

CS-E4510

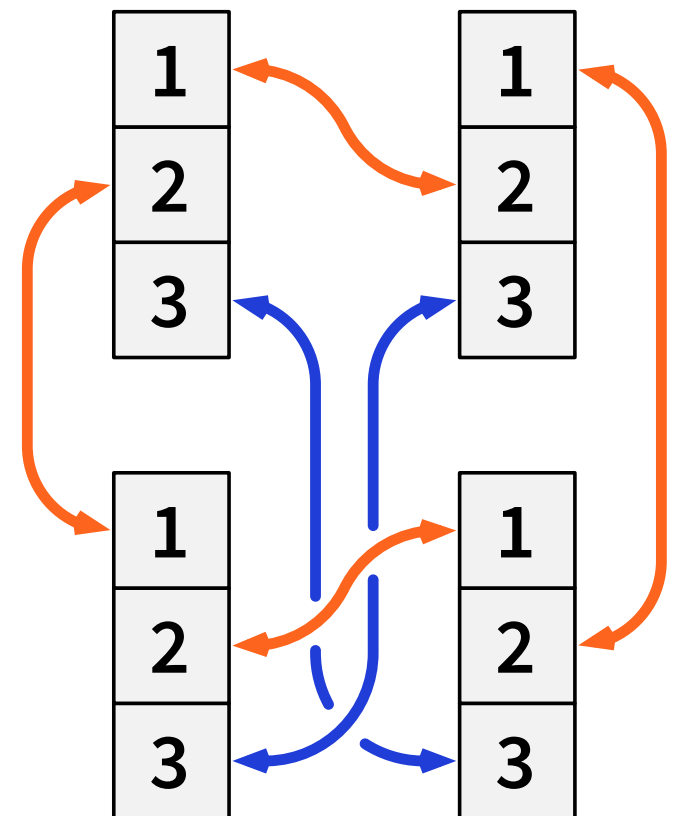
Distributed Algorithms

Jukka Suomela

Aalto University

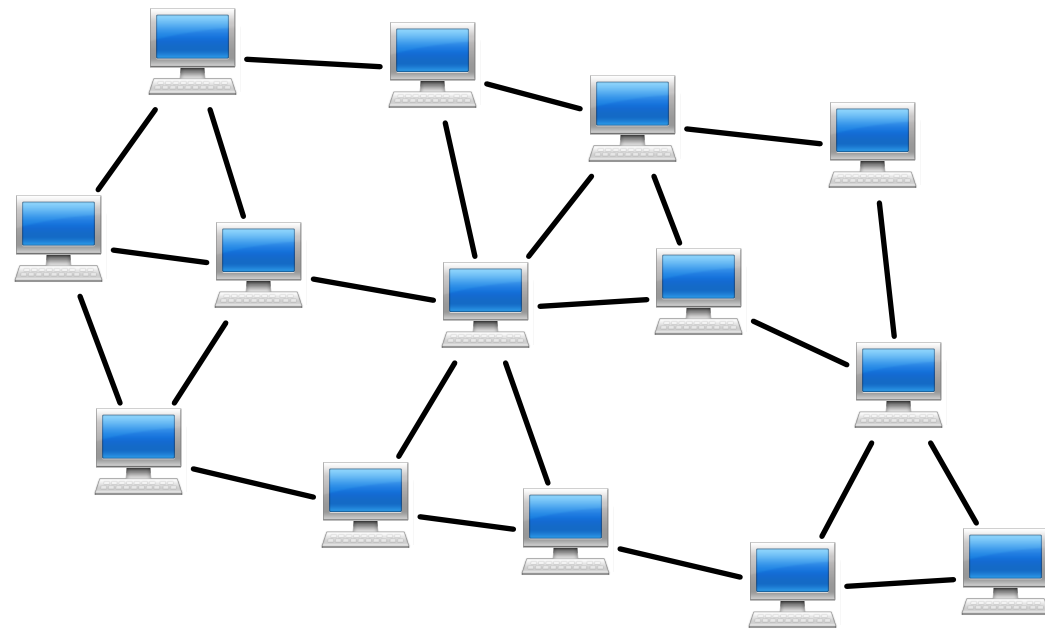
Autumn 2018

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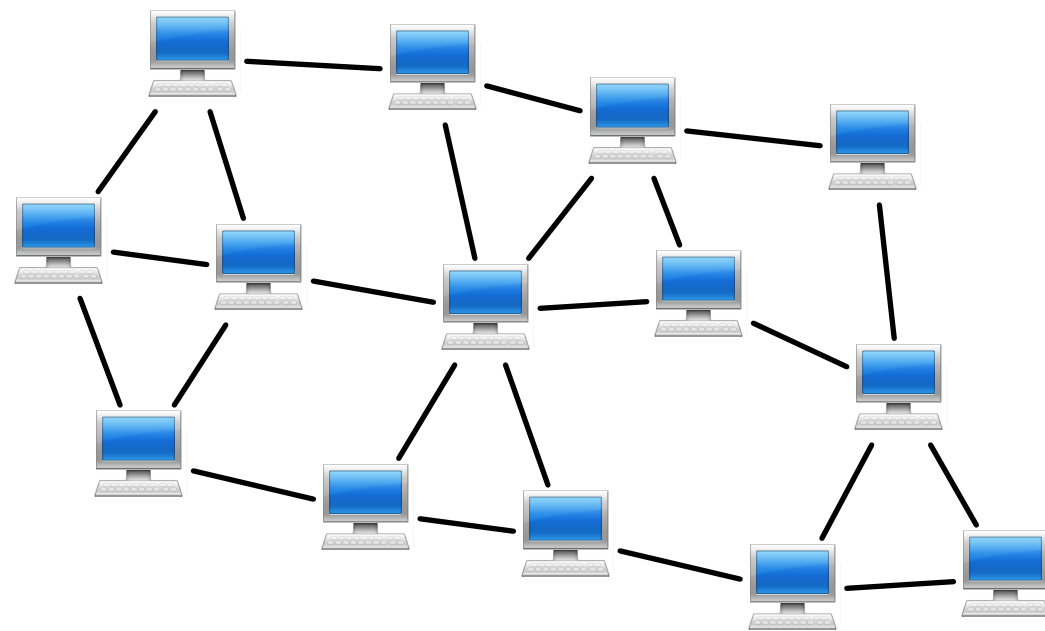
Distributed Algorithms

Algorithms for computer networks



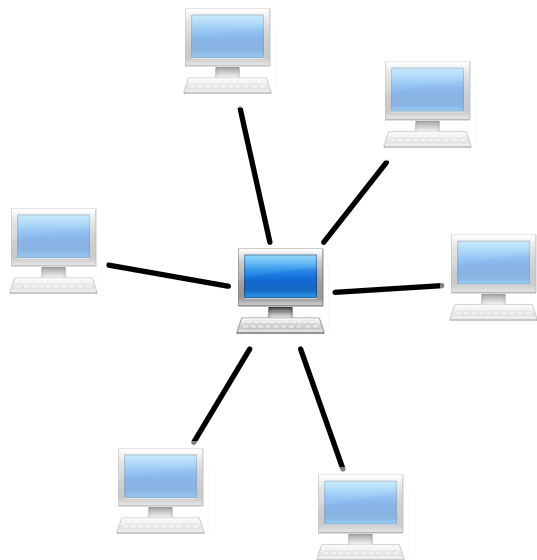
Distributed Algorithms

Identical computers in an unknown network, all running the same algorithm



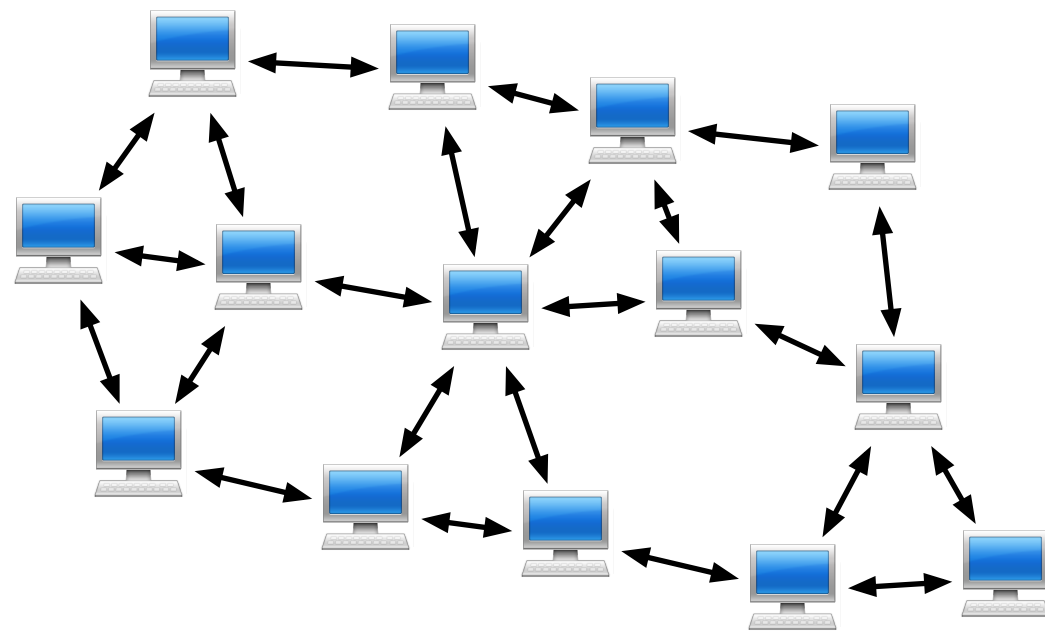
Distributed Algorithms

Initially each computer only aware of its immediate neighbourhood



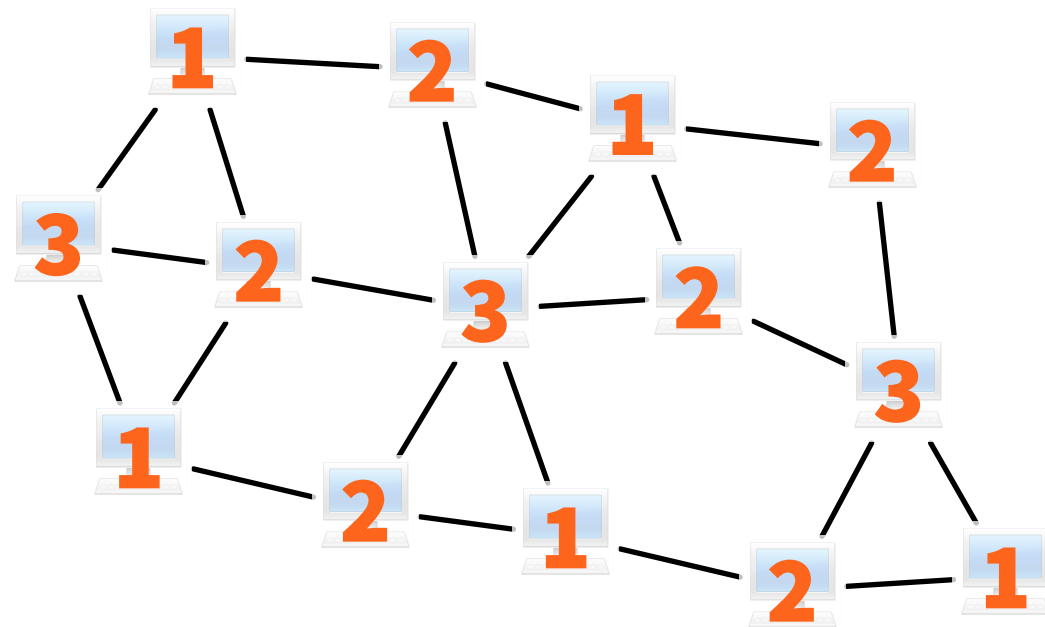
Distributed Algorithms

**Nodes can exchange messages
with their neighbours to learn more...**



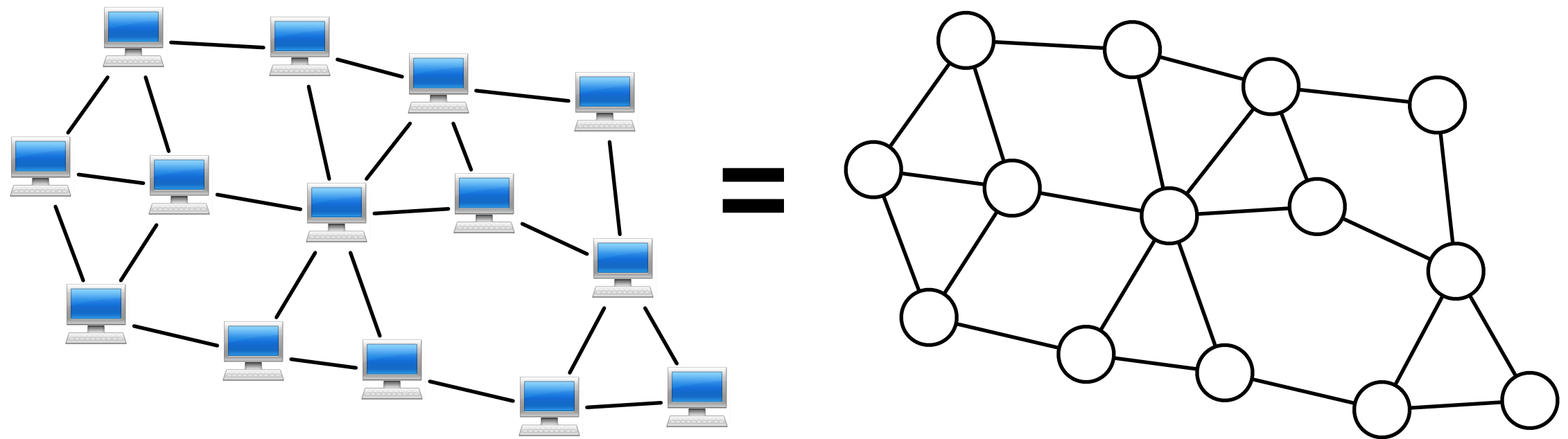
Distributed Algorithms

Finally, each computer has to stop and produce its own **local output**



Distributed Algorithms

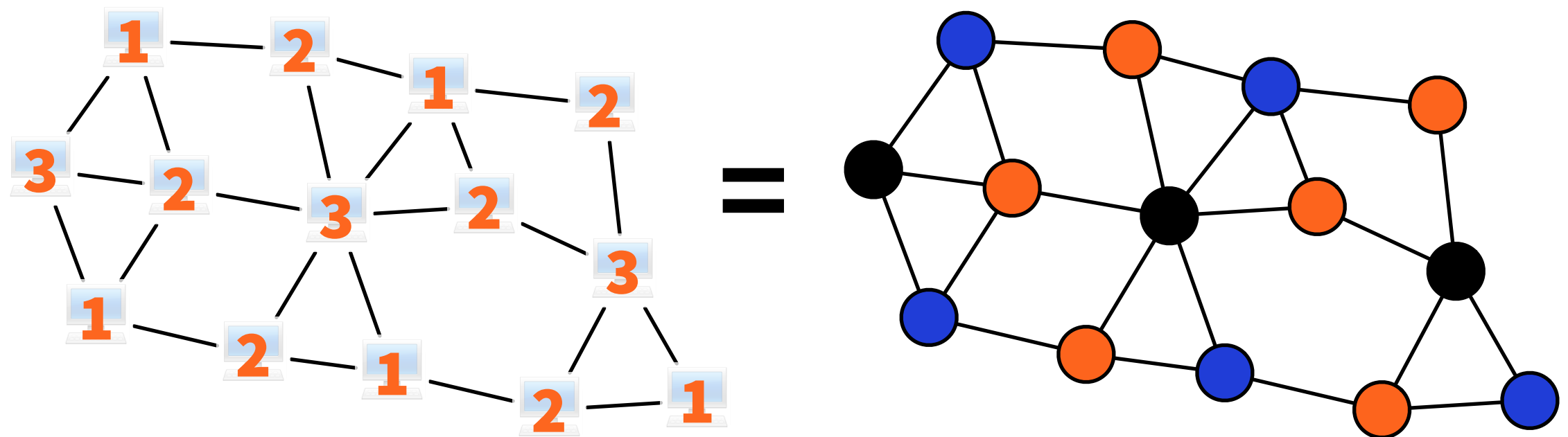
**Focus on graph problems:
network topology = input graph**



Distributed Algorithms

Focus on graph problems:

local outputs = solution (here: graph colouring)



Distributed Algorithms

Typical research question:

“How fast can we solve graph problem X ?”

Time = number of communication rounds

Why?

- 1. Applications in large-scale real-world communication networks**

Why?

- 1. Applications in large-scale real-world communication networks**
- 2. New perspective to theory of computation**

New perspective to theory of computing

- **New kinds of computational resources:**
 - old: **time & space**
 - new: **distance & bandwidth**
- **New kinds of algorithm design challenges:**
 - **parallelism & coordination**

Why?

- 1. Applications in large-scale real-world communication networks**
- 2. New perspective to theory of computation**
- 3. Understanding nature**

Understanding nature: **Algorithmic lens**

- **Distributed systems in different areas:**
 - **sociology**: collaboration networks
 - **economy**: job markets, auctions
 - **ecology**: animal populations
 - **biology**: organs, tissues
 - **chemistry**: chemical reactions ...

Understanding nature:

Algorithmic lens

- **Natural science perspective:**
 - fix a process and analyse it
- **Computational perspective:**
 - fix a goal (“computational problem”)
 - ask if there is **any** process (“algorithm”) that reaches the goal efficiently

Understanding nature: **Algorithmic lens**

- **Model nature as a distributed system**
- **Proving that something cannot be done efficiently with distributed algorithms: discovering fundamental limitations of nature**
 - producing hypotheses: “this process is slow (or our model of nature is wrong)”

Why?

- 1. Applications in large-scale real-world communication networks**
- 2. New perspective to theory of computation**
- 3. Understanding nature**

- **Weeks 1–2: informal introduction**

- network = path



- **Week 3: graph theory**

- **Weeks 4–7: models of computing**

- what can be computed (efficiently)?

- **Weeks 8–11: lower bounds**

- what cannot be computed (efficiently)?

- **Week 12: recap**

Practicalities

- All practical information in **MyCourses**
- **Textbook:**
 - freely available online
- **Exercises:**
 - every week, starting this week!

Weekly exercises

- **Tuesday at noon:** quiz (2 points)
- **Wednesday at midnight:** 1 exercise (2 points)
- **Friday at midnight:** 2 exercises (2+2 points)
- **Whenever you want:** challenging exercises (4 points each)

Grading

- **Two midterm exams:** pass/fail
- **Weekly exercises:** max **96** points (+ extra)
- **Grading:**
 - grade **1/5**: pass exams
 - grade **5/5**: pass exams + at least **80** points

Learning objectives

- **Models of distributed computing**
- **Algorithm design and analysis**
- **Computability and computational complexity**
- **Graph theory**

WARNING: THEORY

100% mathematics

(definitions, theorems, proofs...)

0% practice

(programming, hardware, protocols...)

Week 1

- Warm-up: positive results

Running example: **3-colouring a path**

Given a path:



Output a proper 3-colouring, e.g.:



Model of computing:

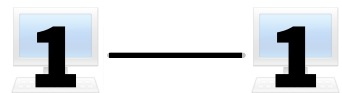
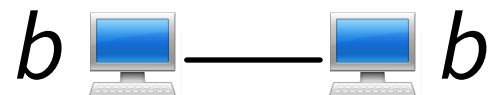
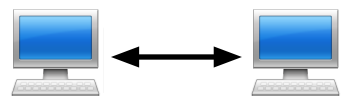
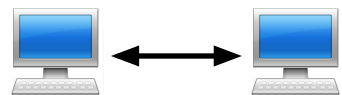
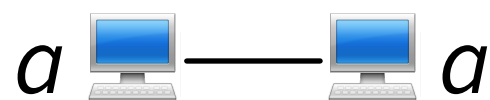
Send, receive, update

- **All nodes in parallel:**
 - send messages to their neighbours
 - receive messages from neighbours
 - update their state
- **Stopping state = final output**
 - can send/receive, but not update any more

Challenge:

Symmetry breaking

- **Identical nodes, everything deterministic and synchronised: cannot break symmetry**



same initial state

same messages sent

same messages received

same new state

...

same output

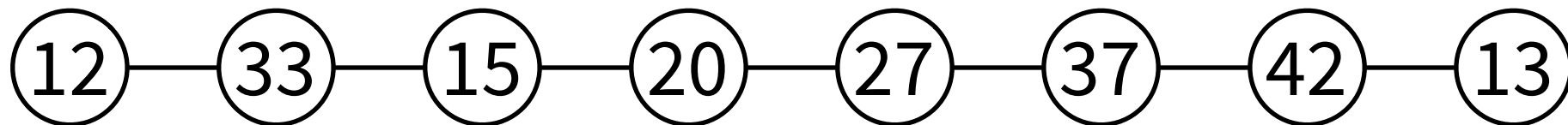
Challenge:

Symmetry breaking

- **Identical nodes, everything deterministic and synchronised: cannot break symmetry**
- **Solutions:**
 - assume unique identifiers
 - use randomised algorithms

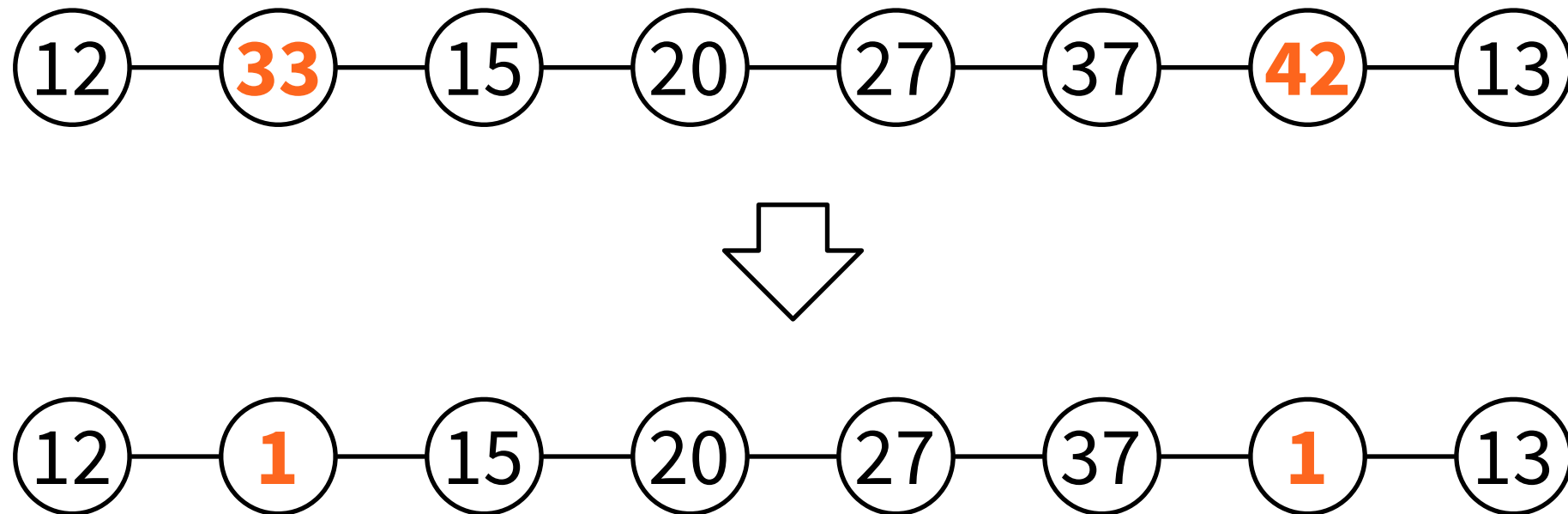
Algorithm P3C: **Using unique IDs**

- **Unique IDs = proper colouring with large number of colours**
- **Goal: reduce the number of colours**



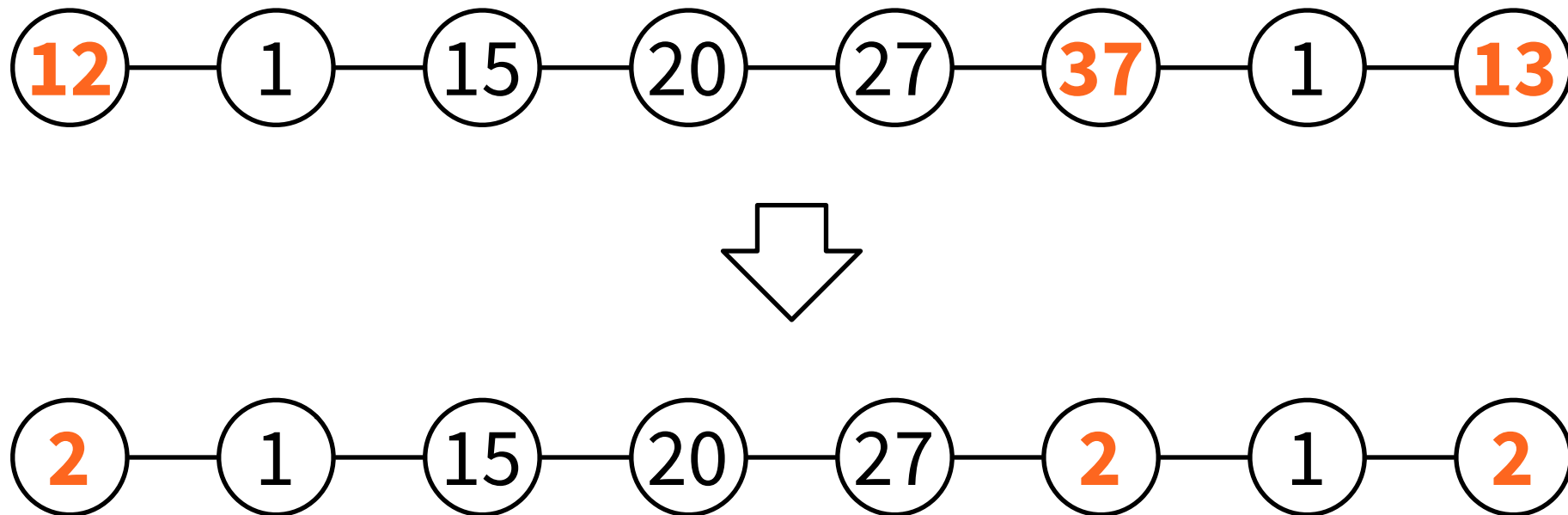
Algorithm P3C: Using unique IDs

- Idea: **local maxima** pick a new colour



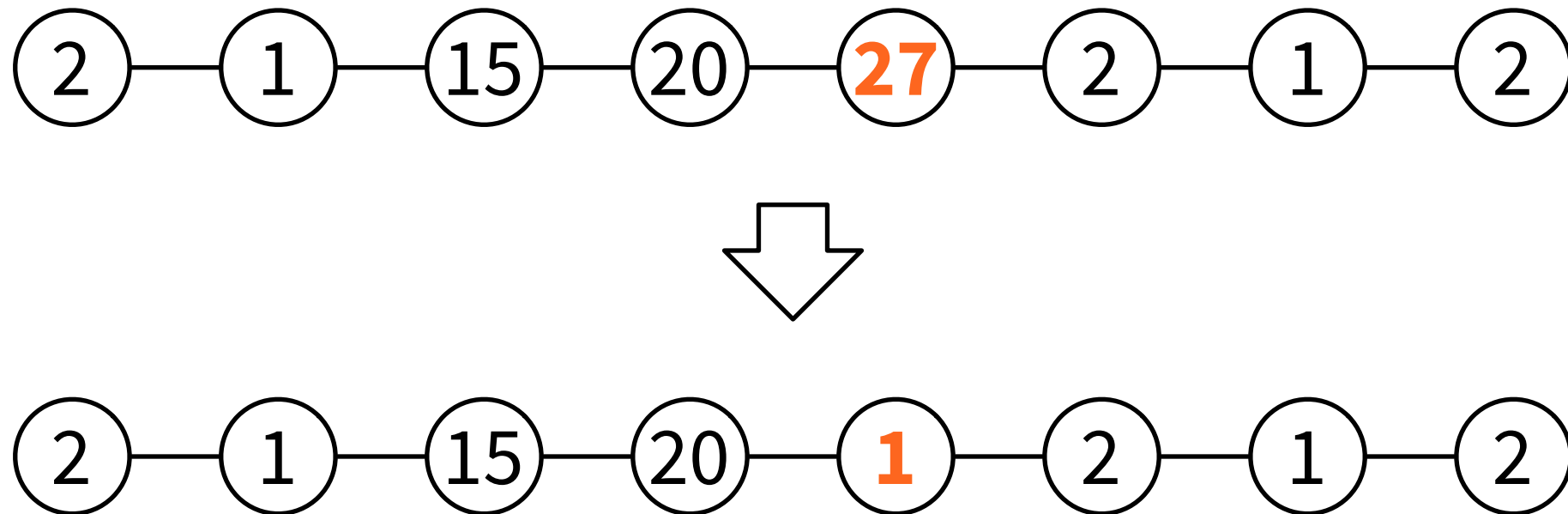
Algorithm P3C: Using unique IDs

- Idea: **local maxima** pick a new colour



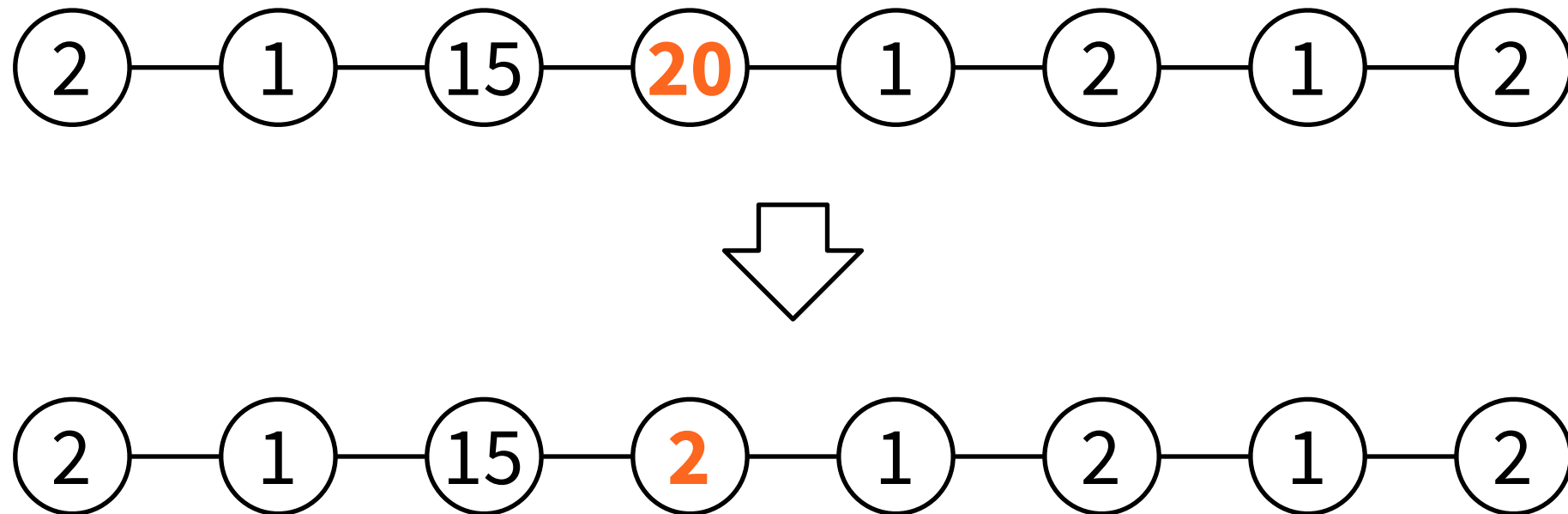
Algorithm P3C: Using unique IDs

- Idea: **local maxima** pick a new colour



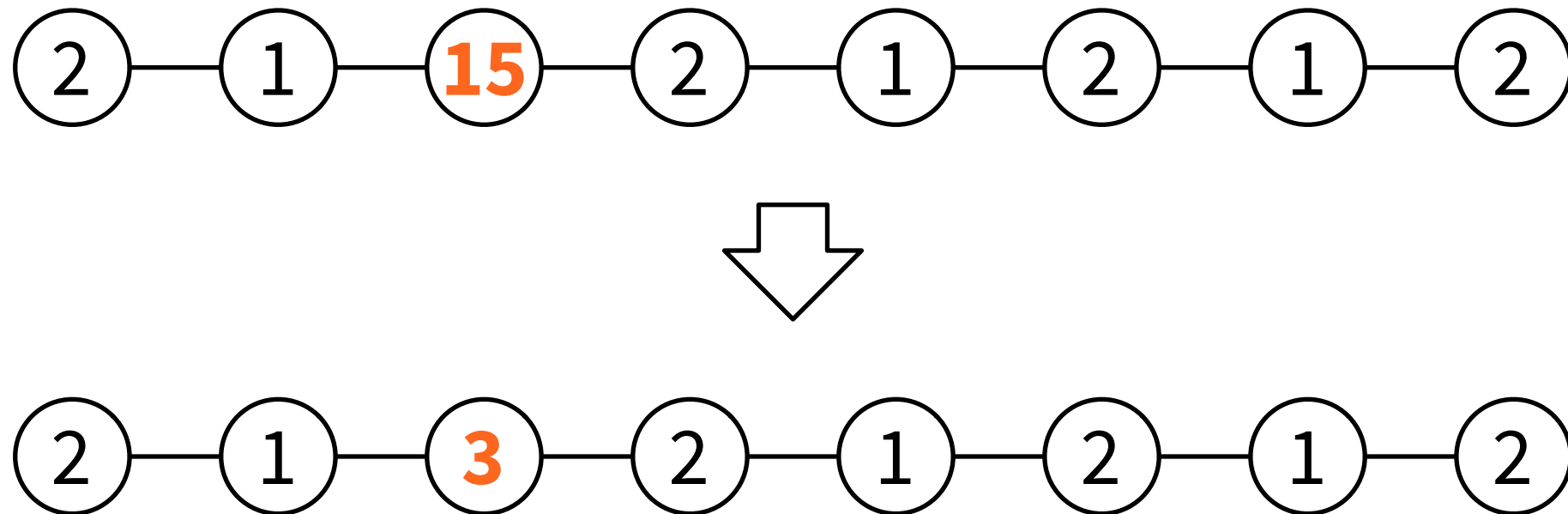
Algorithm P3C: Using unique IDs

- Idea: **local maxima** pick a new colour



Algorithm P3C: Using unique IDs

- Idea: **local maxima** pick a new colour



Algorithm P3C:

Using unique IDs

- **Inform neighbours of your current colour**
- **If your colour $>$ colours of your neighbours:**
 - pick a free colour from $\{1, 2, 3\}$
that is not used by any neighbour
- **Stopping states = $\{1, 2, 3\}$**

Performance

- **P3C: worst case $O(n)$**
- **We can do better!**

Algorithm P3CRand: **Using randomness**

- **Initialise: state = unhappy, colour = 1**
- **While state = unhappy:**
 - pick a new random colour from {1, 2, 3}
 - compare colours with neighbours
 - if different, set state = happy

Performance

- **P3C: worst case $O(n)$**
- **P3CRand: $O(\log n)$ with high probability**
- **We can do better!**
 - and we do not even need randomness

Algorithm P3CBit:

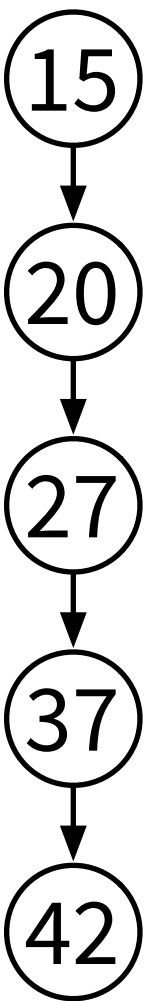
Fast colour reduction

- **Unique IDs = proper colouring with large number of colours**
- **Idea: reduce the number of colours from 2^k to $2k$ in one step**

Algorithm P3CBit:

Fast colour reduction

- **Unique IDs = proper colouring with large number of colours**
- **Idea: reduce the number of colours from 2^k to $2k$ in one step**
- **Note: we will assume a **directed path!****
(general case left as an exercise)



Algorithm P3CBit:

Fast colour reduction

- **Example: 128-bit unique IDs**
 - $2^{128} \rightarrow 2 \cdot 128 = 2^8$ colours
 - $2^8 \rightarrow 2 \cdot 8 = 2^4$ colours
 - $2^4 \rightarrow 2 \cdot 4 = 2^3$ colours
 - $2^3 \rightarrow 2 \cdot 3 = 6$ colours
- **From 2^{128} to 6 colours in 4 steps! How?**

Algorithm P3CBit:

Fast colour reduction

c_0 = my current colour as a k -bit string

c_1 = successor's colour as a k -bit string

i = **index** of a bit that differs between c_0 and c_1

b = **value** of bit i in c_0

$c = 2i + b$ = my new colour

$i \in \{0, \dots, k-1\}, \quad b \in \{0, 1\}, \quad c \in \{0, \dots, 2k-1\}$

Algorithm P3CBit:

Fast colour reduction

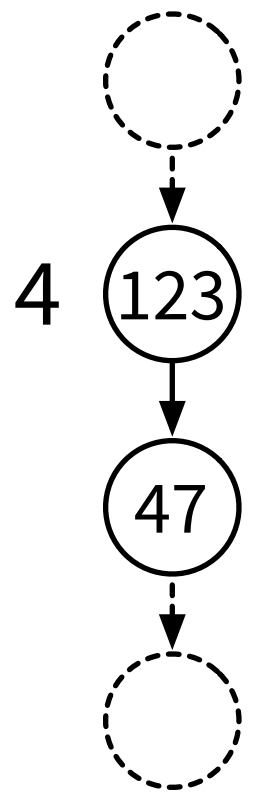
$c_0 = 123 = 01111\mathbf{0}11_2$ (my colour)

$c_1 = 47 = 00101\mathbf{1}11_2$ (successor's colour)

$i = 2$ (bits numbered 0, 1, 2, ... from right)

$b = 0$ (in my colour bit number i was 0)

$c = 2 \cdot 2 + 0 = 4$ (my new colour)



$k = 8$, reducing from $2^8 = 256$ to $2 \cdot 8 = 16$ colours

Algorithm P3CBit:

Fast colour reduction

$c_0 = 123 = 01111\mathbf{0}11_2$ (my colour)

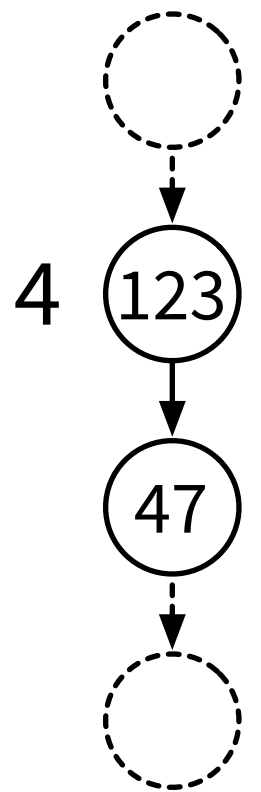
$c_1 = 47 = 0010\mathbf{1}1\mathbf{1}1_2$ (successor's colour)

Successor will pick one of these colours:

$14+\mathbf{0}$, $12+\mathbf{0}$, $10+\mathbf{1}$, $8+\mathbf{0}$, $6+\mathbf{1}$, $4+\mathbf{1}$, $2+\mathbf{1}$, $0+\mathbf{1}$

None of these conflict with my choice:

$4+\mathbf{0}$



Algorithm P3CBit:

Fast colour reduction

i = index of a bit that differs between c_0 and c_1

b = value of bit i in c_0

$c = 2i + b$ = my new colour

Successor picks different $i \rightarrow$ different c

Successor picks same $i \rightarrow$ different $b \rightarrow$ different c

My new colour \neq my successor's new colour

Performance

- **P3C: worst case $O(n)$**
 - assuming unique IDs
- **P3CRand: $O(\log n)$ with high probability**
- **P3CBit: $O(\log^* n)$**
 - assuming unique IDs are polynomial in n

Performance

- **P3CBit:** $O(\log^* n)$
 - assuming unique IDs are polynomial in n
- **Next week:** **this is optimal!**
 - no deterministic distributed algorithm can 3-colour a path in time $o(\log^* n)$