


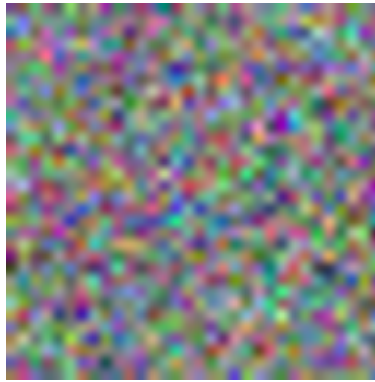









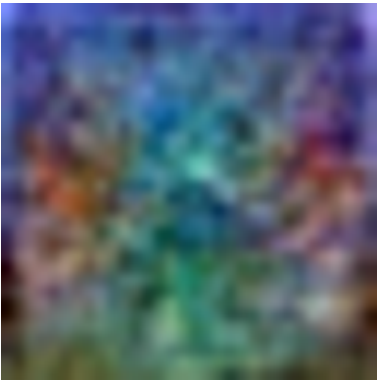









$W_{4 \times 32 \times 32 \times 3}$ ($\tilde{W}_{3072 \times 4}$) $X_{5 \times 32 \times 32 \times 3}$ ($\tilde{X}_{5 \times 3072}$)				
	0.815	1.065	1.069	-0.941
	0.504	1.884	1.517	-1.221
	0.763	1.946	0.956	-0.930
	-0.165	0.905	0.383	0.358
	0.748	1.456	1.781	-0.905

$$\begin{cases} f(X, W) = X \cdot W + b \\ L = \frac{1}{N} \sum_i L_i \left(f(x_i, W), y_i \right) \\ L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \end{cases} \qquad \begin{cases} \frac{\partial L}{\partial W} = \frac{1}{N} \sum_i \left[\frac{\partial L_i(f(x_i, W), y_i)}{\partial f(x_i, W)} \cdot \frac{\partial f(x_i, W)}{\partial W} \right] \\ \frac{\partial L_i}{\partial W_j} = 1(s_j - s_{y_i} + 1 > 0) x_i \\ \frac{\partial L_i}{\partial W_{y_i}} = - \left(\sum_{j \neq y_i} 1(s_j - s_{y_i} + 1 > 0) \right) x_i \end{cases}$$

$$S = f(X, W) = \tilde{X} \cdot \tilde{W} + b = \begin{bmatrix} 0.815 & 1.065 & 1.069 & \textbf{-0.941} \\ 0.504 & 1.884 & \textbf{1.517} & -1.221 \\ \textbf{0.763} & 1.946 & 0.965 & -0.930 \\ -0.165 & \textbf{0.905} & 0.383 & 0.358 \\ \textbf{0.748} & 1.456 & 1.781 & -0.905 \end{bmatrix} \qquad y = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad S_y = \begin{bmatrix} -0.941 \\ 1.517 \\ 0.763 \\ 0.905 \\ 0.748 \end{bmatrix}$$

$$\max(0, S - S_y + 1) = \max \left(0, \begin{bmatrix} 2.756 & 3.006 & 3.010 & \mathbf{1} \\ -0.013 & 1.368 & \mathbf{1} & -1.737 \\ \mathbf{1} & 2.184 & 1.202 & -0.693 \\ -0.070 & \mathbf{1} & 0.477 & 0.452 \\ \mathbf{1} & 1.708 & 2.033 & -0.652 \end{bmatrix} \right) = \begin{bmatrix} 2.756 & 3.006 & 3.010 & \mathbf{1} \\ 0 & 1.368 & \mathbf{1} & 0 \\ \mathbf{1} & 2.184 & 1.202 & 0 \\ 0 & \mathbf{1} & 0.477 & 0.452 \\ \mathbf{1} & 1.708 & 2.033 & 0 \end{bmatrix}$$

$$L = \frac{1}{N} \left(\sum \max(0, S - S_y + 1) - N \right) = 3.639 \qquad \frac{\partial L}{\partial W} = \tilde{X}^T \cdot \left(\begin{bmatrix} 1 & 1 & 1 & \mathbf{1} \\ 0 & 1 & \mathbf{1} & 0 \\ \mathbf{1} & 1 & 1 & 0 \\ 0 & \mathbf{1} & 1 & 1 \\ \mathbf{1} & 1 & 1 & 0 \end{bmatrix} - \begin{bmatrix} & & & \mathbf{4} \\ & 2 & & \\ \mathbf{3} & & & \\ & 3 & & \\ \mathbf{3} & & & \end{bmatrix} \right)$$








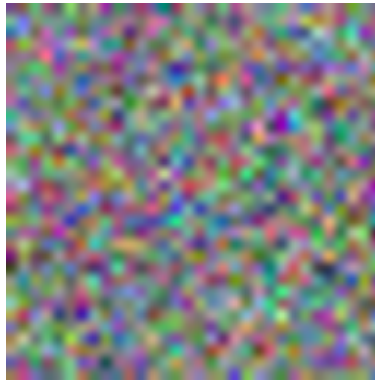





<div> <div>     </div> <div> <div> $W_{4 \times 32 \times 32 \times 3}$ $(\tilde{W}_{3072 \times 4})$ </div> <div> $X_{5 \times 32 \times 32 \times 3}$ $(\tilde{X}_{5 \times 3072})$ </div> </div> </div>				
				
	0.461	-1.016	0.516	0.040
	0.786	-1.502	0.469	0.258
	1.227	-0.985	0.498	-0.732
	0.282	1.573	-1.418	-0.433
	0.896	-0.927	-0.124	0.166

$$\begin{cases} f(X, W) = X \cdot W + b \\ L = \frac{1}{N} \sum_i L_i \left(f(x_i, W), y_i \right) \\ L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \end{cases} \qquad \begin{cases} \frac{\partial L}{\partial W} = \frac{1}{N} \sum_i \left[\frac{\partial L_i(f(x_i, W), y_i)}{\partial f(x_i, W)} \cdot \frac{\partial f(x_i, W)}{\partial W} \right] \\ \frac{\partial L_i}{\partial W_j} = 1(s_j - s_{y_i} + 1 > 0) x_i \\ \frac{\partial L_i}{\partial W_{y_i}} = - \left(\sum_{j \neq y_i} 1(s_j - s_{y_i} + 1 > 0) \right) x_i \end{cases}$$

$$S = f(X, W) = \tilde{X} \cdot \tilde{W} + b = \begin{bmatrix} 0.461 & -1.016 & 0.516 & \mathbf{0.040} \\ 0.786 & -1.502 & \mathbf{0.469} & 0.258 \\ \mathbf{1.227} & -0.985 & 0.498 & -0.732 \\ 0.282 & \mathbf{1.573} & -1.418 & -0.433 \\ \mathbf{0.896} & -0.927 & -0.124 & 0.166 \end{bmatrix} \quad y = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad S_y = \begin{bmatrix} 0.040 \\ 0.469 \\ 1.227 \\ 1.537 \\ 0.896 \end{bmatrix}$$

$$\max(0, S - S_y + 1) = \max \left(0, \begin{bmatrix} 1.421 & -0.057 & 1.475 & \mathbf{1} \\ 1.316 & -0.971 & \mathbf{1} & 0.789 \\ \mathbf{1} & -1.211 & 0.271 & -0.959 \\ -0.291 & \mathbf{1} & -1.992 & -1.006 \\ \mathbf{1} & -0.823 & -0.020 & 0.270 \end{bmatrix} \right) = \begin{bmatrix} 1.421 & 0 & 1.475 & \mathbf{1} \\ 1.316 & 0 & \mathbf{1} & 0.789 \\ \mathbf{1} & 0 & 0.271 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ \mathbf{1} & 0 & 0 & 0.270 \end{bmatrix}$$

$$L = \frac{1}{N} \left(\sum \max(0, S - S_y + 1) - N \right) = 1.109 \qquad \frac{\partial L}{\partial W} = \tilde{X}^T \cdot \left(\begin{bmatrix} 1 & 0 & 1 & \mathbf{1} \\ 1 & 0 & \mathbf{1} & 1 \\ \mathbf{1} & 0 & 1 & 0 \\ 0 & \mathbf{1} & 0 & 0 \\ \mathbf{1} & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} & & & \mathbf{3} \\ & & \mathbf{3} & \\ \mathbf{2} & & & \\ & \mathbf{1} & & \\ \mathbf{2} & & & \end{bmatrix} \right)$$

				
$W_{4 \times 32 \times 32 \times 3}$ ($\tilde{W}_{3072 \times 4}$)				
$X_{5 \times 32 \times 32 \times 3}$ ($\tilde{X}_{5 \times 3072}$)				
	0.267	0.343	0.344	0.046
	0.126	0.503	0.348	0.023
	0.176	0.576	0.216	0.032
	0.136	0.398	0.236	0.230
	0.166	0.337	0.466	0.032














$$\begin{cases} S = f(X, W) = X \cdot W + b \\ P(Y = y_i | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \\ L_i = -\log P(Y = y_i | X = x_i) \end{cases}$$

$$S = f(X, W) = \tilde{X} \cdot \tilde{W} + b = \begin{bmatrix} 0.815 & 1.065 & 1.069 & \textbf{-0.941} \\ 0.504 & 1.884 & \textbf{1.517} & -1.221 \\ \textbf{0.763} & 1.946 & 0.965 & -0.930 \\ -0.165 & \textbf{0.905} & 0.383 & 0.358 \\ \textbf{0.748} & 1.456 & 1.781 & -0.905 \end{bmatrix} \qquad y = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$P(Y = y_i | X = x_i) = \frac{e^{s_{y_i}}}{\sum_j e^{s_j}} = \begin{bmatrix} 0.267 & 0.343 & 0.344 & \textbf{0.046} \\ 0.126 & 0.503 & \textbf{0.348} & 0.023 \\ \textbf{0.176} & 0.576 & 0.216 & 0.032 \\ 0.136 & \textbf{0.398} & 0.236 & 0.230 \\ \textbf{0.166} & 0.337 & 0.466 & 0.032 \end{bmatrix}$$

$$L = \frac{1}{N} \Big(\sum -\log P(Y = y_i | X = x_i) \Big) = 1.717$$

$$\frac{\partial L}{\partial W} = \tilde{X}^T \cdot \left(\begin{bmatrix} 0.267 & 0.343 & 0.344 & \textbf{0.046} \\ 0.126 & 0.503 & \textbf{0.348} & 0.023 \\ \textbf{0.176} & 0.576 & 0.216 & 0.032 \\ 0.136 & \textbf{0.398} & 0.236 & 0.230 \\ \textbf{0.166} & 0.337 & 0.466 & 0.032 \end{bmatrix} - \begin{bmatrix} & & & \textbf{1} \\ & & \textbf{1} & \\ \textbf{1} & & & \\ & \textbf{1} & & \end{bmatrix} \right)$$

				
$W_{4 \times 32 \times 32 \times 3}$ ($\tilde{W}_{3072 \times 4}$)				
$X_{5 \times 32 \times 32 \times 3}$ ($\tilde{X}_{5 \times 3072}$)				
	0.345	0.068	0.392	0.195
	0.533	0.027	0.259	0.180
	0.716	0.038	0.196	0.050
	0.155	0.767	0.019	0.058
	0.541	0.058	0.186	0.215

$$\begin{cases} S = f(X, W) = X \cdot W + b \\ P(Y = y_i | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \\ L_i = -\log P(Y = y_i | X = x_i) \end{cases}$$














$$\left\{ \begin{aligned} \frac{\partial L_i}{\partial W_j} &= -\frac{\sum_j e^{s_j}}{e^{s_{y_i}}} \cdot \frac{0 \cdot \sum_j e^{s_j} - e^{s_{y_i}} e^{s_j}}{(\sum_j e^{s_j})^2} \cdot x_i \\ &= \frac{e^{s_j}}{\sum_j e^{s_j}} \cdot x_i \\ &= P(Y = y_i | X = x_i) \cdot x_i \\ \frac{\partial L_i}{\partial W_{y_i}} &= -\frac{\sum_j e^{s_j}}{e^{s_{y_i}}} \cdot \frac{e^{s_{y_i}} \sum_j e^{s_j} - e^{s_{y_i}} e^{s_{y_i}}}{(\sum_j e^{s_j})^2} \cdot x_i \\ &= \frac{e^{s_{y_i}} - \sum_j e^{s_j}}{\sum_j e^{s_j}} \cdot x_i \\ &= (P(Y = y_i | X = x_i) - 1) \cdot x_i \end{aligned} \right.$$

$$S = f(X, W) = \tilde{X} \cdot \tilde{W} + b = \begin{bmatrix} 0.517 & -1.105 & 0.644 & \textbf{-0.052} \\ 1.203 & -1.767 & \textbf{-0.483} & 0.119 \\ \textbf{-1.727} & -1.204 & 0.431 & -0.929 \\ 0.372 & \textbf{-1.969} & -1.709 & -0.618 \\ \textbf{1.065} & -1.171 & -0.002 & 0.140 \end{bmatrix} \quad y = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$P(Y = y_i | X = x_i) = \frac{e^{s_{y_i}}}{\sum_j e^{s_j}} = \begin{bmatrix} 0.345 & 0.068 & 0.392 & \textbf{0.195} \\ 0.533 & 0.027 & \textbf{0.259} & 0.180 \\ \textbf{0.716} & 0.038 & 0.196 & 0.050 \\ 0.155 & \textbf{0.767} & 0.019 & 0.058 \\ \textbf{0.541} & 0.058 & 0.186 & 0.215 \end{bmatrix}$$

$$L = \frac{1}{N} \Big(\sum -\log P(Y = y_i | X = x_i) \Big) = 0.839$$

$$\frac{\partial L}{\partial W} = \tilde{X}^T \cdot \left(\begin{bmatrix} 0.345 & 0.068 & 0.392 & \textbf{0.195} \\ 0.533 & 0.027 & \textbf{0.259} & 0.180 \\ \textbf{0.716} & 0.038 & 0.196 & 0.050 \\ 0.155 & \textbf{0.767} & 0.019 & 0.058 \\ \textbf{0.541} & 0.058 & 0.186 & 0.215 \end{bmatrix} - \begin{bmatrix} & & & \textbf{1} \\ & & \textbf{1} & \\ \textbf{1} & & & \\ & \textbf{1} & & \end{bmatrix} \right)$$

				
$W_{4 \times 32 \times 32 \times 3}$ ($\tilde{W}_{3072 \times 4}$)				
$X_{5 \times 32 \times 32 \times 3}$ ($\tilde{X}_{5 \times 3072}$)				
	0.408	0.011	0.547	0.033
	0.869	0.004	0.122	0.005
	0.878	0.002	0.117	0.003
	0.128	0.848	0.011	0.013
	0.892	0.012	0.089	0.008

