] Ko, P.C., Lin, P.C., Do, H.T., Kuo, Y.H., Mai, L.M. and Huang, Y.F., 2024. Pairs trading in cryptocurrency markets: A comparative study of statistical methods. Investment Analysts Journal, 53(2), pp.102-119.

[121] Brunetti, M. and De Luca, R., 2023. Pairs trading in the index options market. Eurasian Economic Review, 13(1), pp.145-173.

[122] Das, E.C., Luu, P.T., Tie, J. and Zhang, Q., 2023. Pairs trading under a mean reversion model with regime switching. Numerical Algebra, Control and Optimization, pp.0-0.

[123] Brunetti, M. and De Luca, R., 2023. Pre-selection in cointegration-based pairs trading. Statistical Methods & Applications, 32(5), pp.1611-1640.

[124] Engle, R.F. and Granger, C.W., 1987. Co-integration and error correction: representation, estimation, and testing. Econometrica: journal of the Econometric Society, pp.251-276.

[125] Jurek, J.W. and Yang, H., 2007, April. Dynamic portfolio selection in arbitrage. In EFA 2006 meetings paper.

[126] Alexander, C., 1999. Optimal hedging using cointegration. Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 357(1758), pp.2039-2058.

[127] Gatev, E., Goetzmann, W.N. and Rouwenhorst, K.G., 2006. Pairs trading: Performance of a relative-value arbitrage rule. The Review of Financial Studies, 19(3), pp.797-827.

[128] Vidyamurthy, G., 2004. Pairs trading: Quantitative methods and analysis (Vol. 217). John Wiley & Sons.