VE 492 Homework6

Due: 23:59, July.1st

Q1. Probability

(a) For the following questions, you will be given a set of probability tables and a set of conditional independence assumptions. Given these tables and independence assumptions, write an expression for the requested probability tables. Keep in mind that your expressions cannot contain any probabilities other than the given probability tables. If it is not possible, mark "Not possible."

(i) Using probability tables $\mathbf{P(A)}, \mathbf{P(A \mid C)}, \mathbf{P(B \mid C)}, \mathbf{P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(A, B \mid C)}$.

$\mathbf{P(A, B \mid C)} =$ ____________________ ○ Not possible.

(ii) Using probability tables $\mathbf{P(A)}, \mathbf{P(A \mid C)}, \mathbf{P(B \mid A)}, \mathbf{P(C \mid A, B)}$ and no conditional independence assumptions, write an expression to calculate the table $\mathbf{P(B \mid A, C)}$.

$\mathbf{P(B \mid A, C)} =$ ____________________ ○ Not possible.

(iii) Using probability tables $\mathbf{P(A \mid B)}, \mathbf{P(B)}, \mathbf{P(B \mid A, C)}, \mathbf{P(C \mid A)}$ and conditional independence assumption $\mathbf{A} \perp\!\!\!\perp \mathbf{B}$, write an expression to calculate the table $\mathbf{P(C)}$.

$\mathbf{P(C)} =$ ____________________ ○ Not possible.

(iv) Using probability tables $\mathbf{P(A \mid B, C)}, \mathbf{P(B)}, \mathbf{P(B \mid A, C)}, \mathbf{P(C \mid B, A)}$ and conditional independence assumption $\mathbf{A} \perp\!\!\!\perp \mathbf{B} \mid \mathbf{C}$, write an expression for $\mathbf{P(A, B, C)}$.

$\mathbf{P(A, B, C)} =$ ____________________ ○ Not possible.

(b) For each of the following equations, select the *minimal set* of conditional independence assumptions necessary for the equation to be true.

(i) $\mathbf{P(A, C)} = \mathbf{P(A \mid B)}\ \mathbf{P(C)}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

(ii) $\mathbf{P(A \mid B, C)} = \frac{\mathbf{P(A)\ P(B|A)\ P(C|A)}}{\mathbf{P(B|C)\ P(C)}}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

(iii) $\mathbf{P(A, B)} = \sum_{\mathbf{c}} \mathbf{P(A \mid B, c)\ P(B \mid c)\ P(c)}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp C$
- ☐ $A \perp\!\!\!\perp C \mid B$
- ☐ $B \perp\!\!\!\perp C$
- ☐ $B \perp\!\!\!\perp C \mid A$
- ☐ No independence assumptions needed.

(iv) $\mathbf{P(A, B \mid C, D)} = \mathbf{P(A \mid C, D)\ P(B \mid A, C, D)}$

- ☐ $A \perp\!\!\!\perp B$
- ☐ $A \perp\!\!\!\perp B \mid C$
- ☐ $A \perp\!\!\!\perp B \mid D$
- ☐ $C \perp\!\!\!\perp D$
- ☐ $C \perp\!\!\!\perp D \mid A$
- ☐ $C \perp\!\!\!\perp D \mid B$
- ☐ No independence assumptions needed.

(c) (i) Mark **all** expressions that are equal to $\mathbf{P(A \mid B)}$, **given no independence assumptions.**

- [ ] $\sum_c P(A \mid B, c)$
- [x] $\sum_c P(A, c \mid B)$
- [ ] $\frac{P(B|A)\ P(A|C)}{\sum_c P(B,c)}$
- [x] $\frac{\sum_c P(A,B,c)}{\sum_c P(B,c)}$
- [ ] $\frac{P(A,C|B)}{P(C|B)}$
- [ ] $\frac{P(A|C,B)\ P(C|A,B)}{P(C|B)}$
- [ ] None of the provided options.

(ii) Mark **all** expressions that are equal to $\mathbf{P(A, B, C)}$, given that $\mathbf{A \perp\!\!\!\perp B}$.

- [ ] $P(A \mid C)\ P(C \mid B)\ P(B)$
- [x] $P(A)\ P(B)\ P(C \mid A, B)$
- [ ] $P(C)\ P(A \mid C)\ P(B \mid C)$
- [ ] $P(A)\ P(C \mid A)\ P(B \mid C)$
- [x] $P(A)\ P(B \mid A)\ P(C \mid A, B)$
- [x] $P(A, C)\ P(B \mid A, C)$
- [ ] None of the provided options.

(iii) Mark **all** expressions that are equal to $\mathbf{P(A, B \mid C)}$, given that $\mathbf{A \perp\!\!\!\perp B \mid C}$.

- [x] $P(A \mid C)\ P(B \mid C)$
- [ ] $\frac{P(A)\ P(B|A)\ P(C|A,B)}{\sum_c P(A,B,c)}$
- [ ] $P(A \mid B)\ P(B \mid C)$
- [ ] $\frac{P(C)\ P(B|C)\ P(A|C)}{P(C|A,B)}$
- [ ] $\frac{\sum_c P(A,B,c)}{P(C)}$
- [x] $\frac{P(C,A|B)\ P(B)}{P(C)}$
- [ ] None of the provided options.