In this summer I did two groups of comparison: PDE acceleration and Chambolle-Pock primal-dual on level set segmentation with TV regularization and Beltrami regularization. The level set scheme I used here is called fuzzy region competition, in which the surface is constrain within [0,1] and the contour is 0.5 level set.

For TV regularization, the potential energy function is:

$$\min_{u\in BV[0,1](c1,c2)\in R^2}\left\{Fc(u,c1,c2)=\int_\Omega|\nabla u|+\ \lambda\int_\Omega u(I-c_1)^2\ +\ \lambda\ \int_\Omega(1-u)(I-c_2)^2\right\}$$

The difference from Chan-Vese is that there is no Heaviside function here, *u* is the level set. Check the paper [Mory-Ardon2007] for detail. The primal-dual algorithm is from [Chambolle-Pock2011], the algorithm is shown below.

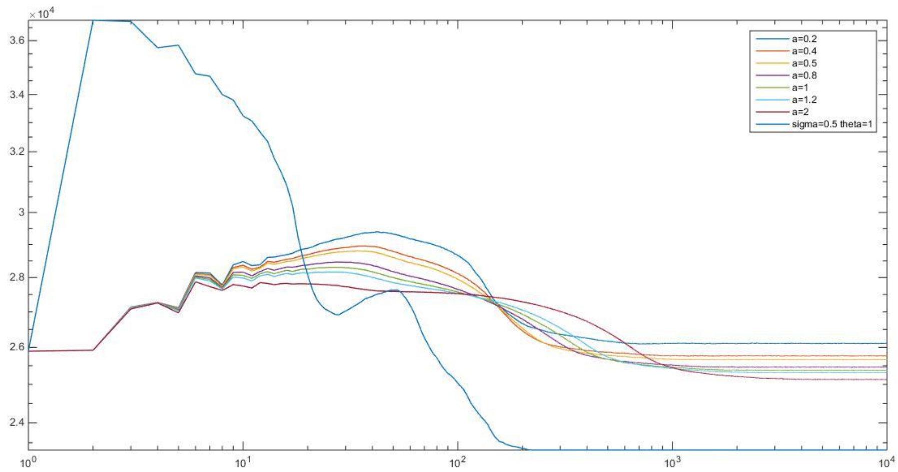
$$\text{Initialize } u^0 = u_0; p^0 = 0$$
$$\begin{cases} \bar{p}^{k+1} = p^k + \sigma\nabla u^k \\ p^{k+1} = \dfrac{\bar{p}^{k+1}}{\max(|\bar{p}^{k+1}|,1)} \\ r = \lambda\,((I-c_1)^2 - (I-c_2)^2) \\ u^{k+1} = \max(\min(u - \tau r + div p^{k+1}, 1), 0) \end{cases}$$

The problem here is I don't know how to choose sigma and tau to have the best performance.

From paper [Chambolle-Pock2011], it says $\tau\sigma L^2<1$, here $L^2 = |\nabla|^2 = |div|^2 \leq 8$,so $\tau\sigma \leq \frac{1}{L^2}$.

So the multiply of sigma and tau is a fixed value, but I don't know what the value is for each instance. I played with those parameters and done lots of experiments. I got the best result with tau = 1, sigma =0.5.

For acceleration, the quantization level Q I use here is 1/255, the acceleration method is forward difference. And Dr.Yezzi told me that there is no optimal damping coefficient for TV, so I tried various of damping coefficients. The energy loglog plot is shown below. Line 1 to line 7 are acceleration, line 8 is primal-dual. All the experiments are done on noisy image.



For the final segmentation result, please check the folder.

For Beltrami regularization, potential energy is

$$\min_{\mathrm{u}\in\mathrm{BV}[0,1](\mathrm{c1},\mathrm{c2})\in\mathrm{R}^2}\Bigg\{\mathrm{Fc}(\mathrm{u},\mathrm{c1},\mathrm{c2})$$

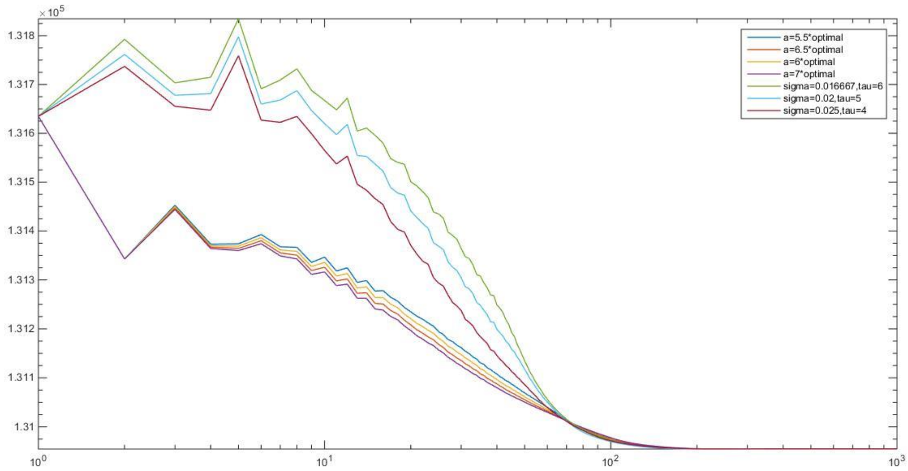
$$= \int_\Omega \frac{1}{\beta}\sqrt{1+\|\beta u\|^2} + \lambda \int_\Omega \mathrm{u}(\mathrm{I}-\mathrm{c}_1)^2 + \lambda \int_\Omega (1-\mathrm{u})(\mathrm{I}-\mathrm{c}_2)^2\Bigg\}$$

The primal-dual Beltrami version algorithm is from [Zosso-Bustin 2014], the algorithm is shown below

$$\text{Initialize } \mathrm{u}^0 = \mathrm{u}_0;\ \mathrm{p}^0 = 0$$

$$\begin{cases} \bar{p}^{k+1} = (1-\sigma\beta)p^k + \beta\sigma\nabla u^k\sqrt{\beta^2 - |p^k|^2} \\ p^{k+1} = \dfrac{\beta\bar{p}^{k+1}}{\max(|\bar{p}^{k+1}|,\beta)} \\ r = \lambda\left((\mathrm{I}-\mathrm{c}_1)^2 - (\mathrm{I}-\mathrm{c}_2)^2\right) \\ u^{k+1} = \max(\min(u-\tau r + div p^{k+1}, 1), 0) \end{cases}$$

In the experiment, beta is 2. For the acceleration, the optimal damping coefficient is $\frac{2\pi}{m}\sqrt{\beta g}$ .But in the experiment the best result occurs when damping coefficient is 6 times of the optimal damping coefficient. The energy loglog plot is shown below. Line 1 to line 4 are acceleration and line 5 to line 7 are primal-dual.



For the visual segmentation results, please check the folder.