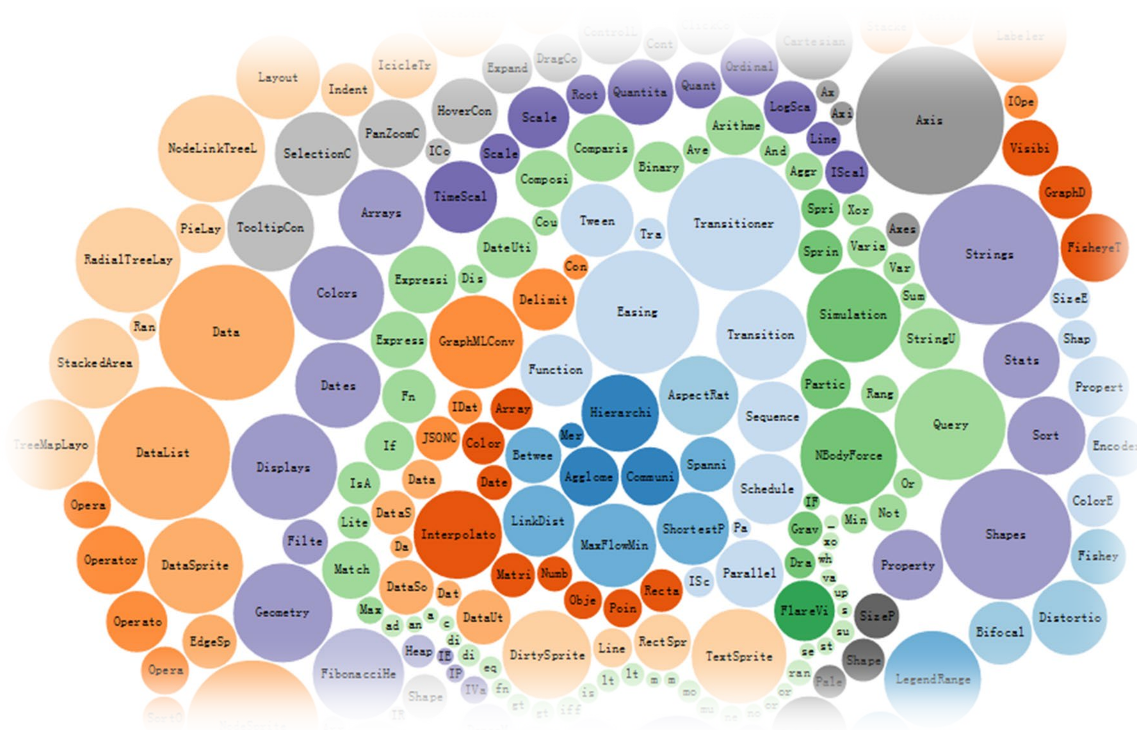

Graph Algorithms

A Brief Introduction

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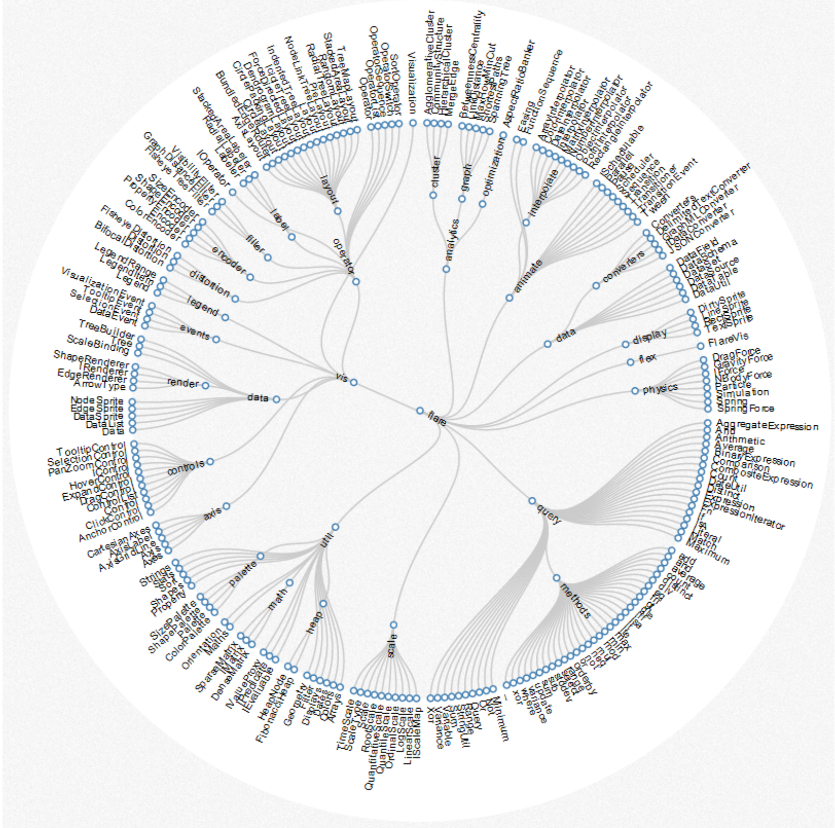
目录



GRAPH AND ITS APPLICATIONS

Definitions and Applications

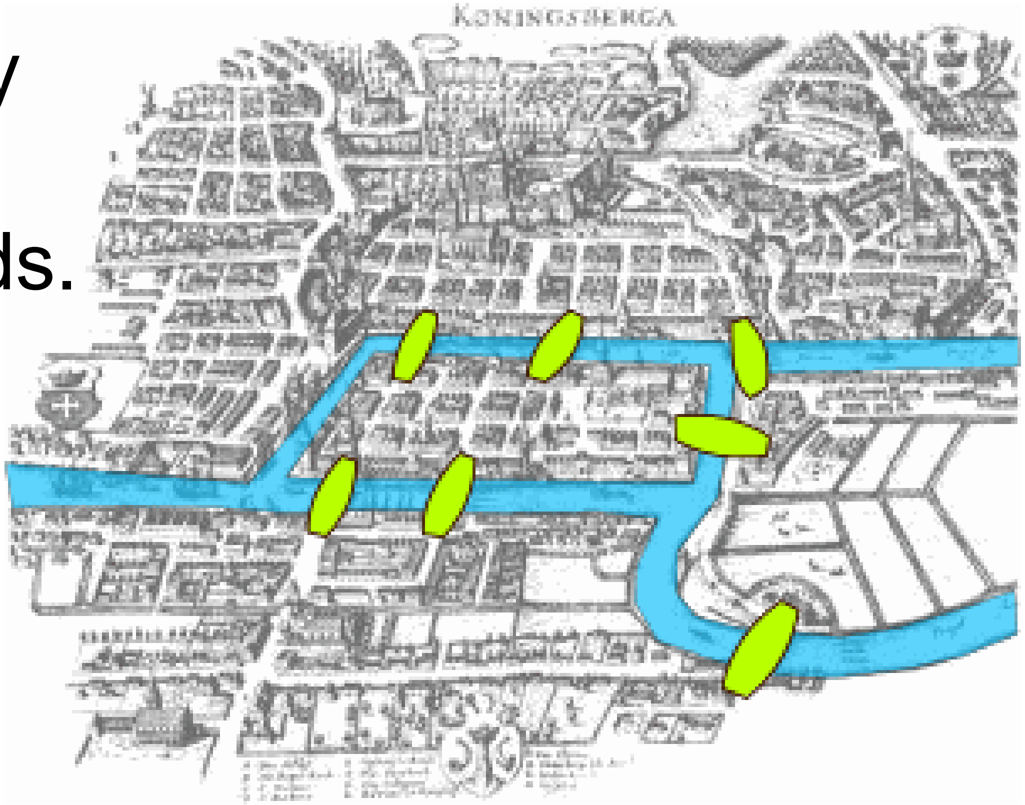
Konigsberg

- Once upon a time there was a city called Konigsberg in Prussia
- The capital of East Prussia until 1945
- Centre of learning for centuries, being home to Goldbach, Hilbert, Kant …

Position of Konigsberg

Seven Bridges

- Pregel river is passing through Konigsberg
- It separated the city into two mainland area and two islands.
- There are seven bridges connecting each area.

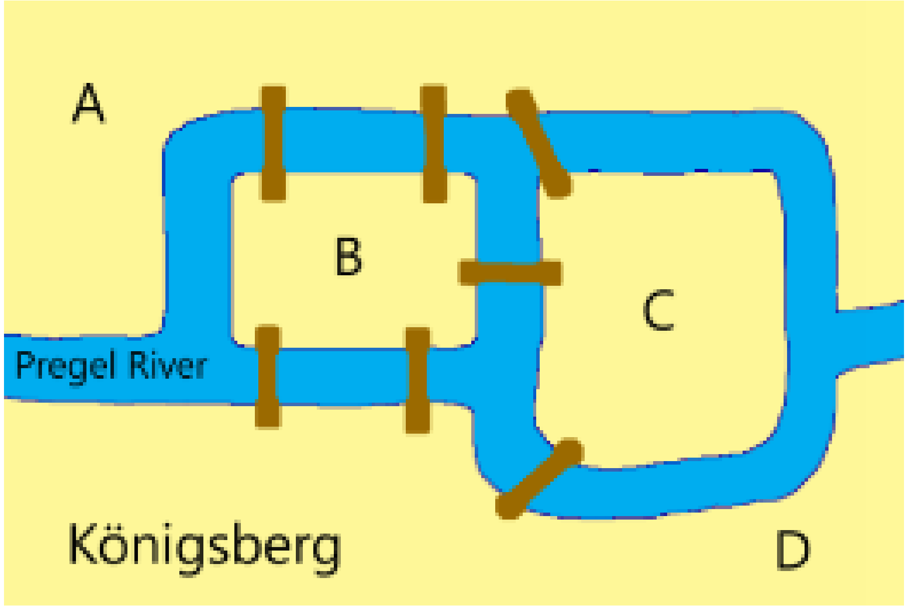


Seven Bridge Problem

- **A Tour Question:**

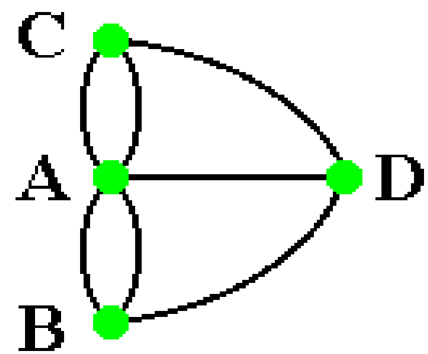
  Can we wander around the city, crossing each bridge once and only once?

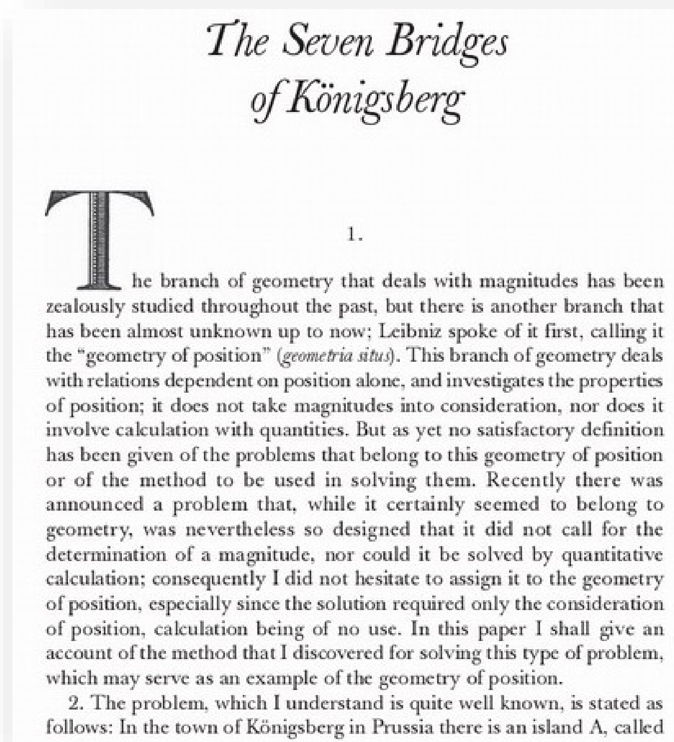
Is there a solution?



Euler's Solution

- Leonhard Euler Solved this problem in 1736
- Published the paper "The Seven Bridges of Konigsbery"
- The first negative solution The beginning of Graph Theory





The Seven Bridges of Königsberg

1.

The branch of geometry that deals with magnitudes has been zealously studied throughout the past, but there is another branch that has been almost unknown up to now; Leibniz spoke of it first, calling it the "geometry of position" (*geometria situs*). This branch of geometry deals with relations dependent on position alone, and investigates the properties of position; it does not take magnitudes into consideration, nor does it involve calculation with quantities. But as yet no satisfactory definition has been given of the problems that belong to this geometry of position or of the method to be used in solving them. Recently there was announced a problem that, while it certainly seemed to belong to geometry, was nevertheless so designed that it did not call for the determination of a magnitude, nor could it be solved by quantitative calculation; consequently I did not hesitate to assign it to the geometry of position, especially since the solution required only the consideration of position, calculation being of no use. In this paper I shall give an account of the method that I discovered for solving this type of problem, which may serve as an example of the geometry of position.

2. The problem, which I understand is quite well known, is stated as follows: In the town of Königsberg in Prussia there is an island A, called

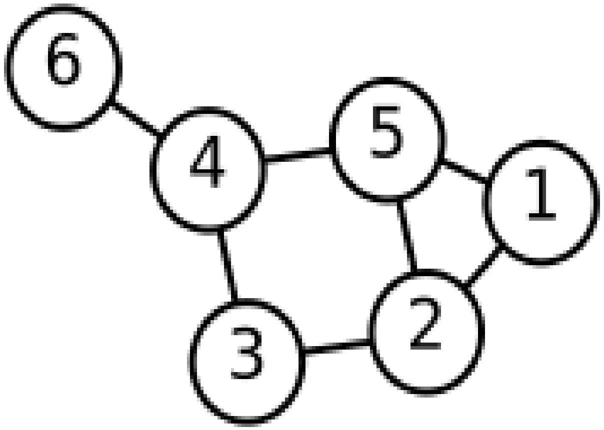
Representing a Graph

- Undirected Graph:

  G=(V, E)

  V: vertex

  E: edges

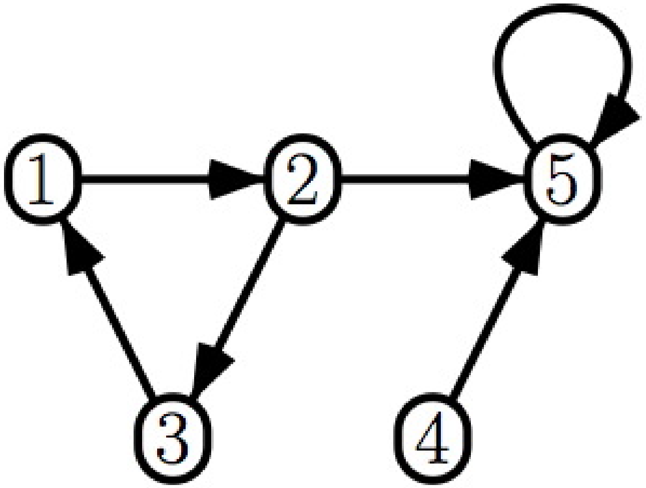


- Directed Graph:

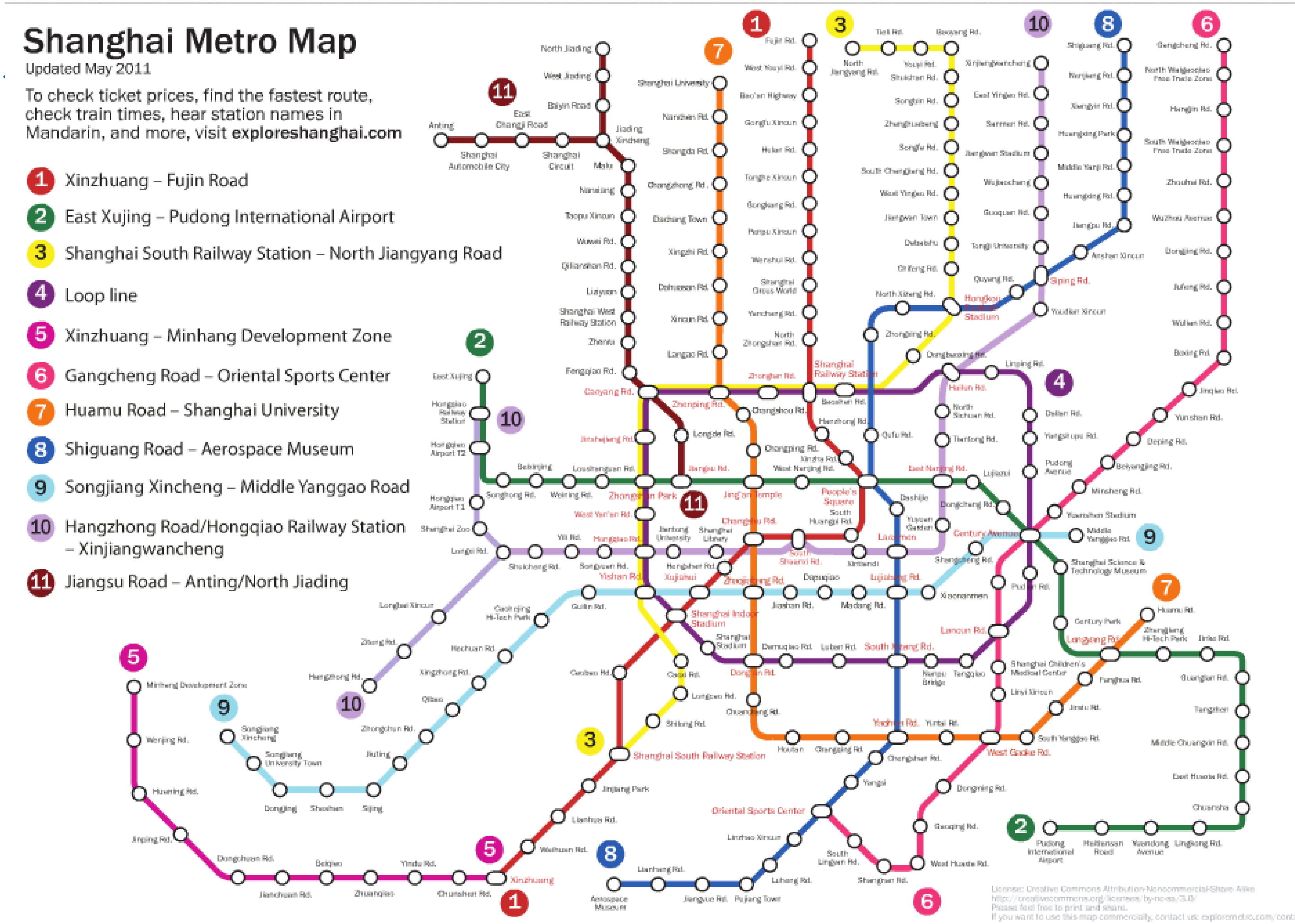
  G=(V, A)

  V: vertex

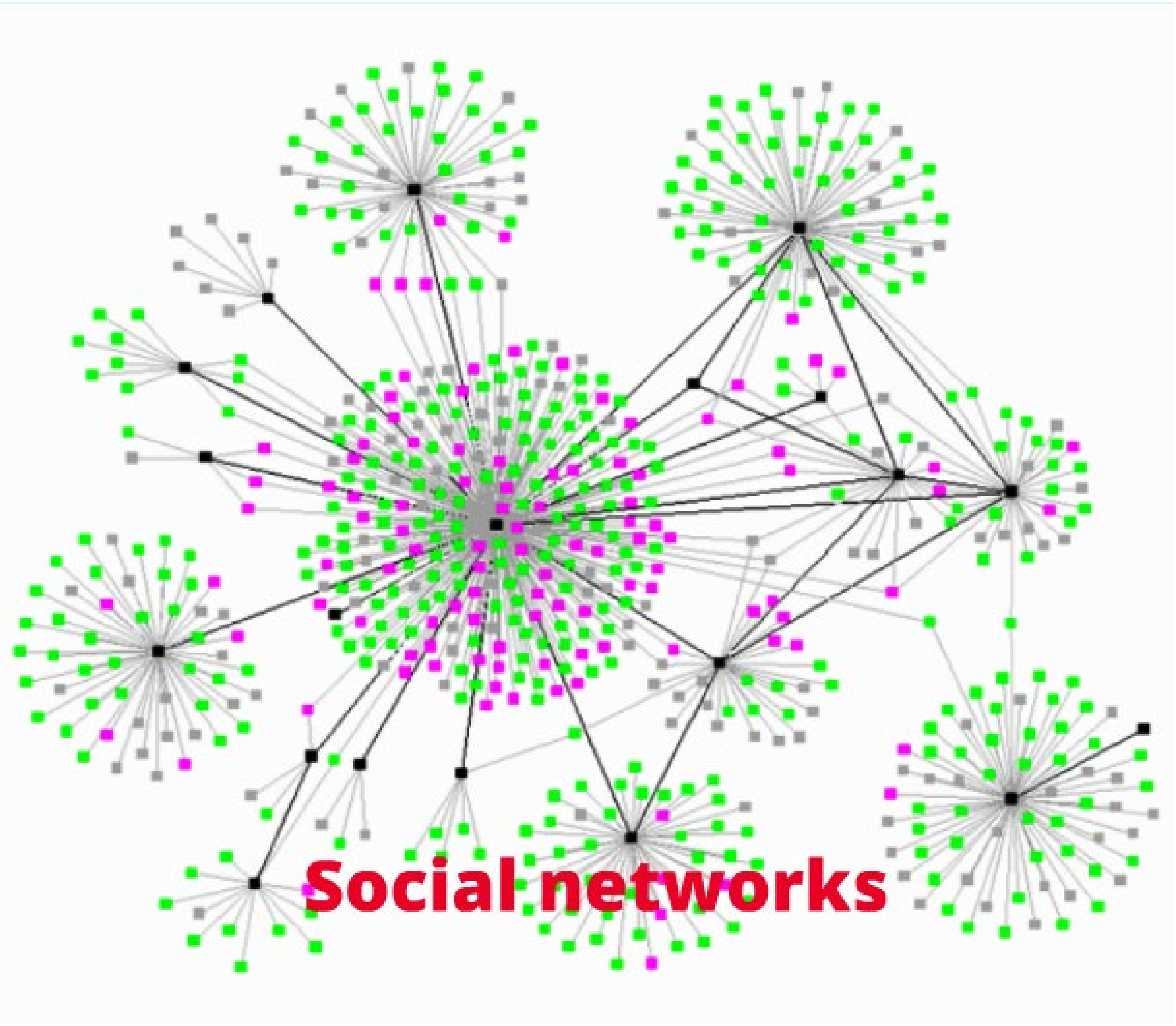
  A: arcs



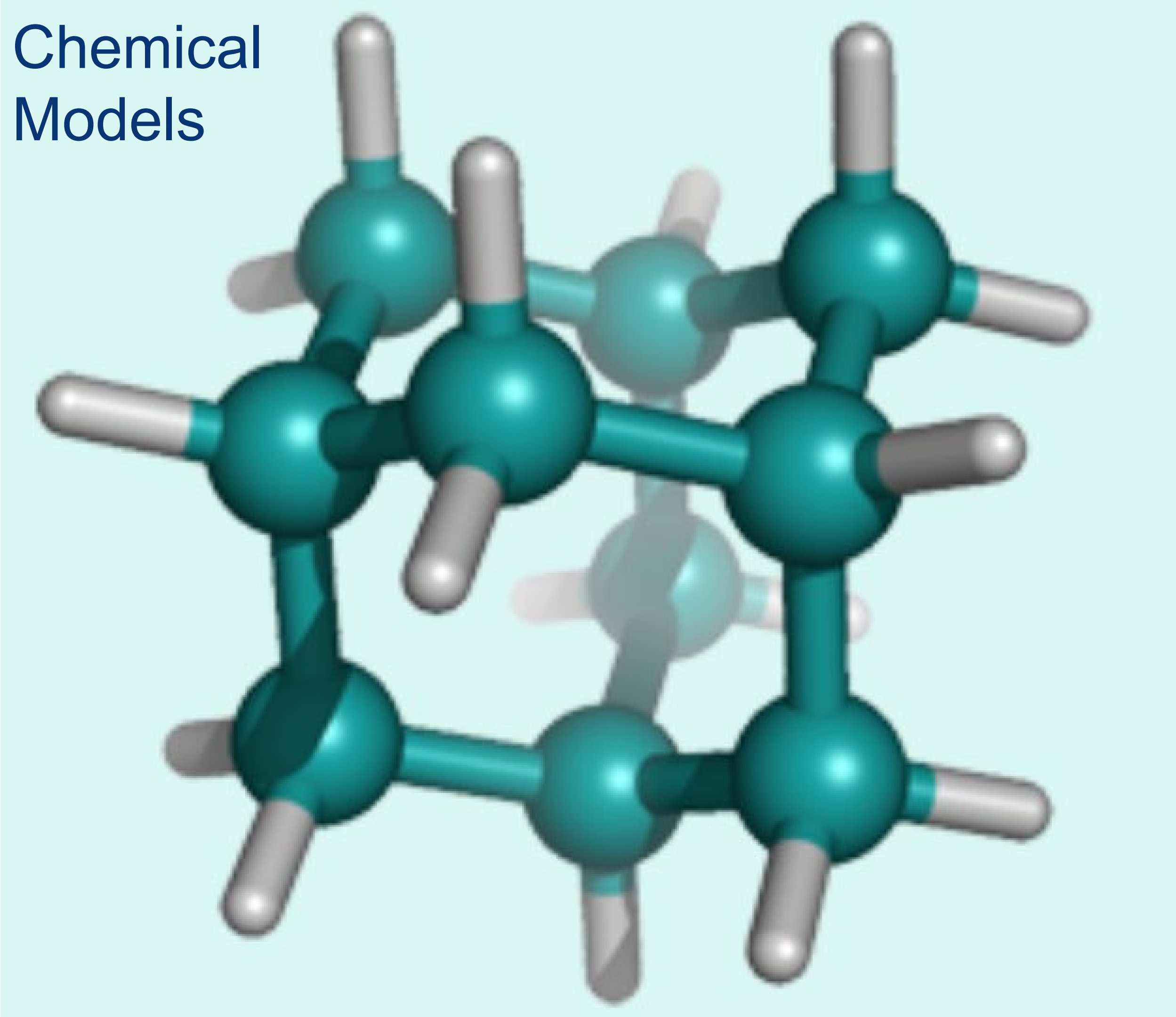
More Examples



More Examples (2)

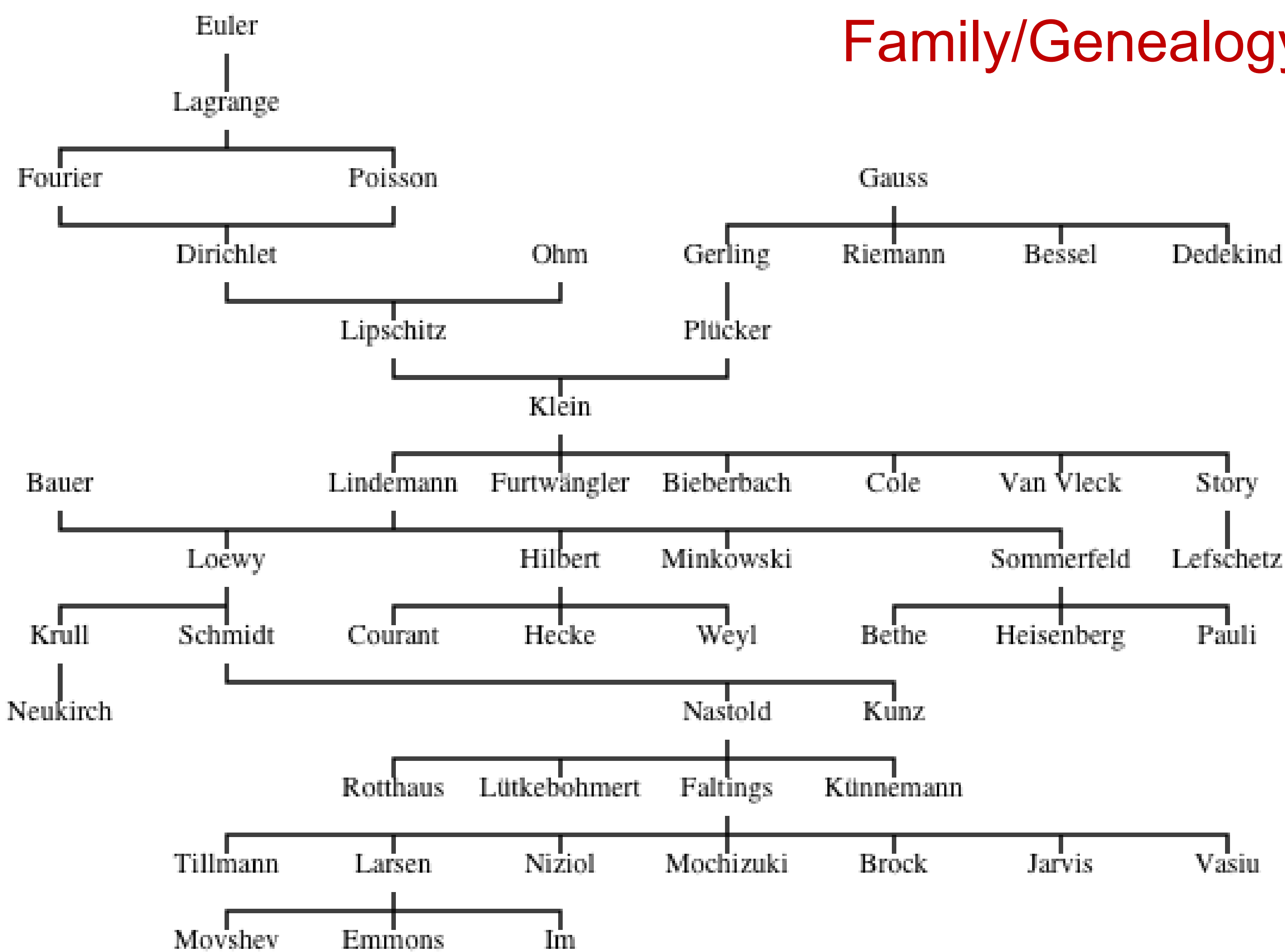


More Examples (3)

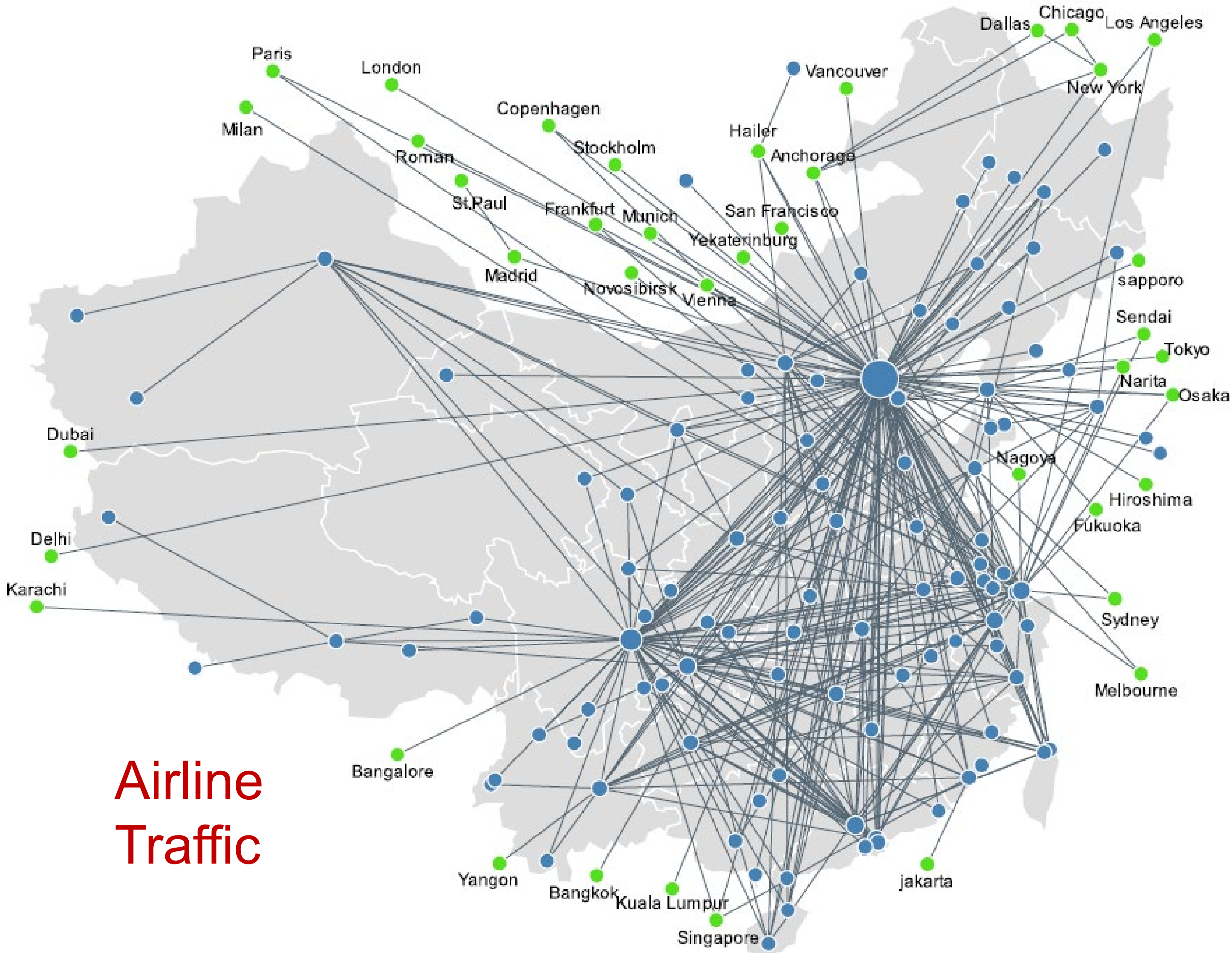


More Examples (4)

Family/Genealogy Tree



More Examples (5)



Airline Traffic

More Examples (6)

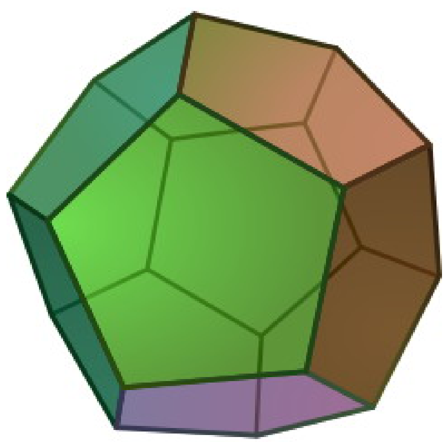
Basketball Pass

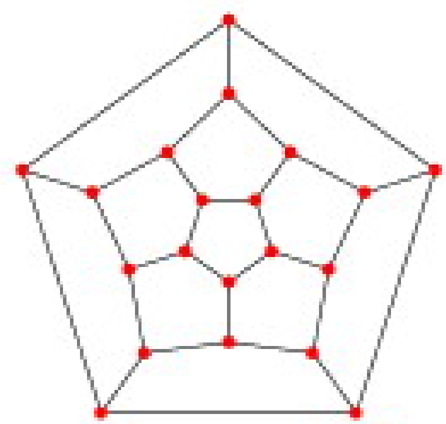
Icosian Game

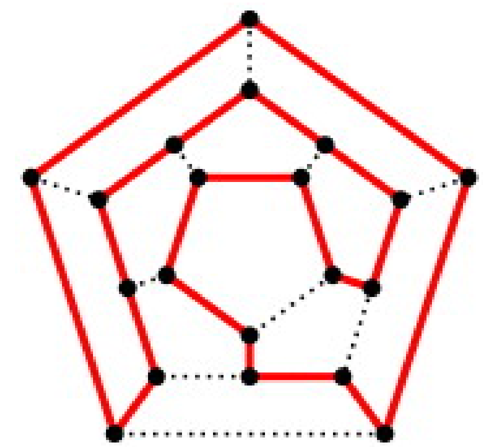
- In 1859, Sir William Rowan Hamilton developed the Icosian Game.
- Traverse the edges of an dodecahedron, i.e., a path such that every vertex is visited a single time, no edge is visited twice, and the ending point is the same as the starting point.
- Also refer as Hamiltonian Game.

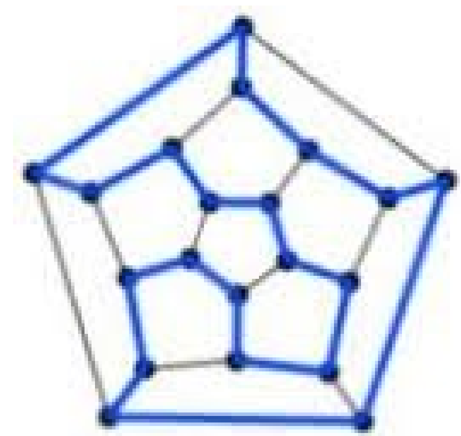
Icosian Game

- Examples









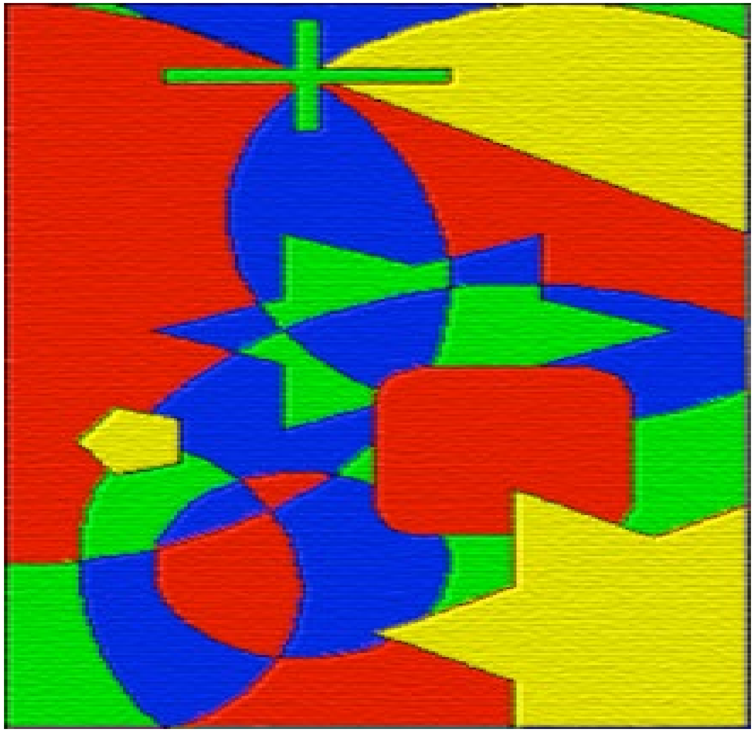
- 3D-Demo
  - http://mathworld.wolfram.com/IcosianGame.html

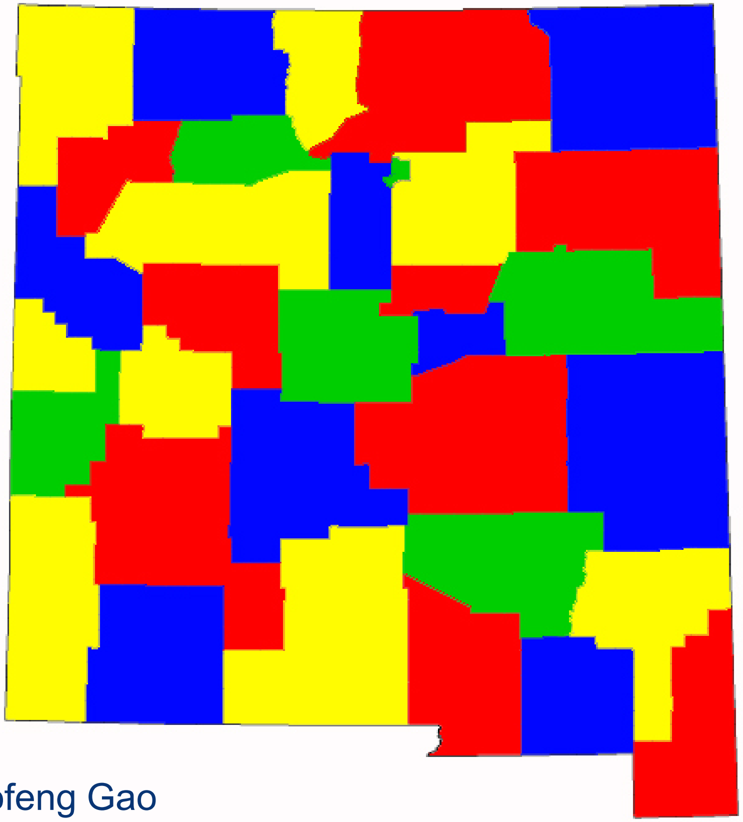
Four Color Theorem

- The four color theorem was stated, but not proved, in 1853 by Francis Guthrie.

Four Color Theorem

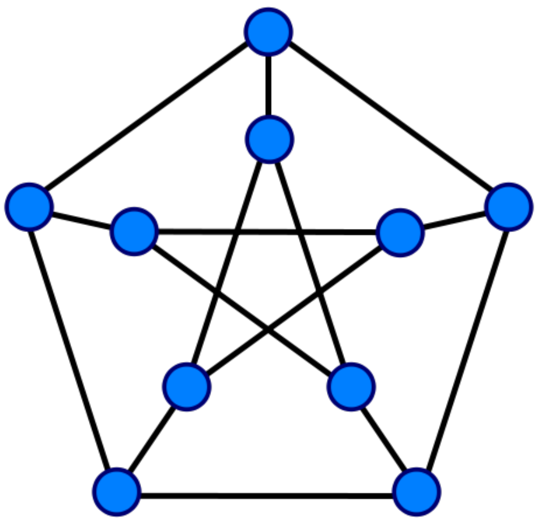
- The theorem asserts that four colors are enough to color any geographical map in such a way that no neighboring two countries are of the same color.



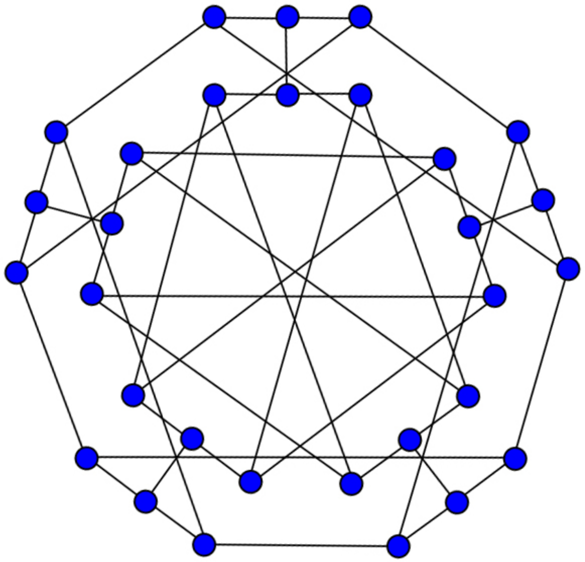


Snark Graph

- Each point has degree 3
- Strong connectivity
  (breaking any three edges will not violating the connectivity of the graph)

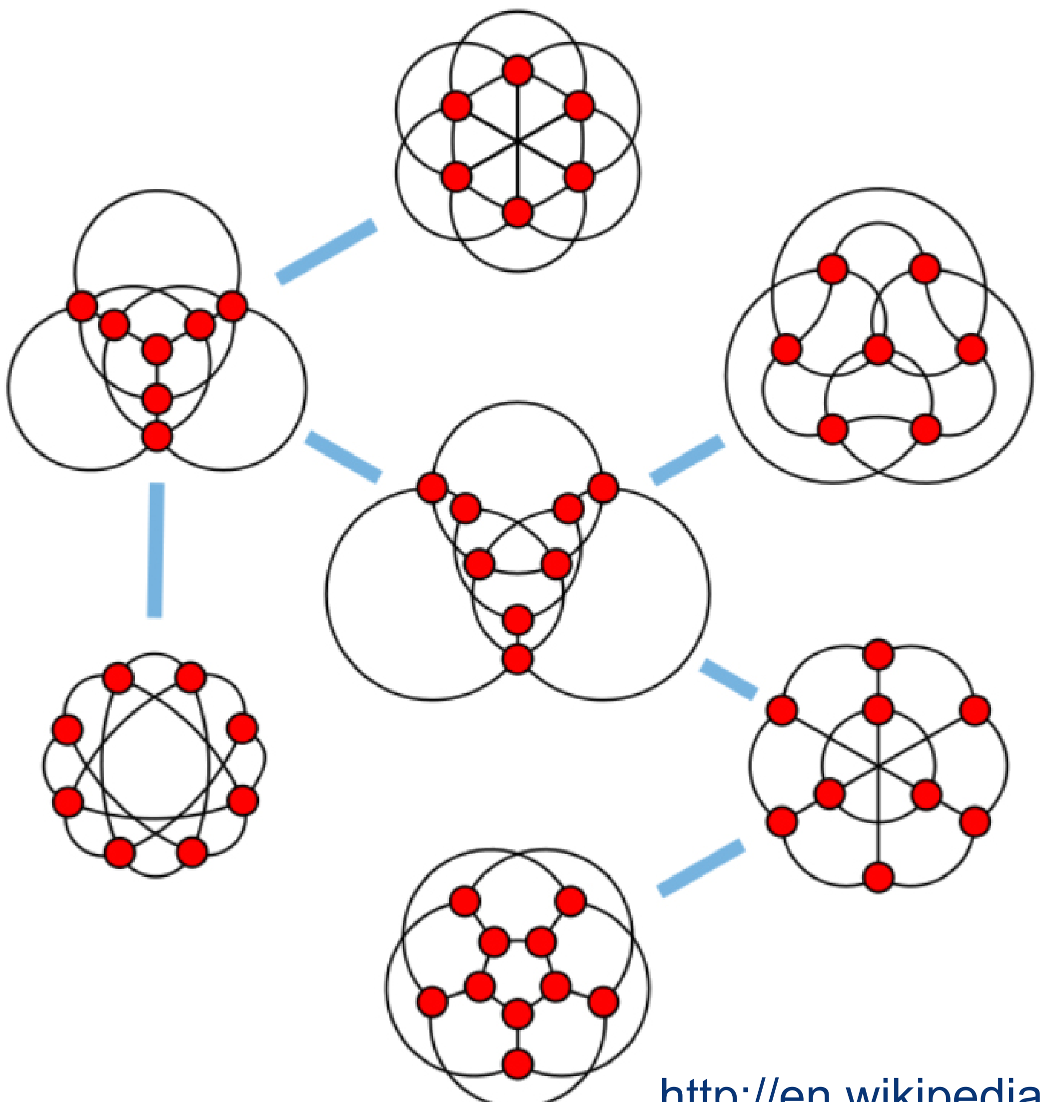


Single Star Snark
(Petersen Graph)



Double Star Snark

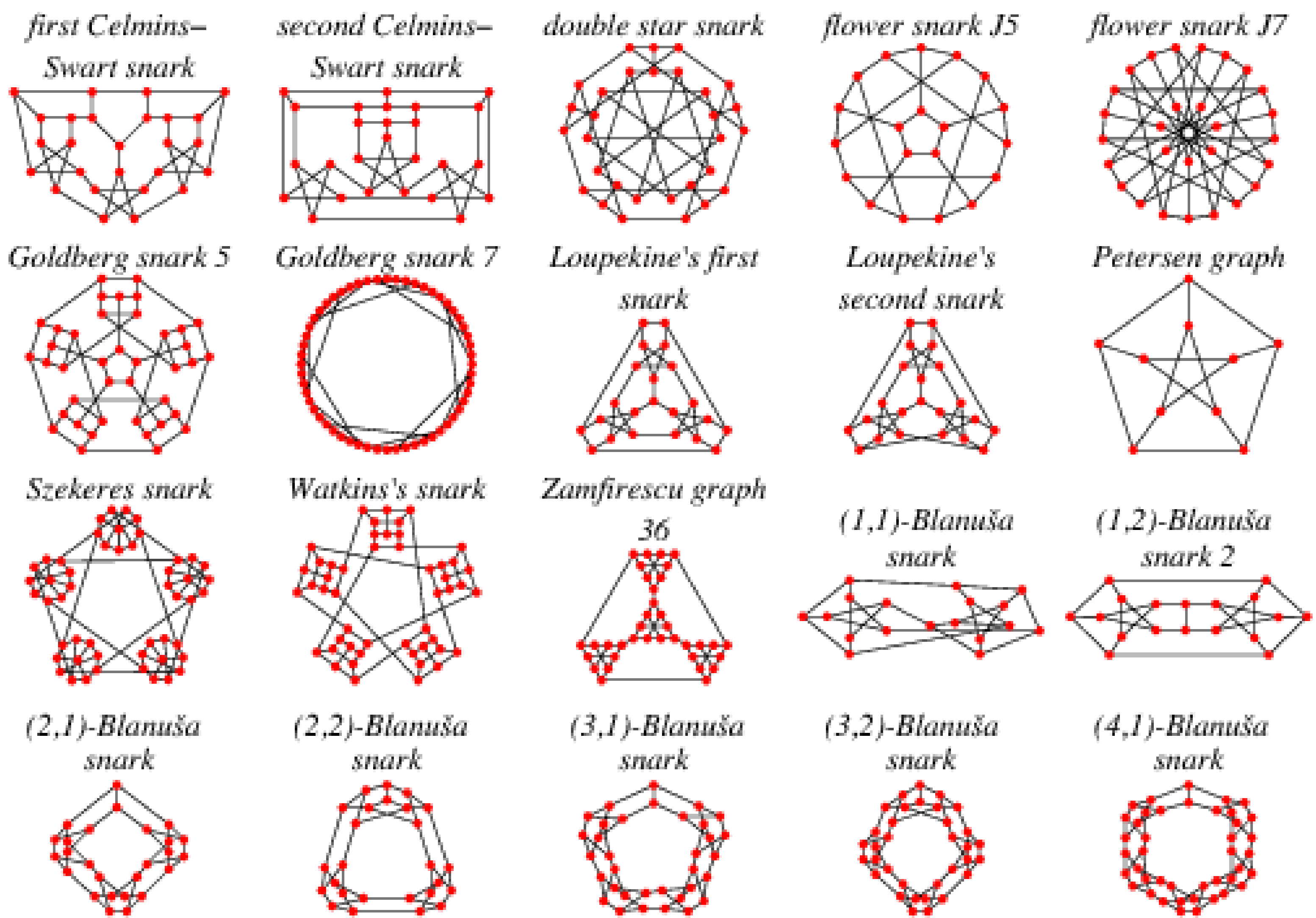
Petersen Family



Y-Δ

http://en.wikipedia.org/wiki/Petersen_graph

Snark Family

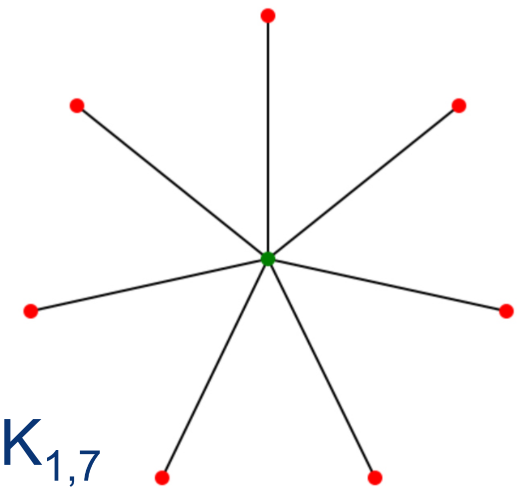


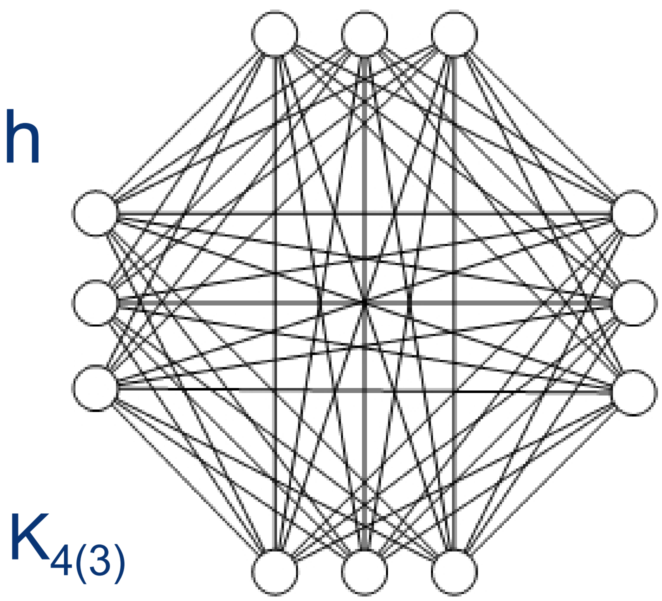
http://en.wikipedia.org/wiki/Snark_(graph_theory)

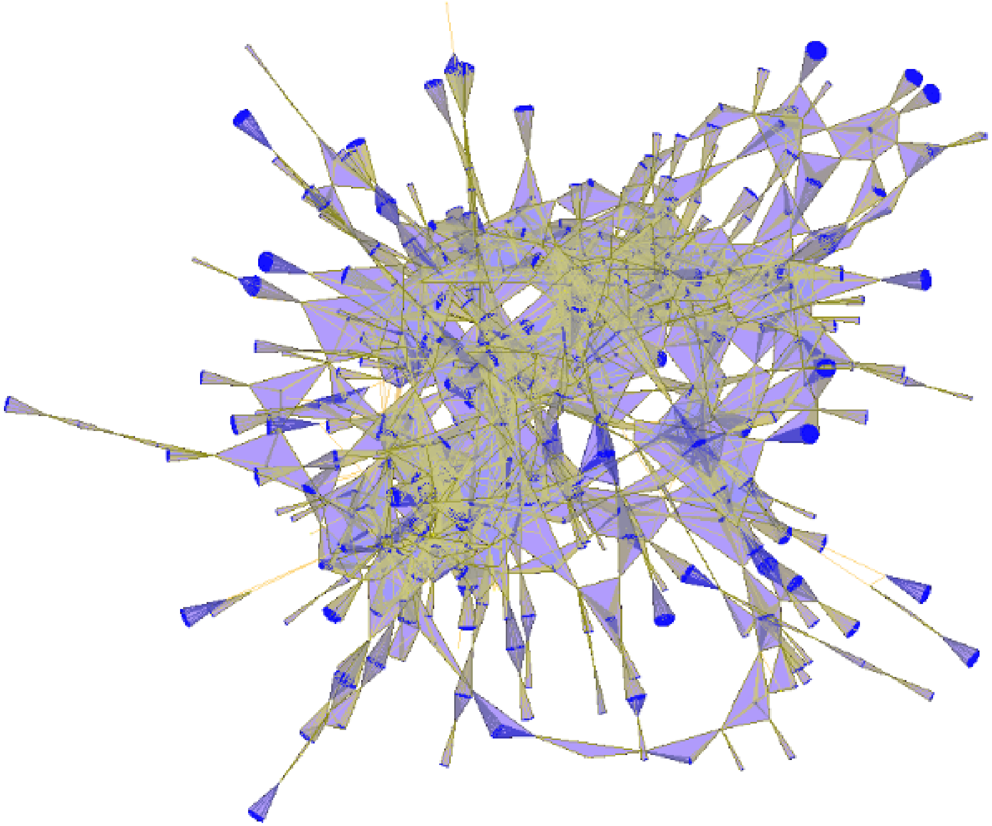
Well-Known Results

- Complete Graph K_n
- Bipartite Graph $K_{m,n}$
- Star $K_{1,n}$
- r-Partite Graph $K_{r(m)}$
- Subgraph $H \subseteq G$
  - Spanning/Induced Subgraph
- Handshaking Theorem

$$\sum_{v \in V} d(v) = 2|E|$$







INTRODUCTION TO GRAPH ALGORITHMS

Three Categories

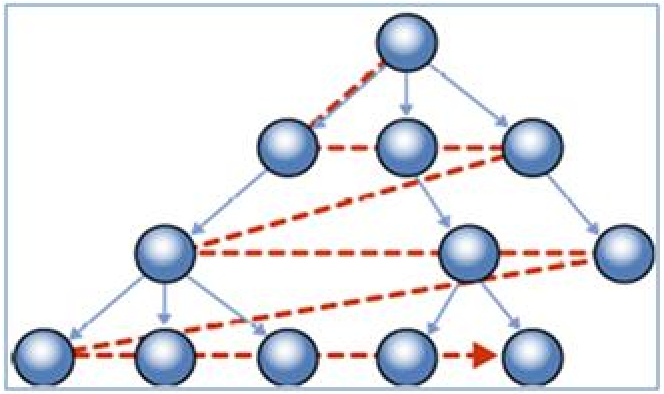
Algorithms on Graphs

- **Graph Exploration**
  - Breadth-First Search
  - Depth-First Search
- **Minimum Spanning Tree**
  - Prim Algorithm
  - Kruskal Algorithm
  - Circle Deletion Algorithm
- **Shortest Path Problem**
  - Single-Source Shortest Path
  - All-Pairs Shortest Path

Breadth-First Search

❖ Basic Strategy

- visit and inspect a node of a graph
- gain access to visit the nodes that neighbor the currently visited node

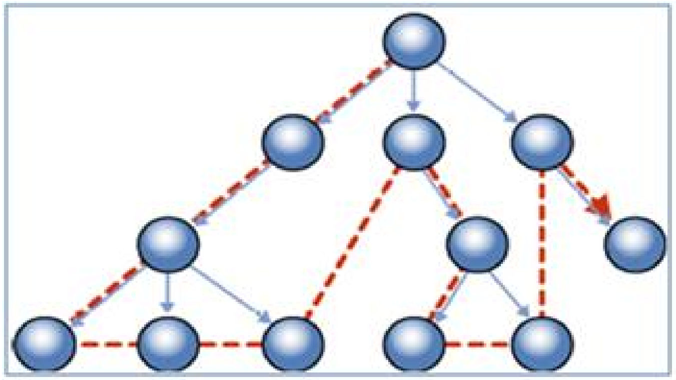


❖ Applications and Theories

- Find nodes within one connected component
- Find shortest path between two nodes u and v
- Ford-Fulkerson method for computing the maximum flow in a flow network

Depth-First Search

- **Basic Strategy**
  - Starts at arbitrary root
  - Explores as far as possible along each branch before backtracking



- **Applications and Theories**
  - Finding (strong) connected components
  - Topological sorting
  - Solving mazes puzzles with only one solution
  - parenthesis theorem

Minimum Spanning Tree

Classical Algorithms

- Prim: maintain an optimal subtree
- Kruskal: maintain min-weight acyclic edge set
- Reverse-Delete: circle-deletion
- Borüvka Algorithm

Fundamental Results

- All greedy Approach with exchange property
- Correctness proof: cycle/cut property
- Efficiency: time complexity → heap

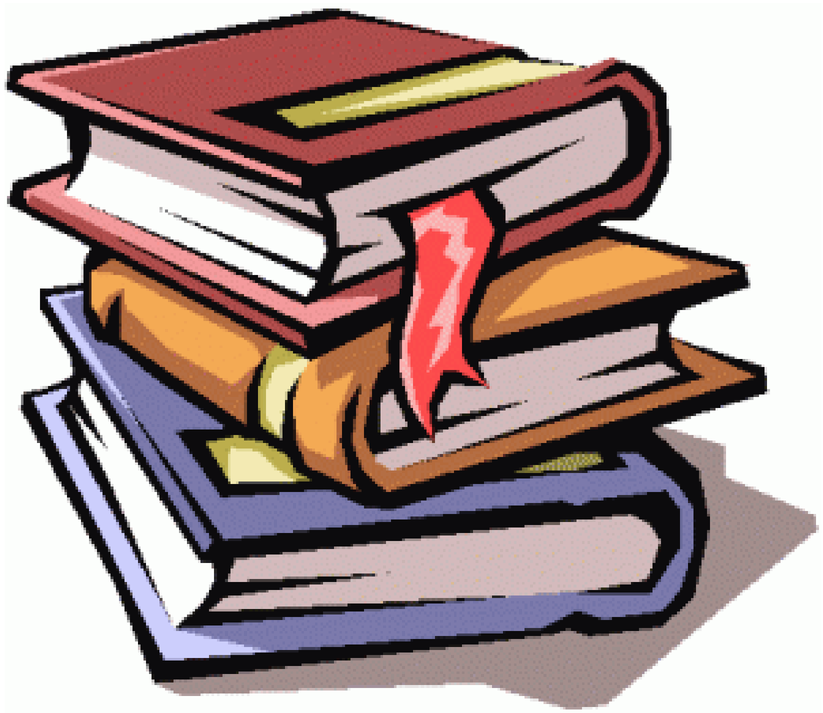
Single-Source Shortest Path

- **Dijkstra’s Algorithm**
  - Greedy Approach
  - Graph with positive weights

- **Bellman-Ford Algorithm**
  - Dynamic Programming
  - Graph with negative weights (without negative-cycle)

All-Pair Shortest Path

- **Basic Dynamic Programming**
  - Matrix multiplication
  - Time Complexity: $\Theta(n^3 \lg n)$.
- **Floyd-Warshall algorithm**
  - Also dynamic programming, but faster
  - Time Complexity: $\Theta(n^3)$
- **Johnson's algorithm**
  - For sparse graph
  - Time Complexity: $O(VE + V^2 \lg V)$.



REFERENCES

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Textbook – CLRS Book

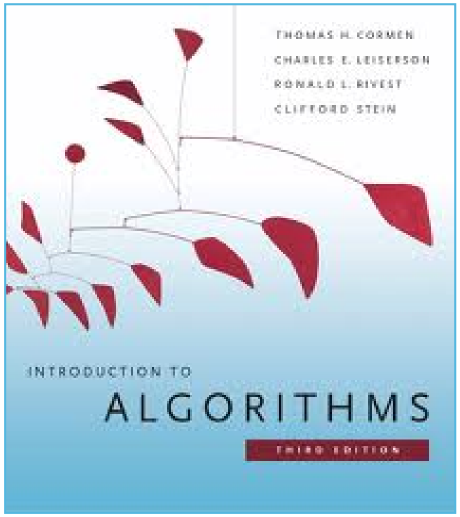
Title: Introduction to Algorithms

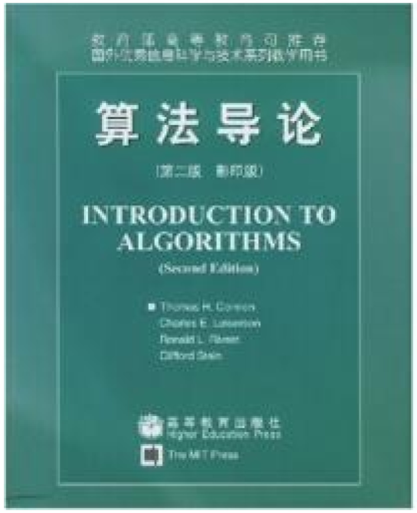
Author: T. Cormen, C. Leiserson, R. Rivest, C. Stein

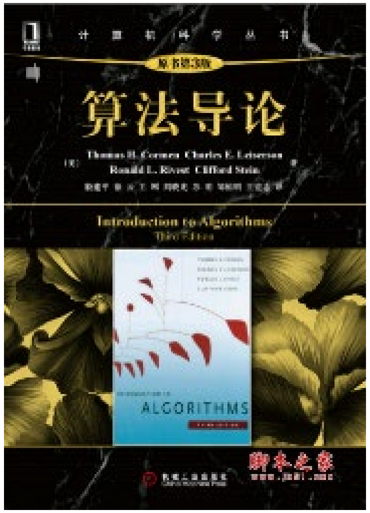
Publisher: The MIT Press, 2018 (First Edition in 1990)

ISBN-10/13: 0262033844 / 978-0262033848

Edition: Second/Third Edition/Forth Edition







The Authors – CLRS Book

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(Ph.D. from MIT)

- 1st Edition: 1990 - CLR Book (The Big White Book)
- 2nd Edition: 2001 - CLRS Book (The Big Book)
- 3rd Edition: 2009 (Has Chinese Version)
- 4th Edition: 2018 (The Latest)

Universities using CLRS Book

- 24 / Top 40 Univ. use CLRS book (33 courses)

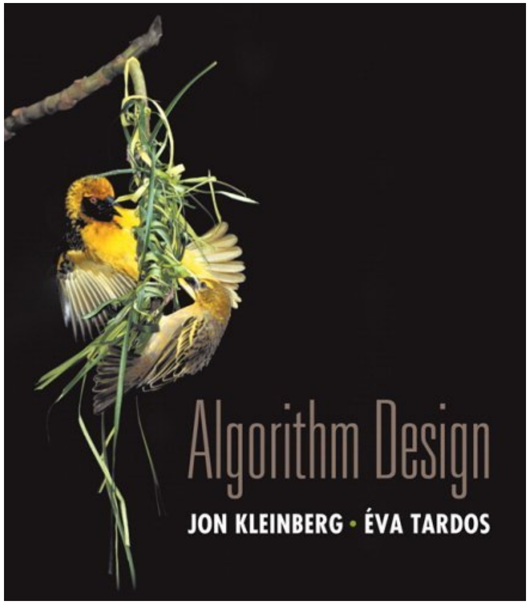
Textbook – Cornell Book

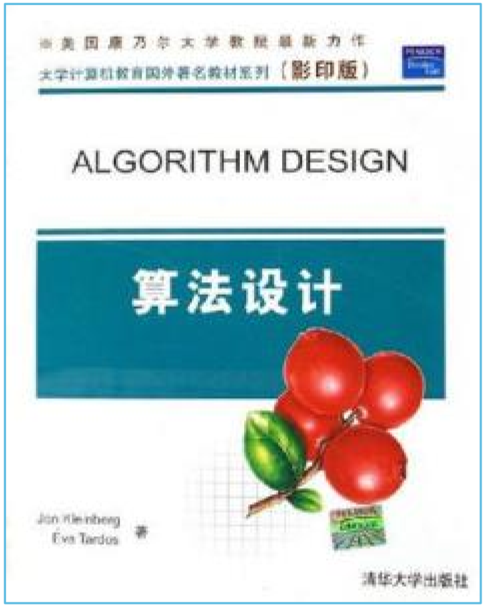
Title: Algorithm Design

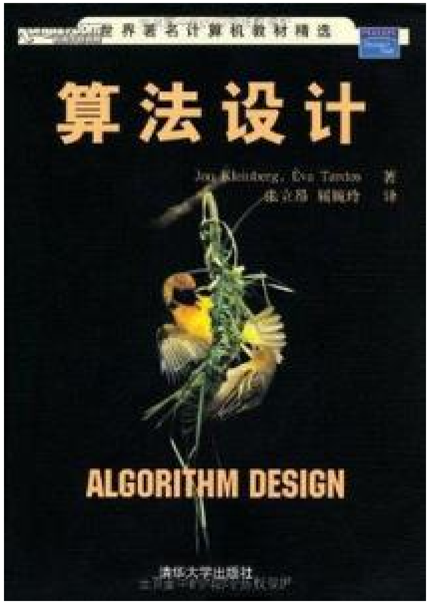
Author: J. Kleinberg, E. Tardos

Publisher: Addison Wesley, 2006.

ISBN-10/13: 0321295358 / 978-0321295354







The Authors – Cornell Book

Jon Kleinberg
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(Ph.D. from MIT)

Éva Tardos
Cornell University
(Hungarian mathematician, Gödel Prize)

- **1st Edition: 2006 – Coauthored with Éva Tardos**
- **2nd Edition: 2013 – Sole Author**

Jon Kleinberg's older brother → Robert Kleinberg@Cornell

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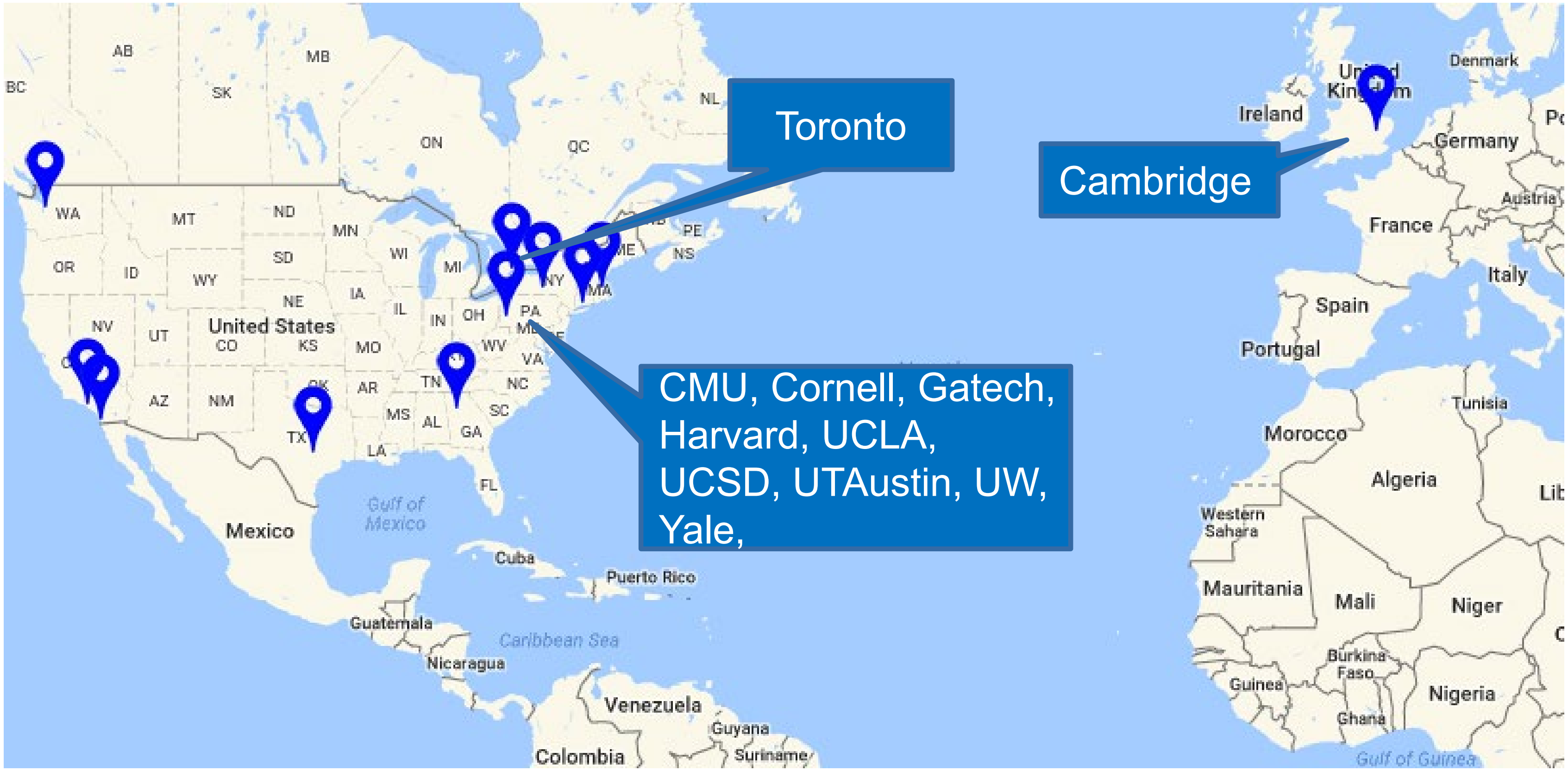


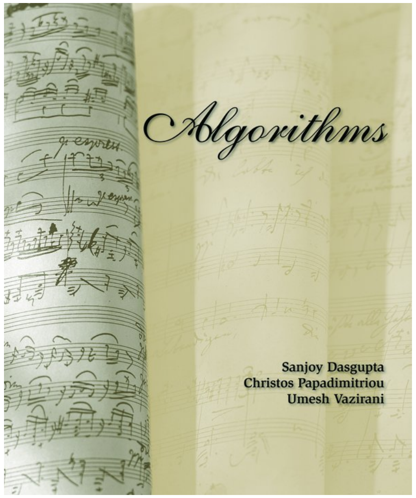
Textbook – Berkeley Book

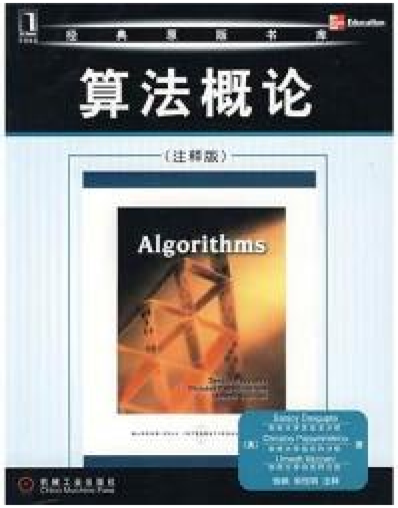
Title: Algorithms

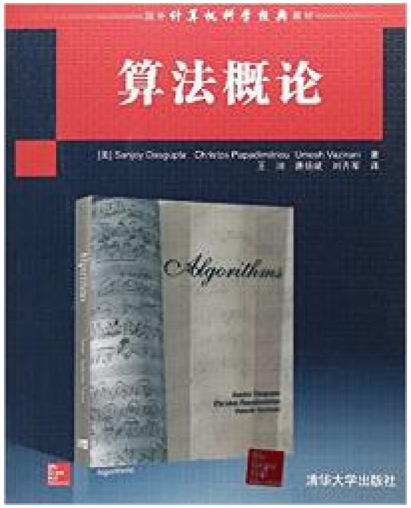
Author: S. Dasgupta, C. Papadimitriou, U. Vazirani

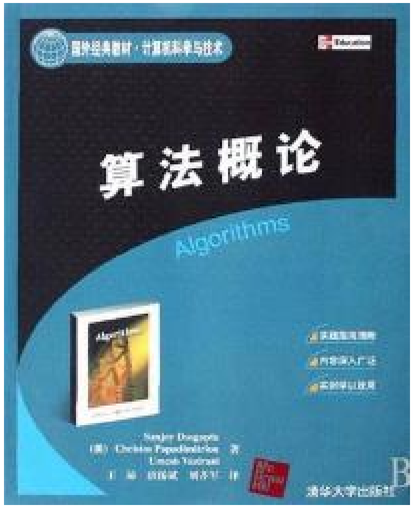
Publisher: McGraw-Hill, 2007.

ISBN-10/13: 0073523402 / 978-0073523408









The Authors – Berkeley Book

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Papadimitriou
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(Greek, Gödel Prize)

Umesh V. Vazirani
UC-Berkeley
(Indian)

- **Dasgupta: Algorithmic Statistics**
- **Papadimitriou: Complexity, Combinatorial Optimization**
- **Umesh Vazirani: Quantum Computing**
  His younger brother → Vijay Vazirani@Gatech

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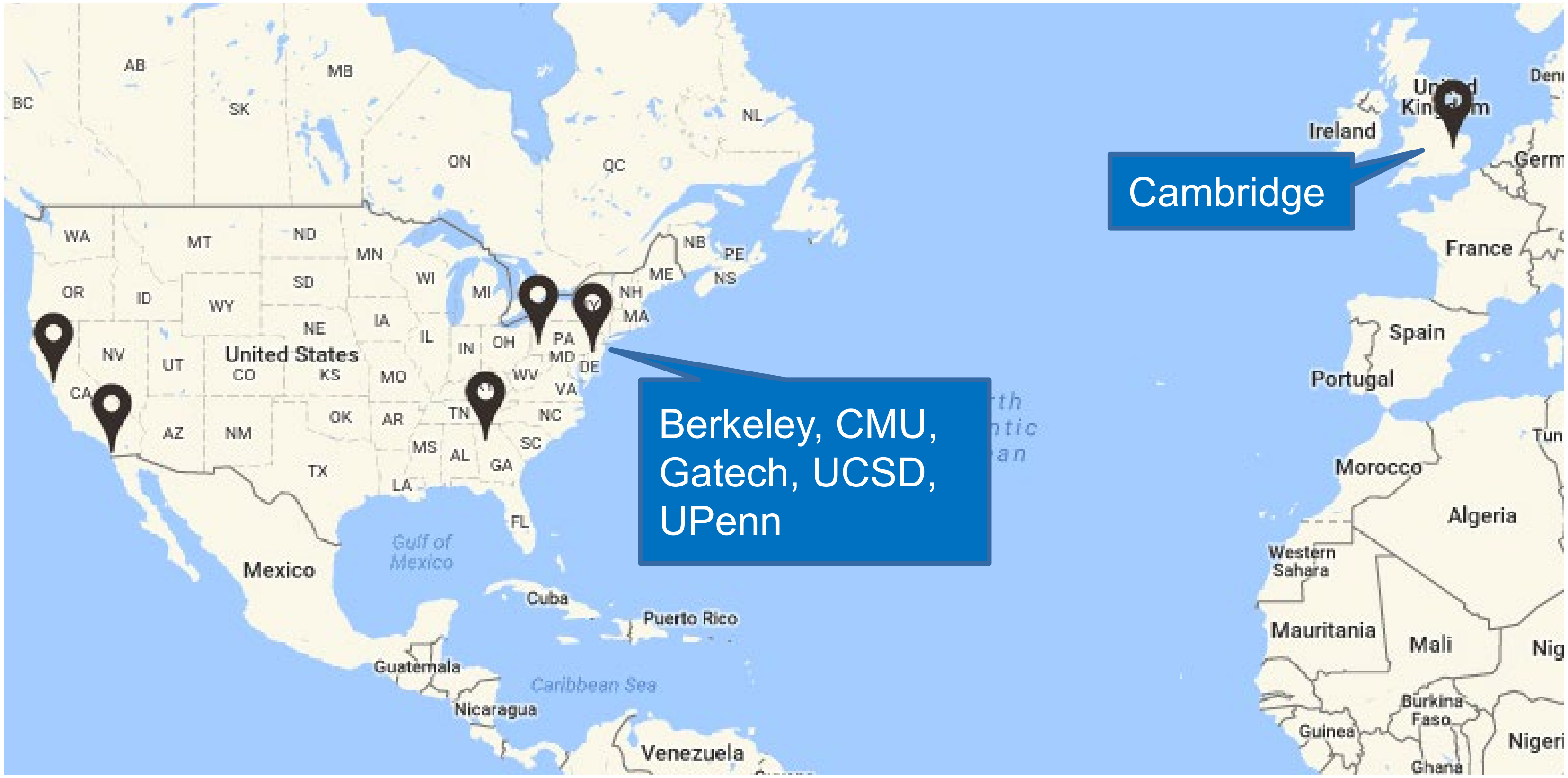

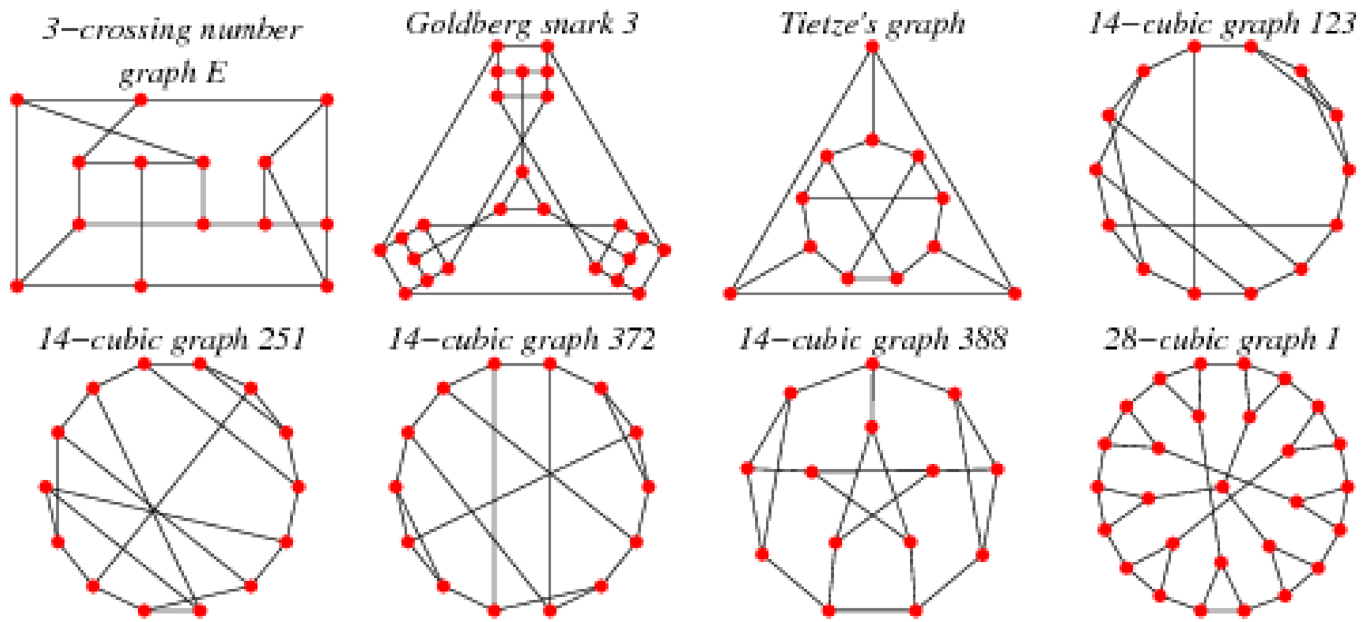

The End !

Xiaofeng Gao