

VE281

Data Structures and Algorithms

Hashing: Basics and Hash Function

Learning Objectives:

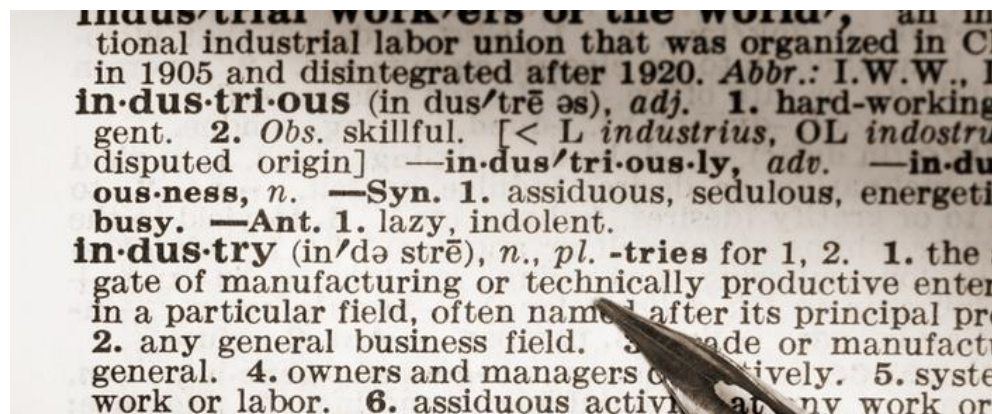
- Know the purpose of hashing
- Understand the collision problem
- Know how to design a good hash function

Outline

- Review of Dictionary
- Hashing Basics
- Hash Function

Dictionary

- How do you use a dictionary?
 - Look up a “word” and find its meaning.
- We also have an abstract data type called dictionary.
 - It is a collection of pairs, each containing a **key** and a **value**
(key, value)
 - **Important**: Different pairs have different keys.



Dictionary

- Key space is usually more regular/structured than value space, so easier to search.
- Dictionary is optimized to quickly **add** (**key**, **value**) pair and **retrieve value** by key.

Methods

- **Value find(Key k)** : Return the value whose key is **k**. Return **Null** if none.
- **void insert(Key k, Value v)** : Insert a pair (**k**, **v**) into the dictionary. If the pair with key as **k** already exists, update its value.
- **Value remove(Key k)** : Remove the pair with key as **k** from the dictionary and return its value. Return **Null** if none.

Runtime for Array Implementation

Pair Array[MAXSIZE] :

| | | | | | | |
|---|---|---|---|--|--|--|
| a | b | c | d | | | |
|---|---|---|---|--|--|--|

- Unsorted array
 - find() $O(n)$
 - insert() $O(n)$: $O(n)$ to verify duplicate, $O(1)$ to put at the end
 - remove() $O(n)$: $O(n)$ to verify existence, $O(1)$ to exchange the “hole” with the last element
- Sorted array
 - find() $O(\log n)$: binary search
 - insert() $O(n)$: $O(\log n)$ to verify duplicate, $O(n)$ to insert
 - remove() $O(n)$: $O(\log n)$ to verify existence, $O(n)$ to remove

Can we do **find**, **insert**, and **remove** in $O(1)$ time?

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Hashing: High-Level Idea

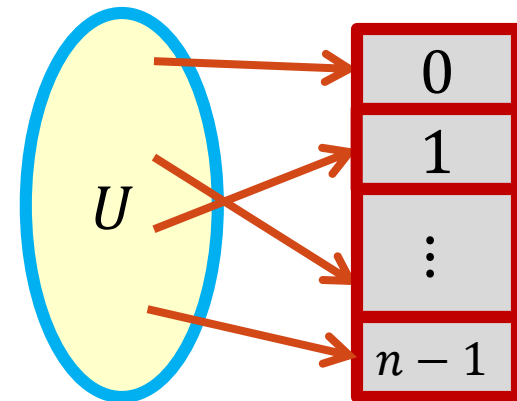
- **Setup:** A universe U of objects
 - E.g., All names, all IP addresses, etc.
 - Generally, very BIG!
- **Goal:** Want to maintain an evolving set $S \subseteq U$
 - E.g., 200 students, 500 IP addresses
 - Generally, of reasonable size.
- Naïve solutions
 1. Array-based solution (index by $u \in U$)
 - $\Theta(1)$ operation time, BUT $\Theta(|U|)$ space.
 2. Linked list-based solution:
 - $\Theta(|S|)$ space, BUT $\Theta(|S|)$ operation time.

Can we get the best of both solutions?

Hashing: High-Level Idea

- Solution:

- Pick an array A of n buckets.
 - $n = c|S|$: a small multiple of $|S|$.
- Choose a hash function $h: U \rightarrow \{0, 1, \dots, n - 1\}$
 - h is fast to compute.
 - The same key is always mapped to the **same** location.
- Store item k in $A[h(k)]$



- The array is called **hash table**

- An array of **buckets**, where each bucket contains items as assigned by a hash function.
- $h[k]$ is called the **home bucket** of key k .

Hashing Example

- Pairs are: (22,a), (33,b), (3,c), (73,d), (85,e)
- Hash table is **A[0:7]** and table size is **M = 8**
- Hash function is **h[key] = key/11**
- Every item with **key** is stored in the bucket **A[h(key)]**

| | | | | | | | |
|-------|-----|--------|--------|-----|-----|--------|--------|
| (3,c) | | (22,a) | (33,b) | | | (73,d) | (85,e) |
| [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] |

Question: What is the time complexity for **find()**, **insert()**, and **remove()**?

O(1)

Achieves both fast runtime and efficient memory usage

What Can Go Wrong?

| | | | | | | | |
|-------|-----|--------|--------|-----|-----|--------|--------|
| (3,c) | | (22,a) | (33,b) | | | (73,d) | (85,e) |
| [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] |

- Where does (35, g) go?
- Problem: The home bucket for (35, g) is already occupied!
 - This is a “**collision**”.

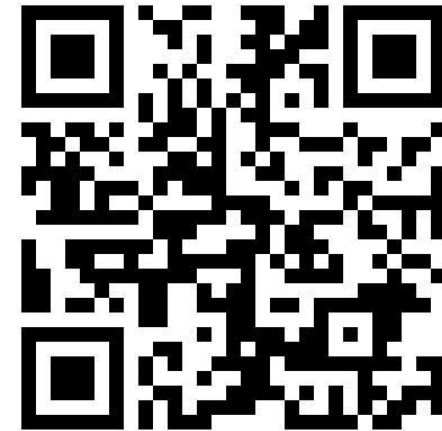
Collision and Collision Resolution

- Collision occurs when the hash function maps two or more items—all having **different** search keys—into the **same** bucket.
- What to do when there is a collision?
 - **Collision-resolution scheme**: assigns distinct locations in the hash table to items involved in a collision.
- Two major schemes:
 - Separate chaining
 - Open addressing



Insight of Collision: Birthday Problem

- Consider n people with random birthdays (i.e., with each day of the year equally likely). What is the smallest n so that there is at least a 50% chance that two people have the same birthday?
 - A. 23
 - B. 57
 - C. 184
 - D. 367



Collision is inevitable!


Hash Table Issues

- Choice of the hash function.
- Collision resolution scheme.
- Size of the hash table and rehashing.

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Hash Function Design Criteria

- Must compute a bucket for every key in the universe.
- Must compute the same bucket for the same key.
- Should be easy and quick to compute.
- Minimizes collision  The hardest criterion
 - Spread keys out evenly in hash table
 - **Gold standard: completely random hashing**
 - The probability that a randomly selected key has bucket i as its home bucket is $1/n$, $0 \leq i < n$.
 - Completely random hashing **minimizes** the likelihood of an collision when keys are selected at random.
 - However, completely random hashing is **infeasible** due to the need to remember the random bucket.

Bad Hash Functions

- Example: keys = phone number in China (11 digits)
 - $|U| = 10^{11}$
 - **Terrible** hash function: $h(key) =$ first 3 digits of key , i.e., area code
 - The keys are not spread out evenly. Buckets 010, 021 may have a lot of keys mapped to them, while some buckets have no keys.
 - **Mediocre** hash function: $h(key) =$ last 3 digits of key .
 - Still vulnerable to patterns in last 3 digits.

Hash Functions

- Hash function ($h(key)$) maps key to buckets in two steps:
 1. Convert key into an integer in case the key is not an integer.
 - A function $t(key)$ which returns an integer value, known as **hash code**.
 2. **Compression map**: Map an integer (hash code) into a home bucket.
 - A function $c(hashcode)$ which gives an integer in the range $[0, n - 1]$, where n is the number of buckets in the table.
- In summary, $h(key) = c(t(key))$, which gives an index in the table.

Map Non-integers into Hash Code

- String: use the ASCII (or UTF-8) encoding of each char and then perform arithmetic on them.
- Floating-point number: treat it as a string of bits.
- Images, (viral) code snippets, (malicious) Web site URLs: in general, treat the representation as a bit-string, using all of it or **extracting** parts of it (i.e., www.sjtu.edu.cn).

Strings to Integers

- Simple scheme: adds up all the ASCII codes for all the chars in the string.
 - Example: $t(\text{"He"}) = 72 + 101 = 173$.
- Not good. Why?
 - Consider English words “post”, “pots”, “spot”, “stop”, “tops”.

Strings to Integers

- A better strategy: Polynomial hash code taking **positional** info into account.

$$t(s[]) = s[0]a^{k-1} + s[1]a^{k-2} + \dots + s[k-2]a + s[k-1]$$

where a is a constant.

- If $a = 33$, the hash codes for “post” and “stop” are
 $t(\text{post}) = 112 \cdot 33^3 + 111 \cdot 33^2 + 115 \cdot 33 + 116 = 4149734$
 $t(\text{stop}) = 115 \cdot 33^3 + 116 \cdot 33^2 + 111 \cdot 33 + 112 = 4262854$

Strings to Integers

$$t(s[]) = s[0]a^{k-1} + s[1]a^{k-2} + \dots + s[k-2]a + s[k-1]$$

- Good choice of a for English words: 31, 33, 37, 39, 41
 - What does it mean for a to be a **good** choice? Why are these particular values **good**?
 - Answer: according to statistics on 50,000 English words, each of these constants will produce less than 7 collisions.
- In Java, its **string** class has a built-in **hashCode()** function. It takes $a = 31$. Why?
 - Multiplication by 31 can be replaced by a shift and a subtraction for **better performance**: $31 * i == (i \ll 5) - i$

Hash function criteria: Should be easy and quick to compute.

Compression Map

- Map an integer (hash code) into a home bucket.
- The most common method is by **modulo arithmetic**.
$$\text{homeBucket} = c(\text{hashcode}) = \text{hashcode} \% n$$
where n is the **number of buckets** in the hash table.
- Example: Pairs are (22,a), (33,b), (3,c), (55,d), (79,e). Hash table size is 7.

| | | | | | | |
|-----|--------|--------|-------|-----|--------|--------|
| | (22,a) | (79,e) | (3,c) | | (33,b) | (55,d) |
| [0] | [1] | [2] | [3] | [4] | [5] | [6] |

Hashing by Modulo

- In practice, keys of an application tend to have a specific pattern
 - For example, memory address in computer is multiple of 4.
- The choice of the hash table size n will affect the distribution of home buckets.

Hashing by Modulo

- Suppose the keys of an application are more likely to be mapped into even integers.
 - E.g., memory address is always a multiple of 4.
- When the hash table size n is an **even** number, **even** integers are hashed into **even** home buckets.
 - E.g., $n = 14$: $20\%14 = 6$, $30\%14 = 2$, $8\%14 = 8$
- The bias in the keys results in a bias toward the **even** home buckets.
 - All **odd** buckets are **guaranteed** to be empty.
 - The distribution of home buckets is not uniform!

Hashing by Modulo

- However, when the hash table size n is **odd**, even (or odd) integers may be hashed into both odd and even home buckets.
 - E.g., $n = 15$: $20\%15 = 5$, $30\%15 = 0$, $8\%15 = 8$
 $15\%15 = 0$, $3\%15 = 3$, $23\%15 = 8$
- The bias in the keys does not result in a bias toward either the odd or even home buckets.
 - Better chance of uniform distribution of home buckets.
- So **do not** use an even hash table size n .

Hashing by Modulo

- Similar **biased** distribution of home buckets happens in practice when the hash table size n is a multiple of small prime numbers.
- The effect of each prime divisor p of n **decreases** as p gets **larger**.
- Ideally, choose the hash table size n as a **large prime number**.