Homework 10 Written

July 31st, 2020 at 11:59pm

1 Propositional Logic 1

A logician tells to his son: "If you don't finish your dinner, you will not play video games afterwards." After the son finishes his meal, he is sent to bed right away.

Which mistake did he make by thinking that he would be able to play video games after dinner?

Solution: Suppose $A =$ "finish dinner" and $B =$ "play video games afterwards", the son made the mistake by assuming $(\neg A \Rightarrow \neg B) \Leftrightarrow (A \Rightarrow B)$, which is wrong.

2 Propositional Logic 2

Write the following sentences in CNF form.

a. $\neg(p \vee (q \wedge r))$

b. $(\neg p \Rightarrow q) \vee \neg(q \wedge r)$

c. $(p \Rightarrow \neg q) \Leftrightarrow ((q \wedge \neg r) \Rightarrow (\neg p))$

Solution:

a $\neg(p \vee (q \wedge r)) = \neg p \wedge \neg(q \wedge r) = \neg p \wedge (\neg q \vee \neg r)$

b $(\neg p \Rightarrow q) \vee \neg(q \wedge r) = (p \vee q) \vee (\neg q \vee \neg r) = T$

c

$$
\begin{aligned}
&(p \Rightarrow \neg q) \Leftrightarrow ((q \wedge \neg r) \Rightarrow (\neg p)) \\
=&(\neg p \vee \neg q) \Leftrightarrow (\neg(q \wedge \neg r) \vee \neg p) \\
=&(\neg(\neg p \vee \neg q) \vee (\neg q \vee r \vee \neg p)) \wedge ((q \wedge \neg r \wedge p) \vee (\neg p \vee \neg q)) \\
=&((p \wedge q) \vee (\neg q \vee r \vee \neg p)) \wedge ((q \wedge \neg r \wedge p) \vee (\neg p \vee \neg q)) \\
=&(p \vee \neg q \vee r \vee \neg p) \wedge (q \vee \neg q \vee r \vee \neg p) \wedge (q \vee \neg p \vee \neg q) \wedge (\neg r \vee \neg p \vee \neg q) \wedge (p \vee \neg p \vee \neg q) \\
=&\neg r \vee \neg p \vee \neg q
\end{aligned}
$$

3 Propositional Logic 3

Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \lor C$.

b. $\neg A \lor \neg B \lor \neg C \lor \neg D$.

c. $(A \Rightarrow B) \land A \land \neg B \land C \land D$.

Solution:

a 4

b 16

c 16

4 Propositional Logic 4

We have defined four binary logical connectives.

a. Are there any others that might be useful?

b. How many binary connectives can there be?

c. Why are some of them not very useful?

Solution:

a Yes. For example, the reverse implication $\Leftarrow$.

b $2^4 - 6 = 10$

c Some of them can be expressed as the negation of $\land$, $\lor$, $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$.

5 Propositional Logic 5

The inference rule *Modus Tollens* is written as follows:

$$\frac{\neg q, p \Rightarrow q}{\neg p}$$

Prove that Modus Tollens is equivalent to Modus Ponens, i.e., the latter can be proved from the former, and the other around.

Solution: Modus Ponens can be expressed as $((p \Rightarrow q) \wedge p) \Rightarrow q$ and Modus Tollens can be expressed as $(\neg p \wedge (p \Rightarrow q)) \Rightarrow \neg p$.

$$
\begin{aligned}
&(((p \Rightarrow q) \wedge p) \Rightarrow q) \Rightarrow ((\neg p \wedge (p \Rightarrow q)) \Rightarrow \neg p) \\
=&(((\neg p \vee q) \wedge p) \Rightarrow q) \Rightarrow ((\neg p \wedge (\neg p \vee q)) \Rightarrow \neg p) \\
=&(\neg((\neg p \vee q) \wedge p) \vee q) \Rightarrow (\neg(\neg p \wedge (\neg p \vee q)) \vee \neg p) \\
=&((\neg(\neg p \vee q) \vee \neg p) \vee q) \Rightarrow ((p \vee \neg(\neg p \vee q)) \vee \neg p) \\
=&(((p \wedge \neg q) \vee \neg p) \vee q) \Rightarrow ((p \vee (p \wedge \neg q)) \vee \neg p) \\
=&(((p \vee \neg p) \wedge (\neg q \vee \neg p)) \vee q) \Rightarrow ((p \vee \neg p) \vee ((p \wedge \neg q) \vee \neg p)) \\
=&((\neg q \vee \neg p) \vee q) \Rightarrow T \\
=&T \Rightarrow T \\
=&T
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&((\neg p \wedge (p \Rightarrow q)) \Rightarrow \neg p) \Rightarrow (((p \Rightarrow q) \wedge p) \Rightarrow q) \\
=&\ldots\ldots \\
=&T \Rightarrow T \\
=&T
\end{aligned}
$$

Therefore, we have proved that Modus Tollens is equivalent to Modus Ponens.

6 First-Order Logic 1

Translate the following sentences in first-order logic:

a. Alice likes everything that Bob dislikes.
b. Bob doesn't like everything Alice likes.
c. Charles doesn't like anything Alice likes.
d. David likes anything everybody else dislikes.
e. I like writing sentences in first-order logic.
f. A parent of my sibling is my parent.
g. A child of my parent, who is not me, is my sibling.

Try to use a minimum number of predicates, functions, and constants.

Solution:

a $\forall x$ thing(x) $\wedge$ $\neg$like(Bob, x) $\Rightarrow$ like(Alice, x)

b $\forall x$ thing(x) $\wedge$ like(Alice, x) $\Rightarrow$ $\neg$like(Bob, x)

c $\forall x$ thing(x) $\wedge$ like(Alice, x) $\Rightarrow$ $\neg$like(Charles, x)

d $\forall x \forall y$ thing(x) $\wedge$ person(y) $\wedge$ $\neg$like(y, x) $\Rightarrow$ like(David, x)

e $\forall x$ sentence(x) $\wedge$ FirstOrderLogic(x) $\Rightarrow$ liketowrite(I, x)

f $\forall x \forall y$ isSibling(I, x) $\wedge$ isParent(x, y) $\Rightarrow$ isParent(I, y)

g $\forall x \forall y$ isParent(x, y) $\wedge$ $\neg$isMe(x) $\Rightarrow$ isSibling(I, x)

7 First-Order Logic 2

Translate into good, natural English (no xs or ys):

$$\forall x, y, l\ SpeaksLanguage(x, l) \wedge SpeaksLanguage(y, l) \Rightarrow Understands(x, y).$$

Solution: Two people who speaks the same language understand each other.

8 First-Order Logic 3

Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., particular physical instances of CDs), The vocabulary contains the following symbols:

- $CopyOf(d, a)$: Predicate. Disk d is a copy of album a.
- $Owns(p, d)$: Predicate. Person p owns disk d.
- $Sings(p, s, a)$: Album a includes a recording of song s sung by person p.
- $Wrote(p, s)$: Person p wrote song s.
- $McCartney$, $Gersharin$, $BHoliday$, Joe, $EleanorRigby$, $TheManILove$, $Revolver$: Constants with the obvious meanings.

Express the following statements in first-order logic:

a. Gershwin wrote "The Man I Love."
b. Gershwin did not write "Eleanor Rigby."
c. Either Gershwin or McCartney wrote "The Man I Love."
d. Joe has written at least one song.
e. Joe owns a copy of Revolver.
f. Every song that McCartney sings on Revolver was written by McCartney.
g. Gershwin did not write any of the songs on *Revolver*.
h. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)
i. There is a single album that contains every song that Joe has written.

j. Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."

k. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)

l. Joe owns a copy of every album on which all the songs are sung by Billie Holiday.

Solution:

a Wrote(Gershwin, "The Man I Love")

b $\neg$Wrote(Gershwin, "Eleanor Rigby")

c Wrote(Gershwin, "Eleanor Rigby") $\vee$ Wrote(McCartney, "Eleanor Rigby")

d $\exists s$ Wrote(Joe, s)

e $\exists d$ CopyOf(d, Revolver) $\wedge$ Owns(Joe, d)

f $\forall s$ Sings(McCartney, s, Revolver) $\Rightarrow$ Wrote(McCartney, s)

g $\forall s \exists p$ Sings(p, s, Revolver) $\Rightarrow \neg$Wrote(Gershwin, s)

h $\forall s \exists a \exists p$ Wrote(Gershwin, s) $\Rightarrow$ Sings(p, s, a)

i $\forall s \exists a \exists p$ Sings(p, s, a) $\wedge$ Wrote(Joe, s)

j $\exists a \exists d$ Sings(Billie Holiday, "The Man I Love", a) $\wedge$ CopyOf(d, a) $\wedge$ Owns(Joe, d)

k $\forall a \exists d \exists s$ Sings(McCartney, s, a) $\wedge$ CopyOf(d, a) $\wedge$ Owns(Joe, d)

l $\forall a \forall s \exists d$ Sings(Billie Holiday, s, a) $\wedge$ CopyOf(d, a) $\wedge$ Owns(Joe, d)