Q1.

a) Domains: $\{1,2,3,4\}$

Constraints: $X_{11} < X_{12}$

$X_{13} < X_{23}$

$X_{14} < X_{24}$

$X_{32} < X_{22}$

$X_{32} < X_{42}$

$\forall i,j,k \quad (X_{ij}, X_{ik}) \notin \{(1,1),(2,2),(3,3),(4,4)\}$

$\forall i,j,k \quad (X_{ij}, X_{kj}) \notin \{(1,1),(2,2),(3,3),(4,4)\}$

b) $X_{14} \in \{1,2\}$ $\quad X_{24} \in \{2,4\}$

c) 4

d) 3

e) No. Because the possible domains for column 2 is

234
234
12
34

If we run arc consistency, we ~~fin~~ will find that X_{32} and X_{i2}, where $i \in \{1,2,4\}$, are all consistent if $X_{32} = 2$. However, when $X_{32}=2$, $(X_{12}, X_{22}) \in \{(3,4),(4,3)\}$ and thus X_{42} can't choose either 3 or 4. The fact that $X_{32}=2$ should be eliminated must be deduced from 4-variable consistency.

Q2. a) T

b) $O(d^n)$

c) $O(n^2 d^3)$

d) $O(nd^2)$

e) Choose the midde variable as a cutset and instantiate the midde variable and prune its neighbors' domains.
Then, this CSP can be solved using backwards root-to-leaf consistency check like a tree-structured CSP.