

# Computer Networks

## Chapter 2: Physical Layer

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# Outline

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- General Architecture of Physical Layer
- Signal Representation
- Digitization of Analog Signals
  - Sampling
  - Quantization
- Digital Signal Transmission and Reception
  - Channel
  - Modulation
  - Forward Error Control
  - Multiplexing
- Transmission Medium



# General Architecture

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- *See class notes*



# Bits, Number, and Information

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- Bit: number with value 0 or 1
  - $n$  bits: digital representation for  $0, 1, \dots, 2^n$
  - Byte or Octet,  $n = 8$
  - Computer word,  $n = 16, 32, \text{ or } 64$
- $n$  bits allows enumeration of  $2^n$  possibilities
  - $n$ -bit field in a header
  - $n$ -bit representation of a voice sample
  - Message consisting of  $n$  bits
- *The number of bits required to represent a message is a measure of its information content*
  - More bits  $\rightarrow$  More content

# Signal Representation: Fourier Analysis

- A periodic signal with period  $T$  can be represented as sum of sinusoids using Fourier Series:

$$x(t) = a_0 + a_1 \cos(2\pi f_0 t + \phi_1) + a_2 \cos(2\pi 2f_0 t + \phi_2) + \dots + a_k \cos(2\pi k f_0 t + \phi_k) + \dots$$

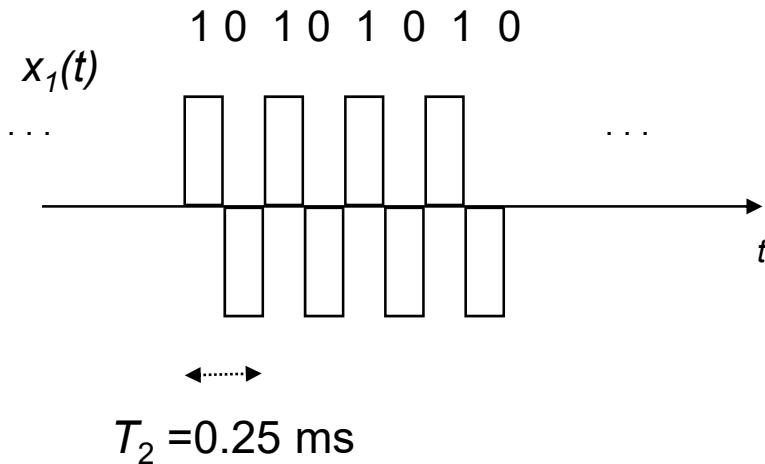
“DC”  
long-term  
average

fundamental  
frequency  $f_0 = 1/T$   
first harmonic

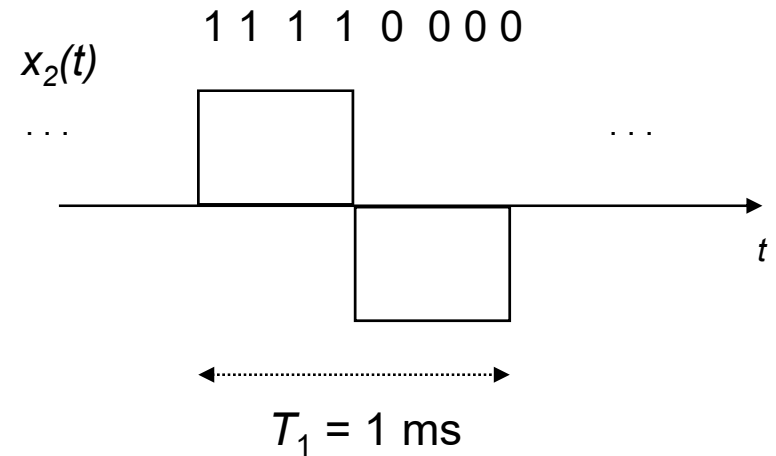
$k$ th harmonic

- $|a_k|$  determines amount of power in  $k$ th harmonic
- Amplitude spectrum  $|a_0|, |a_1|, |a_2|, \dots$

# Example Fourier Series



$$\begin{aligned}
 x_1(t) = & 0 + \frac{4}{\pi} \cos(2\pi 4000t) \\
 & + \frac{4}{3\pi} \cos(2\pi 3(4000)t) \\
 & + \frac{4}{5\pi} \cos(2\pi 5(4000)t) + \dots
 \end{aligned}$$

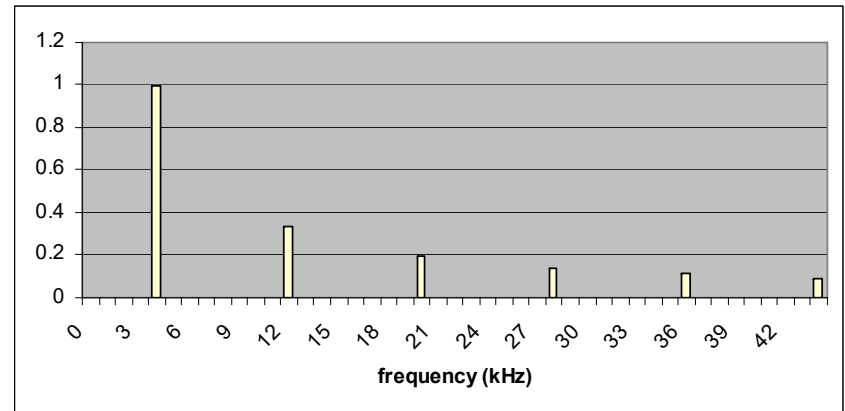


$$\begin{aligned}
 x_2(t) = & 0 + \frac{4}{\pi} \cos(2\pi 1000t) \\
 & + \frac{4}{3\pi} \cos(2\pi 3(1000)t) \\
 & + \frac{4}{5\pi} \cos(2\pi 5(1000)t) + \dots
 \end{aligned}$$

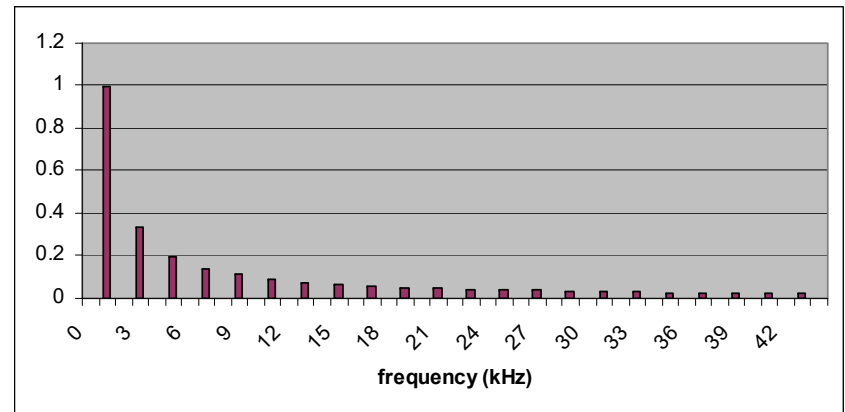
# Spectrum

- Spectrum of a signal: magnitude of amplitudes as a function of frequency
- $x_1(t)$  varies faster in time & has more high frequency content than  $x_2(t)$
- Bandwidth  $W_s$  is defined as range of frequencies where a signal has non-negligible power, e.g. range of band that contains 99% of total signal power

Spectrum of  $x_1(t)$



Spectrum of  $x_2(t)$





# Non-Periodic Signals

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- Fourier Transform





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# Digitization of Analog Signals

# Transmission Impairments



## Communication Channel

- Pair of copper wires
- Coaxial cable
- Radio
- Light in optical fiber
- Light in air
- Infrared

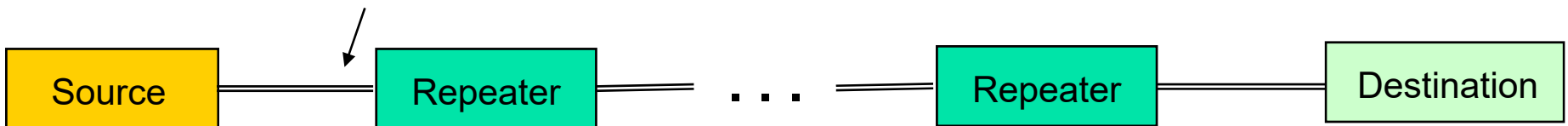
## Transmission Impairments

- Signal attenuation
- Signal distortion
- Spurious noise
- Interference from other signals

# Analog Long-Distance Communications

- Each repeater attempts to restore analog signal to its original form
- Restoration is imperfect
  - Distortion is not completely eliminated
  - Noise & interference is only partially removed
- Signal quality decreases with # of repeaters
- Communications is distance-limited
- Still used in analog cable TV systems
- Analogy: Copy a song using a cassette recorder

Transmission segment



# Analog vs. Digital Transmission

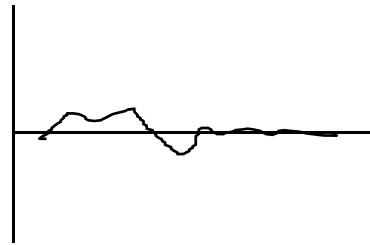
**Analog transmission:** all details must be reproduced accurately

Sent



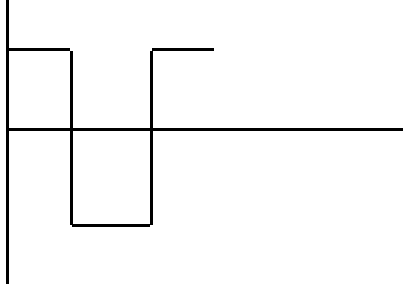
***Distortion***  
***Attenuation***

Received



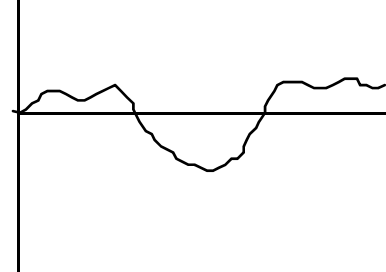
**Digital transmission:** only discrete levels need to be reproduced

Sent



***Distortion***  
***Attenuation***

Received

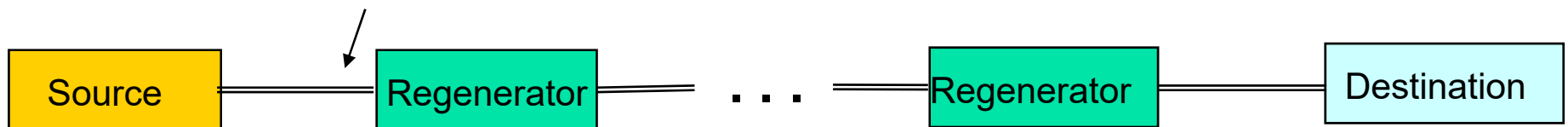


Simple Receiver:  
Was original pulse  
positive or  
negative?

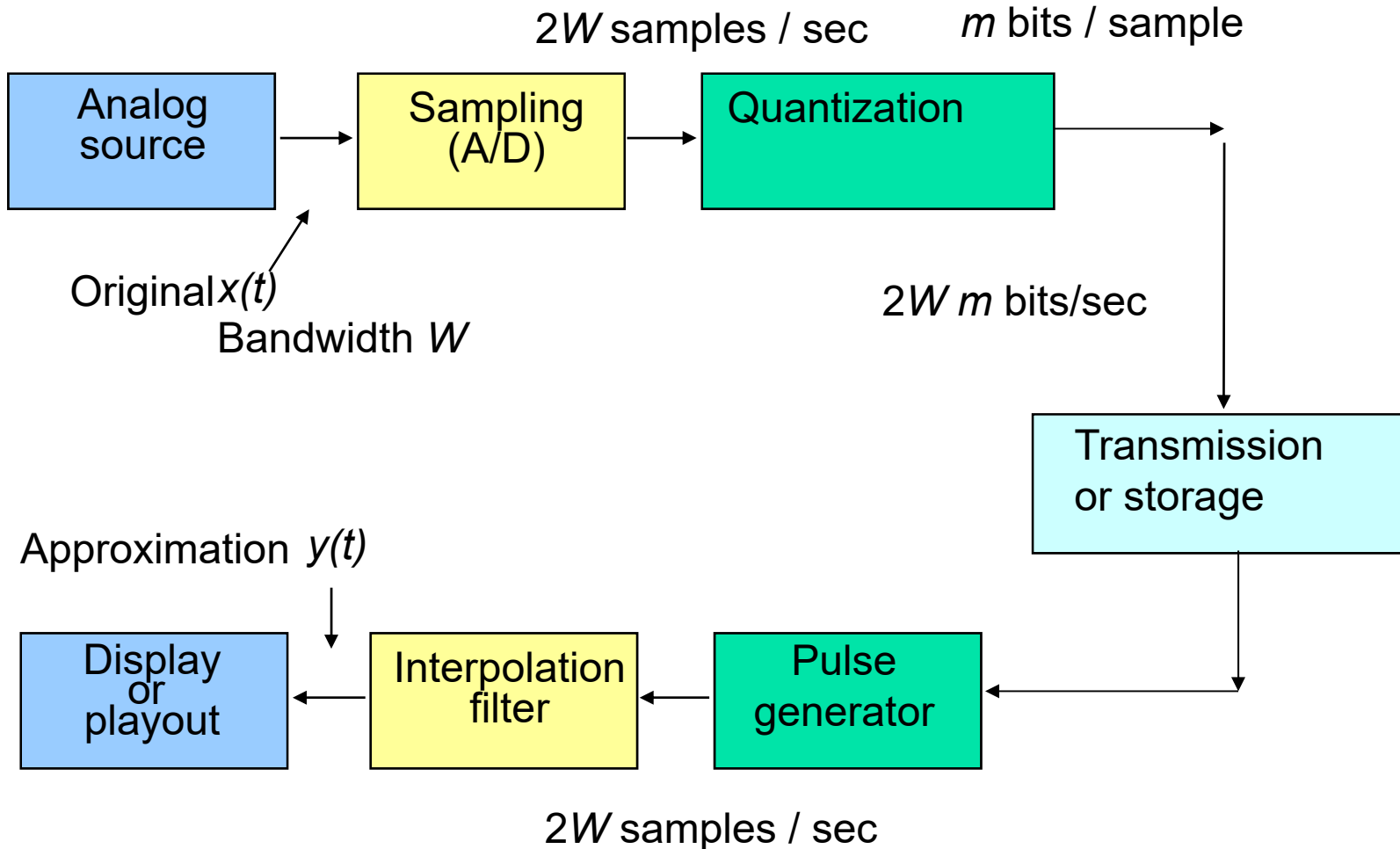
# Digital Long-Distance Communications

- Regenerator recovers original data sequence and retransmits on next segment
- Can design so error probability is very small
- Then each regeneration is like the first time!
- Analogy: copy an MP3 file
- Communications is possible over very long distances
- Digital systems vs. analog systems
  - Less power, longer distances, lower system cost
  - Monitoring, multiplexing, coding, encryption, protocols...

Transmission segment



# System Architecture of Analog Signal Digitization and Transmission



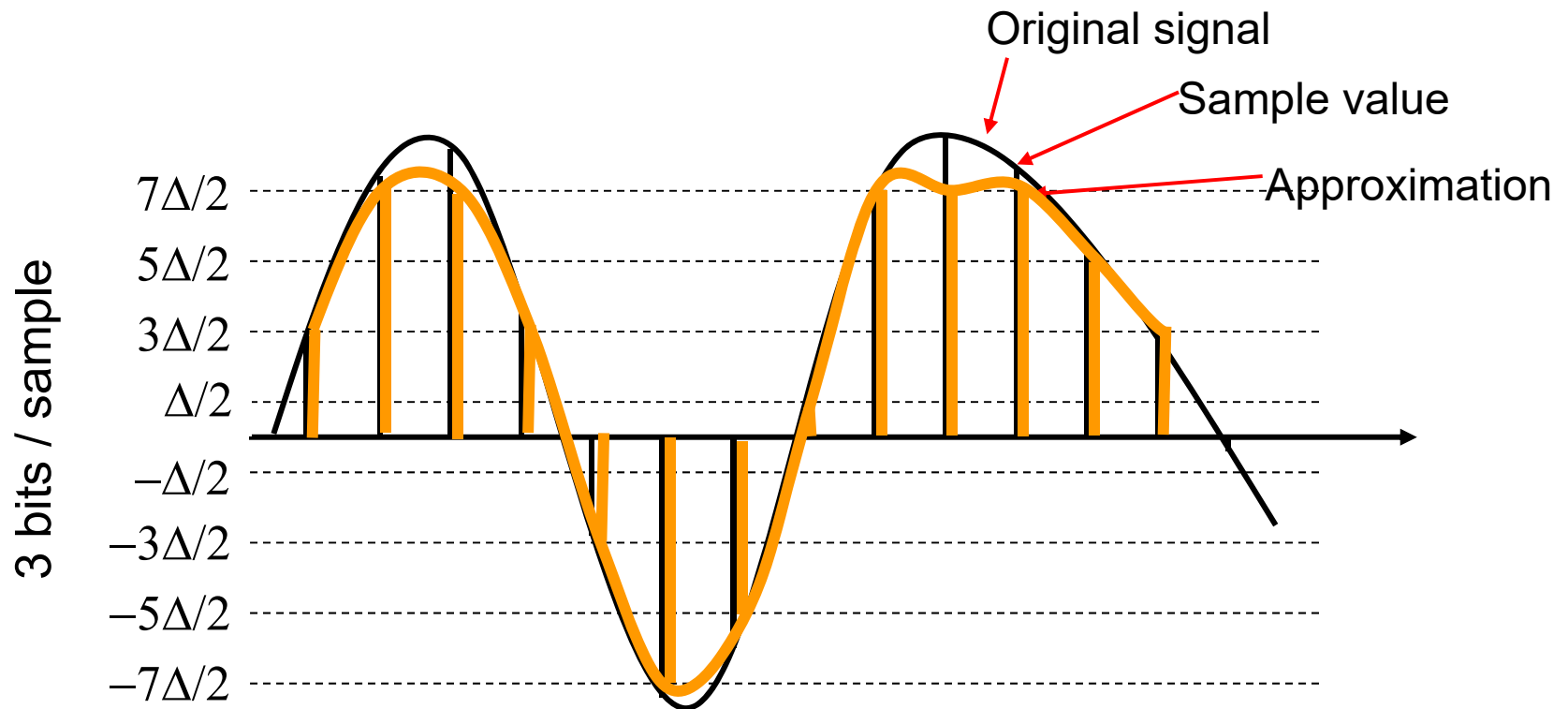


# Digitization of Analog Signals

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1. Sampling: obtain samples of  $x(t)$  at uniformly spaced time intervals
2. Quantization: map each sample into an approximation value of finite precision
  - Pulse Code Modulation: telephone speech
  - CD audio
3. Compression: to lower bit rate further, apply additional compression method
  - Differential coding: cellular telephone speech
  - Subband coding: MP3 audio
  - Other coding schemes

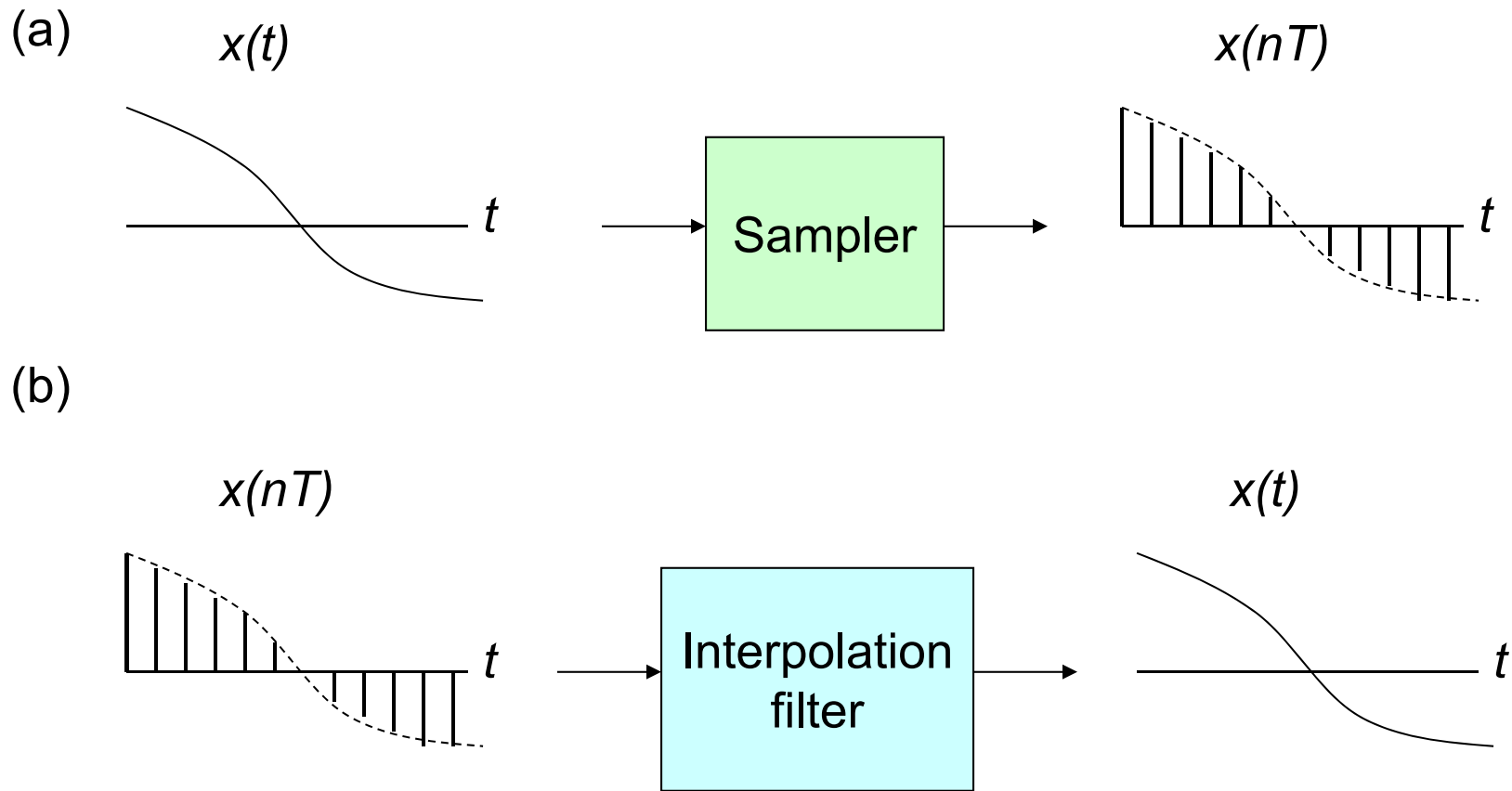
# Sampling of Analog Signal



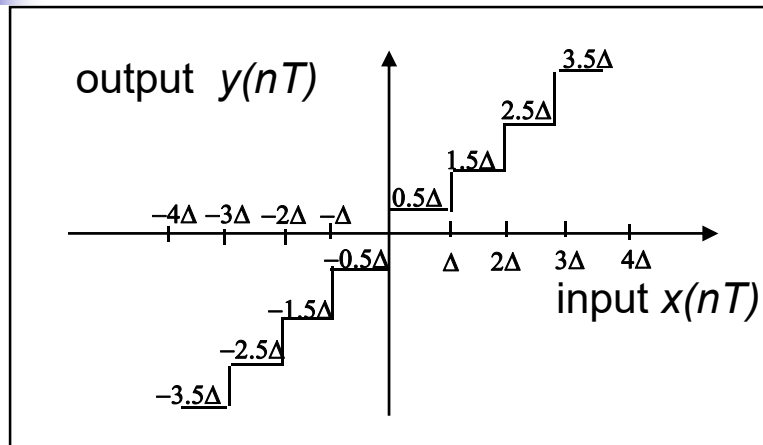


# Sampling Theorem

Nyquist: Perfect reconstruction if sampling rate  $1/T > 2W_s$

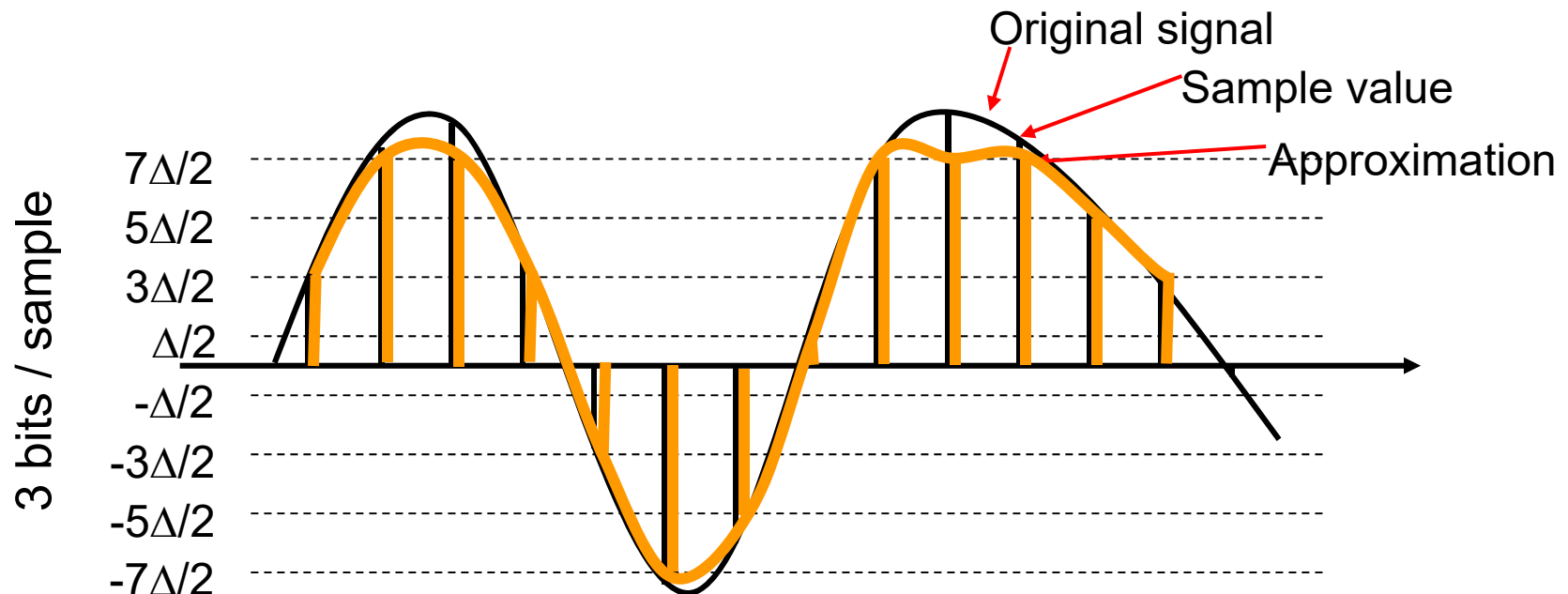


# Quantization of Analog Samples



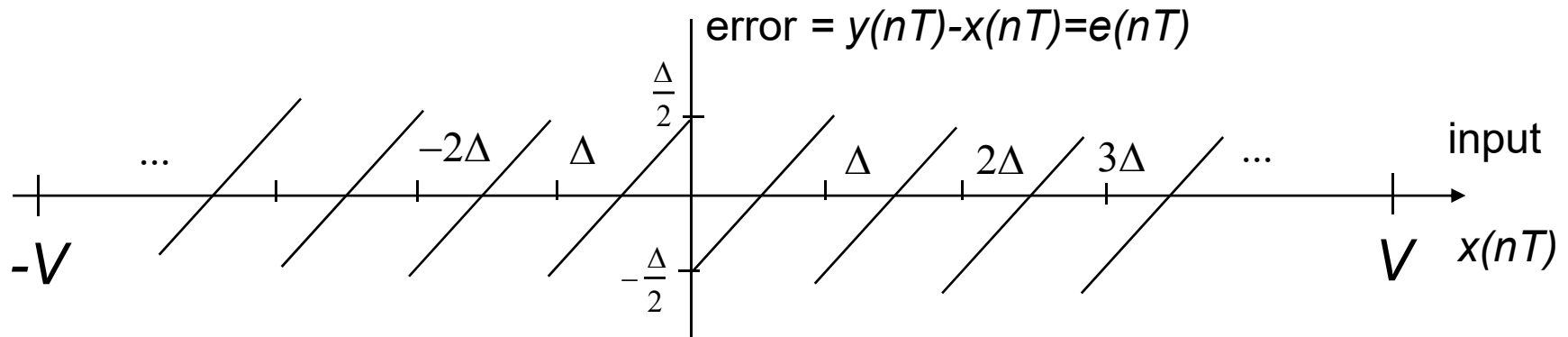
Quantizer maps input into closest of  $2^m$  representation values

Quantization error:  
"noise" =  $x(nT) - y(nT)$



# Quantizer Performance (1)

$M = 2^m$  levels, Dynamic range  $(-V, V)$   $\Delta = 2V/M$



If the number of levels  $M$  is large, then the error is approximately uniformly distributed between  $(-\Delta/2, \Delta/2)$

Average Noise Power = Mean Square Error:

$$\sigma_e^2 = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2 \frac{1}{\Delta} dx = \frac{\Delta^2}{12}$$

## Quantizer Performance (2)

### ■ Figure of Merit:

Signal-to-Noise Ratio = Avg signal power /  
Avg noise power

Let  $\sigma_x^2$  be the signal power, then

$$SNR = \frac{\sigma_x^2}{\Delta^2/12} = \frac{12\sigma_x^2}{4V^2/M^2} = 3 \left(\frac{\sigma_x}{V}\right)^2 M^2 = 3 \left(\frac{\sigma_x}{V}\right)^2 2^{2m}$$

The ratio  $V/\sigma_x \approx 4$

The SNR is usually stated in decibels:

$$SNR \text{ db} = 10 \log_{10} \frac{\sigma_x^2}{\sigma_e^2} = 6m + 10 \log_{10} \frac{3\sigma_x^2}{V^2}$$

$$\mathbf{SNR \text{ db} = 6m - 7.27 \text{ dB}} \quad \text{for } V/\sigma_x = 4.$$



## Example: Telephone Speech

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$W = 4\text{KHz}$ , so Nyquist sampling theorem

$\Rightarrow 2W = 8000$  samples/second

Suppose error requirement = 1% error

$$\text{SNR} = 10 \log(1/.01)^2 = 40 \text{ dB}$$

Assume  $V/\sigma_x = 4$ , then

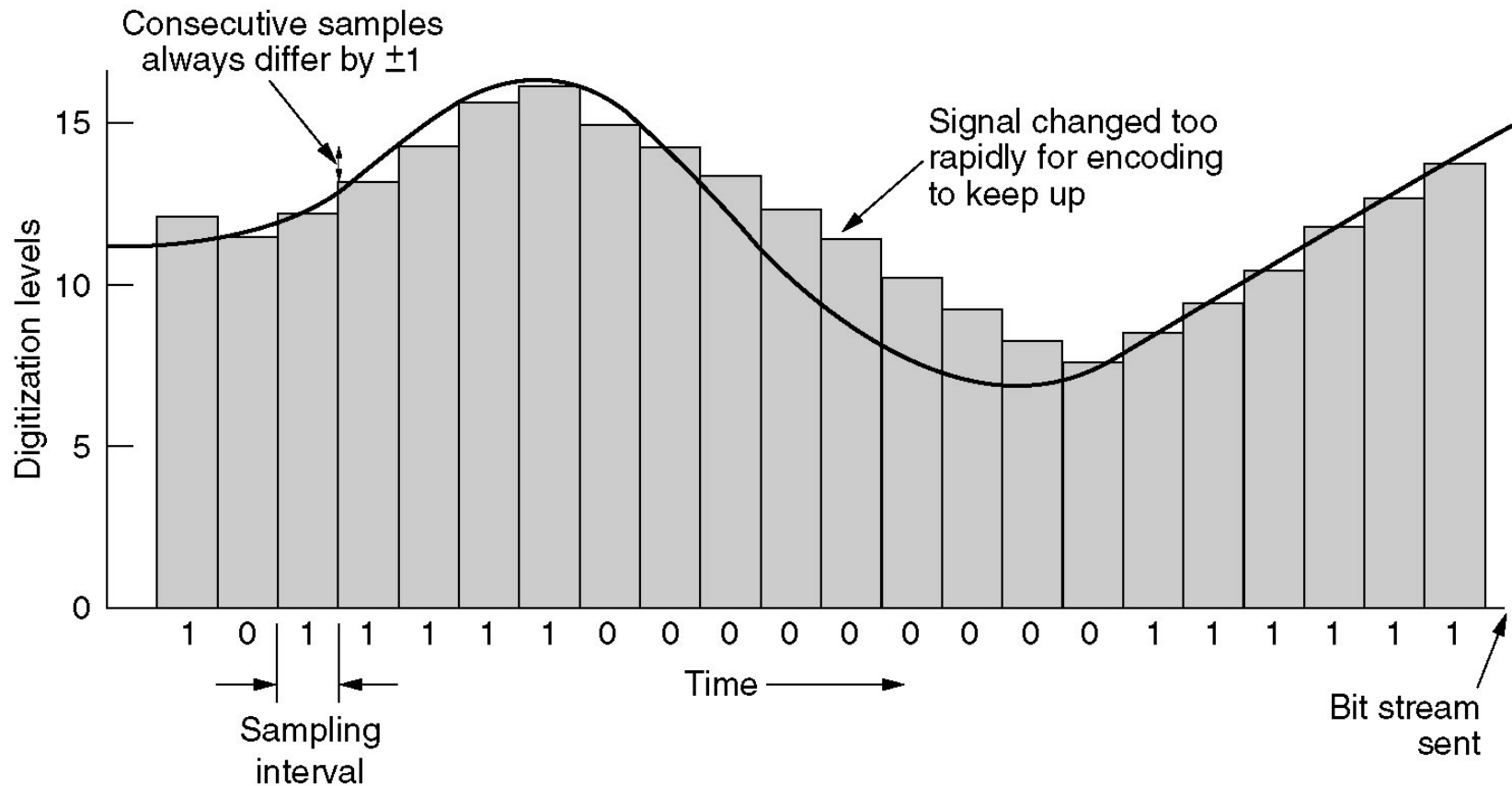
$$40 \text{ dB} = 6m - 7$$

$$\Rightarrow m = 8 \text{ bits/sample}$$

**PCM (“Pulse Code Modulation”) Telephone Speech:**

Bit rate =  $8000 \times 8 \text{ bits/sec} = 64 \text{ kbps}$

# Differential PCM and Delta Modulation

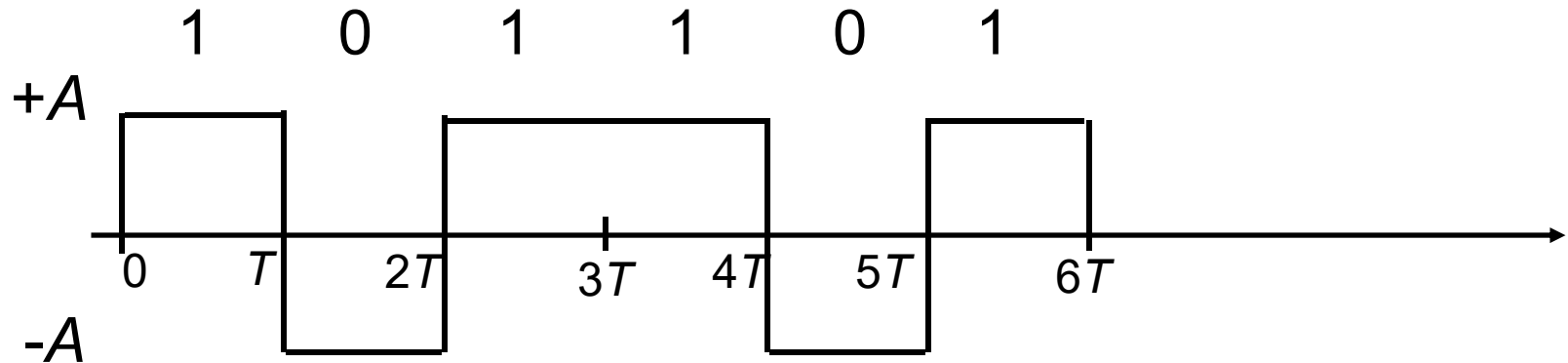




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# Digital Signal Transmission and Reception

# Digital Binary Signal



$$\text{Bit rate} = 1 \text{ bit} / T \text{ seconds}$$

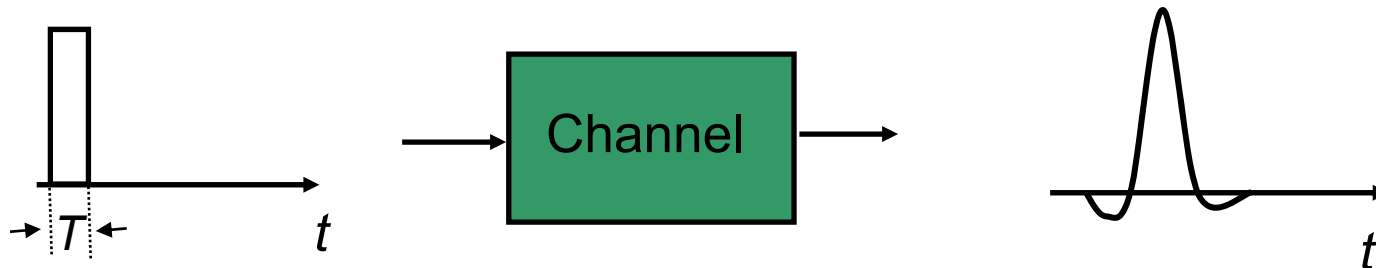
For a given communications medium:

- How do we increase transmission rate?
- How do we achieve reliable communications?
- Are there any limits to rate and reliability?



# Pulse Transmission Rate

- Maximize pulse rate through a channel, that is, make  $T$  as small as possible
- If input is a narrow pulse, then typical output is a spread-out pulse with ringing
- Question: How frequently can these pulses be transmitted without interfering with each other?
- Answer:  $2 \times Wc$  pulses/second, **why???**  
where  $Wc$  is the bandwidth of the channel





# Multilevel Pulse Transmission

- Assume channel of bandwidth  $W_c$  and transmit  $2 W_c$  pulses/sec (without interference)
- If pulses amplitudes are either  $-A$  or  $+A$ , then each pulse conveys 1 bit, so  
**Bit Rate = 1 bit/pulse x  $2 W_c$  pulses/sec =  $2 W_c$  bps**
- If amplitudes are from  $\{-A, -A/3, +A/3, +A\}$ , then bit rate is  $2 \times 2 W_c$  bps
- By going to  $M = 2^m$  amplitude levels, we achieve  
**Bit Rate =  $m$  bits/pulse x  $2 W_c$  pulses/sec =  $2mW_c$  bps**

*In the absence of noise, the bit rate can be increased without limit by increasing  $m$*

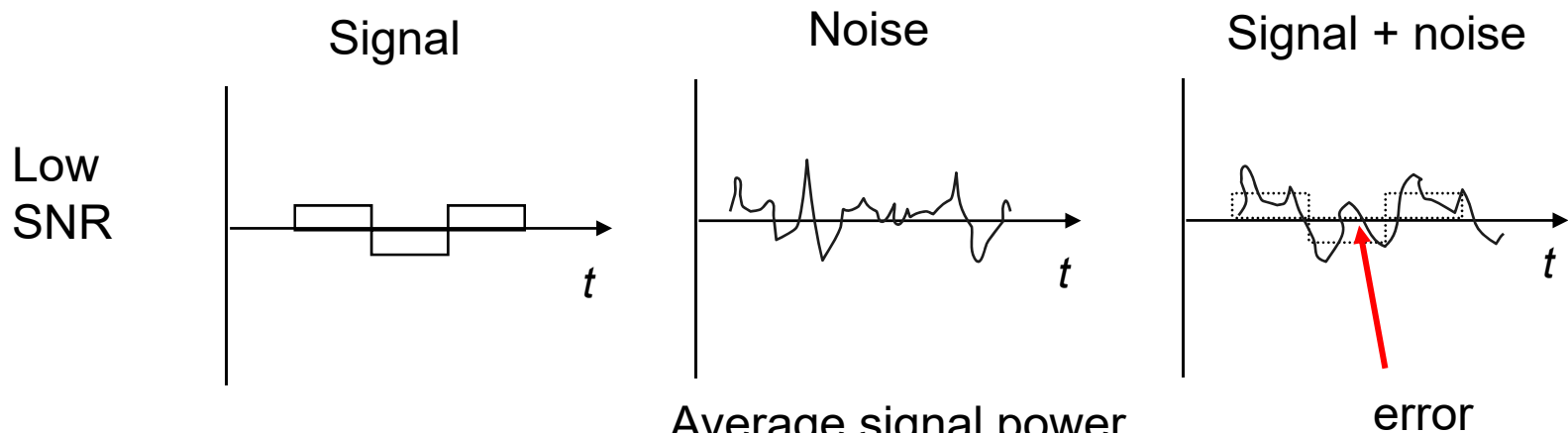
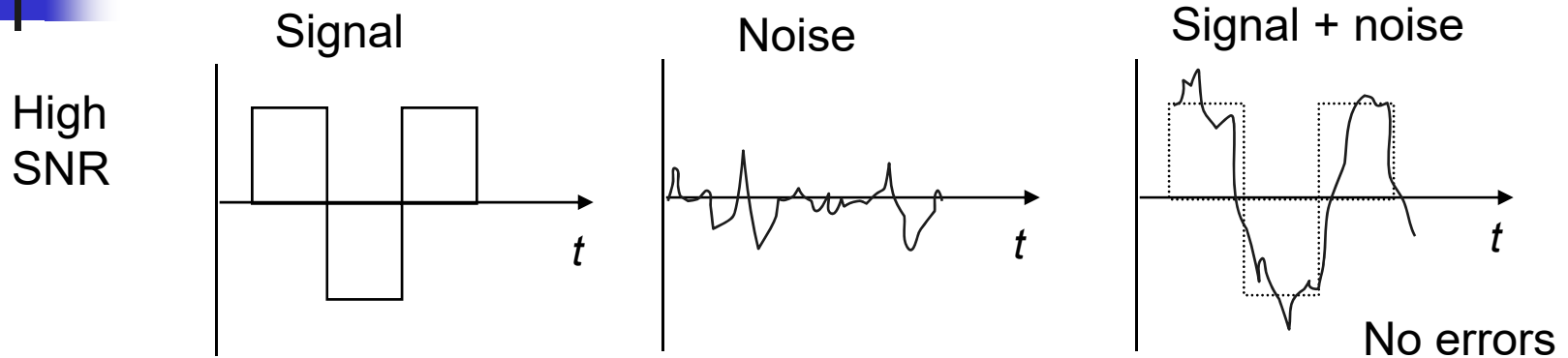


# Noise & Reliable Communications

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- All physical systems have noise
  - Electrons always vibrate at non-zero temperature
  - Motion of electrons induces noise
- Presence of noise limits accuracy of measurement of received signal amplitude
- Errors occur if signal separation is comparable to noise level
- Bit Error Rate (BER) increases with decreasing signal-to-noise ratio
- Noise places a limit on how many amplitude levels can be used in pulse transmission

# Signal-to-Noise Ratio (SNR) Example



$$\text{SNR} = \frac{\text{Average signal power}}{\text{Average noise power}}$$

$$\text{SNR (dB)} = 10 \log_{10} \text{SNR}$$

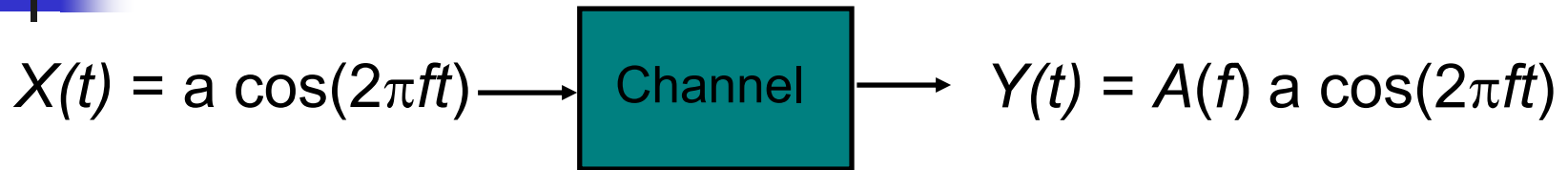


# Channel Capacity: Shannon Theory

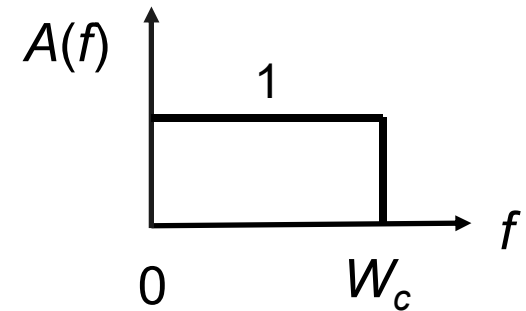
$$C = W_c \log_2 (1 + SNR) \text{ bps}$$

- Arbitrarily reliable communications is possible if the transmission rate  $R < C$ .
- If  $R > C$ , then arbitrarily reliable communications is not possible.
- “Arbitrarily reliable” means the BER can be made arbitrarily small through sufficiently complex coding.
- $C$  can be used as a measure of how close a system design is to the best achievable performance.
- Bandwidth  $W_c$  &  $SNR$  determine  $C$

# Channel Bandwidth



- If input is sinusoid of frequency  $f$ , then
  - output is a sinusoid of the same frequency  $f$
  - Output is attenuated by an amount  $A(f)$  that depends on  $f$
  - $A(f) \approx 1$ , then input signal passes readily
  - $A(f) \approx 0$ , then input signal is blocked
- Bandwidth  $W_c$  is range of frequencies passed by channel

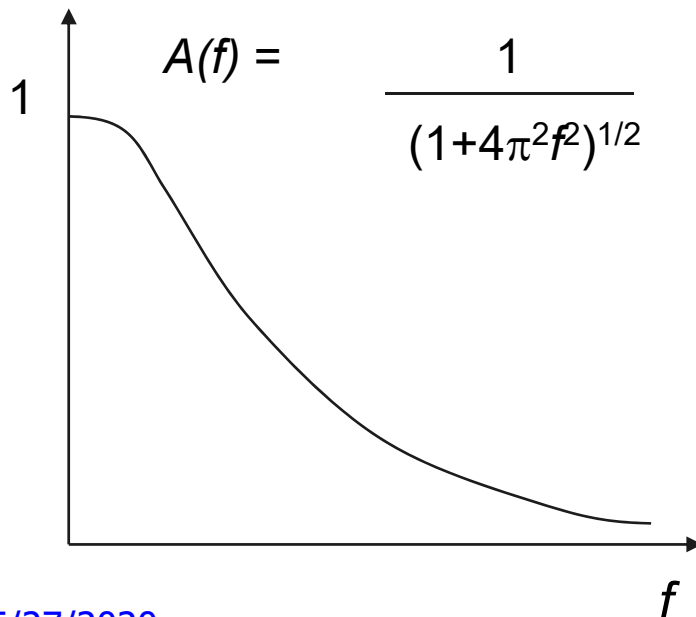


Ideal low-pass channel

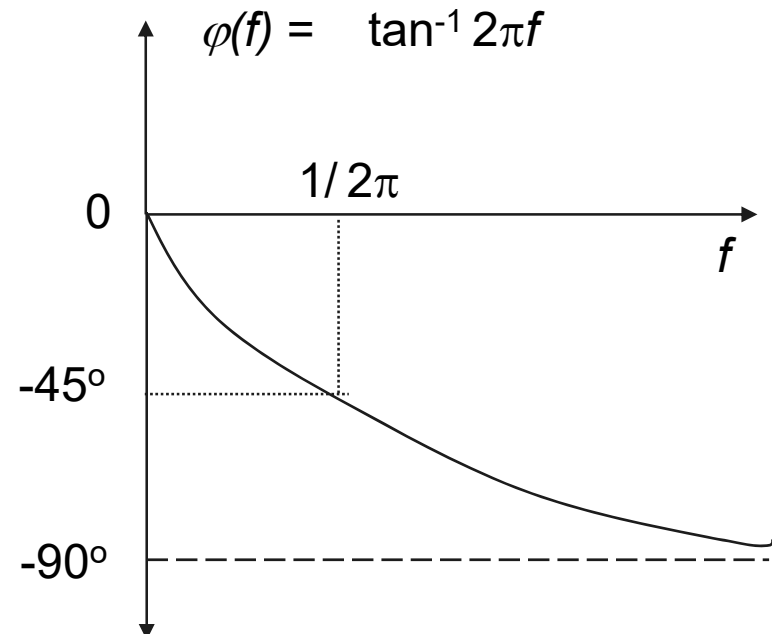
# Example: Low-Pass Filter Channel

- Simplest non-ideal circuit that provides low-pass filtering
  - Inputs at different frequencies are attenuated by different amounts
  - Inputs at different frequencies are delayed by different amounts

Amplitude Response

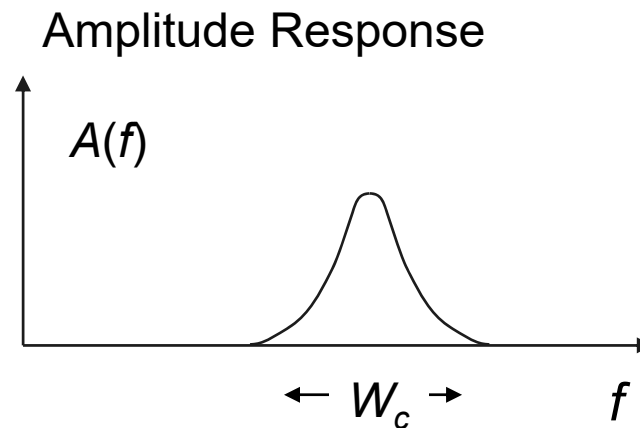


Phase Response



# Example: Band-Pass Filter Channel

- Some channels pass signals within a band that excludes low frequencies
  - Telephone modems, radio systems, ...
- *Channel bandwidth* is the width of the frequency band that passes non-negligible signal power







# How good is a channel?

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- Performance: What is the maximum reliable transmission rate?
  - Rate: Bit rate,  $R$  bps
  - Reliability: Bit error rate,  $\text{BER}=10^{-k}$
- Cost: What is the cost of alternatives at a given level of performance?
  - Wired vs. wireless?
  - Electronic vs. optical?
  - Standard A vs. standard B?

# Channel Characteristics



## Signal Bandwidth

- In order to transfer data faster, a signal has to vary more quickly.

## Channel Bandwidth

- A channel or medium has an inherent limit on how fast the signals it passes can vary
- *Limits how tightly input pulses can be packed*

## Transmission Impairments

- Signal attenuation
- Signal distortion (due to channel)
- Spurious noise
- Interference from other signals
- *Limits accuracy of measurements on received signal*



# How to Increase Communication Rate?

The maximum reliable transmission rate over an ideal channel with bandwidth  $W$  Hz, with Gaussian distributed noise, and with SNR  $S/N$  is

$$C = W \log_2 ( 1 + S/N ) \text{ bits per second}$$

- Given a channel bandwidth, what methods can be used for improving communication rate?
  - Modulation: a way of converting bits into baseband waveforms with different signal levels. Given a SNR, modulation improves bits per pulse
  - Channel coding: improves SNR for higher-rate modulation
  - DSP: e.g., channel equalizer; improves SNR
  - Power amplifier; improves SNR

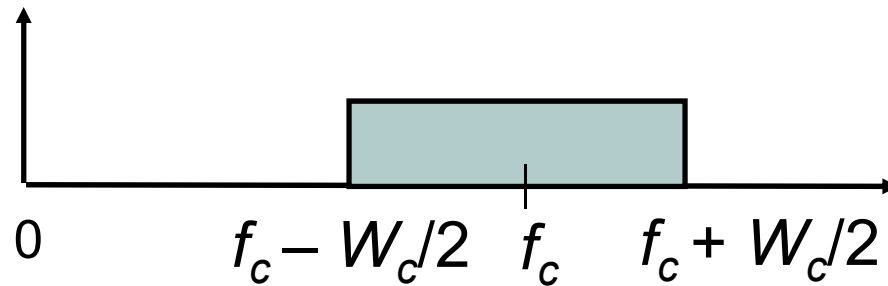


# Modulation: General Procedure

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- Convert bits into symbols
- DAC
- The baseband signal modulates a sinusoid wave (carrier)

# Modulation (on a bandpass channel)



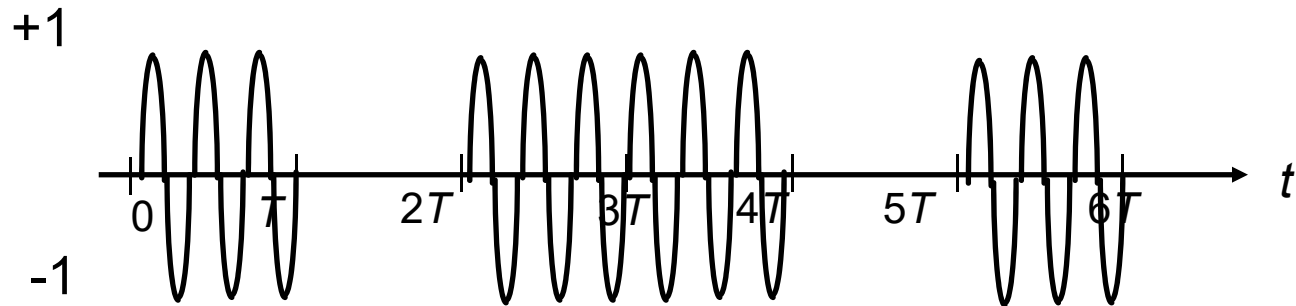
- Bandpass channels pass a range of frequencies around some center frequency  $f_c$ 
  - Radio channels, telephone & DSL modems
- Digital modulators embed information into waveform with frequencies passed by bandpass channel
- Sinusoid of frequency  $f_c$  is centered in middle of bandpass channel
- Modulators embed information into a sinusoid

# Amplitude Modulation and Frequency Modulation

Information

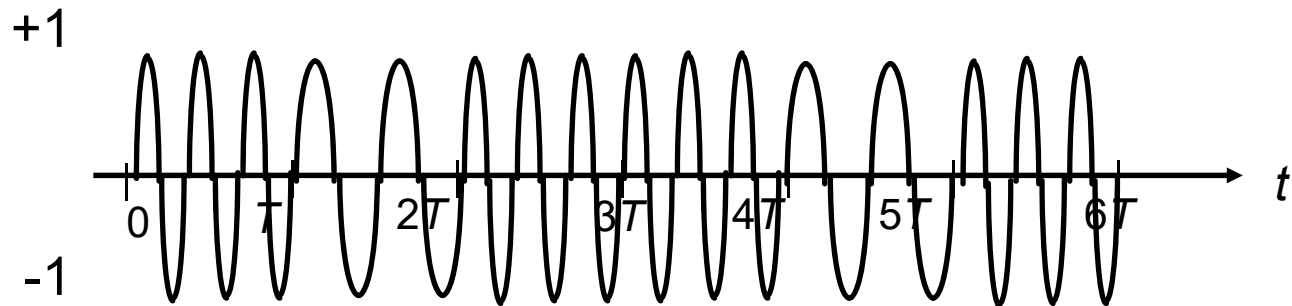
1 0 1 1 0 1

Amplitude  
Shift  
Keying



Map bits into amplitude of sinusoid: “1” send sinusoid; “0” no sinusoid  
Demodulator looks for signal vs. no signal

Frequency  
Shift  
Keying



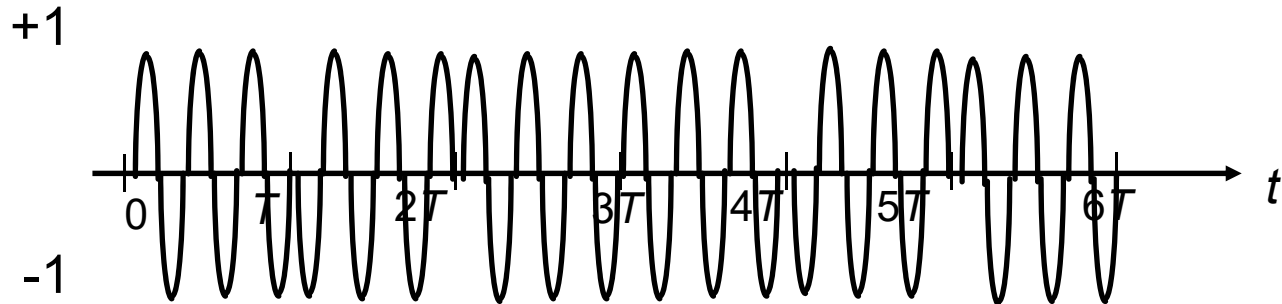
Map bits into frequency: “1” send frequency  $f_c + \delta$  ; “0” send frequency  $f_c - \delta$   
Demodulator looks for power around  $f_c + \delta$  or  $f_c - \delta$

# Phase Modulation

Information

1 0 1 1 0 1

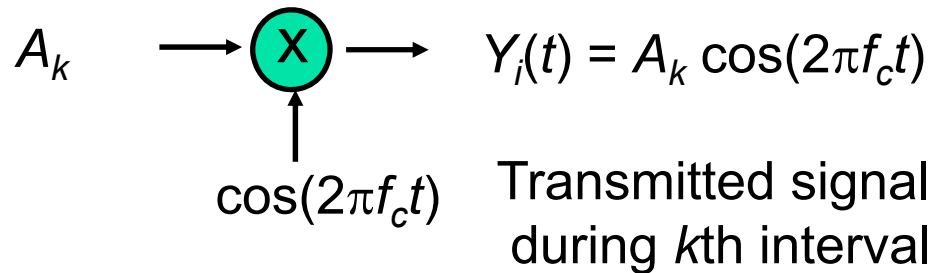
Phase  
Shift  
Keying



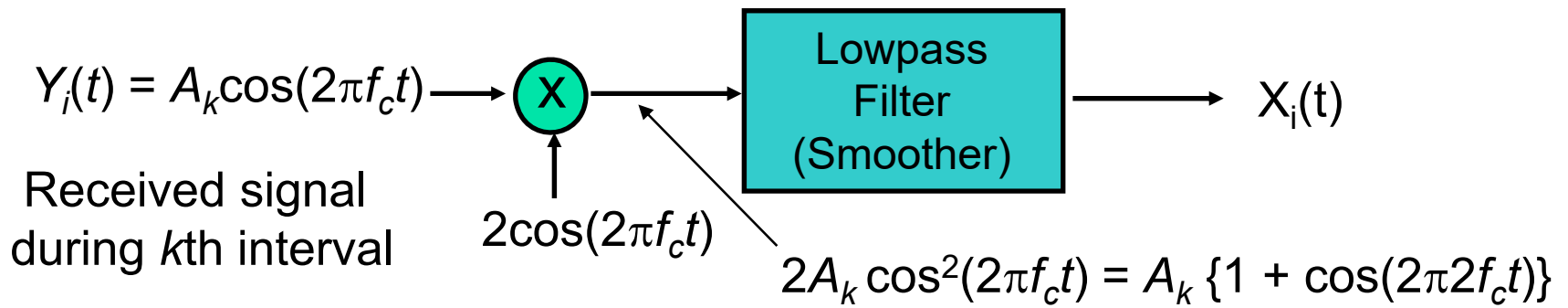
- Map bits into phase of sinusoid:
  - "1" send  $A \cos(2\pi ft)$  , i.e. phase is 0
  - "0" send  $A \cos(2\pi ft + \pi)$  , i.e. phase is  $\pi$
- Equivalent to multiplying  $\cos(2\pi ft)$  by  $+A$  or  $-A$ 
  - "1" send  $A \cos(2\pi ft)$  , i.e. multiply by 1
  - "0" send  $A \cos(2\pi ft + \pi) = -A \cos(2\pi ft)$  , i.e. multiply by -1

# Modulator & Demodulator

Modulate  $\cos(2\pi f_c t)$  by multiplying by  $A_k$  for  $T$  seconds:



Demodulate (recover  $A_k$ ) by multiplying by  $2\cos(2\pi f_c t)$  for  $T$  seconds and lowpass filtering (smoothing):





# Signaling rate and Transmission Bandwidth

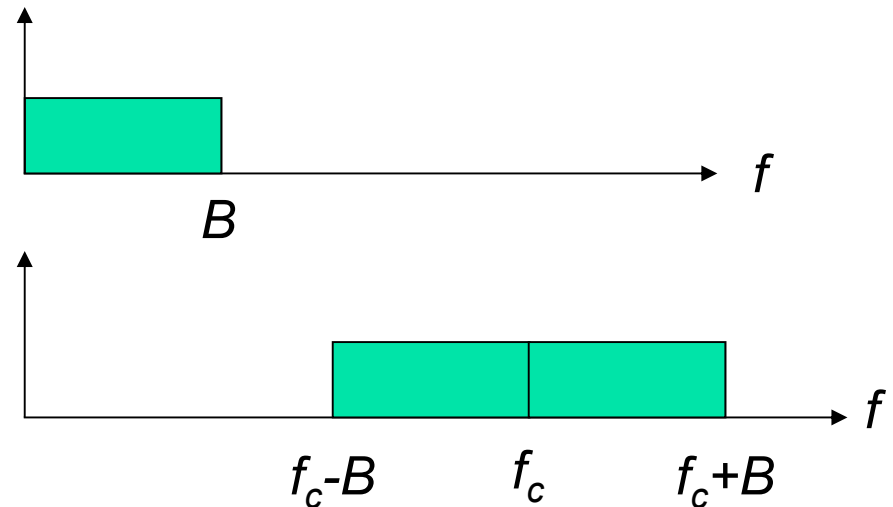
## ■ Fact from modulation theory:

If

Baseband signal  $x(t)$   
with bandwidth  $B$  Hz

then

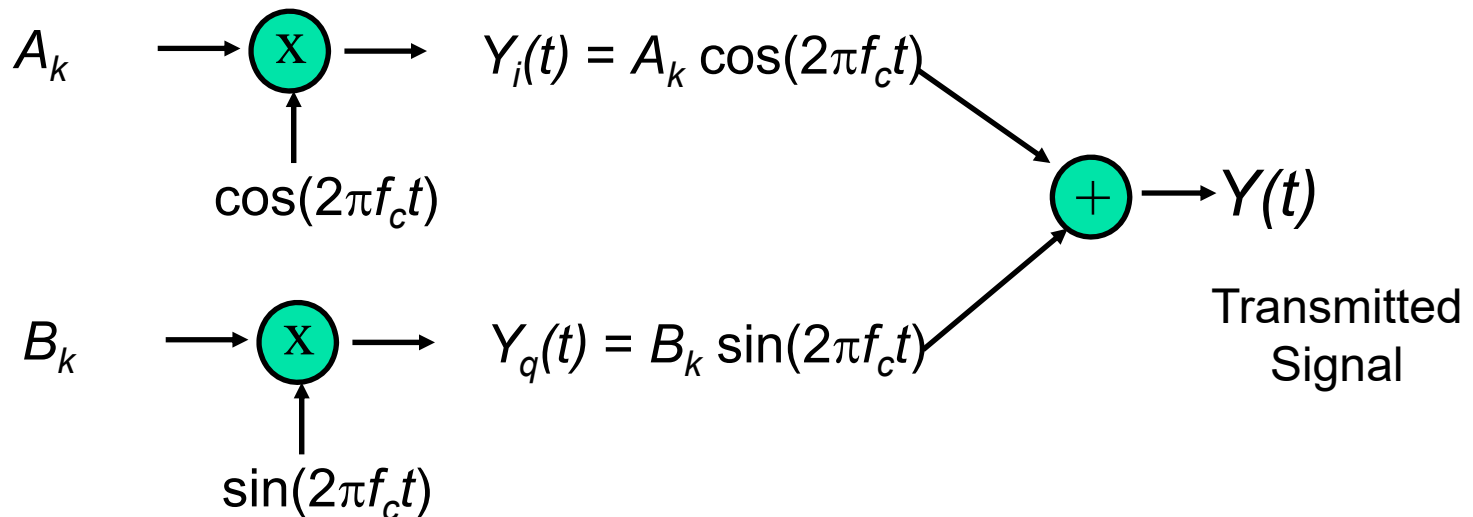
Modulated signal  
 $x(t)\cos(2\pi f_c t)$  has  
bandwidth  $2B$  Hz



- If bandpass channel has bandwidth  $W_c$  Hz,
  - Then baseband channel has  $W_c/2$  Hz available, so
  - modulation system supports  $W_c/2 \times 2 = W_c$  pulses/second
  - That is,  $W_c$  pulses/second per  $W_c$  Hz = 1 pulse/Hz
  - Recall baseband transmission system supports 2 pulses/Hz

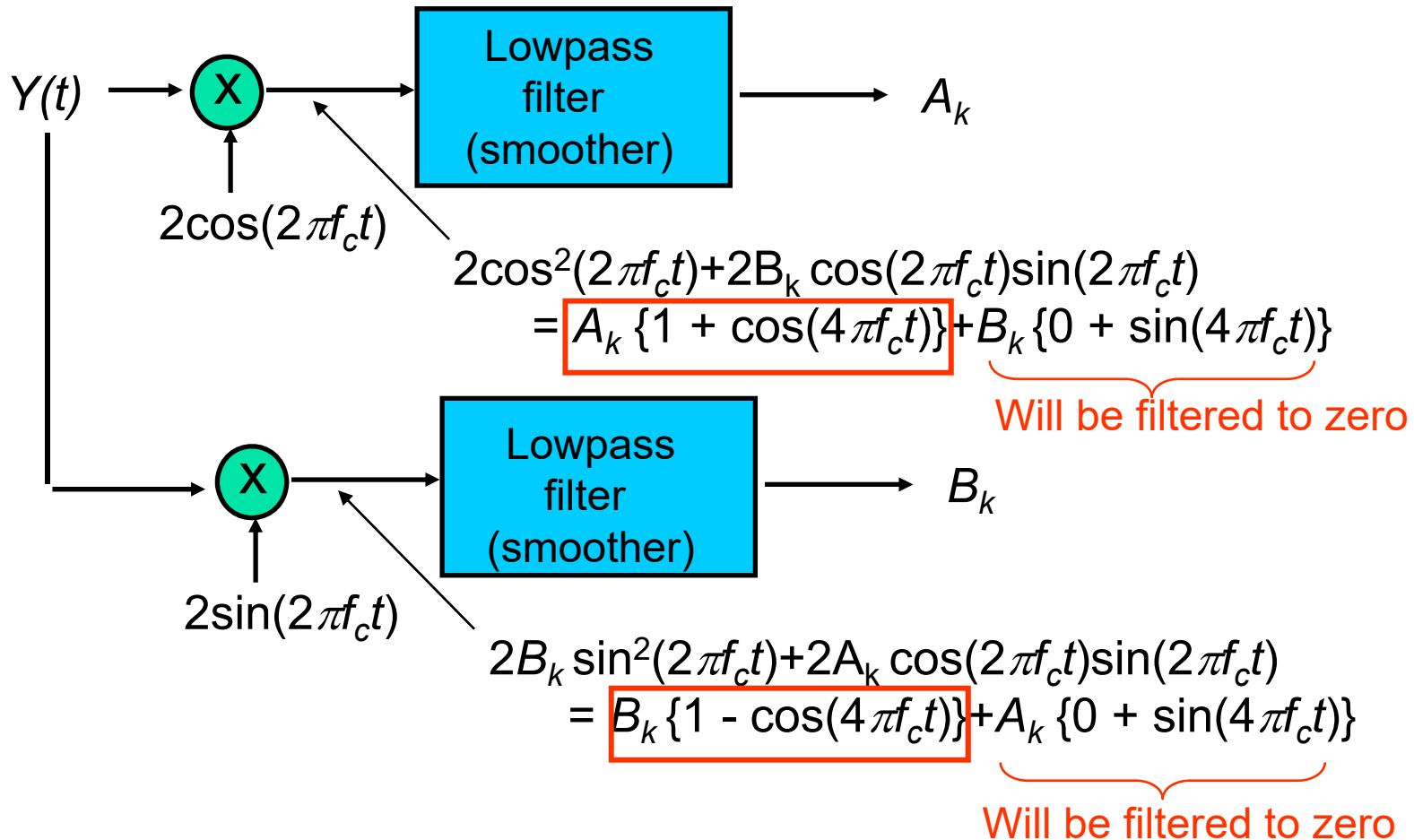
# Quadrature Amplitude Modulation (QAM)

- QAM uses two-dimensional signaling
  - $A_k$  modulates in-phase  $\cos(2\pi f_c t)$
  - $B_k$  modulates quadrature phase  $\cos(2\pi f_c t + \pi/4) = \sin(2\pi f_c t)$
  - Transmit sum of inphase & quadrature phase components



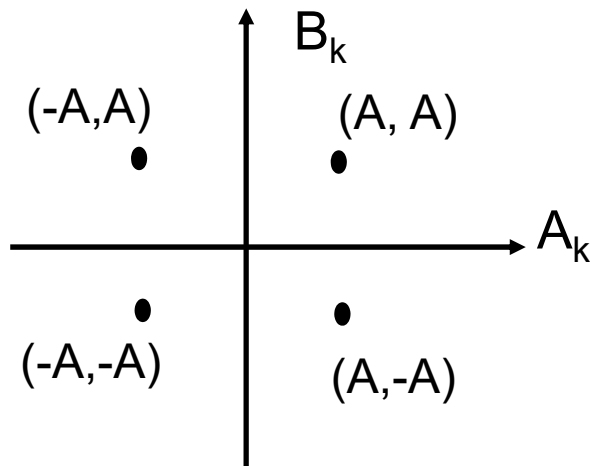
- $Y_i(t)$  and  $Y_q(t)$  both occupy the bandpass channel
- QAM sends 2 pulses/Hz

# QAM Demodulation

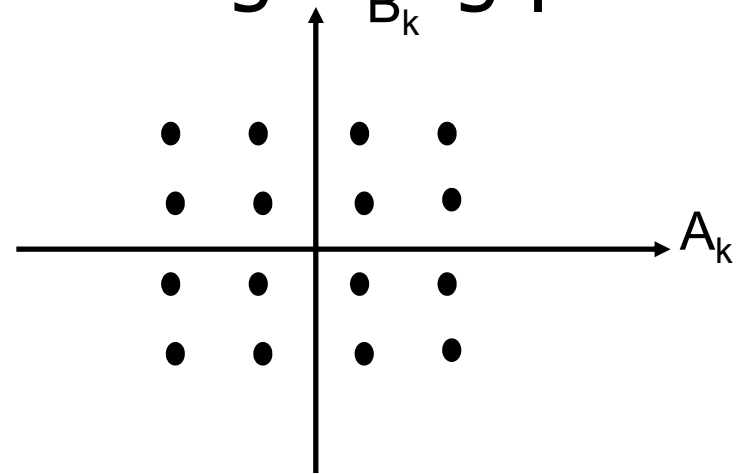


# Signal Constellations

- Each pair  $(A_k, B_k)$  defines a point in the plane
- *Signal constellation* set of signaling points



4 possible points per  $T$  sec.  
2 bits / pulse



16 possible points per  $T$  sec.  
4 bits / pulse



# Telephone Modem Standards

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Telephone Channel for modulation purposes has  
 $W_c = 2400 \text{ Hz} \rightarrow 2400 \text{ pulses per second}$

## Modem Standard V.32bis

- Trellis modulation maps  $m$  bits into one of  $2^{m+1}$  constellation points
- 14,400 bps      Trellis 128      2400x6
- 9600 bps      Trellis 32      2400x4
- 4800 bps      QAM 4      2400x2

## Modem Standard V.34 adjusts pulse rate to channel

- 2400-33600 bps      Trellis 960      2400-3429 pulses/sec



# Error Control

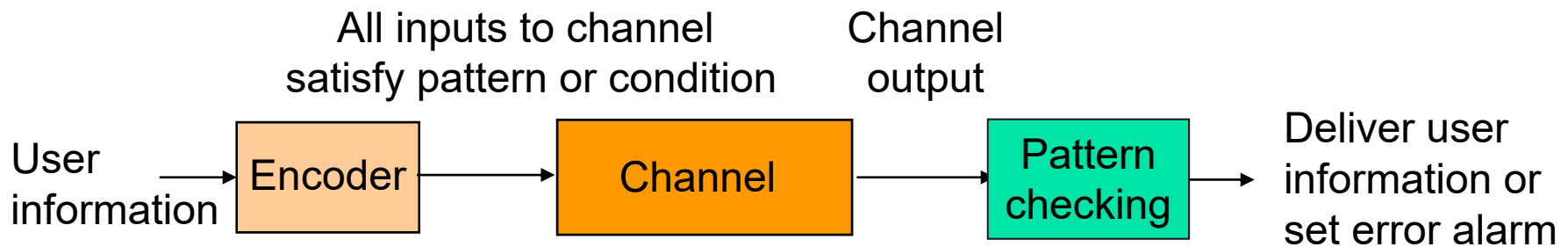
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- Digital transmission systems introduce errors
- Applications require certain reliability level
  - Data applications require error-free transfer
  - Voice & video applications tolerate some errors
- Error control used when transmission system does *not* meet application requirement
- Error control ensures a data stream is transmitted to a certain level of accuracy despite errors
- Two basic approaches:
  - Error ***detection*** & retransmission (ARQ)
  - Forward error ***correction*** (FEC)

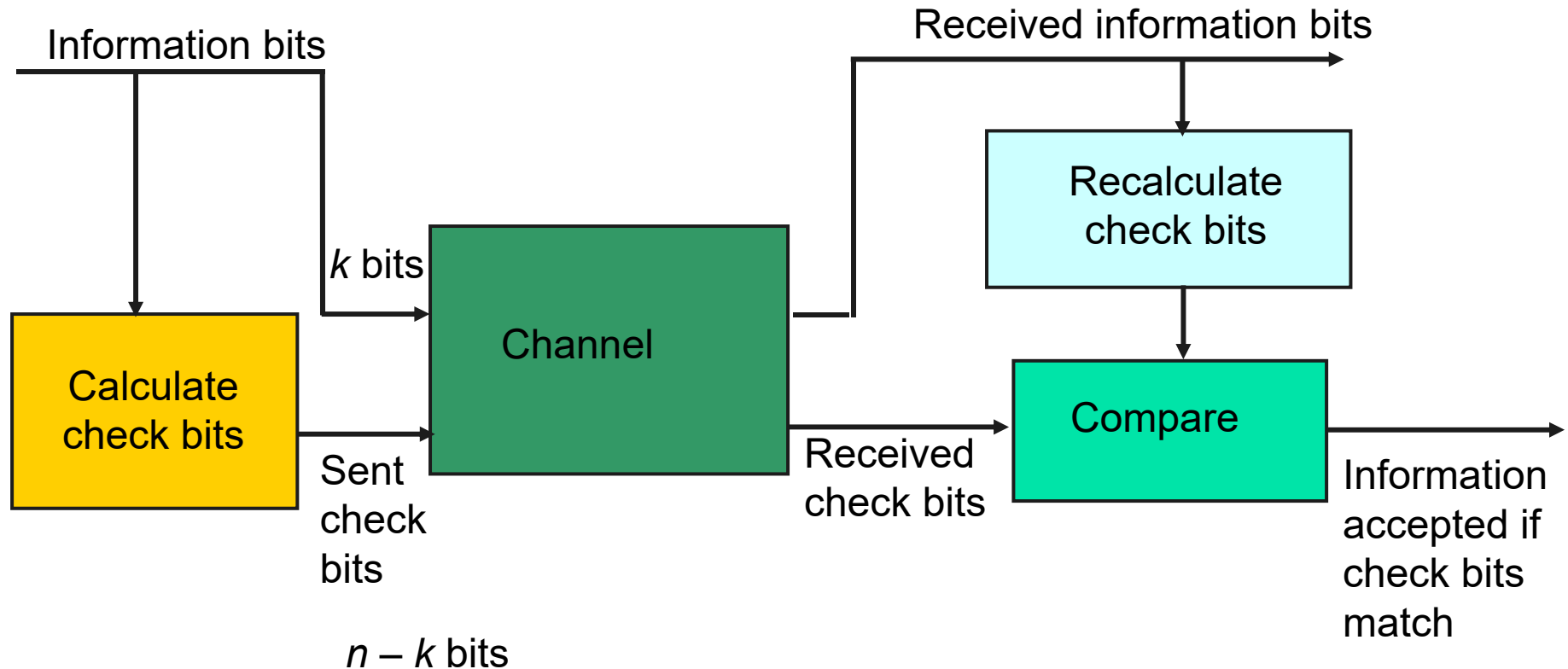
这一章(之后几页介绍的Error Control方法)都是FEC

# Key Concept

- All transmitted data blocks (“codewords”) satisfy a pattern
- If received block doesn’t satisfy pattern, it is in error
- Redundancy: Only a subset of all possible blocks can be codewords
- Blindspot: when channel transforms a codeword into another codeword



# Checksums & Error Detection







# Single Parity Check

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- Append an overall parity check to  $k$  information bits

Info Bits:  $b_1, b_2, b_3, \dots, b_k$

Check Bit:  $b_{k+1} = b_1 + b_2 + b_3 + \dots + b_k \text{ modulo } 2$

Codeword:  $(b_1, b_2, b_3, \dots, b_k, b_{k+1})$

- All codewords have even # of 1s
- Receiver checks to see if # of 1s is even
  - All error patterns that change an odd # of bits are detectable
  - All even-numbered patterns are undetectable
- Parity bit used in ASCII code



# Example of Single Parity Code

- Information (7 bits): (0, 1, 0, 1, 1, 0, 0)
- Parity Bit:  $b_8 = 0 + 1 + 0 + 1 + 1 + 0 + 0 = 1$
- Codeword (8 bits): (0, 1, 0, 1, 1, 0, 0, 1)
  
- If single error in bit 3 : (0, 1, 1, 1, 1, 0, 0, 1)
  - # of 1's = 5, odd
  - Error detected
  
- If errors in bits 3 and 5: (0, 1, 1, 1, 0, 0, 0, 1)
  - # of 1's = 4, even
  - Error not detected



# How good is the single parity check code?

- *Redundancy*: Single parity check code adds 1 redundant bit per  $k$  information bits:  
overhead =  $1/(k + 1)$
- *Coverage*: all error patterns with odd # of errors can be detected
  - An error pattern is a binary  $(k + 1)$ -tuple with 1s where errors occur and 0's elsewhere
  - Of  $2^{k+1}$  binary  $(k + 1)$ -tuples,  $1/2$  are odd, so 50% of error patterns can be detected
- Is it possible to detect more errors if we add more check bits?
- Yes, with the right codes

# Two-Dimensional Parity Check

- More parity bits to improve coverage
- Arrange information as columns
- Add single parity bit to each column
- Add a final “parity” column
- Used in early error control systems

1	0	0	1	0	0	0
0	1	0	0	0	0	1
1	0	0	1	0	0	0
1	1	0	1	1	1	0
1	0	0	1	1	1	1

Last column consists of check bits for each row

Bottom row consists of check bit for each column

# Error-detecting capability

1	0	0	1	0	0	0
0	0	0	0	0	0	1
1	0	0	1	0	0	0
1	1	0	1	1	1	0
<hr/>						
1	0	0	1	1	1	1

One error

↑

1	0	0	1	0	0	0
0	0	0	0	0	0	1
1	0	0	1	0	0	0
1	0	0	1	1	0	0
<hr/>						
1	0	0	1	1	1	1

Two errors

1, 2, or 3 errors  
can always be  
detected; Not all  
patterns  $\geq 4$   
errors can be  
detected

1	0	0	1	0	0	0
0	0	0	0	0	0	1
1	0	0	1	0	0	0
1	0	0	1	1	0	0
<hr/>						
1	0	0	1	1	1	1

Three errors

↑

1	0	0	1	0	0	0
0	0	0	0	0	0	1
1	0	0	1	0	0	0
1	0	0	1	0	0	0
<hr/>						
1	0	0	1	1	1	1

Four errors  
(undetectable)



# Other Error Detection Codes

---

- Many applications require very low error rate
- Need codes that detect the vast majority of errors
- Single parity check codes do not detect enough errors
- Two-dimensional codes require too many check bits
- The following error detecting codes used in practice:
  - Internet Check Sums
  - CRC Polynomial Codes



# Internet Checksum

---

- Several Internet protocols (e.g. IP, TCP, UDP) use check bits to detect errors in the *IP header* (or in the header and data for TCP/UDP)
- A checksum is calculated for header contents and included in a special field.
- Checksum recalculated at every router, so algorithm selected for ease of implementation in software
- Let header consist of  $L$ , 16-bit words,  
 $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_{L-1}$
- The algorithm appends a 16-bit checksum  $\mathbf{b}_L$



# Checksum Calculation

---

The checksum  $\mathbf{b}_L$  is calculated as follows:

- Treating each 16-bit word as an integer, find
$$\mathbf{x} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \dots + \mathbf{b}_{L-1} \text{ modulo } 2^{16}-1$$
- The checksum is then given by:
$$\mathbf{b}_L = -\mathbf{x} \text{ modulo } 2^{16}-1$$

Thus, the headers must satisfy the following ***pattern***:

$$\mathbf{0} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \dots + \mathbf{b}_{L-1} + \mathbf{b}_L \text{ modulo } 2^{16}-1$$

- The checksum calculation is carried out in software using one's complement arithmetic





# Internet Checksum Example

## Use Modulo Arithmetic

- Assume 4-bit words
- Use mod  $2^4-1$  arithmetic
- $\underline{b}_0 = 1100 = 12$
- $\underline{b}_1 = 1010 = 10$
- $\underline{b}_0 + \underline{b}_1 = 12 + 10 = 7 \text{ mod } 15$
- $\underline{b}_2 = -7 = 8 \text{ mod } 15$
- Therefore
- $\underline{b}_2 = 1000$

## Use Binary Arithmetic

- Note  $16 = 1 \text{ mod } 15$
- So:  $10000 = 0001 \text{ mod } 15$
- leading bit wraps around
- $b_0 + b_1 = 1100 + 1010$
- $= 10110$
- $= 10000 + 0110$
- $= 0001 + 0110$
- $= 0111$
- $= 7$
- Take 1s complement
- $b_2 = -0111 = 1000$



# Error Correction

---

- Error correction not only detects errors but also corrects
  - Error detection only schemes need ARQ to correct error
  - Error correction code can directly detect and correct codes → Forward error correction (FEC)
- Examples
  - Hamming codes
  - Convolutional codes
  - Turbo codes
  - LDPC (low-density parity check codes)

# Hamming Codes

- Class of *error-correcting* codes
- Capable of correcting all *single-error* patterns
- For each  $m \geq 2$ , there is a Hamming code of length  $n = 2^m - 1$  with  $n - k = m$  parity check bits

Redundancy

$m$	$n = 2^m - 1$	$k = n - m$	$m/n$
3	7	4	3/7
4	15	11	4/15
5	31	26	5/31
6	63	57	6/63



## $m = 3$ Hamming Code

---

- Information bits are  $b_1, b_2, b_3, b_4$
- Equations for parity checks  $b_5, b_6, b_7$

$$b_5 = b_1 + b_3 + b_4$$

$$b_6 = b_1 + b_2 + b_4$$

$$b_7 = b_2 + b_3 + b_4$$

- There are  $2^4 = 16$  codewords
- $(0,0,0,0,0,0,0)$  is a codeword

# Hamming (7,4) code

Information				Codeword							Weight
$b_1$	$b_2$	$b_3$	$b_4$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$w(\underline{b})$
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	1	1	1	4
0	0	1	0	0	0	1	0	1	0	1	3
0	0	1	1	0	0	1	1	0	1	0	3
0	1	0	0	0	1	0	0	0	1	1	3
0	1	0	1	0	1	0	1	1	0	0	3
0	1	1	0	0	1	1	0	1	1	0	4
0	1	1	1	0	1	1	1	0	0	1	4
1	0	0	0	1	0	0	0	1	1	0	3
1	0	0	1	1	0	0	1	0	0	1	3
1	0	1	0	1	0	1	0	0	1	1	4
1	0	1	1	1	0	1	1	1	0	0	4
1	1	0	0	1	1	0	0	1	0	1	4
1	1	0	1	1	1	0	1	0	1	0	4
1	1	1	0	1	1	1	0	0	0	0	3
1	1	1	1	1	1	1	1	1	1	1	7

# Parity Check Equations

- Rearrange parity check equations:

$$0 = b_5 + b_5 = b_1 + b_3 + b_4 + b_5$$

$$0 = b_6 + b_6 = b_1 + b_2 + b_4 + b_6$$

$$0 = b_7 + b_7 = b_2 + b_3 + b_4 + b_7$$

- In matrix form:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} = \mathbf{H} \underline{b}^t = \underline{0}$$

- All codewords must satisfy these equations
- Note: each nonzero 3-tuple appears once as a column in **check matrix H**

# Error Detection with Hamming Code

$$\underline{s} = \mathbf{H} \underline{e} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Single error detected and can be corrected.

**Error syndrome is shown in H**

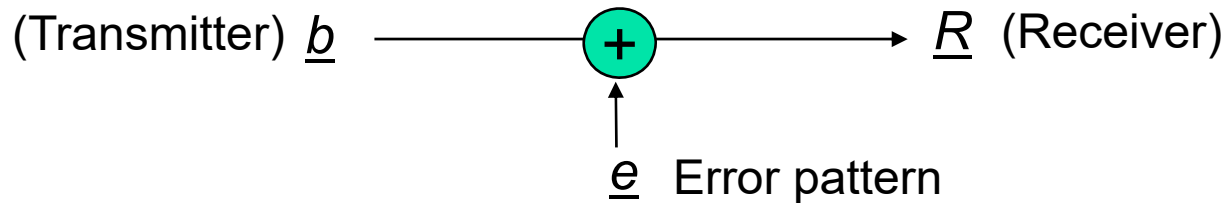
$$\underline{s} = \mathbf{H} \underline{e} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Double error detected

$$\underline{s} = \mathbf{H} \underline{e} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \underline{0}$$

Triple error not detected

# Error-correction using Hamming Codes



- The receiver first calculates the syndrome:
$$\underline{s} = H\underline{R} = H(\underline{b} + \underline{e}) = H\underline{b} + H\underline{e} = H\underline{e}$$
- If  $\underline{s} = \underline{0}$ , then the receiver accepts  $\underline{R}$  as the transmitted codeword
- If  $\underline{s}$  is nonzero, then an error is detected
  - Hamming decoder *assumes* a single error has occurred
  - Each single-bit error pattern has a unique syndrome
  - The receiver matches the syndrome to a single-bit error pattern and corrects the appropriate bit
- Minimum distance of Hamming codes





# General Hamming Codes

---

- For  $m \geq 2$ , the Hamming code is obtained through the check matrix  $H$ :
  - Each nonzero  $m$ -tuple appears once as a column of  $H$
  - The resulting code corrects all single errors
- For each value of  $m$ , there is a polynomial code with  $g(x)$  of degree  $m$  that is equivalent to a Hamming code and corrects all single errors
  - For  $m = 3$ ,  $g(x) = x^3 + x + 1$



# Polynomial Codes

---

- Polynomials instead of vectors for codewords
- Polynomial arithmetic instead of check sums
- Implemented using shift-register circuits
- Also called *cyclic redundancy check (CRC)* codes
- Most data communications standards use polynomial codes for error detection
- Polynomial codes also basis for powerful error-correction methods



# Binary Polynomial Arithmetic

- Binary vectors map to polynomials

$$(i_{k-1}, i_{k-2}, \dots, i_2, i_1, i_0) \rightarrow i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \dots + i_2x^2 + i_1x + i_0$$

Addition:

$$\begin{aligned}(x^7 + x^6 + 1) + (x^6 + x^5) &= x^7 + x^6 + x^6 + x^5 + 1 \\ &= x^7 + (1+1)x^6 + x^5 + 1 \\ &= x^7 + x^5 + 1 \quad \text{since } 1+1=0 \text{ mod } 2\end{aligned}$$

Multiplication:

$$\begin{aligned}(x + 1)(x^2 + x + 1) &= x(x^2 + x + 1) + 1(x^2 + x + 1) \\ &= x^3 + x^2 + x + x^2 + x + 1 \\ &= x^3 + 1\end{aligned}$$

# Binary Polynomial Division

## ■ Division with Decimal Numbers

$$\begin{array}{r}
 34 \leftarrow \text{quotient} \\
 35 \overline{) 1222} \leftarrow \text{dividend} \\
 \underline{105} \phantom{00} \\
 172 \\
 \underline{140} \\
 32 \leftarrow \text{remainder}
 \end{array}$$

divisor

dividend = quotient x divisor + remainder

$$1222 = 34 \times 35 + 32$$

## ■ Polynomial Division

$$\begin{array}{r}
 x^3 + x^2 + x \phantom{+ 0} = q(x) \text{ quotient} \\
 x^3 + x + 1 \overline{) x^6 + x^5} \leftarrow \text{dividend} \\
 \underline{x^6 + \phantom{x^5} x^4 + x^3} \\
 x^5 + x^4 + x^3 \\
 \underline{x^5 + \phantom{x^4} x^3 + x^2} \\
 x^4 + \phantom{x^3} x^2 \\
 \underline{x^4 + \phantom{x^3} x^2 + x} \\
 x = r(x) \text{ remainder}
 \end{array}$$

divisor

*Note: Degree of  $r(x)$  is less than degree of divisor*

# Polynomial Coding

- Code has binary *generating polynomial* of degree

$$n-k \quad g(x) = x^{n-k} + g_{n-k-1}x^{n-k-1} + \dots + g_2x^2 + g_1x + 1$$

- $k$  *information bits* define polynomial of degree  $k-1$

$$i(x) = i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \dots + i_2x^2 + i_1x + i_0$$

- Find *remainder polynomial* of at most degree  $n-k-1$

$$\begin{array}{r} q(x) \\ g(x) \overline{) x^{n-k} i(x)} \\ \underline{\phantom{q(x)} r(x)} \end{array} \quad x^{n-k}i(x) = q(x)g(x) + r(x)$$

- Define the *codeword polynomial* of degree  $n-1$

$$\underbrace{b(x)}_{n \text{ bits}} = \underbrace{x^{n-k}i(x)}_{k \text{ bits}} + \underbrace{r(x)}_{n-k \text{ bits}}$$

# Polynomial example: $k = 4, n-k = 3$

Generator polynomial:  $g(x) = x^3 + x + 1$

Information:  $(1, 1, 0, 0)$

$$i(x) = x^3 + x^2$$

Encoding:  $x^3 i(x) = x^6 + x^5$

$$\begin{array}{r}
 x^3 + x + 1 \overline{) x^6 + x^5} \\
 \underline{x^6 + \phantom{x^5} x^4 + x^3} \phantom{+ x^2 + x} \\
 x^5 + x^4 + x^3 \\
 \underline{x^5 + \phantom{x^4} x^3 + x^2} \\
 x^4 + \phantom{x^3} x^2 \\
 \underline{x^4 + \phantom{x^3} x^2 + x} \\
 x
 \end{array}$$

$$\begin{array}{r}
 1110 \\
 1011 \overline{) 1100000} \\
 \underline{1011} \\
 1110 \\
 \underline{1011} \\
 1010 \\
 \underline{1011} \\
 010
 \end{array}$$

Transmitted codeword:

$$b(x) = x^6 + x^5 + x$$

$$\underline{b} = (1, 1, 0, 0, 0, 1, 0)$$



## The *Pattern* in Polynomial Coding (Decoding)

- All codewords satisfy the following **pattern**:

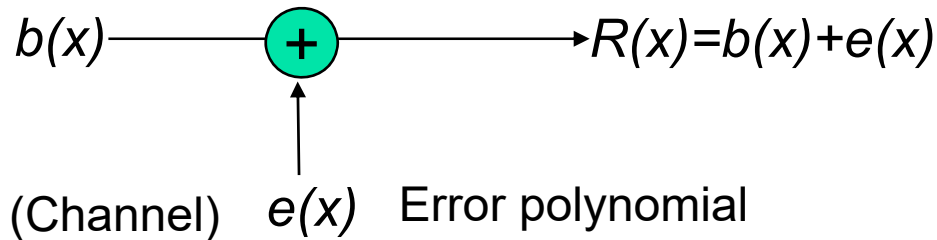
$$b(x) = x^{n-k}i(x) + r(x) = q(x)g(x) + r(x) + r(x) = q(x)g(x)$$

- All codewords are a multiple of  $g(x)$ !
- Receiver should divide received n-tuple by  $g(x)$  and check if remainder is zero
- If remainder is nonzero, then received n-tuple is not a codeword

# Undetectable error patterns

(Transmitter)

(Receiver)



- $e(x)$  has 1s in error locations & 0s elsewhere
- Receiver divides the received polynomial  $R(x)$  by  $g(x)$
- Blindspot: If  $e(x)$  is a multiple of  $g(x)$ , that is,  $e(x)$  is a nonzero codeword, then
$$R(x) = b(x) + e(x) = q(x)g(x) + q'(x)g(x)$$
- *The set of undetectable error polynomials is the set of nonzero code polynomials*
- *Choose the generator polynomial so that selected error patterns can be detected.*





# Designing good polynomial codes (1)

- Select generator polynomial so that likely error patterns are not multiples of  $g(x)$
- *Detecting Single Errors*
  - $e(x) = x^i$  for error in one location
  - If  $g(x)$  has more than 1 term, it cannot divide  $x^i$
- *Detecting Double Errors*
  - $e(x) = x^i + x^j = x^i(x^{j-i} + 1)$  where  $j > i$
  - If  $g(x)$  has more than 1 term, it cannot divide  $x^i$
  - If  $g(x)$  is a *primitive* polynomial, it cannot divide  $x^m + 1$  for all  $m < 2^{n-k} - 1$  (Need to keep codeword length  $n$  less than  $2^{n-k} - 1$ )
  - Primitive polynomials can be found by consulting coding theory books



## Designing good polynomial codes (2)

### ■ *Detecting Odd Numbers of Errors*

- Suppose all codeword polynomials have an even # of 1s, then all odd numbers of errors can be detected
- As well,  $b(x)$  evaluated at  $x = 1$  is zero because  $b(x)$  has an even number of 1s
- This implies  $x + 1$  must be a factor of all  $b(x)$
- Pick  $g(x) = (x + 1) p(x)$  where  $p(x)$  is primitive

### ■ *Bursty Errors*

- All bursts of length  $n-k$  or less can be detected

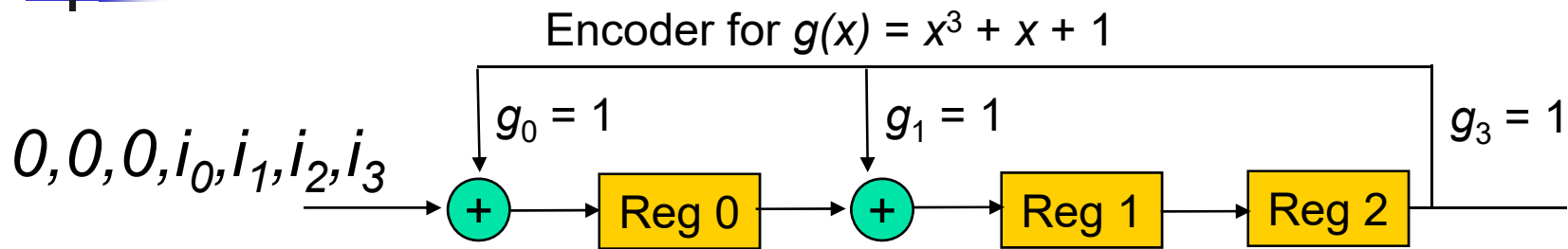


# Shift-Register Implementation of Polynomial Codes

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1. Accept information bits  $i_{k-1}, i_{k-2}, \dots, i_2, i_1, i_0$
2. Append  $n - k$  zeros to information bits
3. Feed sequence to shift-register circuit that performs polynomial division
4. After  $n$  shifts, the shift register contains the remainder

# Division Circuit



Clock	Input	Reg 0	Reg 1	Reg 2
0	-	0	0	0
1	$1 = i_3$	1	0	0
2	$1 = i_2$	1	1	0
3	$0 = i_1$	0	1	1
4	$0 = i_0$	1	1	1
5	0	1	0	1
6	0	1	0	0
7	0	<b>0</b>	<b>1</b>	<b>0</b>

**Check bits:**

$$\implies r_0 = 0 \implies r(x) = x$$

$$r_1 = 1$$

$$r_2 = 0$$



# Standard Generator Polynomials

---

CRC = cyclic redundancy check

- **CRC-8:**

$$= x^8 + x^2 + x + 1$$

ATM

- **CRC-16:**

$$= x^{16} + x^{15} + x^2 + 1$$

$$= (x + 1)(x^{15} + x + 1)$$

Bisync

- **CCITT-16:**

$$= x^{16} + x^{12} + x^5 + 1$$

HDLC, XMODEM, V.41

- **CCITT-32:**

$$= x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$$

IEEE 802, DoD, V.42

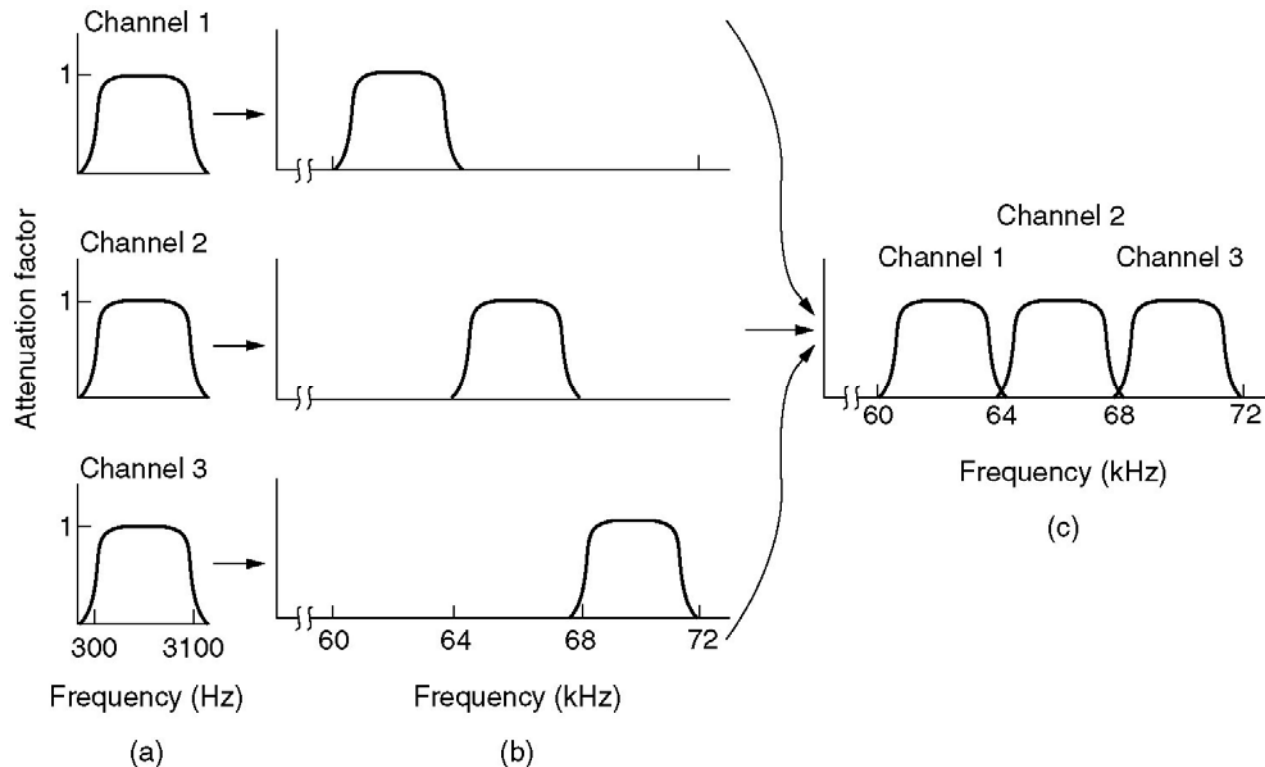


# Multiplexing

---

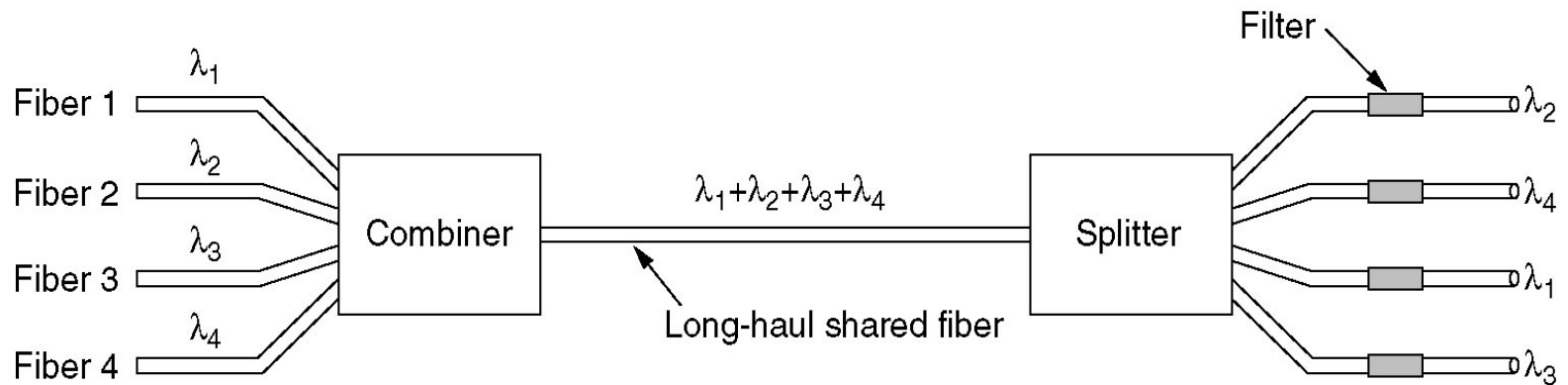
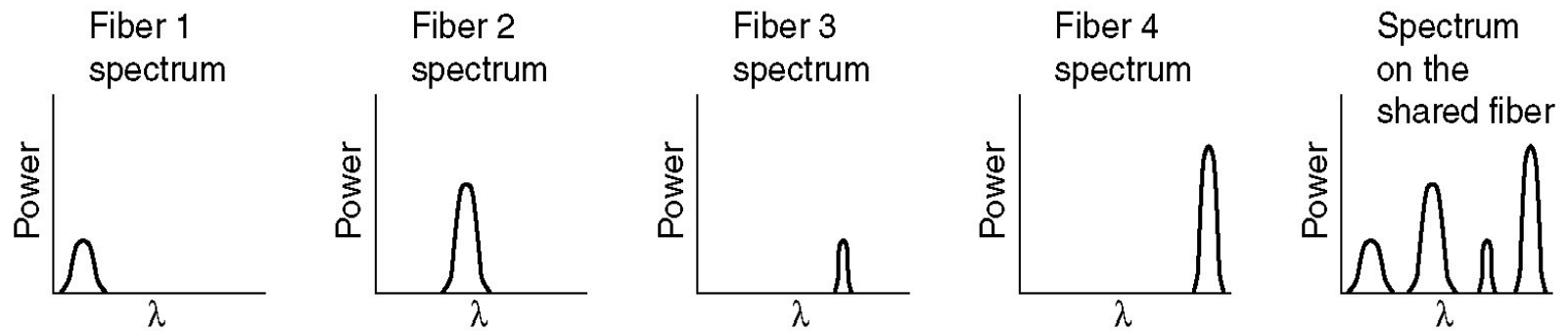
- Goal
  - Improve efficiency by filling the pipe (transmission medium)
- Approaches
  - Frequency division
  - Time division
  - Code division
  - Wavelength division

# Frequency Division Multiplexing



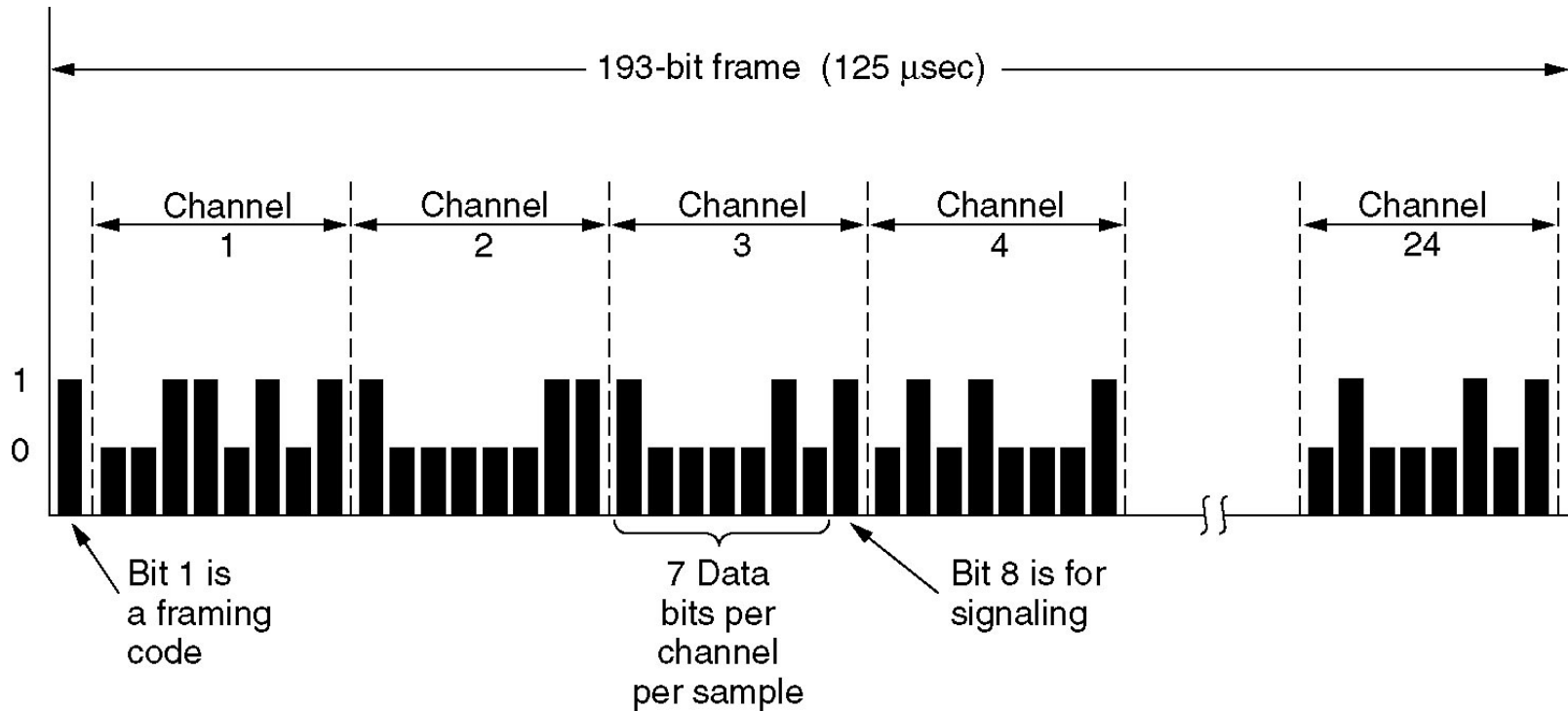
- (a) The original bandwidths.**
- (b) The bandwidths raised in frequency.**
- (b) The multiplexed channel.**

# Wavelength Division Multiplexing



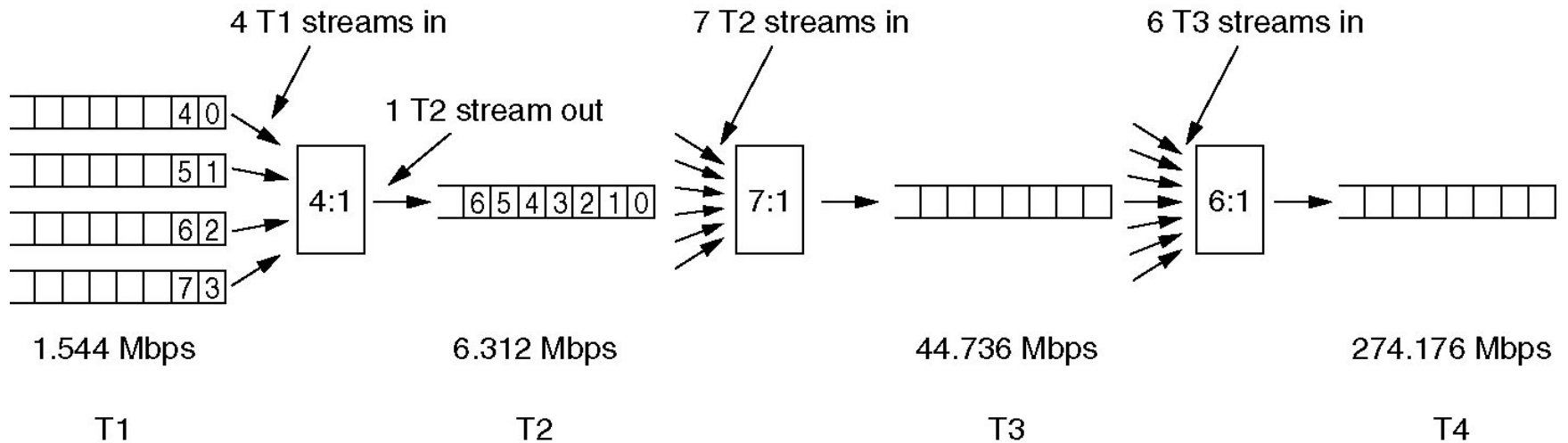


# Time Division Multiplexing (1)



The T1 carrier (1.544 Mbps).

# Time Division Multiplexing (2)



Multiplexing T1 streams into higher carriers.

# Code Division Multiplexing

■ (a) Binary chip sequences for four stations

A: 0 0 0 1 1 0 1 1  
B: 0 0 1 0 1 1 1 0  
C: 0 1 0 1 1 1 0 0  
D: 0 1 0 0 0 0 1 0

(a)

A: (-1 -1 -1 +1 +1 -1 +1 +1)  
B: (-1 -1 +1 -1 +1 +1 +1 -1)  
C: (-1 +1 -1 +1 +1 +1 -1 -1)  
D: (-1 +1 -1 -1 -1 -1 +1 -1)

(b)

■ (b) Bipolar chip sequences

■ (c) Six examples of transmissions

■ (d) Recovery of station C's signal

Six examples:

-- 1 --	<b>C</b>	$S_1 = (-1 +1 -1 +1 +1 +1 -1 -1)$
- 1 1 -	<b>B + <math>\overline{C}</math></b>	$S_2 = (-2 \ 0 \ 0 \ 0 +2 +2 \ 0 -2)$
1 0 --	<b>A + <math>\overline{B}</math></b>	$S_3 = ( \ 0 \ 0 -2 +2 \ 0 -2 \ 0 +2)$
1 0 1 -	<b>A + B + C</b>	$S_4 = (-1 +1 -3 +3 +1 -1 -1 +1)$
1 1 1 1	<b>A + B + C + D</b>	$S_5 = (-4 \ 0 -2 \ 0 +2 \ 0 +2 -2)$
1 1 0 1	<b>A + B + <math>\overline{C}</math> + D</b>	$S_6 = (-2 -2 \ 0 -2 \ 0 -2 +4 \ 0)$

(c)

$S_1 \bullet C = (1 +1 +1 +1 +1 +1 +1 +1)/8 = 1$   
 $S_2 \bullet C = (2 +0 +0 +0 +2 +2 +0 +2)/8 = 1$   
 $S_3 \bullet C = (0 +0 +2 +2 +0 -2 +0 -2)/8 = 0$   
 $S_4 \bullet C = (1 +1 +3 +3 +1 -1 +1 -1)/8 = 1$   
 $S_5 \bullet C = (4 +0 +2 +0 +2 +0 -2 +2)/8 = 1$   
 $S_6 \bullet C = (2 -2 +0 -2 +0 -2 -4 +0)/8 = -1$

(d)



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# Transmission Media



# Wired or Wireless Media

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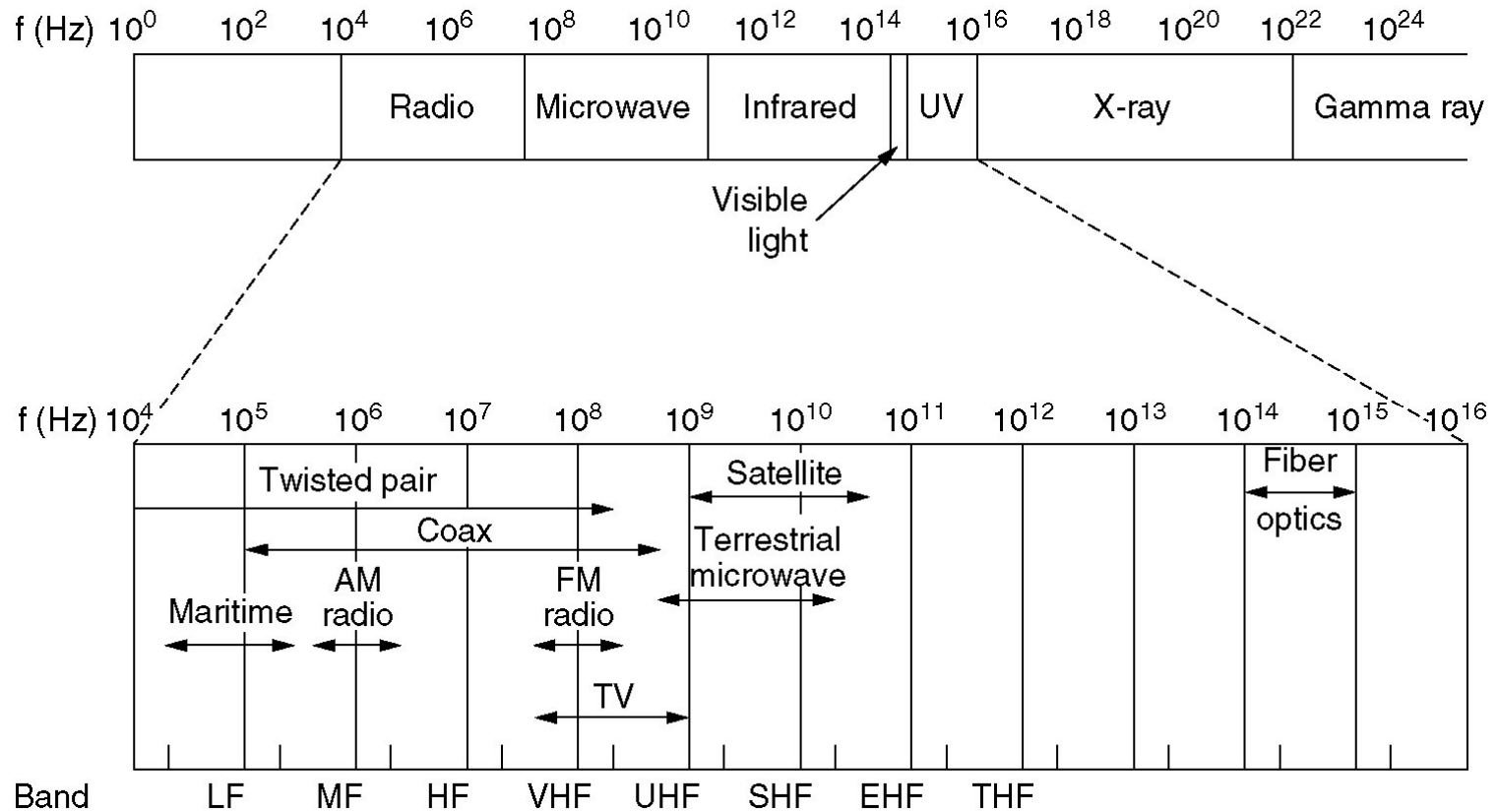
## Wired Media

- Signal energy contained & guided within medium
- Spectrum can be re-used in separate media (wires or cables), more scalable
- Extremely high bandwidth
- Complex infrastructure: ducts, conduits, poles, right-of-way

## Wireless Media

- Signal energy propagates in space, limited directionality
- Interference, so spectrum regulated
- Limited bandwidth
- Simple infrastructure: antennas & transmitters
- No physical connection between network & user
- Users can move

# The Electromagnetic Spectrum



- Magnetic Media
- Twisted Pair
- Coaxial Cable
- Fiber Optics

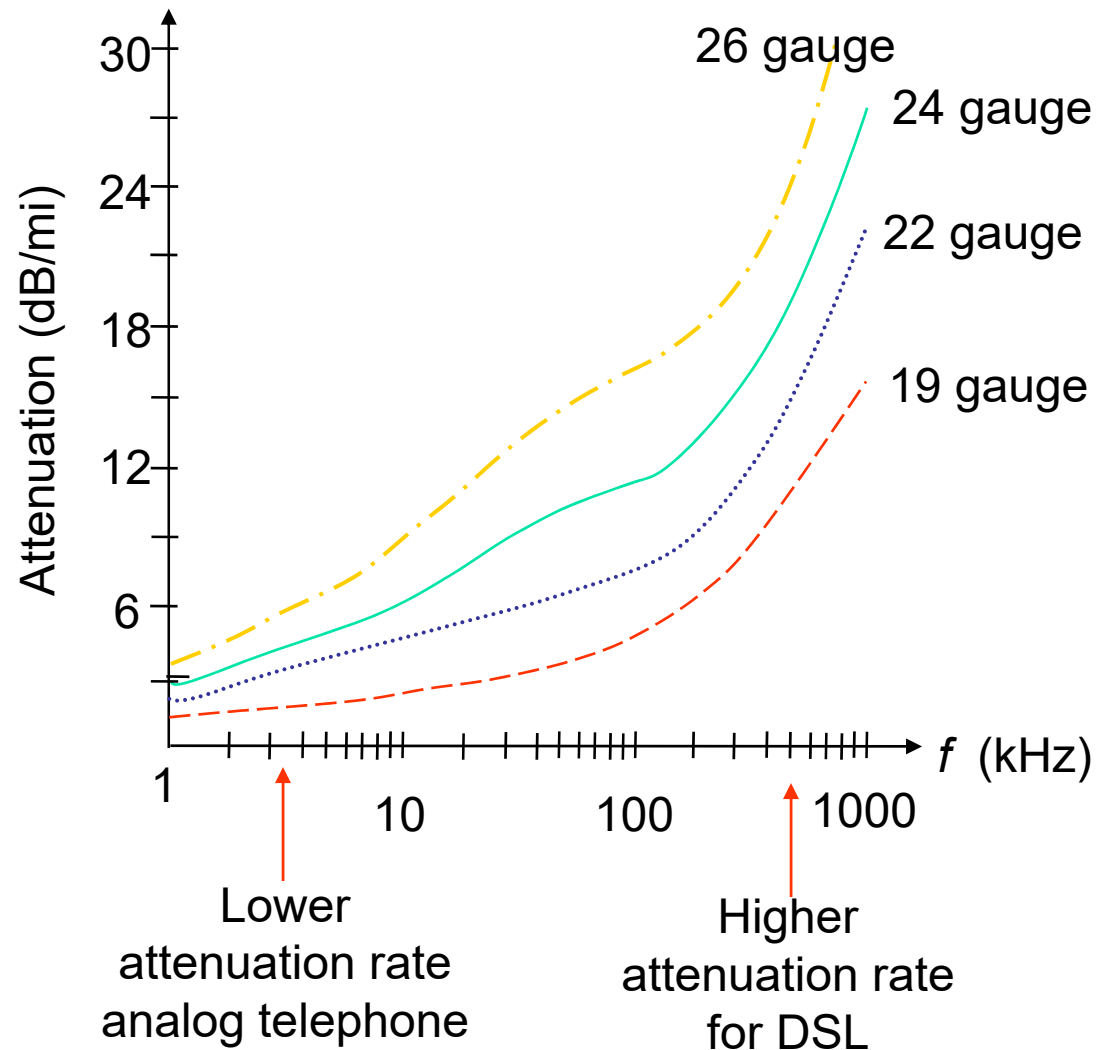
# Twisted Pair



## Twisted pair

- Two insulated copper wires arranged in a regular spiral pattern to minimize interference
- Various thicknesses, e.g. 0.016 inch (24 gauge)
- Low cost
- Telephone subscriber loop from customer to CO
- Old trunk plant connecting telephone COs
- Intra-building telephone from wiring closet to desktop
- In old installations, loading coils added to improve quality in 3 kHz band, but more attenuation at higher frequencies

5/27/2020







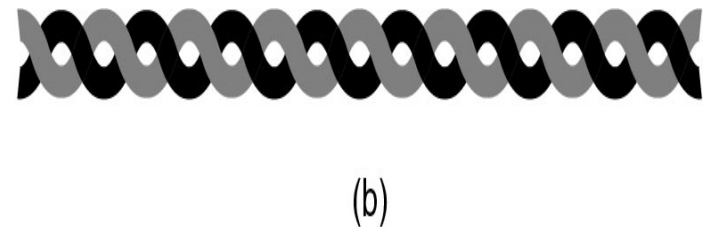
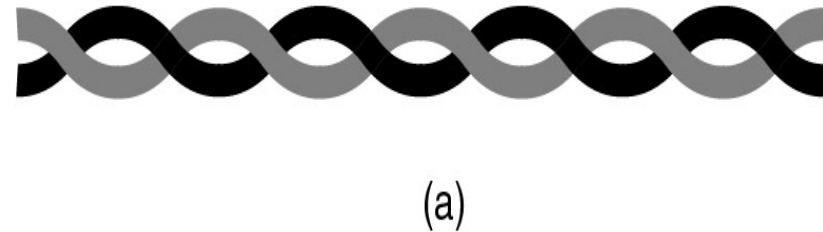
# Twisted Pair Bit Rates

Standard	Data Rate	Distance
T-1	1.544 Mbps	18,000 feet, 5.5 km
DS2	6.312 Mbps	12,000 feet, 3.7 km
1/4 STS-1	12.960 Mbps	4500 feet, 1.4 km
1/2 STS-1	25.920 Mbps	3000 feet, 0.9 km
STS-1	51.840 Mbps	1000 feet, 300 m

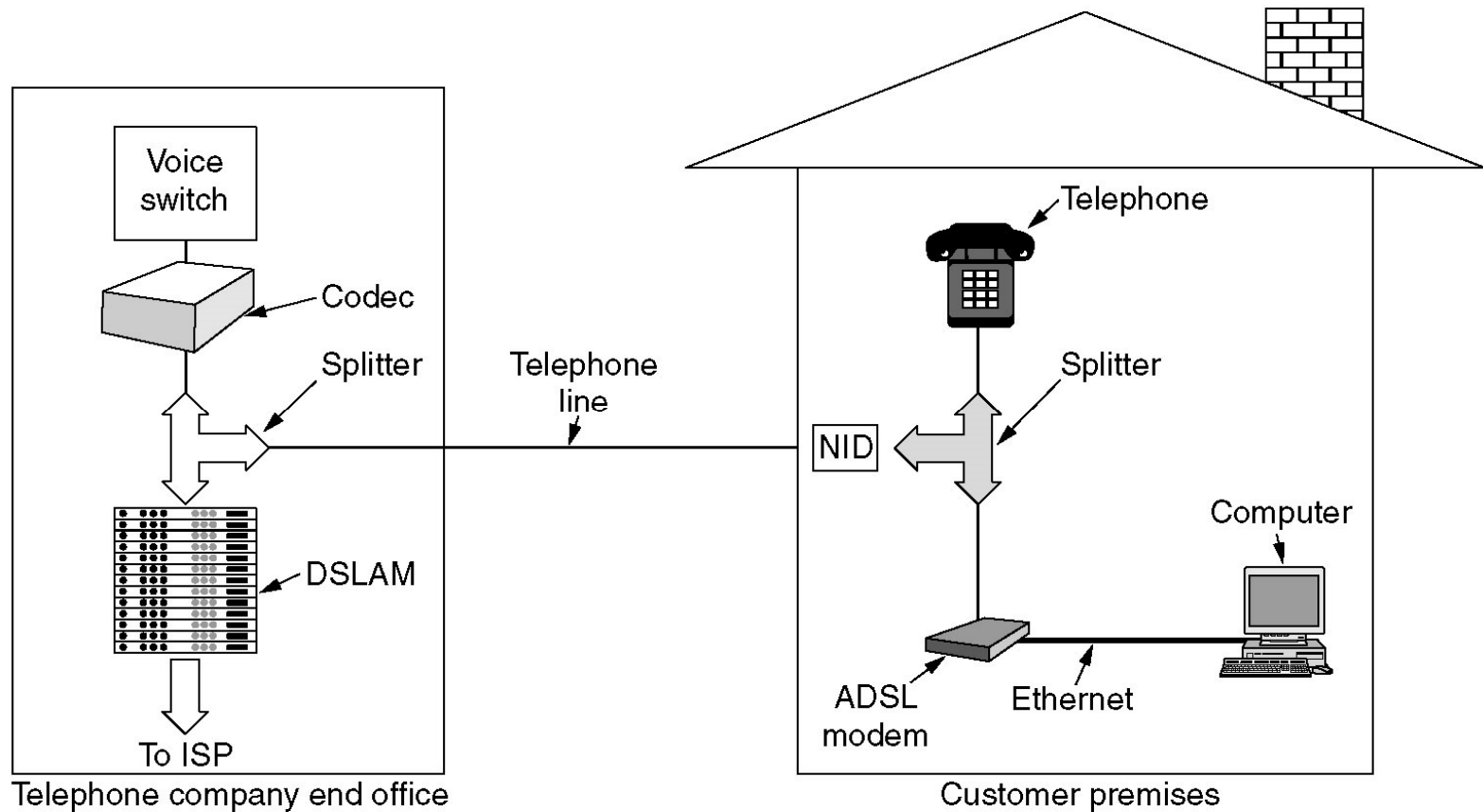
- Twisted pairs can provide high bit rates at short distances
- Asymmetric Digital Subscriber Loop (ADSL)
  - High-speed Internet Access
  - Lower 3 kHz for voice
  - Upper band for data
- Much higher rates possible at shorter distances
  - Strategy for telephone companies is to bring fiber close to home & then twisted pair
  - Higher-speed access + video

# Cat3, Cat5, and UTP

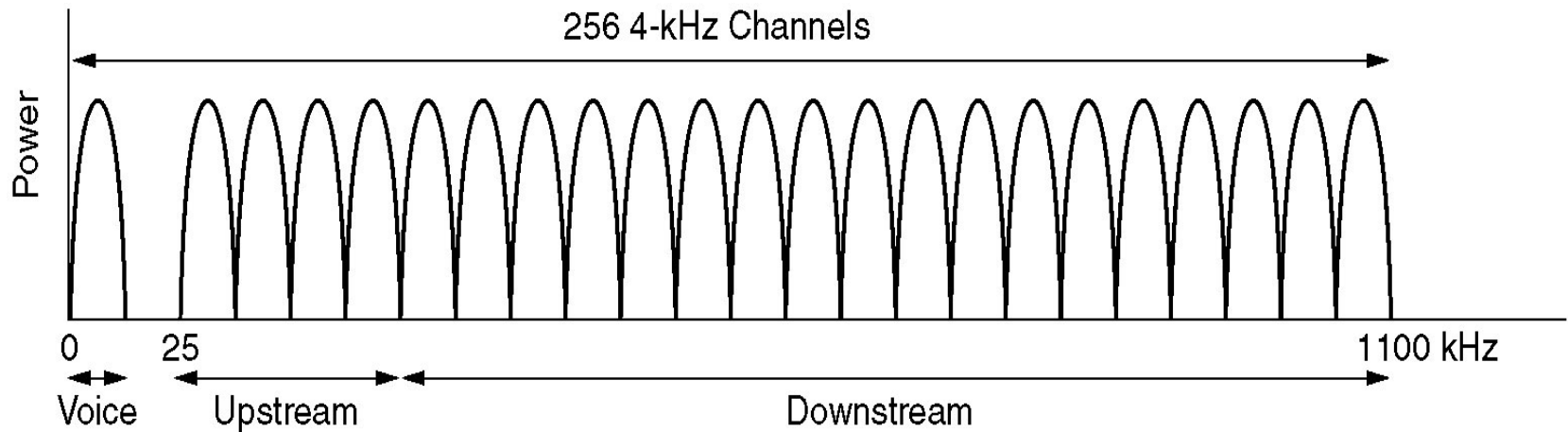
- Category 3 unshielded twisted pair (UTP): ordinary telephone wires
- Category 5 UTP: tighter twisting to improve signal quality
- Shielded twisted pair (STP): to minimize interference; costly
- 10BASE-T Ethernet
  - 10 Mbps, Baseband, Twisted pair
  - Two Cat3 pairs
  - Manchester coding, 100 meters
- 100BASE-T4 *Fast* Ethernet
  - 100 Mbps, Baseband, Twisted pair
  - Four Cat3 pairs
  - Three pairs for one direction at-a-time
  - 100/3 Mbps per pair;
  - 3B6T line code, 100 meters
- Cat5 & STP provide other options



# Digital Subscriber Line (1)

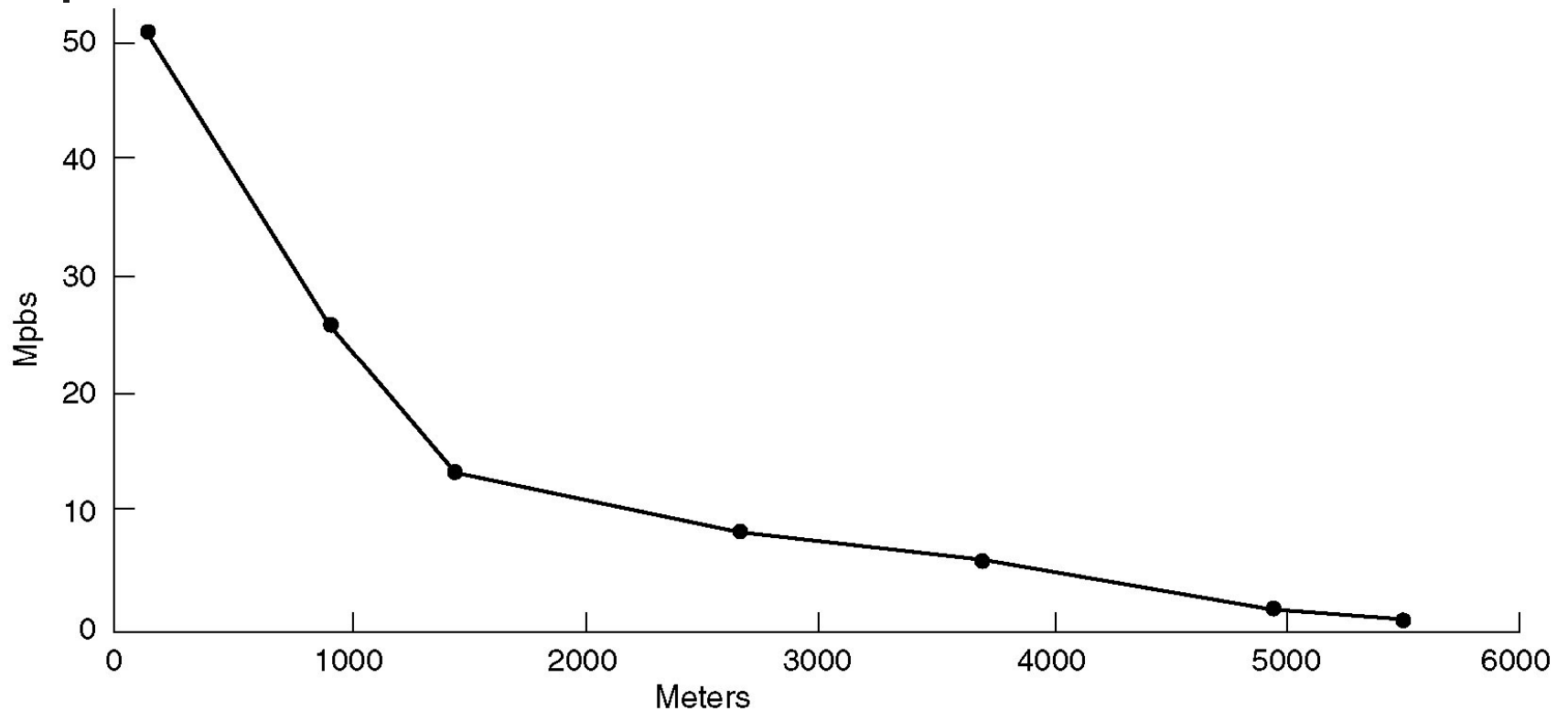


## DSL (2)



**Operation of ADSL using discrete multitone modulation.**

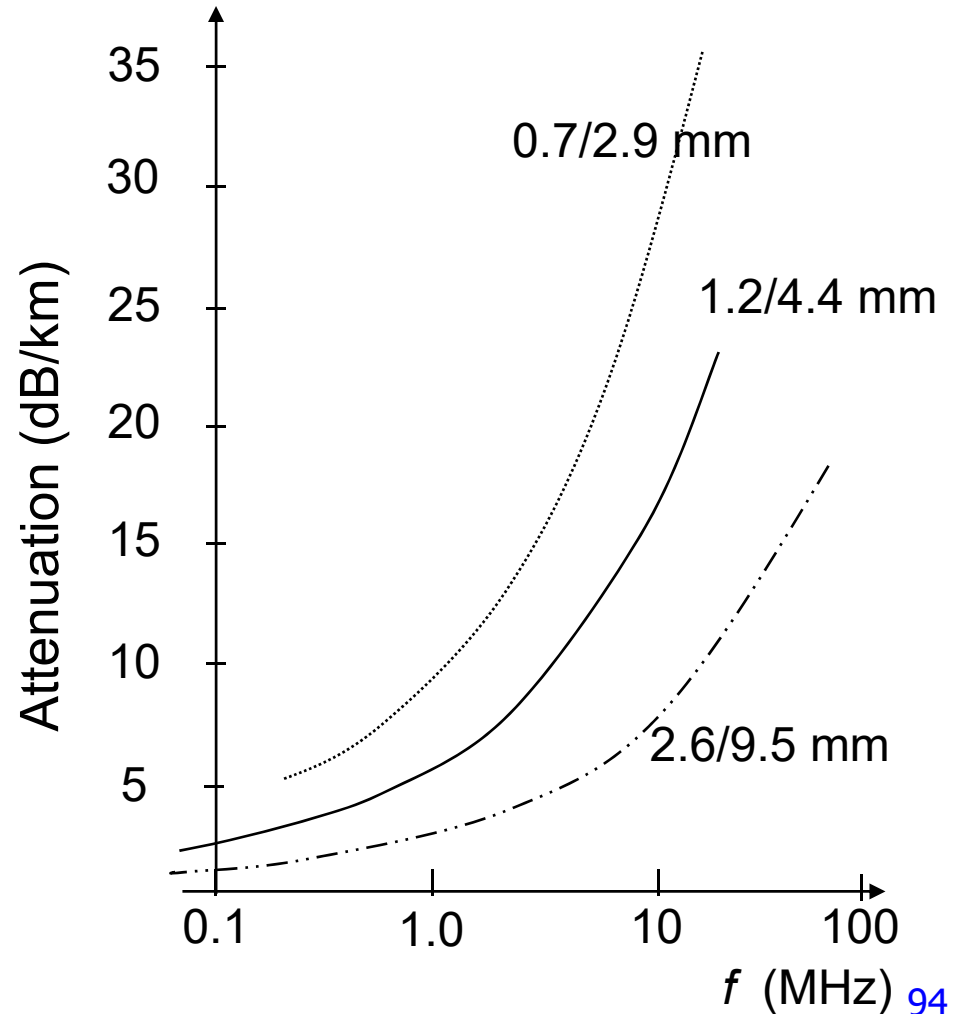
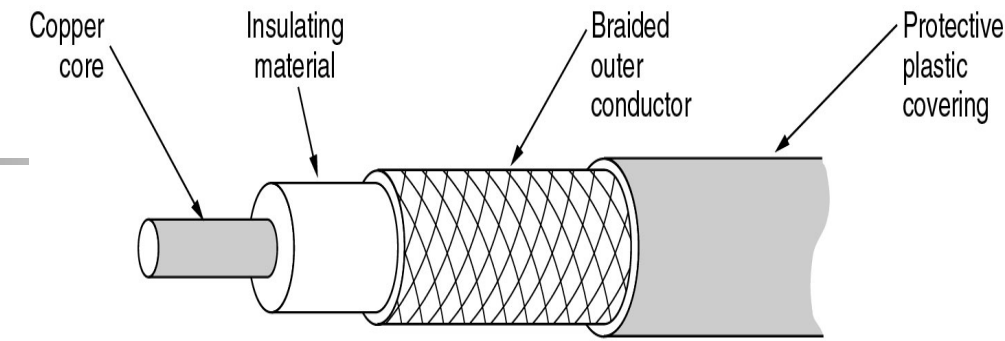
## DSL (3)



Bandwidth versus distanced over category 3 UTP for DSL.

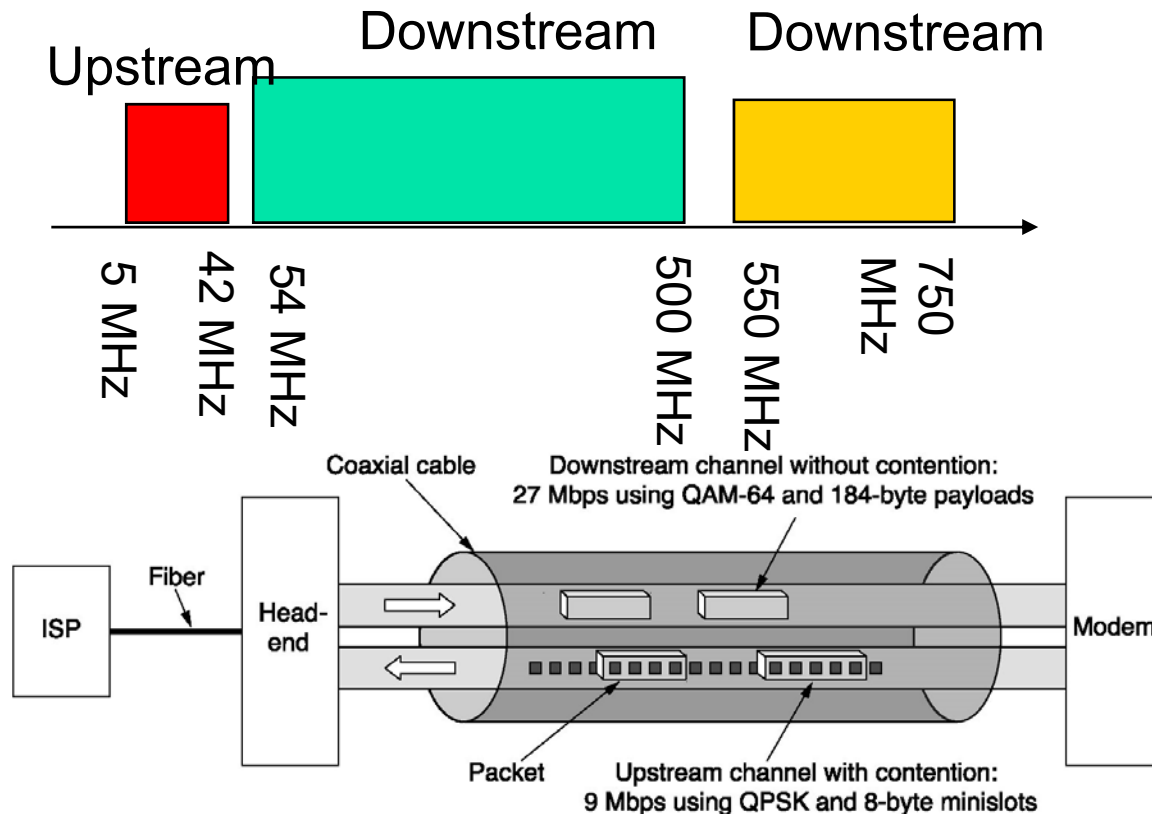
# Coaxial Cable

- Cylindrical braided outer conductor surrounds insulated inner wire conductor
- High interference immunity
- Higher bandwidth than twisted pair
- Hundreds of MHz
- Cable TV distribution
- Long distance telephone transmission
- Original Ethernet LAN medium

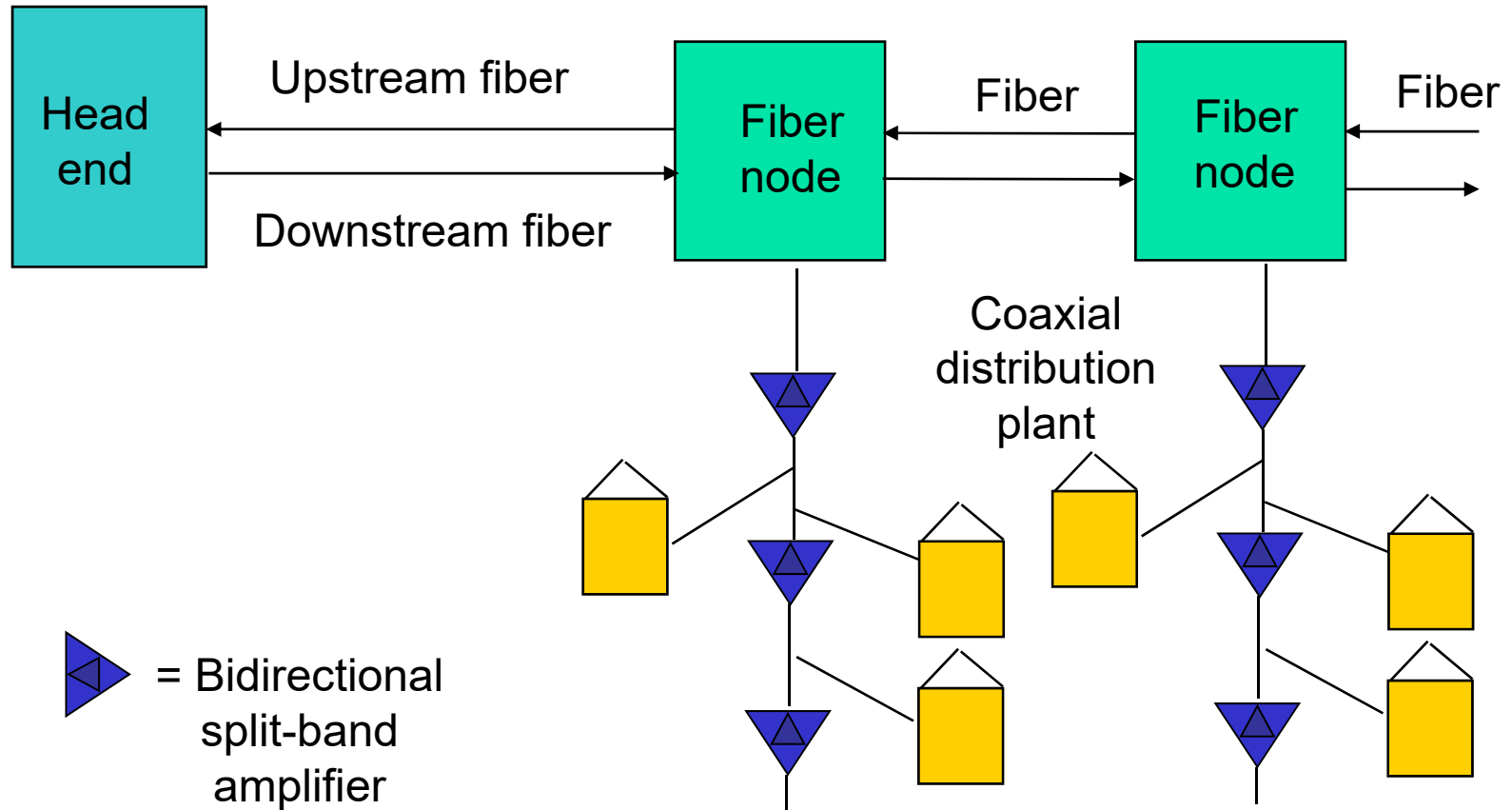


# Cable Modem & TV Spectrum

- Cable TV network originally unidirectional
- Cable plant needs upgrade to bidirectional
- 1 analog TV channel is 6 MHz, can support very high data rates
- Cable Modem: *shared* upstream & downstream
  - 5-42 MHz upstream into network; 2 MHz channels; 500 kbps to 4 Mbps
  - >550 MHz downstream from network; 6 MHz channels; 36 Mbps

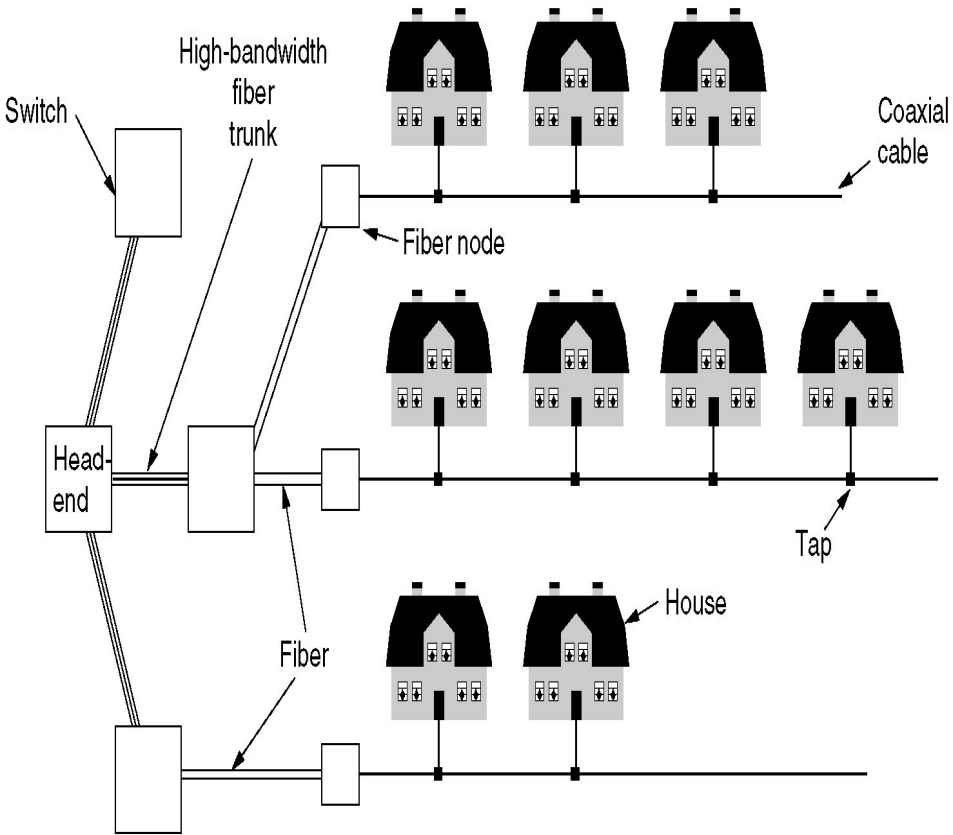


# Cable Network Topology

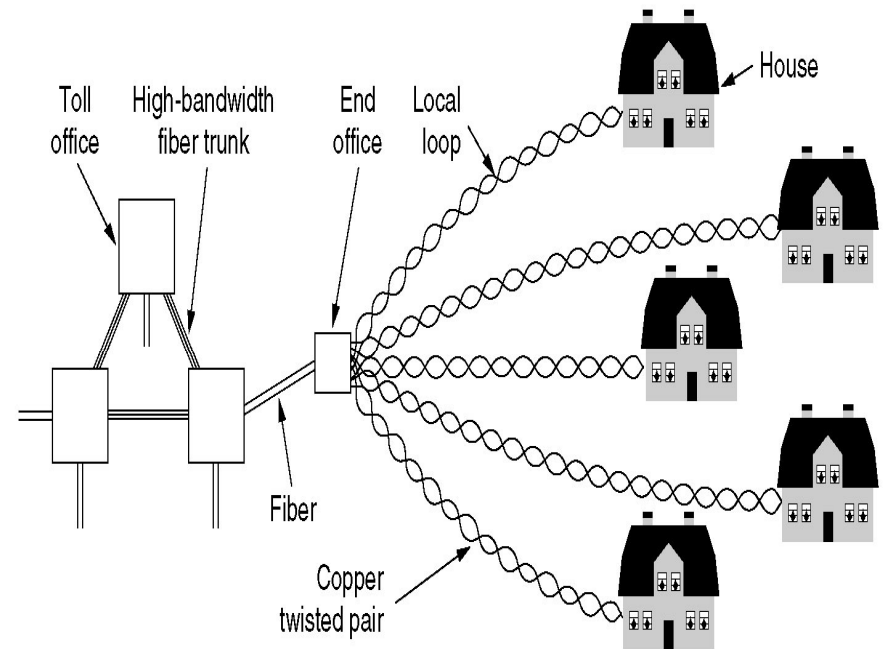




# Internet over Cable



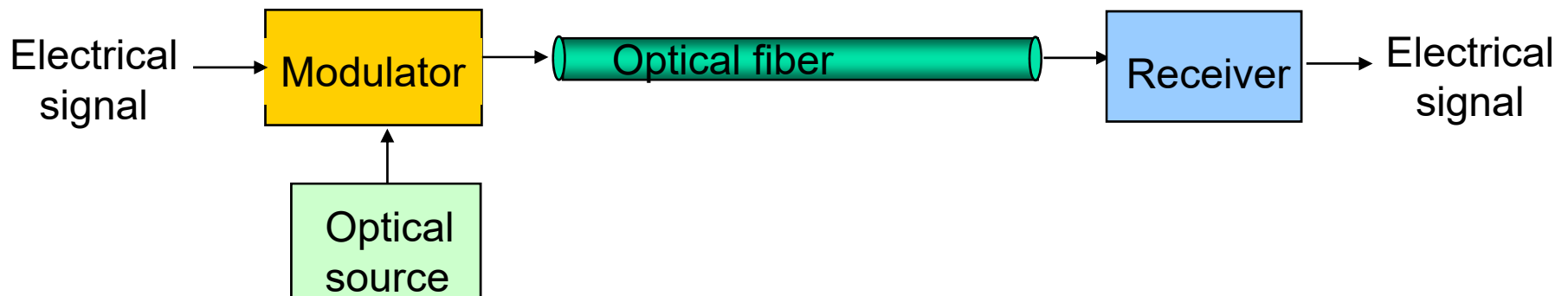
(a)



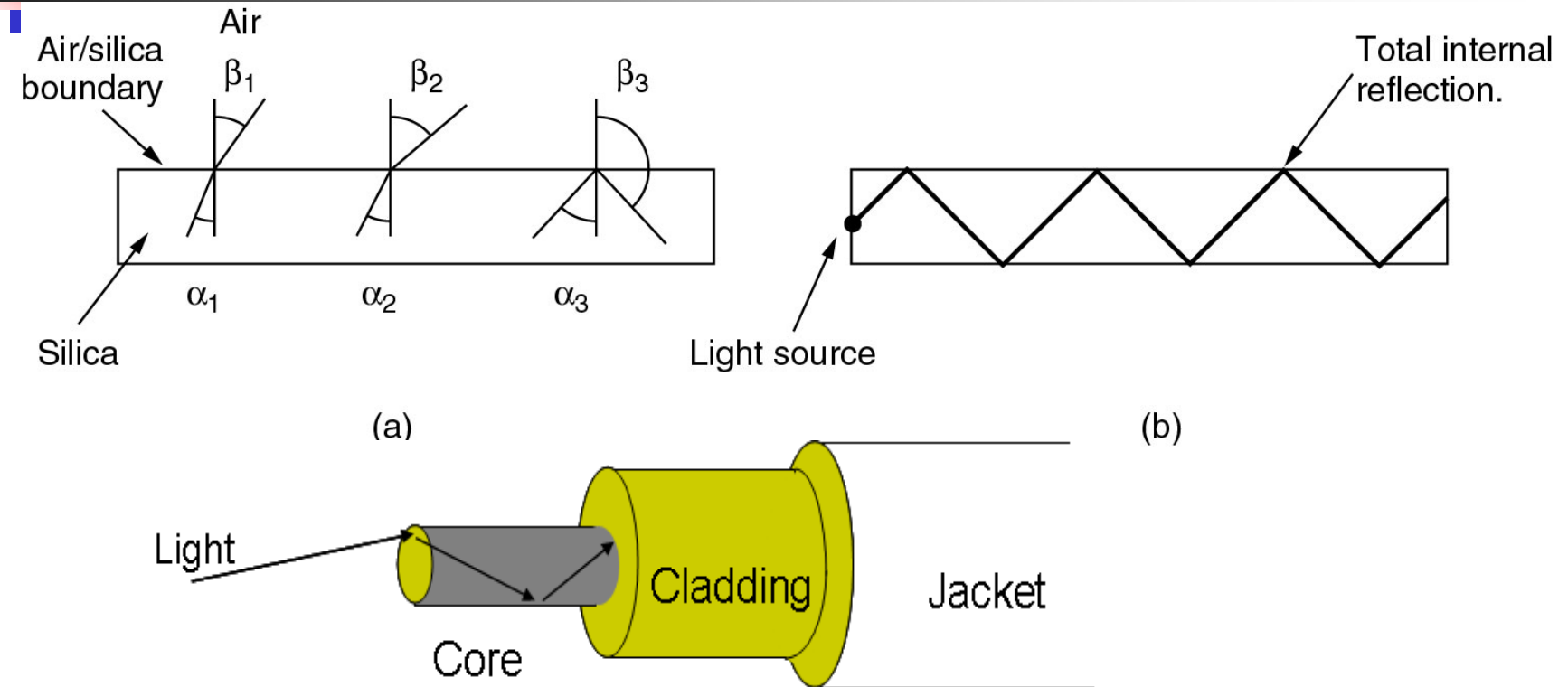
(b)

# Optical Fiber

- Light sources (lasers, LEDs) generate pulses of light that are transmitted on optical fiber
  - Very long distances ( $>1000$  km)
  - Very high speeds ( $>40$  Gbps/wavelength)
  - Nearly error-free (BER of  $10^{-15}$ )
- Profound influence on network architecture
  - Dominates long distance transmission
  - Distance less of a cost factor in communications
  - Plentiful bandwidth for new services



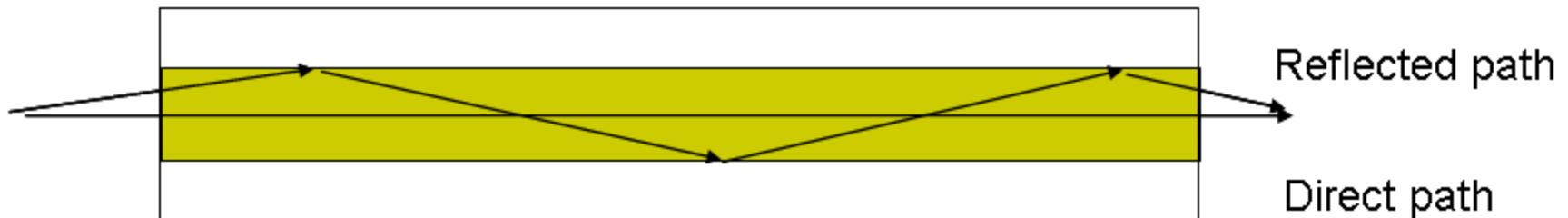
# Transmission in Optical Fiber



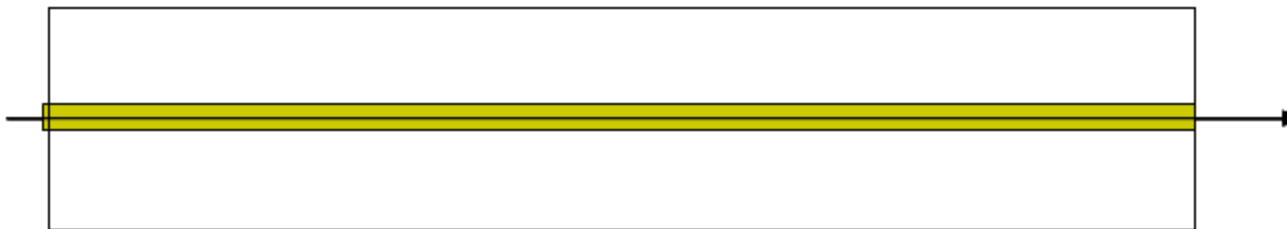
- Very fine glass cylindrical core surrounded by concentric layer of glass (cladding)
- Core has higher index of refraction than cladding
- Light rays incident at less than critical angle  $\theta_c$  is completely reflected back into the core

# Multimode & Single-mode Fiber

Multimode fiber: multiple rays follow different paths

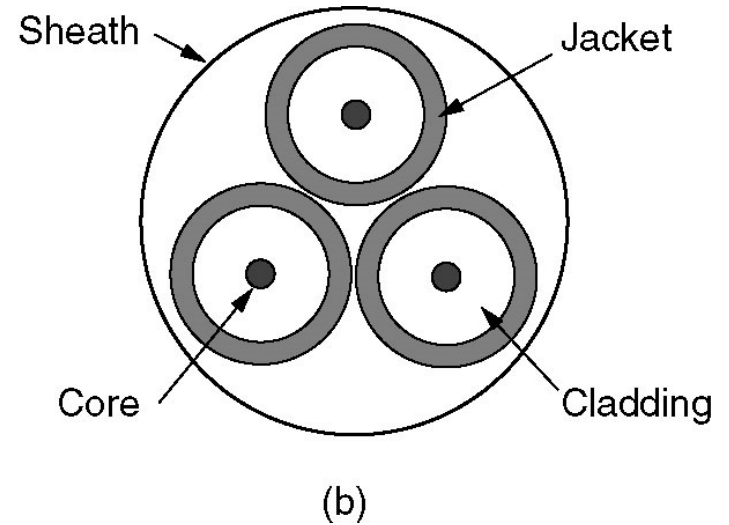
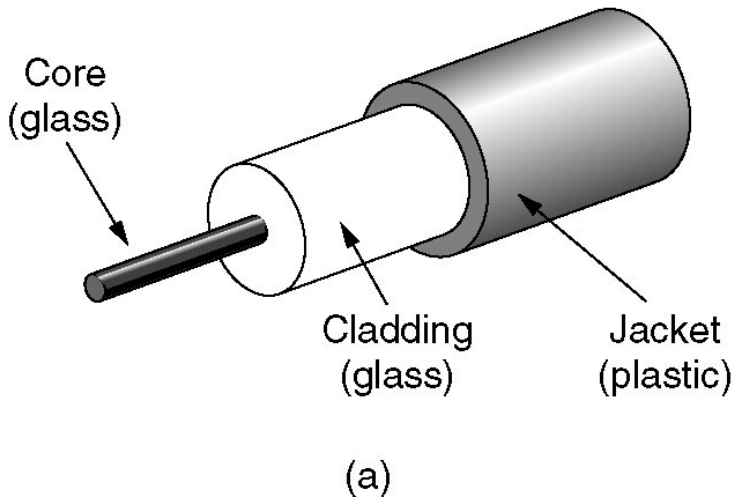


Single-mode fiber: only direct path propagates in fiber



- Multimode: Thicker core, shorter reach
  - Rays on different paths interfere causing dispersion & limiting bit rate
- Single mode: Very thin core supports only one mode (path)
  - More expensive lasers, but achieves very high speeds

# Fiber Cables



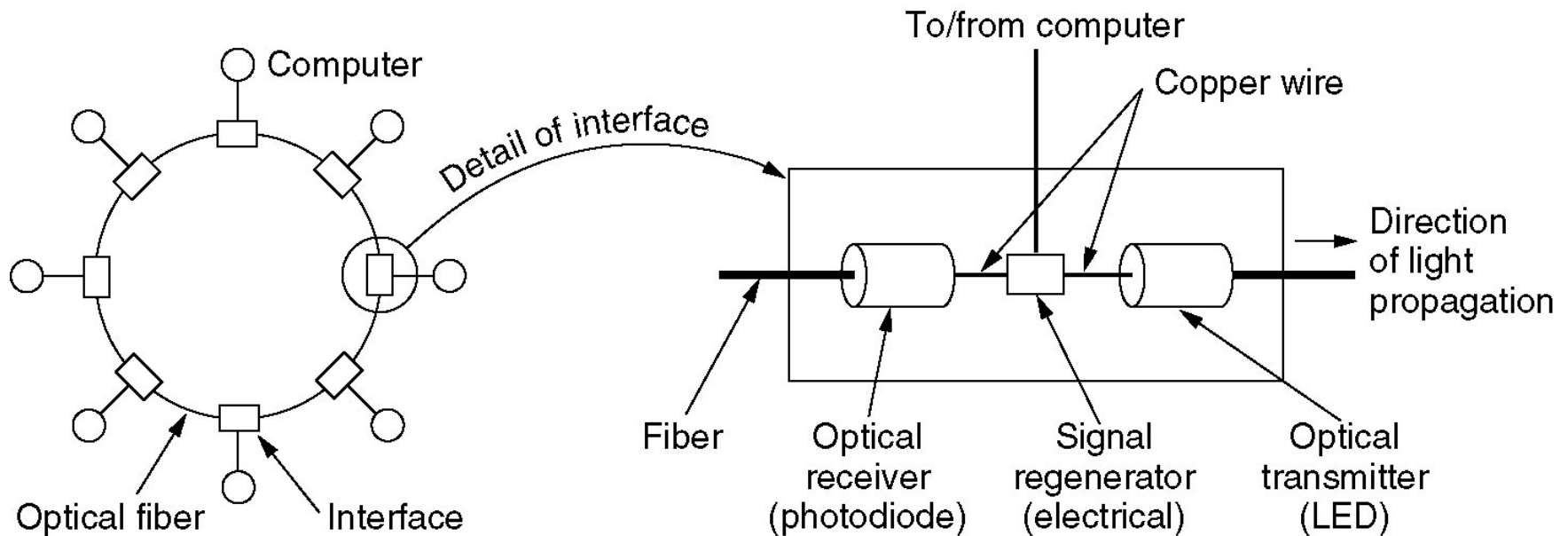
- (a) Side view of a single fiber.
- (b) End view of a sheath with three fibers.



## Fiber Cables (2)

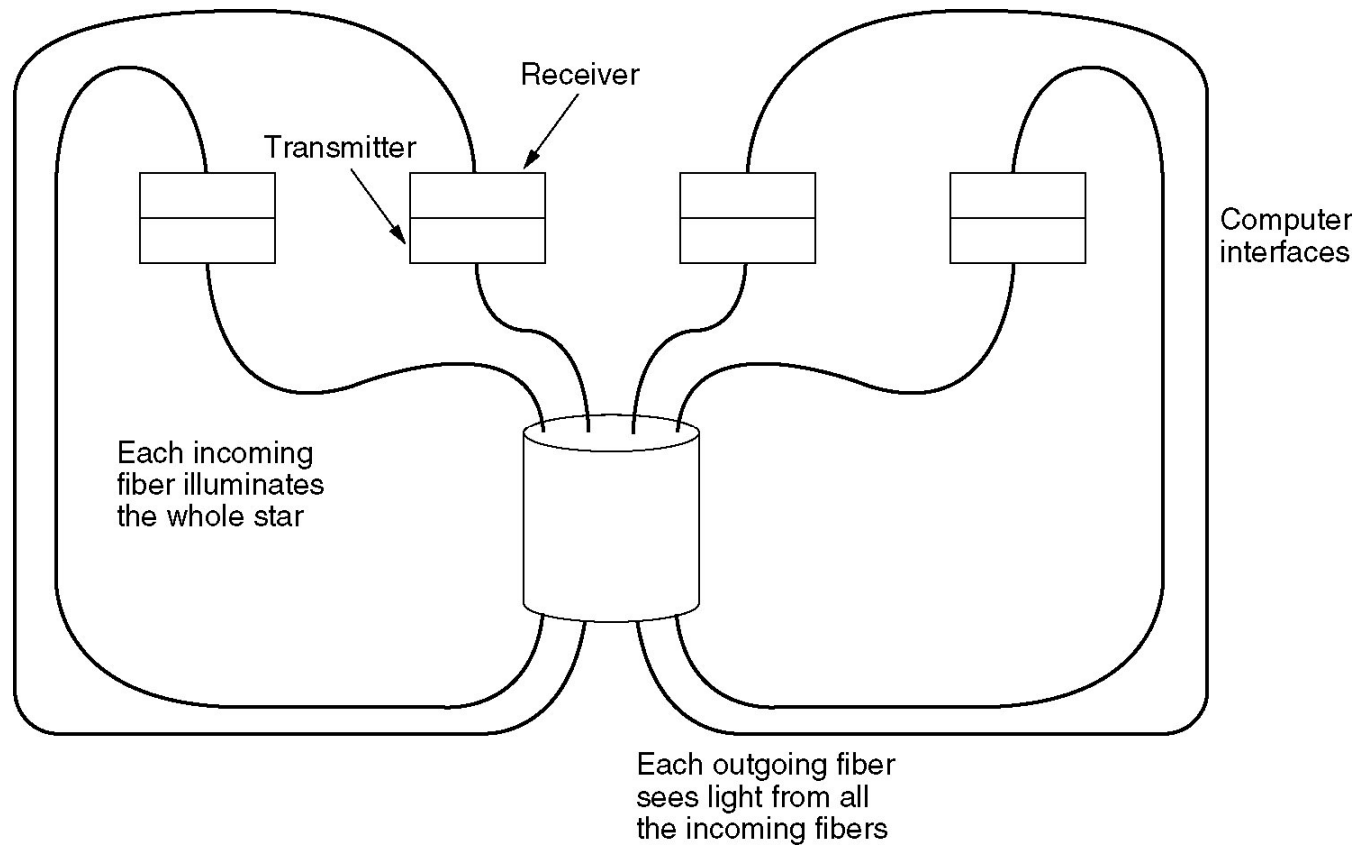
Item	LED	Semiconductor laser
Data rate	Low	High
Fiber type	Multimode	Multimode or single mode
Distance	Short	Long
Lifetime	Long life	Short life
Temperature sensitivity	Minor	Substantial
Cost	Low cost	Expensive

# Fiber Optic Networks (1)



A fiber optic ring with active repeaters.

## Fiber Optic Networks (2)



A passive star connection in a fiber optics network.



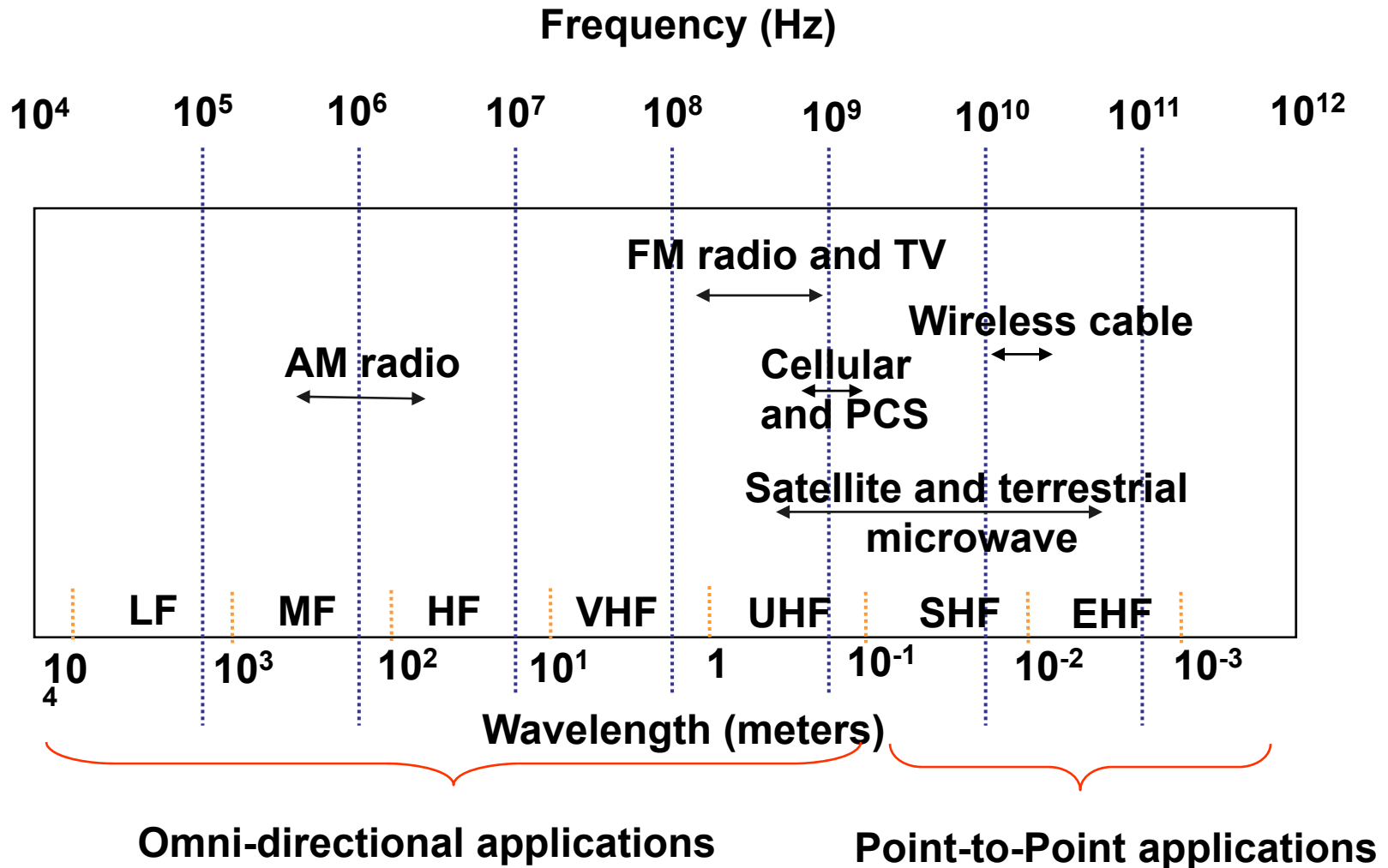


# Wireless Transmission

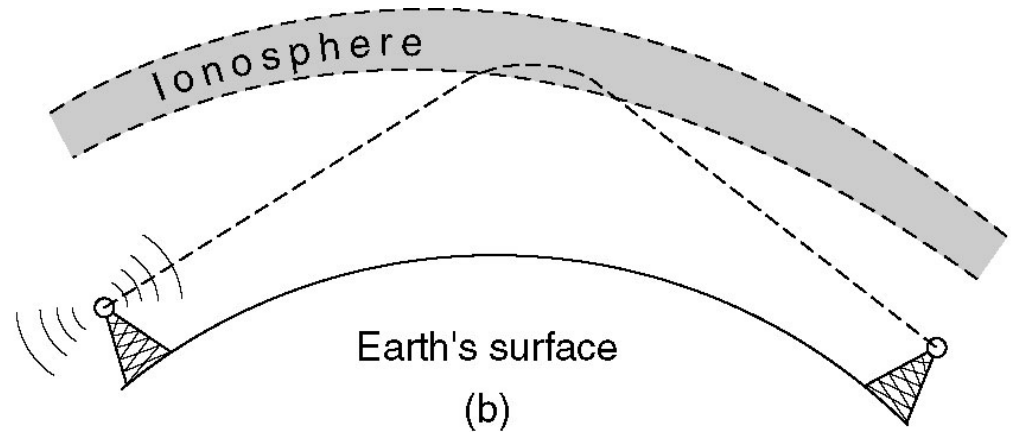
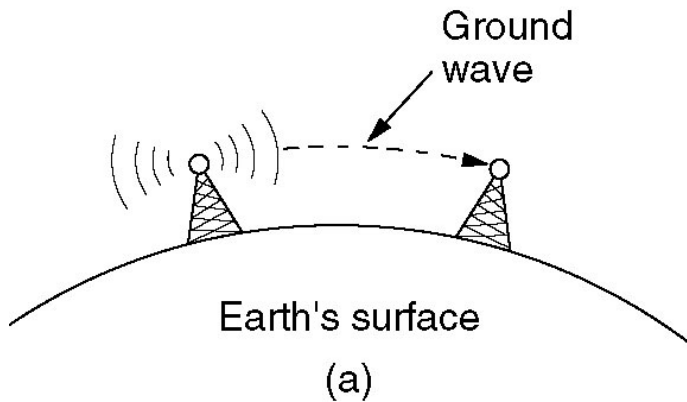
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- Radio Transmission
- Microwave Transmission
- Infrared and Millimeter Waves
- Lightwave Transmission

# Radio Spectrum



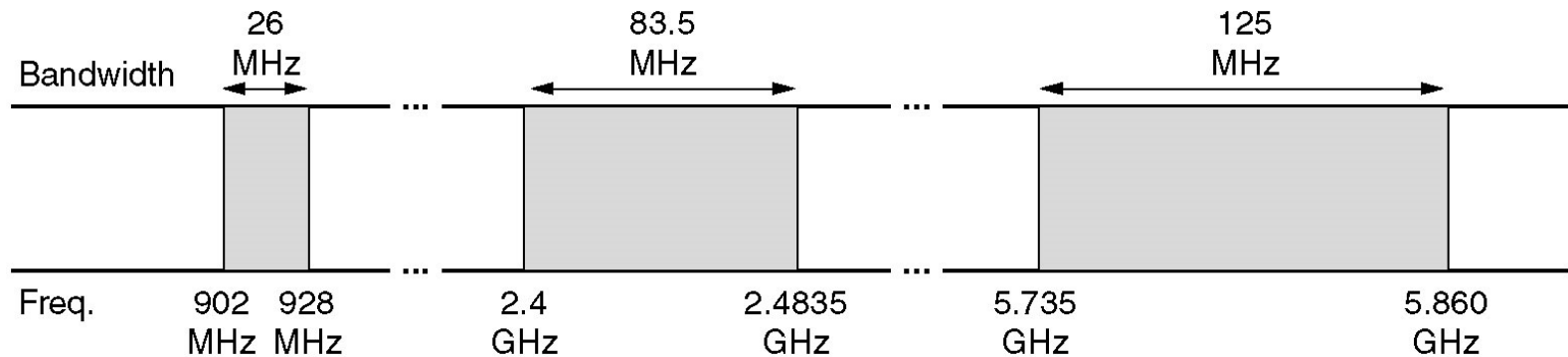
# Radio Transmission



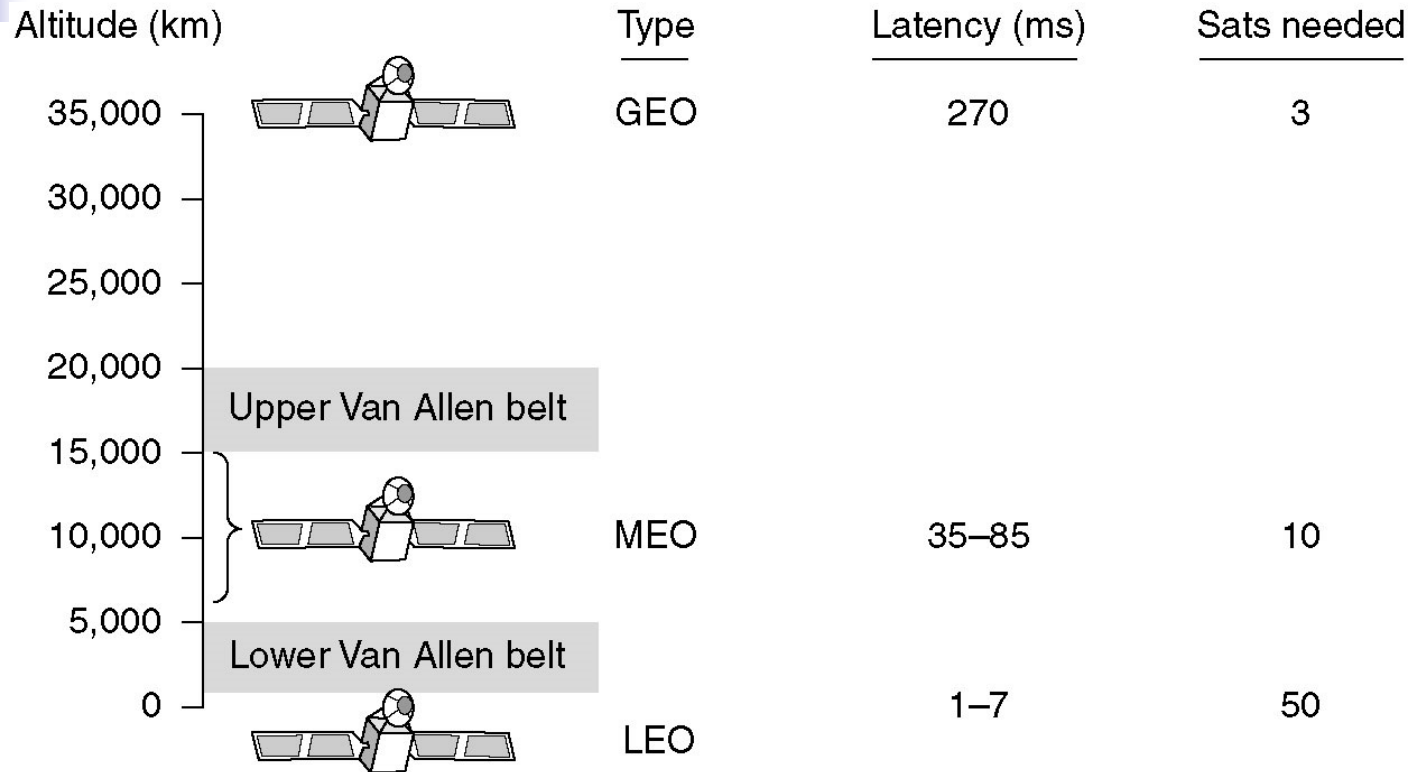
- (a) In the VLF, LF, and MF bands, radio waves follow the curvature of the earth.
- (b) In the HF band, they bounce off the ionosphere.

# Politics of the Electromagnetic Spectrum

## The ISM bands in the United States.



# Communication Satellites



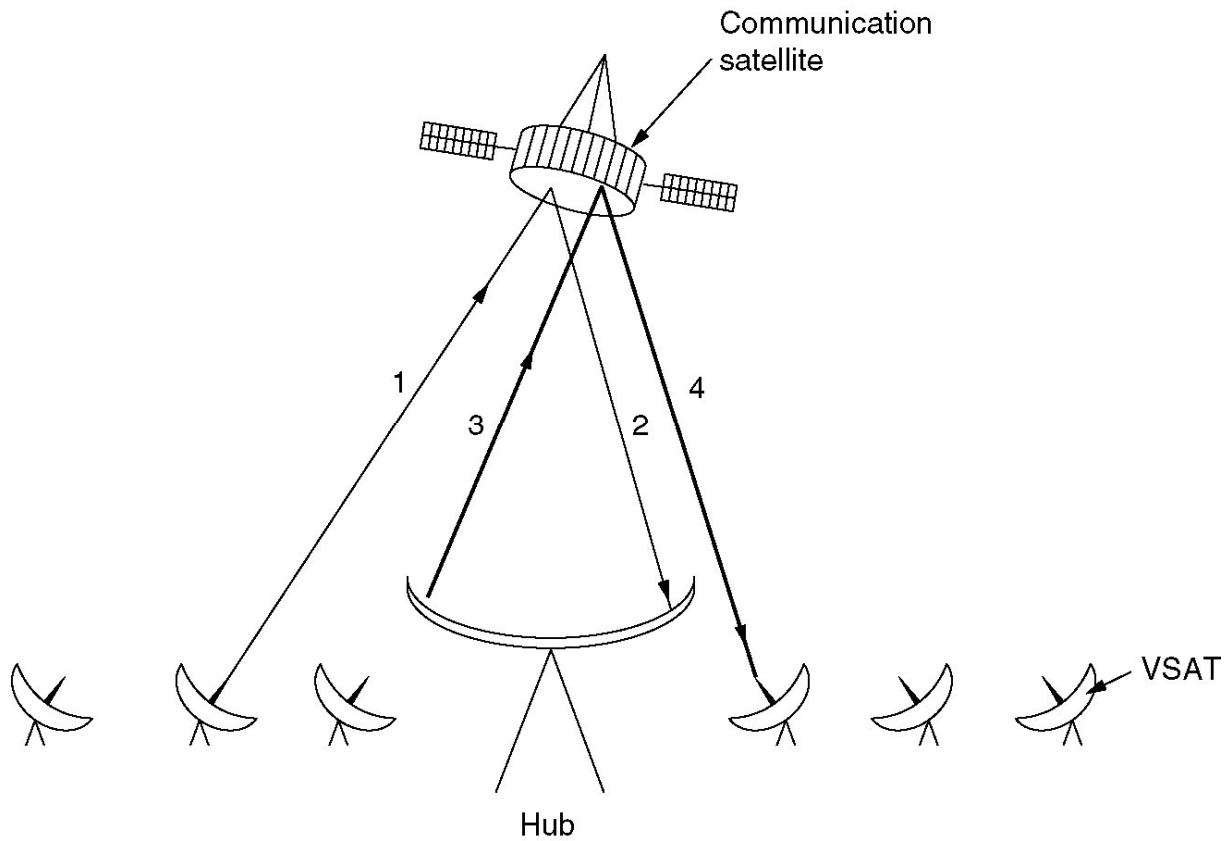
Communication satellites and some of their properties, including altitude above the earth, round-trip delay time and number of satellites needed for global coverage.

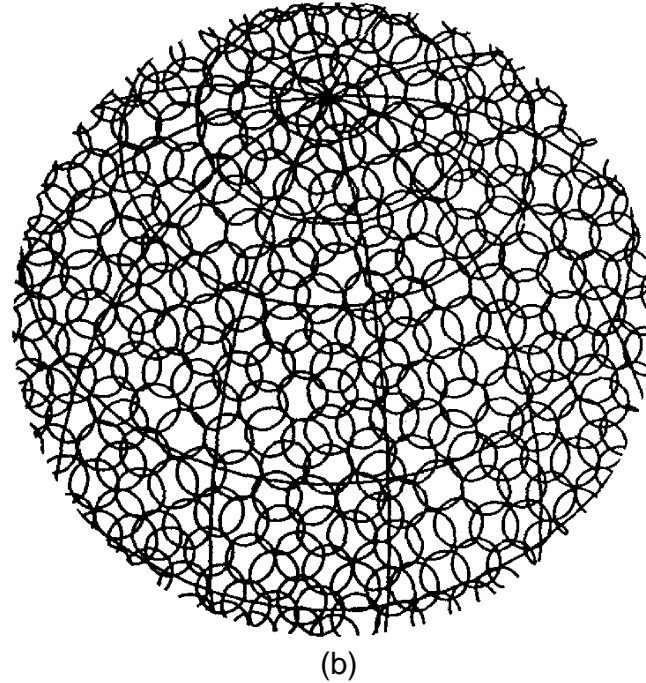
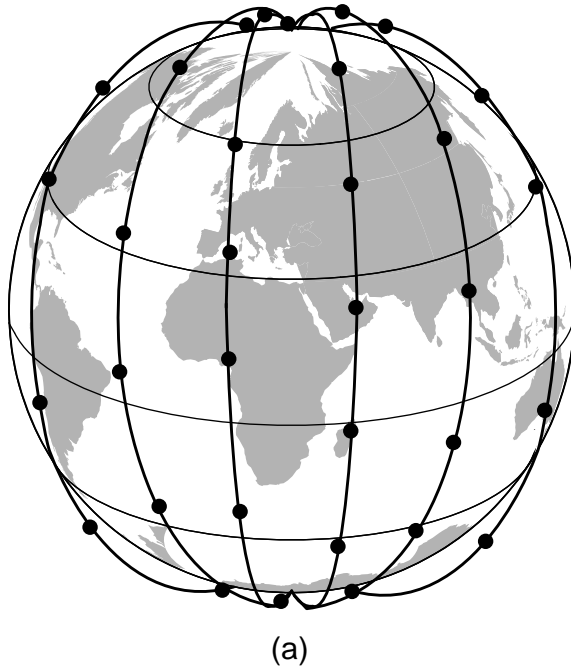


# Principal Satellite Bands

Band	Downlink	Uplink	Bandwidth	Problems
L	1.5 GHz	1.6 GHz	15 MHz	Low bandwidth; crowded
S	1.9 GHz	2.2 GHz	70 MHz	Low bandwidth; crowded
C	4.0 GHz	6.0 GHz	500 MHz	Terrestrial interference
Ku	11 GHz	14 GHz	500 MHz	Rain
Ka	20 GHz	30 GHz	3500 MHz	Rain, equipment cost

# Satellite Hub

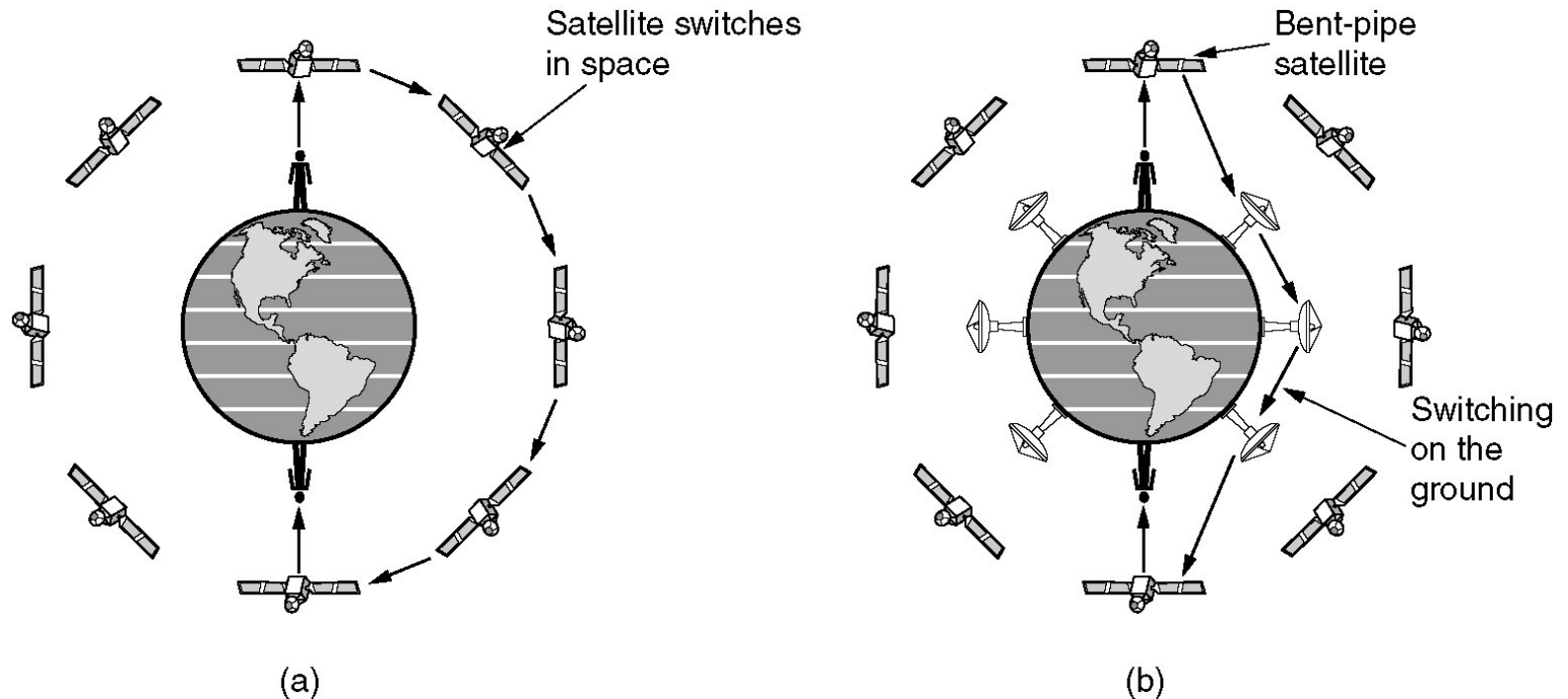




- (a) The Iridium satellites from six necklaces around the earth.
- (b) 1628 moving cells cover the earth.



# Globalstar



- (a) Relaying in space.
- (b) Relaying on the ground.

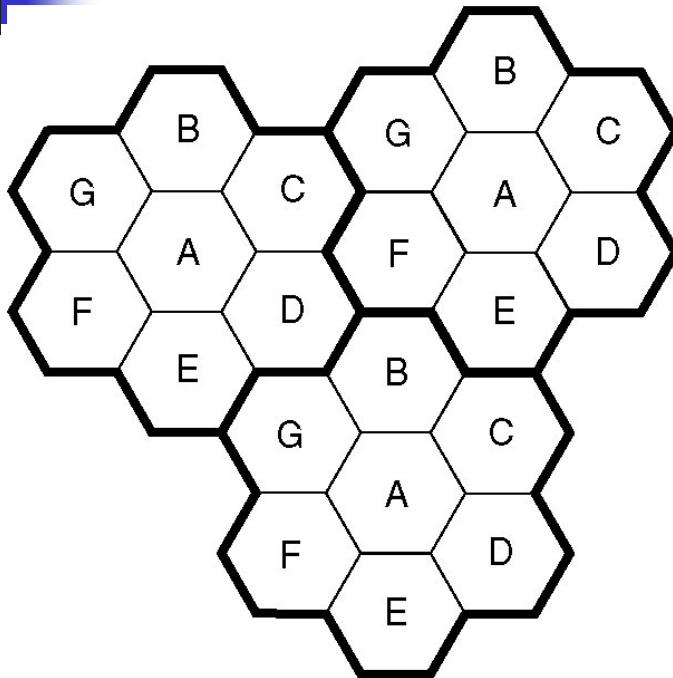


# The Mobile Telephone System

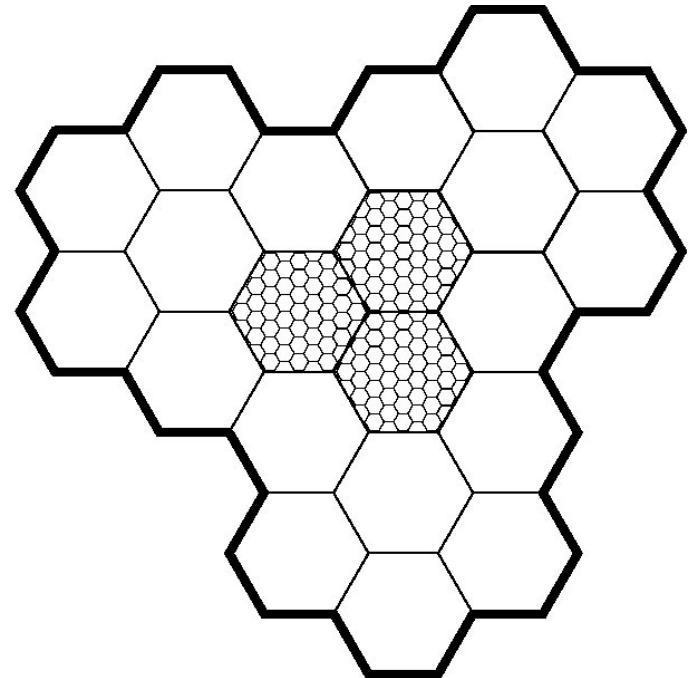
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- First-Generation Mobile Phones:  
Analog Voice
- Second-Generation Mobile Phones:  
Digital Voice
- Third-Generation Mobile Phones:  
Digital Voice and Data
- Fourth-Generation

# Advanced Mobile Phone System



(a)



(b)

- (a) Frequencies are not reused in adjacent cells.
- (b) To add more users, smaller cells can be used.



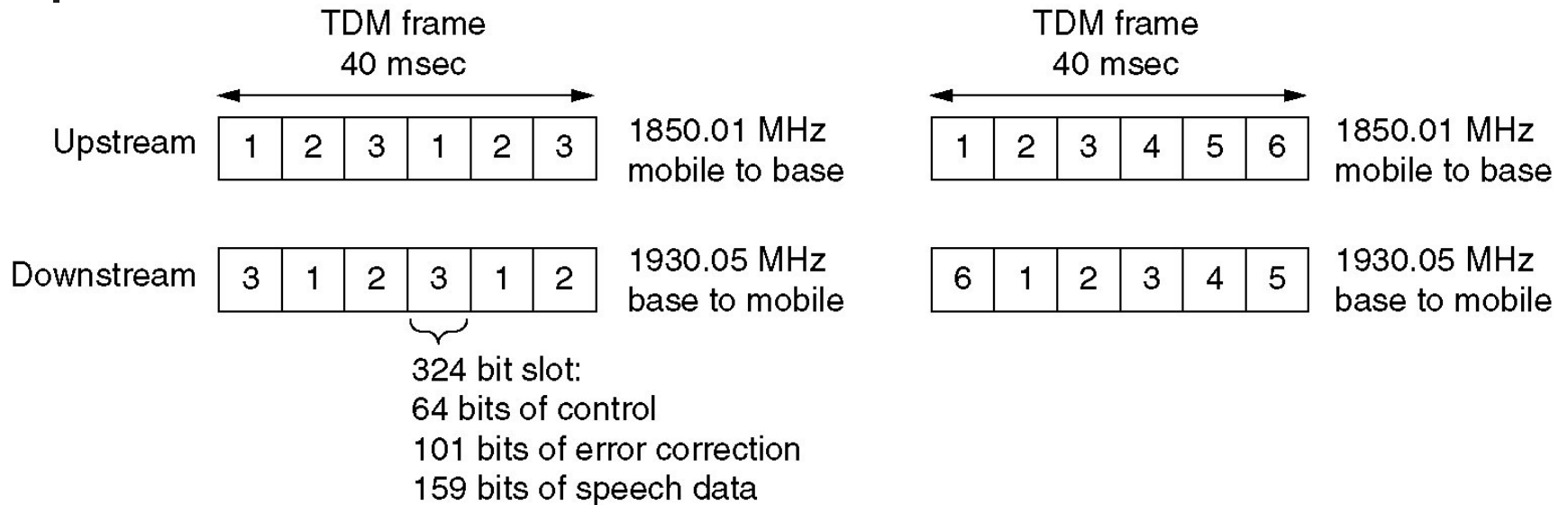
# Channel Categories

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The 832 channels are divided into four categories:

- Control (base to mobile) to manage the system
- Paging (base to mobile) to alert users to calls for them
- Access (bidirectional) for call setup and channel assignment
- Data (bidirectional) for voice, fax, or data

# Digital Advanced Mobile Phone System

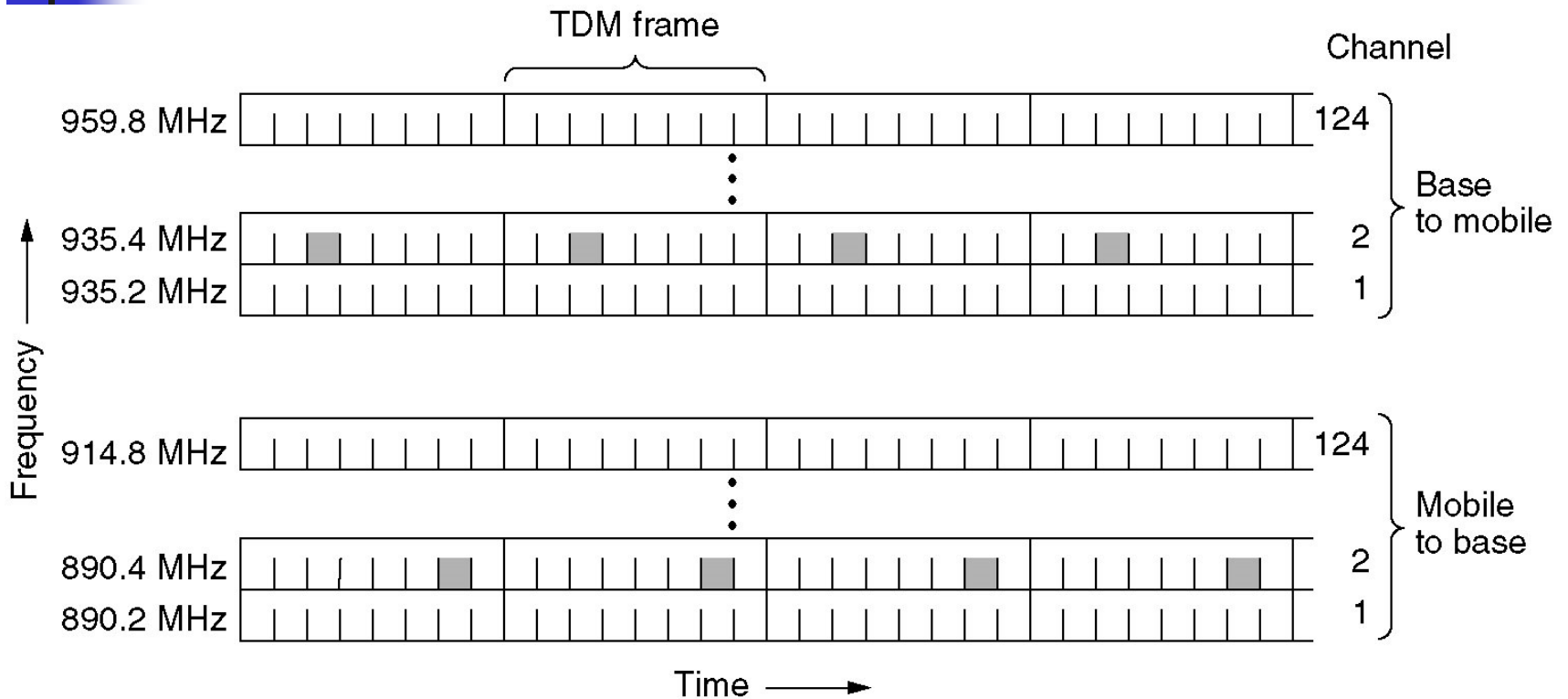


(a)

(b)

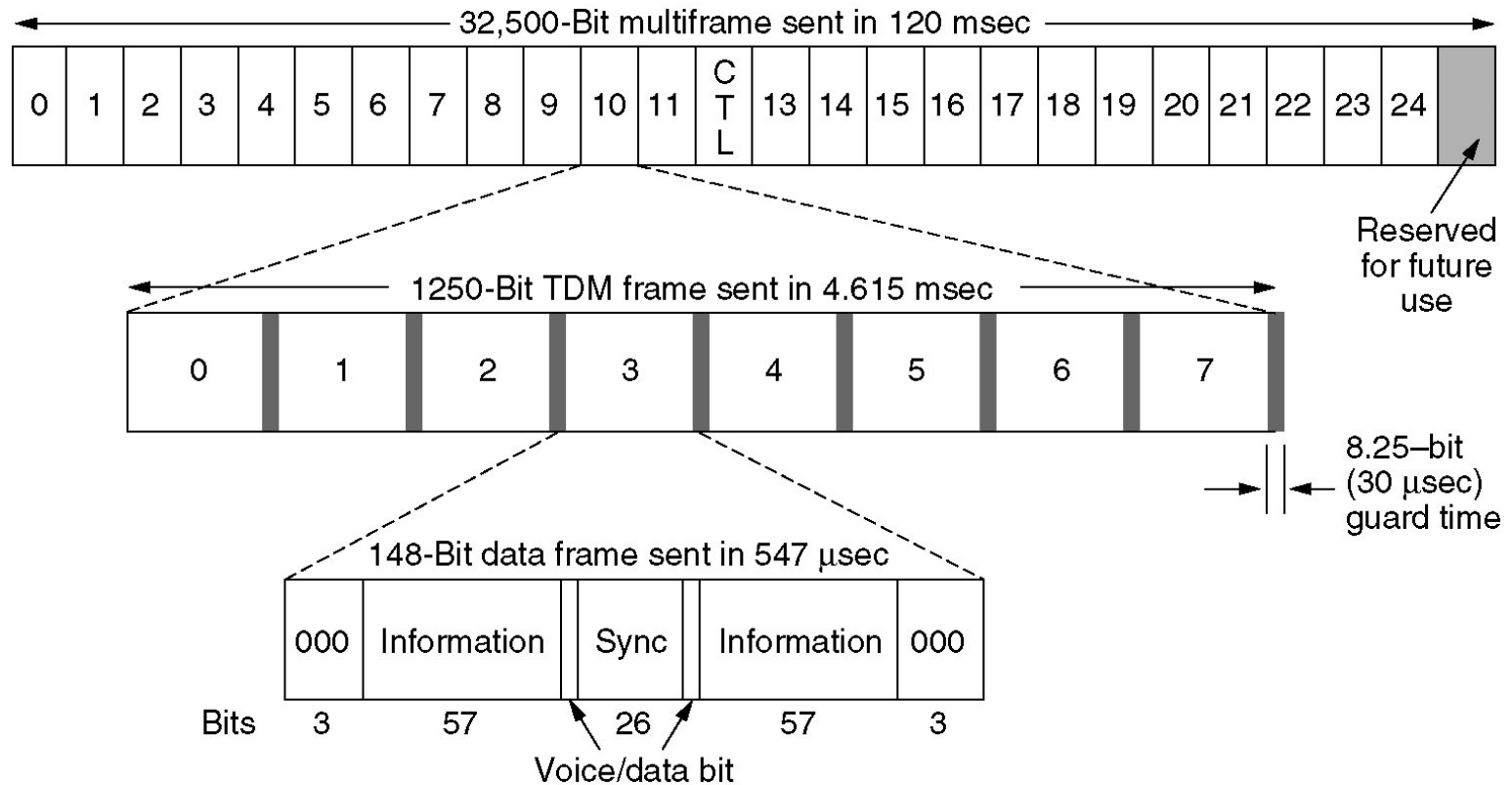
- (a) A D-AMPS channel with three users.
- (b) A D-AMPS channel with six users.

# Global System for Mobile Communications (GSM)



GSM uses 124 frequency channels, each of which uses an eight-slot TDM system

# GSM



A portion of the GSM framing structure.

# CDMA

A: 0 0 0 1 1 0 1 1  
 B: 0 0 1 0 1 1 1 0  
 C: 0 1 0 1 1 1 0 0  
 D: 0 1 0 0 0 0 1 0

(a)

A: (-1 -1 -1 +1 +1 -1 +1 +1)  
 B: (-1 -1 +1 -1 +1 +1 +1 -1)  
 C: (-1 +1 -1 +1 +1 +1 -1 -1)  
 D: (-1 +1 -1 -1 -1 -1 +1 -1)

(b)

Six examples:

- - 1 -	<b>C</b>	$S_1 = (-1 +1 -1 +1 +1 +1 -1 -1)$
- 1 1 -	<b>B + <math>\bar{C}</math></b>	$S_2 = (-2 \ 0 \ 0 \ 0 +2 +2 \ 0 -2)$
1 0 - -	<b>A + <math>\bar{B}</math></b>	$S_3 = (0 \ 0 -2 +2 \ 0 -2 \ 0 +2)$
1 0 1 -	<b>A + B + C</b>	$S_4 = (-1 +1 -3 +3 +1 -1 -1 +1)$
1 1 1 1	<b>A + B + C + D</b>	$S_5 = (-4 \ 0 -2 \ 0 +2 \ 0 +2 -2)$
1 1 0 1	<b>A + B + <math>\bar{C}</math> + D</b>	$S_6 = (-2 -2 \ 0 -2 \ 0 -2 +4 \ 0)$

(c)

$S_1 \bullet C = (1 +1 +1 +1 +1 +1 +1 +1)/8 = 1$   
 $S_2 \bullet C = (2 +0 +0 +0 +2 +2 +0 +2)/8 = 1$   
 $S_3 \bullet C = (0 +0 +2 +2 +0 -2 +0 -2)/8 = 0$   
 $S_4 \bullet C = (1 +1 +3 +3 +1 -1 +1 -1)/8 = 1$   
 $S_5 \bullet C = (4 +0 +2 +0 +2 +0 -2 +2)/8 = 1$   
 $S_6 \bullet C = (2 -2 +0 -2 +0 -2 -4 +0)/8 = -1$

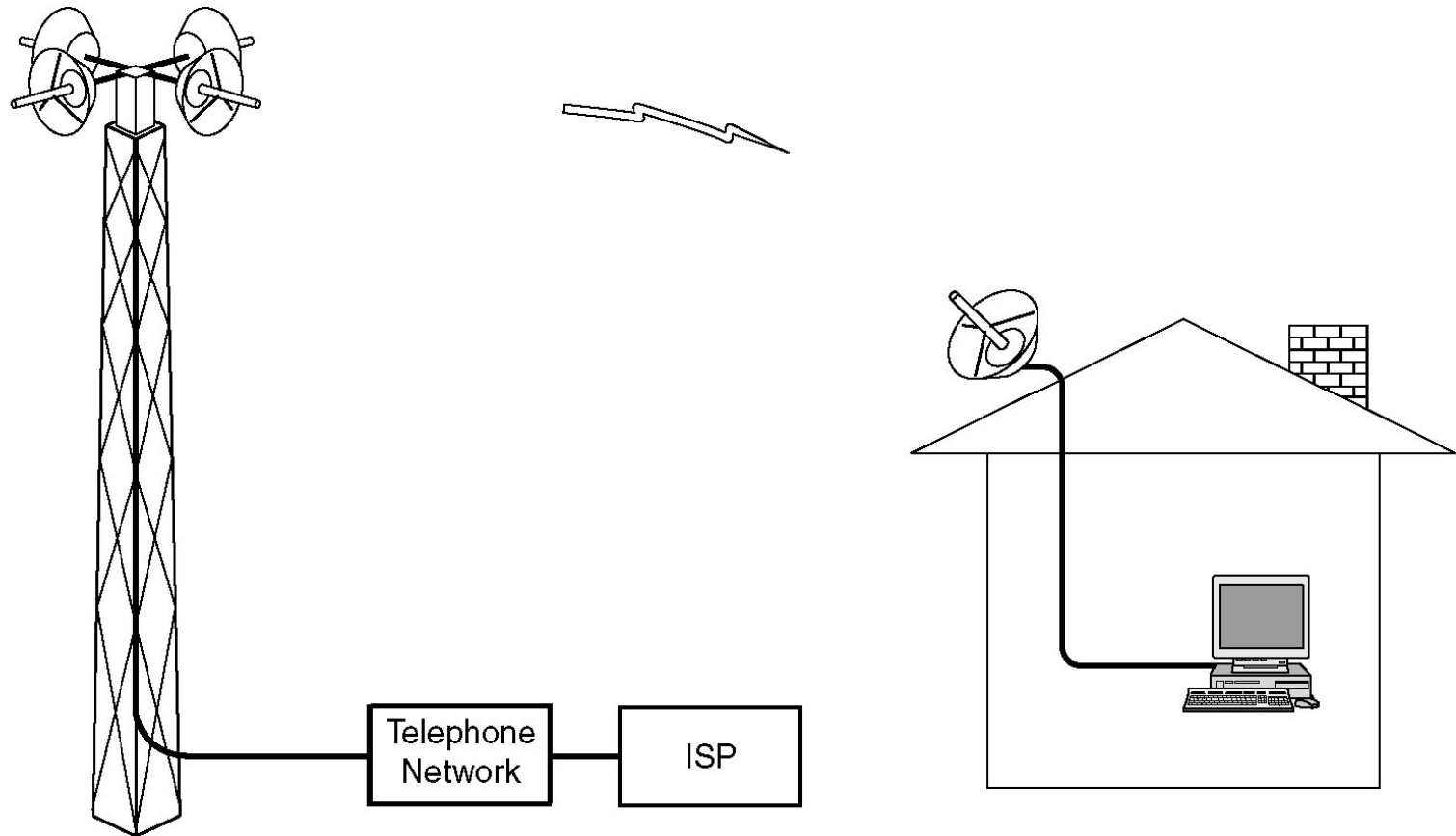
(d)



Basic services an IMT-2000 network should provide

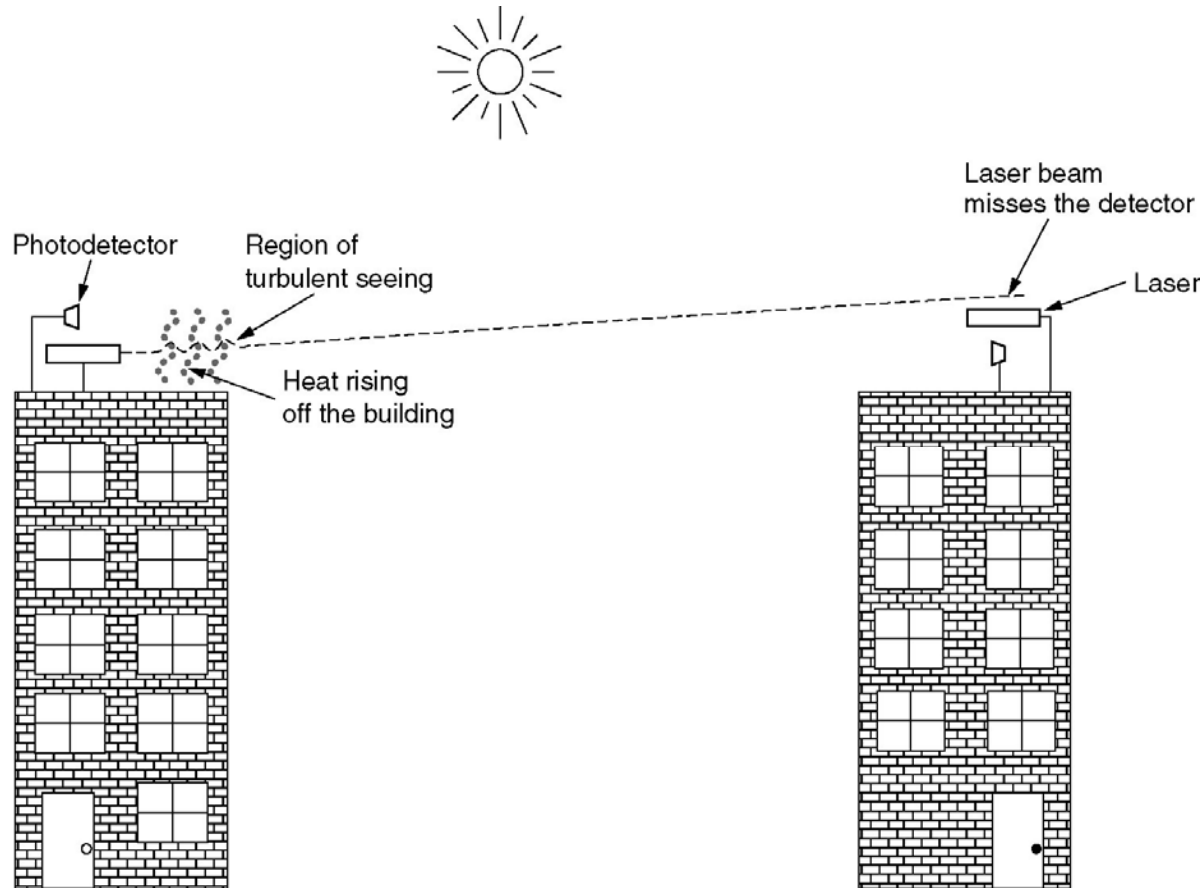
- High-quality voice transmission
- Messaging (replace e-mail, fax, SMS, chat, etc.)
- Multimedia (music, videos, films, TV, etc.)
- Internet access (web surfing, w/multimedia.)

# Wireless Local Loops



Architecture of an LMDS system.

# Lightwave Transmission



Convection currents can interfere with laser communication systems.  
A bidirectional system with two lasers is pictured here.