Gender Effects on Law Firm Networks

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1 Abstract

Gender biases in the workforce have been a strong point of contention for the past few decades. This paper, in particular, focuses on gender inequality within law firms by examining the effects that gender has on forming relationships and ties in 3 law firm networks in the Lazega dataset. In the friendship network, it was found that a directed friendship tie is most likely to form with other attorneys of the same gender and status and least likely to form with other attorneys of the opposite gender and status. In the coworker network, we observed that a coworker relationship is most likely to occur between two attorneys if they are an associate and a partner of the same gender. Finally, in the advice network, it was found that attorneys (regardless of gender) are more likely to want/be given advice from another attorney if the other attorney is male. Thus it was found that gender did have an effect on the ties and formations within the 3 law firm networks.

2 Introduction

Despite the great efforts made toward gender equality in the workforce over the last century, there are still a number of fields where women are underrepresented. Currently, Law is one of those industries. While studies have shown that women make up around 50% of law students in America and that many of these women go on to become associates (regular firm employees) at law firms, only around 20% of all partners (attorney with partial ownership of the law firm) in private practices are women (American Bar Association 2016).

To get a closer examination of some of the work dynamics that women in law face, we can take a look at the Lazega law firm networks dataset (Lazega 2001). The dataset includes 3 different relationship networks among the 71 attorneys (partners and associates) of a firm:

- Directed friendship network: employees are asked to name friends within the firm that they socialize with often even outside of work-related situations and functions
- Undirected coworker network: employees are asked to name people within the firm that they have worked with at some (spent time together/assigned to work on the same case, if one or both of them have read or used each other's work product, etc.)
- Directed advice network: employees are asked to name people within the firm that they have gone to for basic professional advice. This includes asking about if a case is being handled correctly, making a proper decision, or consulting with someone whose professional opinions they respect (not simply technical advice).

The dataset also contains attributes for each of the 71 attorneys (nodes):

- Status: partner or associate
- Gender: men or women

- Office: Boston, Hartford, or Providence
- Seniority: years within the firm
- Age: age of attorney
- Practice: litigation or corporate
- Law school: (Harvard/Yale), UConn, or other

The goal of this paper is to analyze all three of these networks and see how relationship dynamics and connections within the networks differ between female and male attorneys. The friendship and advice networks are particularly of interest, as forming friendships and connections with people of higher status can have a significant (positive) impact on one's career.

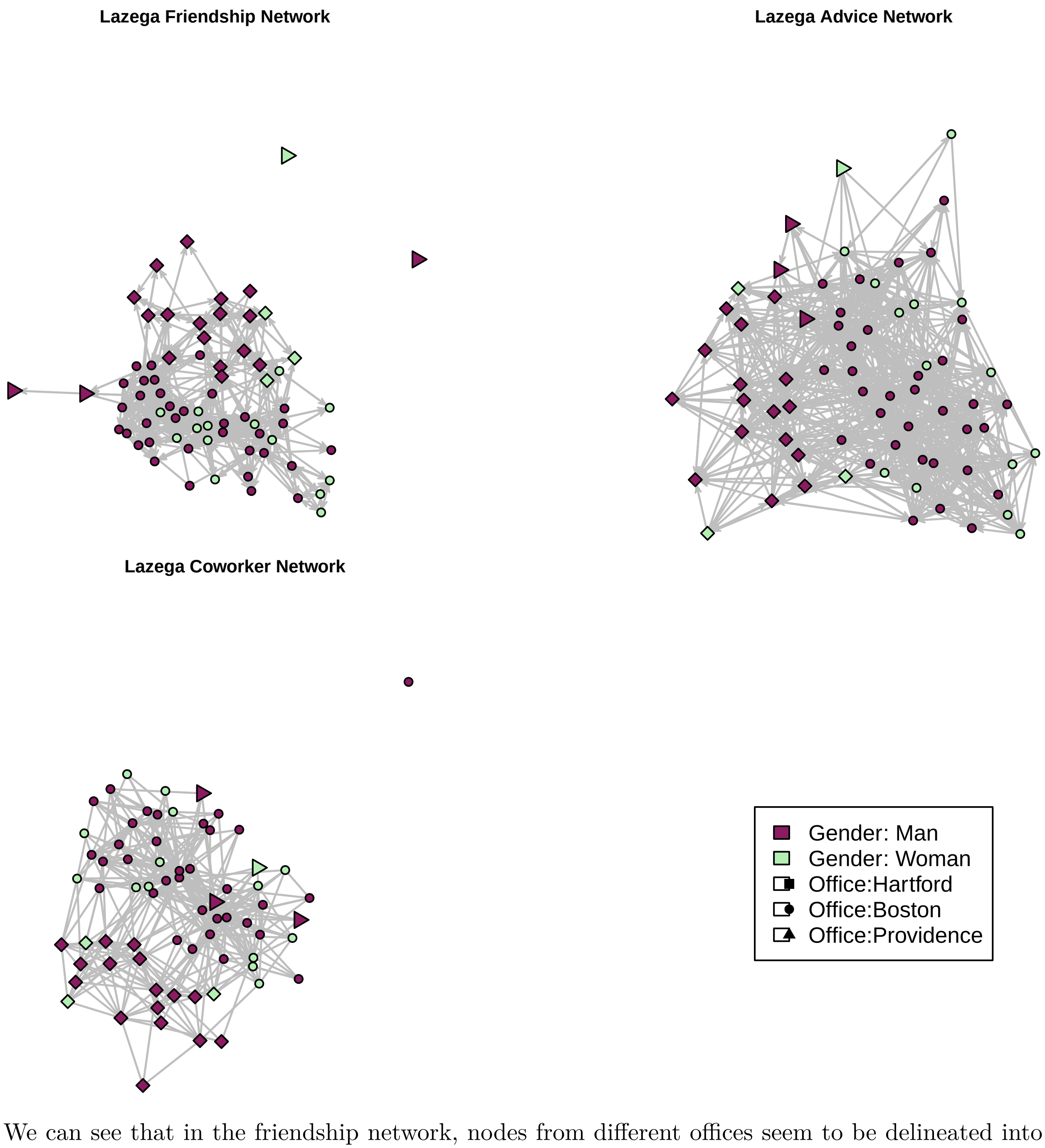
3 Analysis and Results

3.1 Network Overview

To start with just a cursory glance at the networks, we note that out of the 71 nodes (attorneys of the firm), only 18 are women, indicating that only about 25.35% of the attorneys in the firm are female. While women make up around 42.9% of the associates in the firm (15 out of 35), only around 8.33% of the partners in the firm are women (3 out of 36).

Thus these networks seem to support the findings of the American Bar Association report in 2016 in that there is a large gender inequality for higher status attorneys (partners). In the next few subsections, we will take a more detailed look at the dataset to get an overview of other aspects of the networks.

3.1.1 Visalization



We can see that in the friendship network, nodes from different offices seem to be delineated into different clusters in the plot (square, triangle, and circle nodes are all clustered into their own groups). In addition, female and male clusters seem to be forming within the different office clusters. This could indicate that friendship ties are more likely to occur for attorneys in the same office, and within the same office, ties occur more commonly between the same gender.

The nodes from different offices in the coworker and advice networks also both seem to be delineated into different clusters but there does not seem to be any evident signs of further clustering by gender. Thus this could indicate that coworker and advice ties are more likely to occur for attorneys in the same office, but the effects of gender on these ties are unclear just by visual analysis. These effects can be further explored when we examine the densities of ties between nodes of different attributes in the next section.

We also note that with the exception of one or two nodes, the rest of the nodes in all the networks seem to be connected and part of the largest component. Thus further analysis of subgroups and components seems unnecessary for this paper and the questions it is trying to answer.

3.1.2 Density of Ties

Examining the densities in the networks can help us understand the impact of the covariates on the patterns of ties.

Friendship Network

The overall density of the friendship network is 0.2313883. However, since we want to understand the impact of the covariates on the patterns of ties in the network, we should also look at the densities of ties between nodes with different attributes. The tables below show the density of ties *from* nodes with attributes given in the **row** *to* nodes with attributes given in the **column** since friendship is a directed network.

Note that network density is calculated by dividing the number of total ties in the network by all possible ties. However, since we are looking at the ties between nodes with certain attributes, we are dividing the ties found by number of possible ties *given* certain specified attributes. So for example,

$$\text{Dens. of friend ties from male to female } = \frac{\text{\# ties from male to female in the network}}{\text{\# possible ties from male to female}} = 0.0985$$

Since there are much more male nodes in the network than female nodes, it would be natural for ties involving male nodes to occur more often overall in the network. Thus calculating densities of ties for nodes with certain attributes like we did above helps standardize everything.

	Male	Female
Male	0.1288	0.0985
Female	0.0755	0.1765

	Partner	Associate
Partner	0.2060	0.0452
Associate	0.0548	0.1486

	Male Associate	Fem. Associate	Male Partner	Fem. Partner
Male Associate	0.1868	0.1367	0.0591	0.0500
Fem. Associate	0.1067	0.1810	0.0424	0.1333
Male Partner	0.0394	0.0505	0.2074	0.2525
Fem. Partner	0.0000	0.1333	0.1919	0.6667

Table 1: Densities of ties between nodes of different attributes in friendship network

We see from the tables above that we can infer that within this network that friendship ties seem more likely to occur between attorneys of the same gender, particularly for women. Similarly, attorneys are more likely to form friendship ties with people of the same status (associate or partner). In general, the trend in the firm seems to be that a directed friendship tie is *most* likely to form with other attorneys of the *same* gender and status and *least* likely to form with other attorneys of the *opposite* gender and status. The only exception seems to be for male partners as the likelihood of a directed friendship tie forming between them and a female partner is the highest, and the likelihood of a directed friendship tie forming between them and a male associate is the lowest.

Coworker Network

The overall density of the coworker network is 0.3042254. Once again, the tables below show the density of ties *from* nodes with attributes given in the **row** *to* nodes with attributes given in the **column**. However, since the coworker network is undirected, the matrix entries in the tables are symmetric.

	Male	Female
Male	0.1734	0.1289
Female	0.1289	0.1046

	Partner	Associate
Partner	0.1775	0.1778
Associate	0.1778	0.0637

	Male Associate	Fem. Associate	Male Partner	Fem. Partner
Male Associate	0.0789	0.0600	0.1909	0.1500
Fem. Associate	0.0600	0.0571	0.1616	0.2000
Male Partner	0.1909	0.1616	0.1856	0.1616
Fem. Partner	0.0119	0.2000	0.1616	0.3333

Table 2: Densities of ties between nodes of different attributes in coworker network

From the tables above, we can infer that within this network, a coworker tie seems more likely to occur if both attorneys are male and least likely to occur if both attorneys are female. It also seems that a coworker tie is equally likely to occur between two partners or a partner and an associate, but least likely to occur between two associates. More specifically, a coworker relationship is most likely to occur if the two nodes are an associate and a partner of the same gender. There is an exception in that within the network, a coworker tie is most likely to occur if both attorneys are female partners. It is important to note that because lawyers are often assigned to certain cases, these coworker ties are not necessarily always formed because of personal choices, unlike in the advice and friendship network.

Advice Network

The overall density of the advice network is 0.3589537 (highest of all the networks).

	Male	Female
Male	0.2101	0.1059
Female	0.1719	0.1569

	Partner	Associate
Partner	0.3048	0.0468
Associate	0.1921	0.1600

We see that in the advice network, attorneys (regardless of gender) are more likely to want/be given advice from another attorney if the other attorney is male. In addition, both partners and associates are more likely to want/be given advice if the other attorney is a partner. Partners, in turn, are very unlikely to want/be given advice from an associate. More specifically, male attorneys seem most likely to receive advice if the other attorney is a male partner. For female attorneys, they are most likely to receive advice if the other attorney is a female partner.

	Male Associate	Fem. Associate	Male Partner	Fem. Partner
Male Associate	0.1895	0.1233	0.2182	0.2167
Fem. Associate	0.1733	0.1667	0.1556	0.1778
Male Partner	0.0470	0.0505	0.3144	0.2626
Fem. Partner	0.0333	0.0222	0.3333	0.6667

Table 3: Densities of ties between nodes of different attributes in advice network

Professional advice (especially from those of high standing in the firm like partners) is probably most valuable to new workers or workers of lower standing (like associates). Thus it is interesting to note that female associates seem less likely to receive advice from partners compared to male associates. In fact, they are the least likely attorneys to receive advice from partners in general.

3.1.3 Actor Centrality

Actor centrality gives us a good idea of who or what type of nodes are important or influential in a network based on their network positions. This analysis can be useful in determining if female or male attorneys have more or less influence in the different networks (and thus also in the firms) compared to each other. Since these centralities are calculated based on the ties and position of nodes in the networks, the prevalence of certain nodes in the networks (more male than female) will not influence our calculations in determining which types of nodes are more influential.

In-Degree

In-degree calculates the number of ties pointing inward onto each node in a directed graph. In an undirected graph, in-degree = out-degree = degree. In this section, we will examine the differences in the mean in-degree between male and female nodes for each of the three networks.

Table 4: In-Degree Summary for Female

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	4	7.500	8.220	12.750	17
Coworkers	4	5.500	8	8.610	10.750	17
Advice	0	2.250	8	8.280	10.750	25

Table 5: In-Degree Summary for Male

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	4	7	8.060	11	22
Coworkers	0	7	10	11.340	14	28
Advice	1	8	13	14.020	20	37

For the friendship network, having more in-degrees indicates that the attorney was named as a friend by many other attorneys in the firm. The average in-degree for female attorneys is 8.220 and for male attorneys is 8.060. They also have similar medians (not as susceptible to outliers) so these numbers are fairly similar, seemingly indicating that female and male attorneys are both similarly likely to be named as a friend. In the coworker network, both the mean and median in-degree (which is really just degree since it's undirected) for males is higher, which could potentially mean that on average, male attorneys are more likely to be assigned to cases with others compared to female attorneys. The difference between the mean and median in-degree for male and female nodes are even higher in the advice network, potentially indicating that on average, men attorney are more likely to give advice than female attorneys.

Out-Degree

Out-degree calculates the number of ties pointing outward from each node in a directed graph. In an undirected graph, in-degree = out-degree = degree. In this section, we will examine the differences in the mean out-degree between male and female nodes for each of the three networks.

Table 6: Out-Degree Summary for Female

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	4	6	7	8.750	19
Coworkers	4	5.500	8	8.610	10.750	17
Advice	3	7.250	12	11.780	14	23

Table 7: Out-Degree Summary for Male

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	4	8	8.470	12	25
Coworkers	0	7	10	11.340	14	28
Advice	0	7	11	12.830	18	30

For both the friendship network and advice network, we note that the mean and median out-degree for male attorneys are slightly higher compared to females in both networks, but this difference seems small enough to be potentially negligible. So we may infer that on average, male and female attorneys are generally similarly likely to both name other attorneys as friends and also to receive advice from other attorneys. Since in-degree and out-degree are the same for undirected networks, in-degree conclusions about the coworker network hold here as well.

Eigenvalue

Eigenvalue centrality is a measure of the influence of a node. A node can be connected to many other nodes but those nodes may not be highly connected themselves. This would imply a low eigenvalue centrality.

Table 8: Eigen Centr. Summary for Female

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	0.020	0.070	0.080	0.100	0.190
Coworkers	0.040	0.070	0.090	0.090	0.110	0.150
Advice	0.020	0.060	0.110	0.100	0.130	0.180

Table 9: Eigen Centr. Summary for Male

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	0.040	0.080	0.100	0.140	0.300
Coworkers	0	0.080	0.100	0.110	0.140	0.260
Advice	0	0.060	0.090	0.100	0.150	0.240

For all 4 networks, the mean and median eigenvalue centralities for male attorneys are slightly higher than those of female nodes. It is thus difficult to say whether male attorneys are on average truly more influential then female attorneys. However, the maximum eigenvalue centralities in all 3 networks for males is consistently higher than the maximum eigenvalue centralities for females by a more significant amount. Thus, while we cannot really determine if male attorneys are *on average* truly more influential then female attorneys by just looking at this, we can say that these values imply the most influential attorneys at the firm in all three networks are male.

Betweeness

Betweeness of a node is measured as how many pairs of other nodes would have to go through it in order to reach one another in the minimum number of connections and quantifies the extent to which the node serves as a bridge to other nodes.

Table 10: Betweenness Centr. Summary for Female

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	14.990	58.020	94.160	88.400	652.390
Coworkers	0.560	1.760	8.180	16.890	20.790	67.070
Advice	0	7.910	33.830	55.740	56.430	353.590

Table 11: Betweenness Centr. Summary for Male

	Min.	X1st.Qu.	Median	Mean	X3rd.Qu.	Max.
Friends	0	13.130	56.910	89.910	120.780	502.460
Coworkers	0	6.930	21.930	44.550	45.430	289.810
Advice	0	21.420	45.570	88.200	117.290	386.320

The mean and median betweeness of female nodes is a little higher than that of male nodes in the friendship network. This could imply that on average, female attorneys serve as bridges in friendship more than male attorneys. Conversely, the mean and median betweeness of female nodes is significantly lower than that of male nodes in the advice and coworker network. This implies that on average, male attorneys have more influence and act more frequently as bridges compared to female attorneys when it comes to working together on cases and giving/receiving advice.

3.2 Triad Census

Triadic configurations are important in the analysis of social networks as they can evaluate the structural balance in a communication network (such as the spread of useful advice from one attorney to another in the advice network) and friendship networks (Zvereva 2016). In addition, it can reveal information about the transitivity of a network. Transitivity of a relationship means that when there is a tie from node i to node j, and also from node j to node h, then there is also a tie from node i to node h. Triad census finds the counts of 16 different triad types within the network:

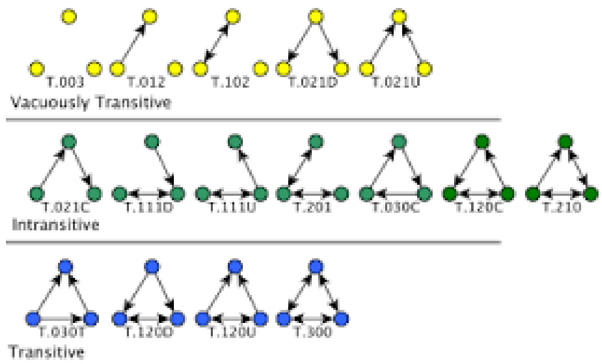


Figure 1: Source: Alhazmi 2015

Since this paper is mainly focused on the different effects of gender on forming ties in the network, but R does not provide a function to find triad census with nodal attributes factored in, we will have to manually do this. A way to look at the effects of gender on triad formation is to first find the triad census of all the male nodes and female nodes separately in the observed networks. Then we can permutate the gender attribute of the nodes by randomly reassigning gender to different nodes (while keeping the count of female and male node count the same). We can do this permutation several times (5,000 in this case) and run a triad census on all the (reassigned) males nodes and female nodes separately again and average these values. Since there was clustering for `office` in all 3 networks, I only permutated the gender within each office to keep this from impacting the triad census results of the gender-permutated networks.

If gender does not have an effect on triad formations in the network, then permutating the gender for the nodes should not matter, and the average triad census (for the separate male and female nodes of the network) after the permutations should be similar to the separate male and female triad census of the original observed networks. However, after doing this process for all three networks, there is a fairly significant difference between the average triad census (for the separate male and female nodes of the network) after the permutations and that of the original observed networks. These results will be more carefully examined below.

Friendship Network

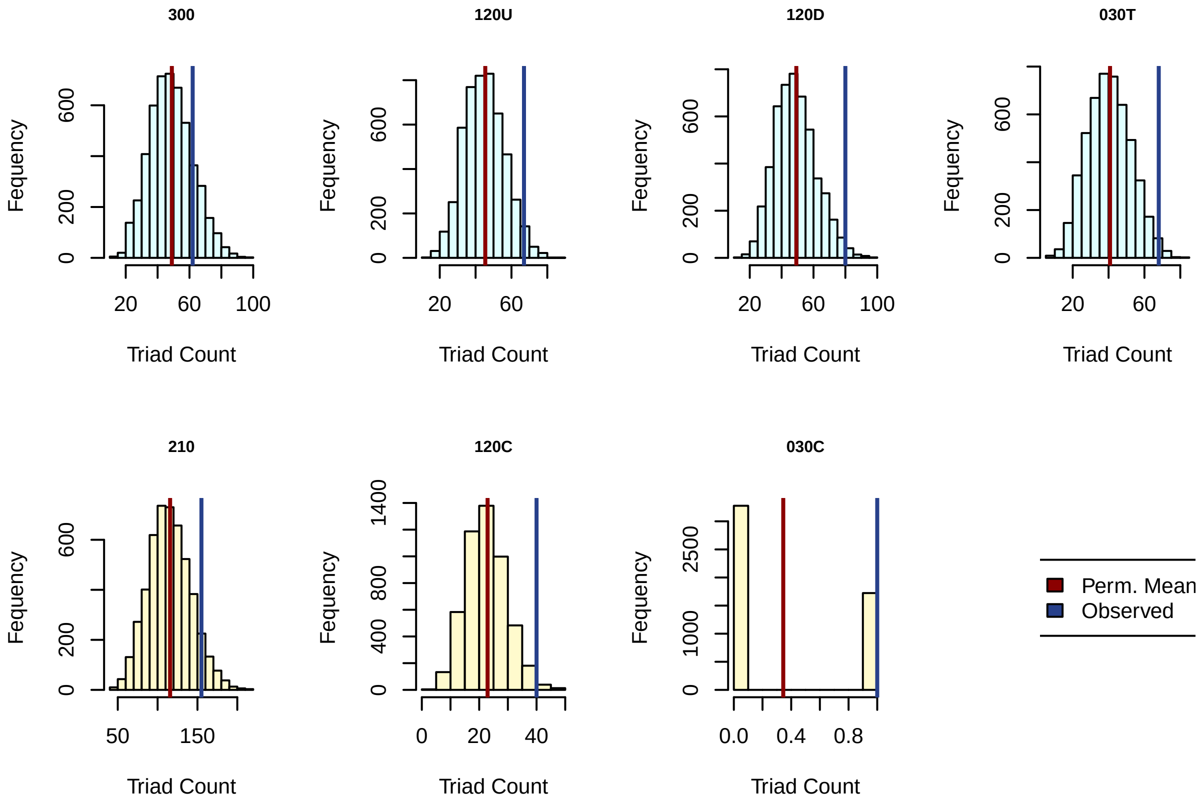


Table 12: Friend Net: Triads vs. Male

	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
Observed	240	164	207	297	434	68	1	223	80	67	40	155	62
Perm. Ave	176.0	115.9	142.7	252.8	344.4	40.8	0.3	200.5	49.2	45.4	22.9	115.7	48.9

We see that when compared to the original observed friendship network, the triad census for all the male nodes in the network seems to produce some significant difference in results after the permutation of gender. In particular, all the intransitive triangle triads (030C, 120C, 210) and the transitive triangle triads (030T, 120D, 120U, 300) decreased by a statistically significant amount. This indicates that in the friendship network, gender does have an effect on the triad formations in the network. Male attorneys have a tendency to form male-exclusive triangle triads, which indicate that there is a gender preference in friendship groups in the network. Having so many male-exclusive triangle triads could indicate that female attorneys might have a tendency to be excluded from friendship groups that are primarily male.

When compared to the original observed friendship network, the triad census for all the female nodes in the network also produced some differences as all the intransitive triangle triads (030C, 120C, 210) and the transitive triangle triads (030T, 120D, 120U, 300) also decreased after the permutation. However, since the original counts were already too small to begin with, the counts

Table 13: Friend Net: Triads vs. Female

	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
Observed	9	2	7	15	31	1	0	20	0	3	2	11	3
Perm. Ave	7.6	5.0	5.4	10.9	13.3	2.0	0.02	8.1	2.2	2.2	1.2	5.3	2.1

only decrease by around 1 or 2 units. However, this could still indicate that female attorneys also potentially have the tendency to form female exclusive friendship triads.

Coworker Network

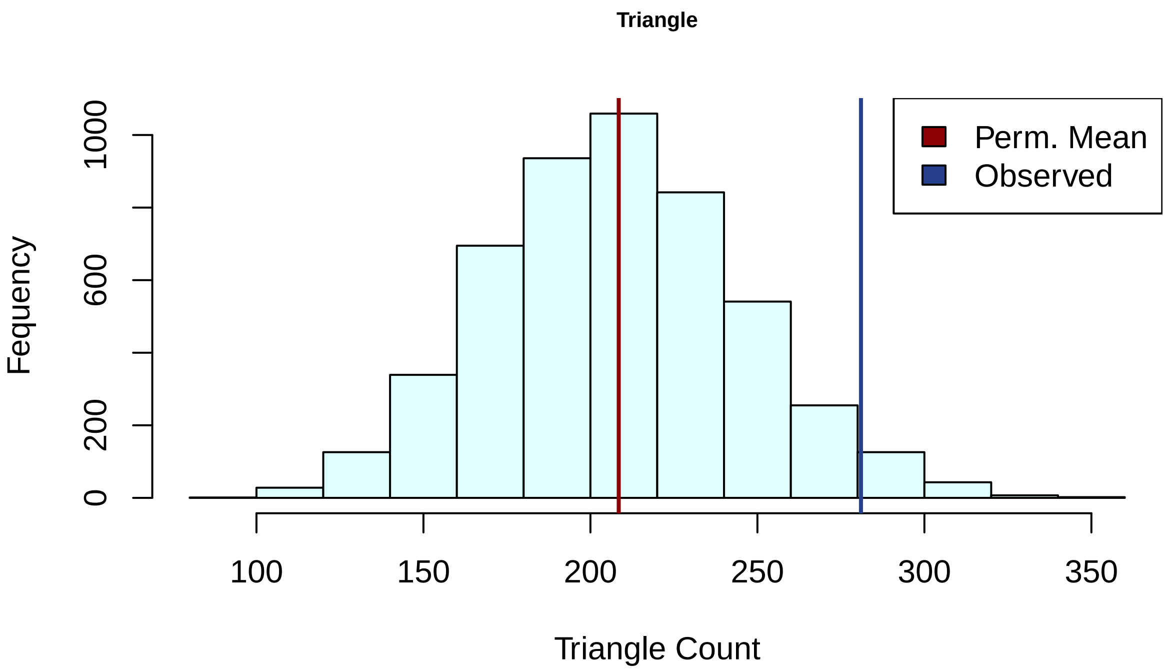


Table 14: Coworker Net: Triads vs.Male

	0	1	2	3
Observed	$40,746$	$12,017$	$1,696$	281
Perm. Ave	$43,581.6$	$11,066.8$	$1,330.5$	208.5

Since the coworker network is undirected, the main triad census we're looking at is undirected triangle triads. Once again, compared to the original observed coworker network, the triangle triad census for all the male nodes in the network showed there was a large drop in triangle triad formations when gender was not a factor in the network. This indicates that in the coworker network, gender does have an effect on the triad formations. Male attorneys have a tendency to form male-exclusive triangle triads, which indicate that male attorneys tend to work with each other more than with female attorneys.

When compared to the original observed coworker network, the triad census for all the female nodes in the network also produced some differences as the undirected triangle triad counts surprisingly

Table 15: Coworker Net: Triads vs. Female

	0	1	2	3
Observed	56,078	1,053	21	3
Perm. Ave	49,002.7	1,425.0	52.0	6.9

increased. This could imply that female attorneys do not work with each other as often as they do with male attorneys.

Advice Network

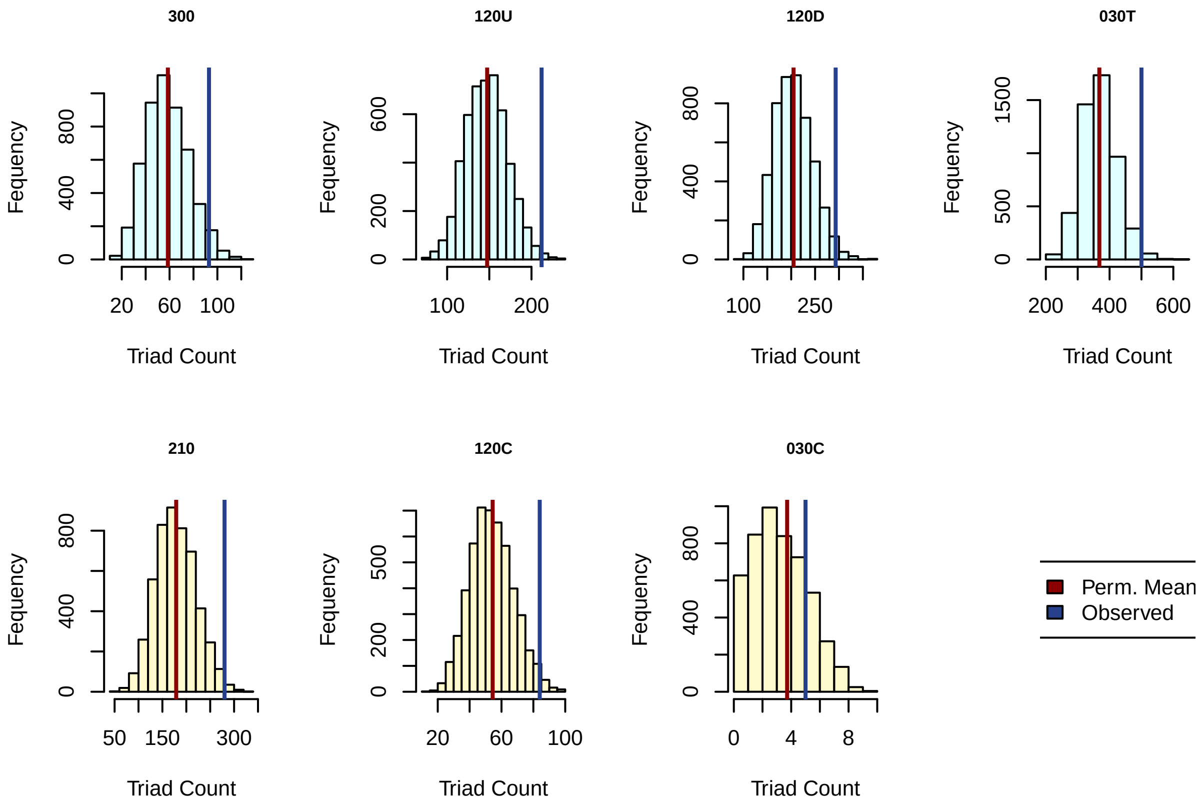


Table 16: Advice Net: Triads vs. Male

	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
Observed	757	825	919	836	709	500	5	275	293	212	84	280	93
Perm. Ave	595.8	708.6	709.7	625.5	550.5	367.9	3.7	191.1	204.9	147.5	54.5	178.9	58.6

When compared to the original observed advice network, the triad census for all the male nodes in the network seem to produce some significant difference in results after the permutation of gender. In particular, all the intransitive triangle triads (030C, 120C, 210) and the transitive triangle triads (030T, 120D, 120U, 300) decreased by a statistically significant amount. This indicates that in the advice network, gender does have an effect on the triad formations. Male attorneys have a tendency to form male-exclusive triangle triads, which indicate that there is a gender factor involved when sharing advice among attorneys. Having so many male-exclusive triangle triads could indicate that when male attorneys pass down advice they heard from others, they might tend to pass these down to other male attorneys rather than female attorneys.

Table 17: Advice Net: Triads vs. Female

	021D	021U	021C	111D	111U	030T	030C	201	120D	120U	120C	210	300
Observed	26	36	12	14	20	9	1	2	5	4	0	1	0
Perm. Ave	22.8	26.6	27.9	24.7	19.9	15.6	0.2	6.0	9.1	5.7	2.3	6.7	2.2

Compared to the original observed advice network, the triad census for all the female nodes in the network also produced some differences as once again, all the different exclusively female triangle triads increased. This could imply that when female attorneys pass down advice they heard from others, they might not have a tendency to pass it down and share advice with other female attorneys.

3.3 Fitting ERGM to Data

After a fairly detailed look at the networks, we want to try to fit an ERGM to the network datasets.

3.3.1 Model Fitting

We will try several different ERGM fits and decide the best one for each model and take a look at the important predictors in the best fit model. In order to take into account the interaction between gender and status (which we believe could be important from our analysis of densities in section 3.1.2), the `gender` and `status` covariates/attributes of the node were multiplied together to get a new covariate/attribute named `StatGen`.

Friendship Network

After adding in different covariates/attributes and homophily effects one by one and running an ANOVA on the models, it seems that `edges`, `age`, and homophily for `gender`, `status`, `office`, `practice`, and `school` are the important predictors in the model.

In addition, I tried to include variables from `ergm-temrs` to try and decrease the AIC, BIC, and Residual Deviance of the models. I tried adding in mutual, ctriple, transitiveties, dgwdsp (with decay 1, 0.5, 0.3, 0.25, and 0.35), and dgswd, but the only terms that worked well to improve the model was `mutual`, `transitiveties`, and `dgwesp(0.36, fixed = TRUE)`. The results of the best model fit are shown in figure 2.

We see that people of the same gender have a higher probability of forming friendship ties. In addition, attorneys of the same status, office, and school also have a higher probability of forming friendship ties. For the nodal covariate `age`, however, we see that it has a slightly negative coefficient. This means that there is a negative effect from increasing age on the probability of a node to form ties.

```
Iterations:  8 out of 20

Monte Carlo MLE Results:
                     Estimate Std. Error MCMC % z value Pr(>|z|)
edges               -4.820893   0.285050      0 -16.912  < 1e-04 ***
nodematch.gender.1   0.224696   0.074050      0   3.034  0.00241 **
nodematch.gender.2   0.422248   0.136233      0   3.099  0.00194 **
nodematch.status.1   0.884015   0.079151      0  11.169  < 1e-04 ***
nodematch.status.2   0.475025   0.079078      0   6.007  < 1e-04 ***
nodematch.office     0.686896   0.082198      0   8.357  < 1e-04 ***
nodematch.practice   0.320631   0.071511      0   4.484  < 1e-04 ***
nodematch.school.1   0.272745   0.149507      0   1.824  0.06811 .
nodematch.school.2   0.269074   0.086714      0   3.103  0.00192 **
nodematch.school.3   0.068239   0.108182      0   0.631  0.52819
nodecov.age         -0.008249   0.002546      0  -3.240  0.00119 **
gwesp.OTP.fixed.0.36  2.327847  0.228897      0  10.170  < 1e-04 ***
mutual               2.185307   0.163983      0  13.326  < 1e-04 ***
transitiveties      -1.734838   0.350917      0  -4.944  < 1e-04 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

     Null Deviance: 6890  on 4970  degrees of freedom
 Residual Deviance: 2370  on 4956  degrees of freedom

AIC: 2398    BIC: 2490    (Smaller is better.)
```

Figure 2: Summary of model fit

Coworker Network

Once again, by adding in different covariates/attributes and homophily effects one by one and running an ANOVA on the models, it seems that `edges`, `age`, and homophily for `gender`, `status`, `office`, and `practice` are the important predictors in the model.

In addition, I tried to include variables such as triangle, density, gwesp, and gwdsp, but only the terms `gwesp(1,fixed=T)` and `dgwesp(0.36, fixed = TRUE)` seemed to have improved the model. In addition, once these two terms were introduced, homophily on gender was no longer an important predictor. The results of the model are shown in figure 3.

```
Iterations:  3 out of 20

Monte Carlo MLE Results:
                     Estimate Std. Error MCMC % z value Pr(>|z|)
edges               -4.813757   0.434812      0 -11.071   <1e-04 ***
nodematch.status.1  -0.344483   0.166351      0  -2.071   0.0384 *
nodematch.status.2  -1.237804   0.225087      0  -5.499   <1e-04 ***
nodematch.office.1   1.184782   0.143863      0   8.235   <1e-04 ***
nodematch.office.2   2.253984   0.247379      0   9.111   <1e-04 ***
nodematch.office.3   4.008420   0.815475      0   4.915   <1e-04 ***
nodematch.practice.1  1.298450  0.157613      0   8.238   <1e-04 ***
nodematch.practice.2  1.694714  0.183842      0   9.218   <1e-04 ***
nodecov.age         -0.009220   0.003891      0  -2.369   0.0178 *
gwesp.fixed.1        0.575212   0.073781      0   7.796   <1e-04 ***
gwdsp.fixed.1        0.065438   0.014118      0   4.635   <1e-04 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

     Null Deviance: 3445  on 2485  degrees of freedom
 Residual Deviance: 1583  on 2474  degrees of freedom

AIC: 1605    BIC: 1669    (Smaller is better.)
```

Figure 3: Summary of model fit

We see that attorneys of the same office and practice have a higher probability of forming coworker ties. However, from the negative coefficients on the status homophily terms, it seems like attorneys of the same status are less likely to work together. The nodal covariate `age`, is once again slightly negative. This means that there is a negative effect from increasing age on the probability of a node to form coworker ties.

Advice Network

Again by adding in different covariates/attributes and homophily effects one by one and running an ANOVA on the models, it seems that `nodemix("StatGen")`, `age`, and homophily for, `school`, `office`, and `practice` are the important predictors in the model.

In addition, I tried to include variables from `ergm-temrs` but only the terms `mutual`, `transitiveties`, `ctriple`and `dgwesp(0.72, fixed = T)` seemed to have improved the model. The results of the model are shown in figure 4.

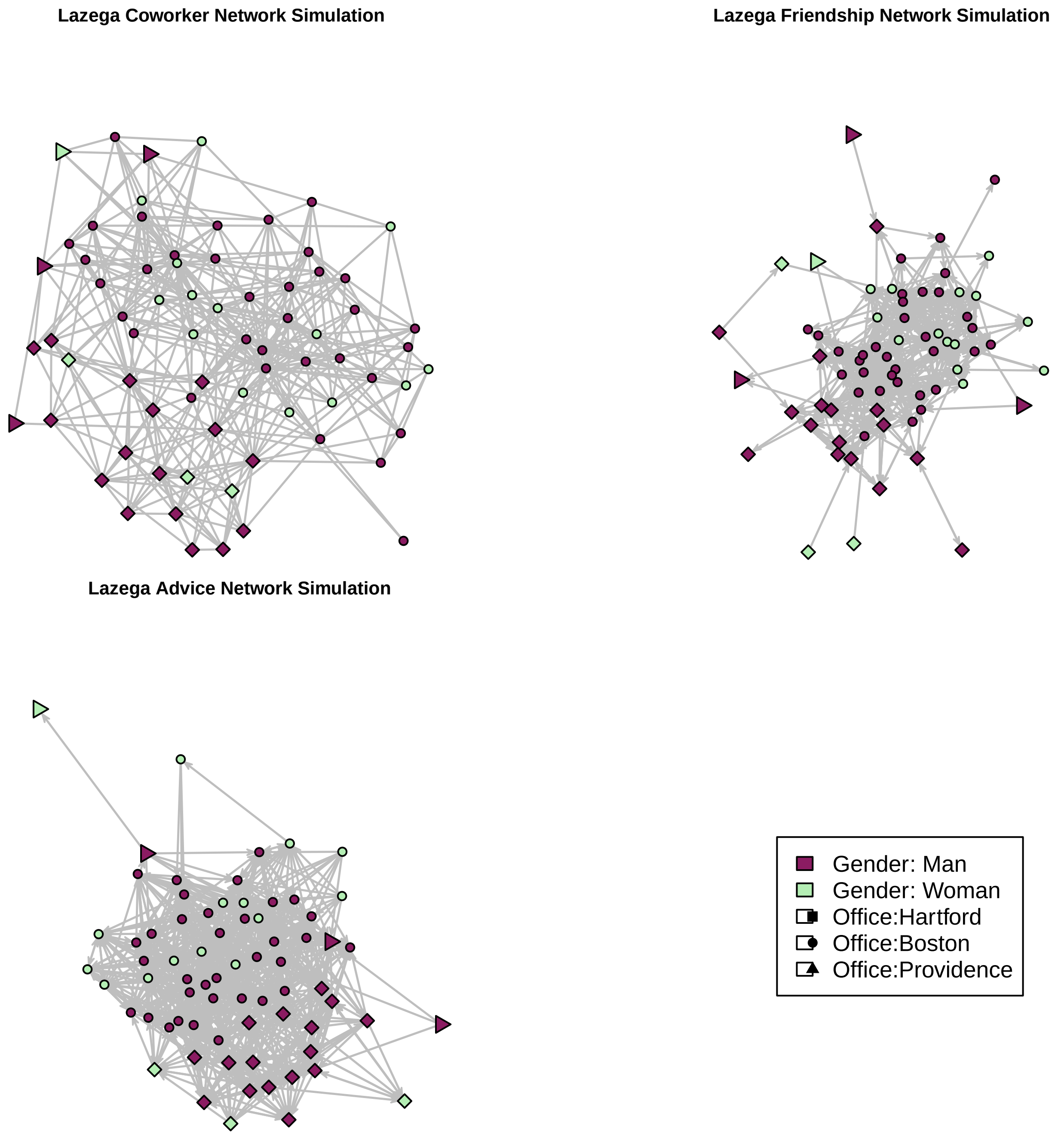
```
Iterations:  6 out of 20

Monte Carlo MLE Results:
                                 Estimate Std. Error MCMC % z value Pr(>|z|)
mix.StatGen.Stat1Fem.Stat1Fem   -2.520858   1.005029      0  -2.508 0.012133 *
mix.StatGen.Stat1Male.Stat1Fem  -3.746662   0.445357      0  -8.413  < 1e-04 ***
mix.StatGen.Stat2Fem.Stat1Fem   -4.199963   0.518579      0  -8.099  < 1e-04 ***
mix.StatGen.Stat2Male.Stat1Fem  -4.219365   0.460678      0  -9.159  < 1e-04 ***
mix.StatGen.Stat1Fem.Stat1Male  -3.401219   0.444758      0  -7.647  < 1e-04 ***
mix.StatGen.Stat1Male.Stat1Male -3.339032   0.422285      0  -7.907  < 1e-04 ***
mix.StatGen.Stat2Fem.Stat1Male  -4.195728   0.366887      0 -11.436  < 1e-04 ***
mix.StatGen.Stat2Male.Stat1Male -3.899109   0.364439      0 -10.699  < 1e-04 ***
mix.StatGen.Stat1Fem.Stat2Fem   -5.687749   1.032289      0  -5.510  < 1e-04 ***
mix.StatGen.Stat1Male.Stat2Fem  -4.722443   0.391501      0 -12.062  < 1e-04 ***
mix.StatGen.Stat2Fem.Stat2Fem   -3.926417   0.356727      0 -11.007  < 1e-04 ***
mix.StatGen.Stat2Male.Stat2Fem  -4.584701   0.360358      0 -12.723  < 1e-04 ***
mix.StatGen.Stat1Fem.Stat2Male  -5.320719   0.731385      0  -7.275  < 1e-04 ***
mix.StatGen.Stat1Male.Stat2Male -4.872974   0.384577      0 -12.671  < 1e-04 ***
mix.StatGen.Stat2Fem.Stat2Male  -4.204175   0.351472      0 -11.962  < 1e-04 ***
mix.StatGen.Stat2Male.Stat2Male -4.135415   0.330929      0 -12.496  < 1e-04 ***
nodematch.office.1               0.932462   0.094013      0   9.918  < 1e-04 ***
nodematch.office.2               1.454525   0.133082      0  10.930  < 1e-04 ***
nodematch.office.3               3.296202   0.362830      0   9.085  < 1e-04 ***
nodematch.practice.1             0.898261   0.086103      0  10.432  < 1e-04 ***
nodematch.practice.2             1.176415   0.094144      0  12.496  < 1e-04 ***
nodecov.age                     -0.012622   0.003362      0  -3.754 0.000174 ***
nodematch.school.1               0.560485   0.163468      0   3.429 0.000606 ***
nodematch.school.2               0.145726   0.098587      0   1.478 0.139367
nodematch.school.3               0.035761   0.116836      0   0.306 0.759546
mutual                           0.672216   0.159934      0   4.203  < 1e-04 ***
transitiveties                  -1.440039   0.291775      0  -4.935  < 1e-04 ***
gwesp.OTP.fixed.0.72             1.815694   0.132852      0  13.667  < 1e-04 ***
ctriple                         -0.265128   0.038565      0  -6.875  < 1e-04 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

     Null Deviance: 6890  on 4970  degrees of freedom
 Residual Deviance: 3331  on 4941  degrees of freedom

AIC: 3389    BIC: 3578    (Smaller is better.)
```

Figure 4: Summary of model fit

We see that attorneys of the same office, school, and practice have a higher probability of forming advice ties. The nodal covariate `age`, is once again slightly negative, meaning there is a negative effect from increasing age on the probability of a node to form advice ties. An interesting thing to note is that unlike the previous two models, the interaction between gender and status is an important predictor in the model.

3.3.2 Diagnostics

One way to look at how well the models performed is to simulate another network using the model and checking if the resulting simulated network look similar. The results of the simulated network are shown below:

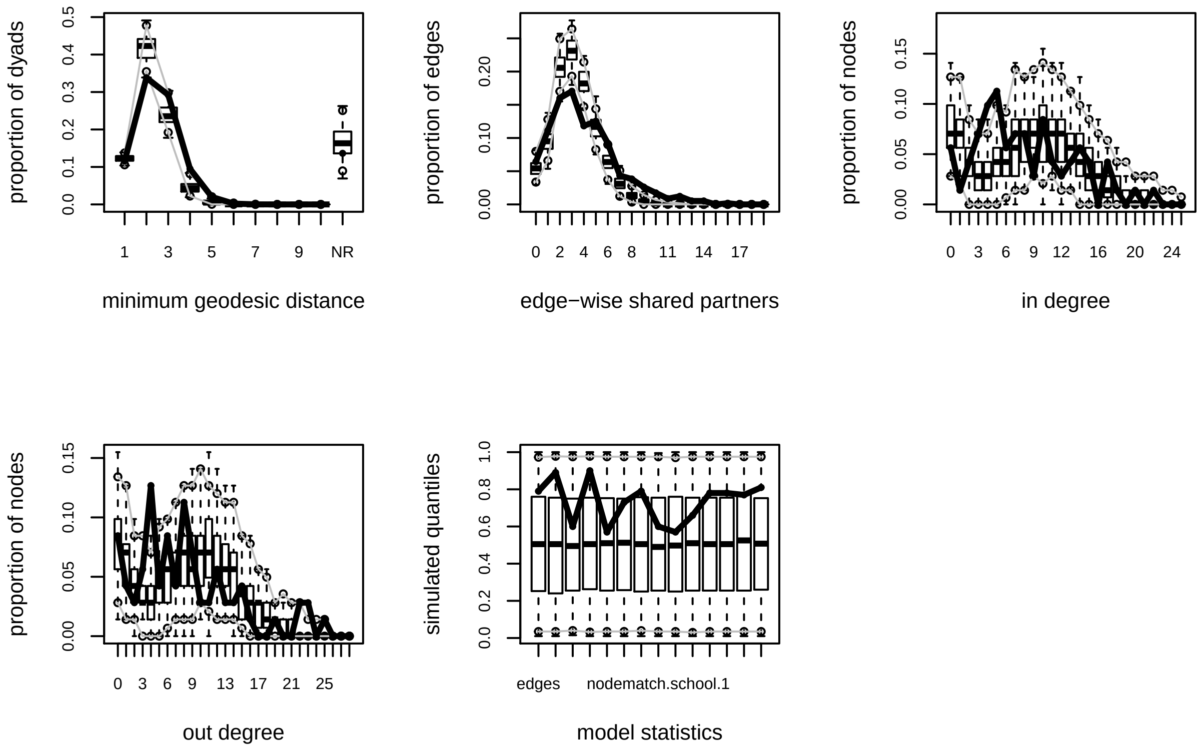


We see from the simulated networks above that they matched the observed graphs in section 3.1.1 pretty well. The clustering from the observed network is also correctly present in these simulated models. We also want to look at the diagnostics (MCMC diagnostics and goodness-of-fit) of the best fit model for each network to ensure the models are valid.

The results of the MCMC diagnostics (trace plots for all variables) of all 3 networks are much too long to include in this paper but for each predictor in the models of the 3 networks, the chains in each trace plot are all more or less centered around one value and varying randomly around it. In addition, all the chains appear to be overlapping one another in the trace plot. This indicates that the chains are stable and well mixed, meaning convergence has occurred in MCMC for our models.

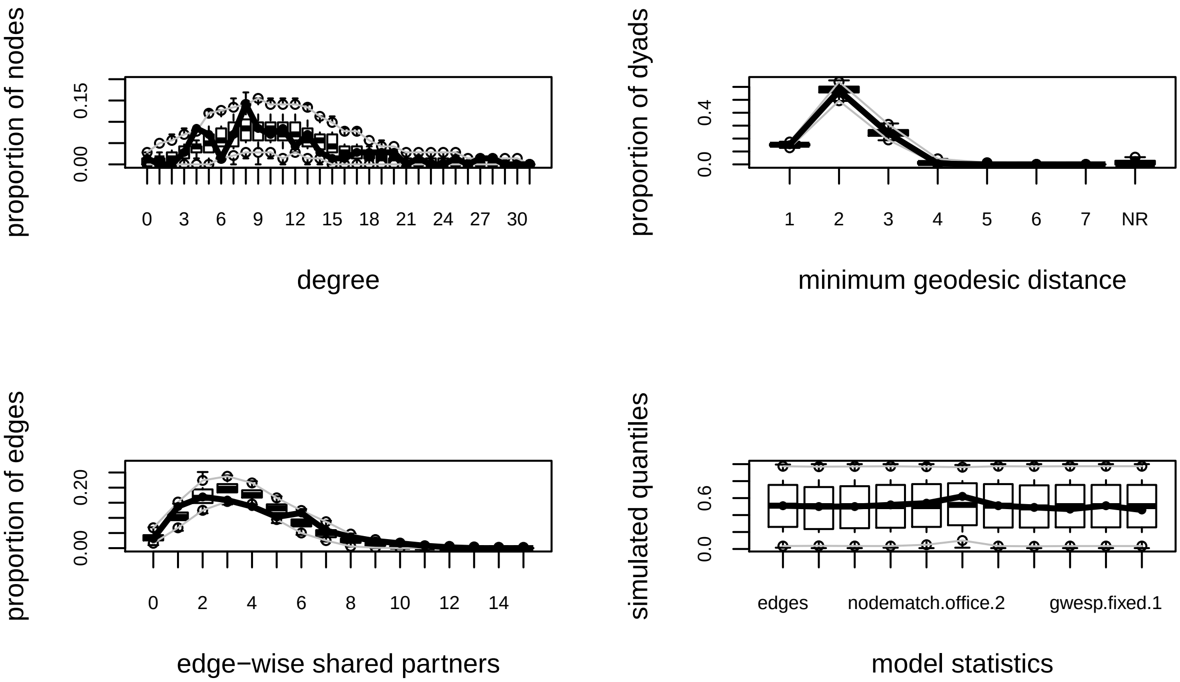
We now want to look at the goodness of fit for each of the models we made for our 3 networks. The results are shown in the plots below.

Friendship Network



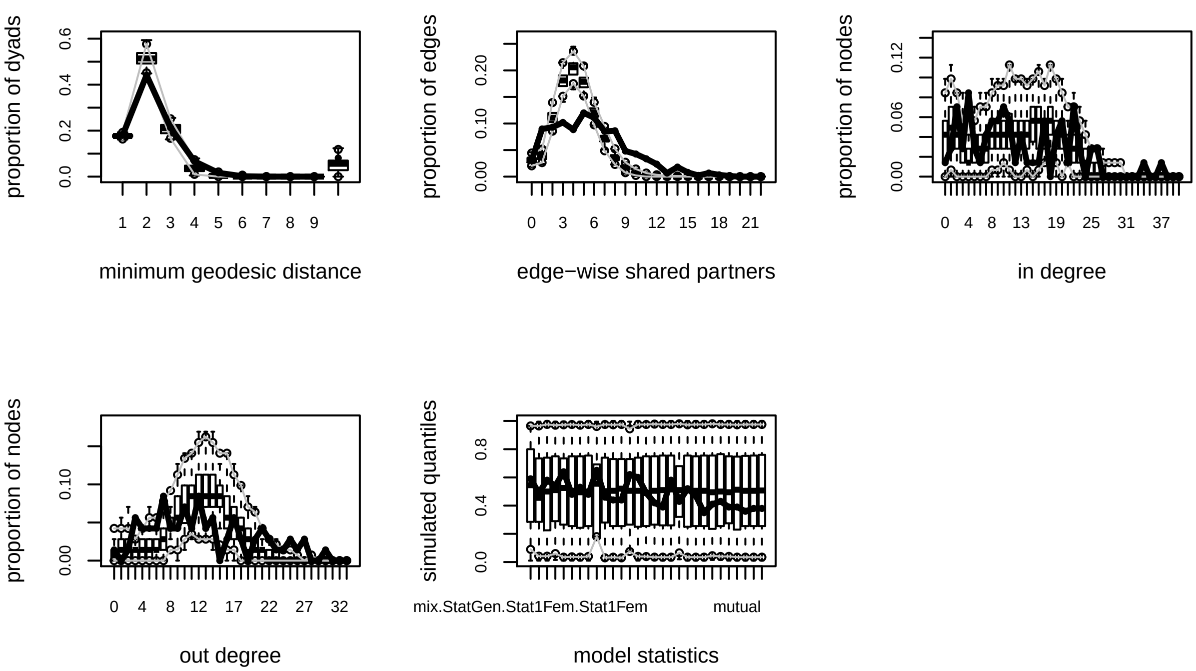
The solid line in each plot represents the observed statistics for the friendship network, and the boxplots summarize the statistics for the model we fit in section 3.3.1 resulting from the MLE estimates. We see that with there is some slight issue in our model with overestimating the edge-wise shared partners from the original friendship network (but only around values 3-8), but other elements such as in-degree, out-degree, minimum geodesic distance, and overall model statistics seem to match the original friendship network fairly well. So we can assume that the ERMG we fit for the friendship network is a fairly good and valid model.

Coworker Network



In the plots above, we see that the summary statistics from our model matched the observed statistics of the coworker network very well. So we can assume that the ERMG we fit for the coworker network was a very good and valid model. In fact, it seems like the network we were able to fit the best ERGM for is this coworker network.

Advice Network



The results above show that once again, there is some slight issue in our model with overestimating the edge-wise shared partners from the original advice network (but only around values 3-8 again), but other elements such as in-degree, out-degree, minimum geodesic distance, and overall model statistics seem to match the original advice network fairly well. Thus we can assume that the ERGM we fit for the advice network is a fairly good and valid model.

4 Discussion

Although we have analyzed the 3 networks in quite some detail in this paper, there are still limitations to the analysis that was conducted in this paper. One thing, in particular, that would have been beneficial to re-examine is instead of looking at the changes in the triad census of only male nodes and only female nodes after permutations, it would have been better to analyze how the census triad of the entire networks changed after gender permutations, and how nodes of different gender were positioned differently afterward.

However, R does not have a function that conducts triad census that takes into account nodal attributes. I was not able to manually write a triad census function that takes into account the effects of gender by adding code to an already existing R function because all the triad census functions already in R only produces the count of each triad formations in the network. They do not give the list of nodes involved in each triad. We cannot look at the effects of nodal attributes on triads formations if we do not know the nodes involved in the different triads. Since I did not have enough time to write a triad census function from scratch that outputs all the nodes and their positions in the triad, I simply looked at differences in triad formations for male nodes and female nodes separately. This way we can be certain of the distribution of gender in the different triads (because they're all one gender), without having to know which specific nodes are involved in each triadic formations.

Given more time, however, I think implementing and looking at how the census triad of the entire networks changed after gender permutations would be very beneficial to the analysis.

5 References

Alhazmi, Huda et al. "Understanding social effects in online networks." 2015 International Conference on Computing, Networking and Communications (ICNC) (2015): 863-868.

American Bar Association, "A Current Glance at Women in the Law," May 2016.

Emmanuel Lazega, The Collegial Phenomenon: The Social Mechanisms of Cooperation Among Peers in a Corporate Law Partnership, Oxford University Press (2001).

Zvereva, Olga M.. "Triad Census Usage for Communication Network Analysis." AIST (2016).