Modern Spatial Economics: A Primer*

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Abstract

We present a primer of the basic framework and solution techniques for models of the spatial economy with mobility of trade and workers flows. We document a number of facts about the flow of people and goods both within and across countries. We then show how these empirical patterns are consistent with a simple version of one of the most successful theories in modern economics: the gravity model. In this framework we highlight the importance of mobility and trade frictions by considering the impact of the Interstate Highway System under different scenarios. Our approach is designed for teaching this material at an advanced level but is still accessible to audiences without expertise on the topic.

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1 Introduction

Space - Economic Science's final frontier. Over the past twenty years a revolution in the exploration of space in Economics led by a combined effort from geographers, trade, and urban economists has finally brought the required technology to analyze space in all its glory: An armada of tools from mathematics, physics, cartography, and computer science has gradually formed the necessary equipment to explore space and its consequence for growth and allocation of economic activity.

Spearheading this exploration is spatial theory's dreadnought: The gravity model. This mathematical design strikes a careful balance between two key ingredients behind any successful theory. It is intuitive and pedagogical while at the same time rich enough to provide an incredibly good fit to the empirical observations. In other words, it is beautiful and it works like a charm!

The key force that spatial models with mobility of goods and people need to harness is space itself. In an environment with N locations, mobility of goods and people implies that N^2 number of trade interactions and N^2 migration flows have to be modeled. These many complex interactions could potentially obscure the main forces that determine economic activity across space. The resolution that the gravity model provides cuts right through these complexities: It allows for as many (exongeous) frictions as relationships, but assumes that the elasticity of the flows to (endogenous) push and pull factors are governed by only a single parameter. This lets the theory capture the first-order impacts of the effects of geography on economic outcomes while ignoring the (potentially) less important impact of varying bilateral elasticities.

To understand the model and the impact of space on economic activity we present a generalized, yet simple, version of the gravity model that allows for mobility of goods and people across space limited by frictions specific to these flows.[1] In particular, we employ an extension of the Allen and Arkolakis (2014) framework with an exogenous population in each location and mobility frictions across locations. Variations of this extension have been more formally modeled by Tombe, Zhu, et al. (2015), Bryan and Morten (2015), Caliendo, Dvorkin, and Parro (2015), Desmet, Nagy, and Rossi-Hansberg (2016), Faber and Gaubert (2016), Allen, Morten, and Dobbin (2017), and Allen and Donaldson (2017). Within this

[1] Examples of gravity trade models included in our framework are perfect competition models such as Anderson (1979), Anderson and Van Wincoop (2003), Eaton and Kortum (2002), Caliendo and Parro (2015) monopolistic competition models such as Krugman (1980), Melitz (2003) as specified by Chaney (2008), Arkolakis, Demidova, Klenow, and Rodríguez-Clare (2008), Di Giovanni and Levchenko (2008), Dekle, Eaton, and Kortum (2008), and the Bertrand competition model of Bernard, Eaton, Jensen, and Kortum (2003). Economic geography models incorporated in our framework include Allen and Arkolakis (2014) and Redding (2016) and the geography-trade framework of Allen, Arkolakis, and Takahashi (2014).

environment we can answer a number of important questions: What is the allocation of economic activity across space and how is it determined by location fundamentals or bilateral frictions of mobility. When does a solution of the model exists and when is it unique? Do different assumptions on the frictions of moving goods and people imply that policies have different implications?

Finally, we should note that the brief literature review of gravity models above is by no means complete and refer the interested reader to the excellent review articles by Baldwin and Taglioni (2006), Head and Mayer (2013), Costinot and Rodriguez-Clare (2013) and Redding and Rossi-Hansberg (2017), where the latter two focus especially on quantitative spatial models.

2 Gravity: The Evidence

We begin by explaining and documenting the "gravity" relationship. Empirically, the notion of gravity introduced by Tinbergen (1962) postulates that flows decline with distance. We illustrate that both the flow of goods (trade) and people (migration) exhibit gravity. Moreover, this gravity relationship is robust to different scales of distance: it is prevalent for the flow of people and goods both across countries and within countries. Finally, gravity has been present in the data for (at least) the past fifty years, and shows no signs of attenuating over time.

2.1 Gravity in the flow of goods

We first examine the flow of goods (trade). We use two different data sets: the first, from Head, Mayer, and Ries (2010), comprise the value of trade between between countries from 1948 to 2006; the second, from (CFS, 2007, 2012), are the Commodity Flow Surveys, comprise the value of trade between U.S. states for 2007 and 2012. To reduce concerns of selection bias, we constrain each sample to be balanced by only including origin-destination pairs that reported positive trade flows for each year in the sample; moreover, to avoid having to take a stand on what constitutes the "distance-to-self", in each sample we exclude own trade flows.

Let X_{ijt} be the value of trade flows from location i to location j in time t. The gravity relationship postulates that the (log) of the value of trade flows declines in the distance between locations, conditional on (endogenous) origin-specific push factors γ_{it}^T and destination-specific pull factors δ_{jt}^T:

$$\ln X_{ijt} = f\left(\ln dist_{ij}\right) + \gamma_{it}^T + \delta_{jt}^T, \tag{1}$$

where $\frac{\partial f}{\partial \ln dist_{ij}} < 0$. Figure 1 overlays a non-parametric function $f\left(\cdot\right)$ on top of a scatter plot of the relationship between log trade flows and log distance after partitioning out the origin-year and destination-year fixed effects for international trade in both 1950 and 2000; as is evident, there is a strong negative (and approximately log-linear) relationship in both years, with the negative effect being especially pronounced in the year 2000. In Figure 2, we impose a linear function $f(\ln dist_{ij}) = \gamma_t \ln dist_{ij}$ and estimate the coefficient γ_t separately for each year; we find a precisely estimated negative relationship that appears to be getting stronger over time, with a coefficient γ_t of between -0.5 and -1.5.

This strong gravity relationship in the flow of goods is also prevalent within countries. Figure 3 is the analog of Figure 1 for trade between U.S. states. In both 2007 and 2012 there is a strong negative (and approximately log-linear) relationship between log trade flows and log distance. As with the international trade flows, the trade coefficient is about -1 , showing that the effect of distance is similar within and across countries.

We can also examine how the origin push factor γ_{it} and destination pull factor δ_{jt} are correlated with various observables. Figure 4 shows that both the push and pull factors in international trade are strongly positively correlated with GDP – even after partitioning out time-invariant country effects and country-invariant year effects. Put another way, changes in GDP within a country over time (relative to total world GDP) are strongly positively correlated with both a country's imports and exports. This strong positive correlation is also present in within country trade flows, as is evident in Figure 5. Moreover, Figures 6 and 7 show a strong positive correlation between the push and pull factors both across and within countries. As we will see below, this positive correlation will be predicted by a gravity spatial model with balanced trade and symmetric trade costs.

2.2 Gravity in the flow of people

The flow of labor (migration) exhibits similar – but not identical – patterns as the flow of goods. We analyze the flow of labor both across and within countries. For international migration, we turn to the WBG (2011) dataset, which provides bilateral flows of people across countries every ten years from 1960 to 2010. For intranational migration, we construct flows of people across U.S. states using their state of birth and current location for each decennial census from 1850 to 2000 from Ruggles, Fitch, Kelly Hall, and Sobek (2000). As with the trade data, we consider a balanced sample of location pairs for which there is a positive flow of people in all years and exclude own-flows of people (i.e. those that do not migrate).

As with the flow of goods, we can construct a simple empirical gravity specification for the flow of people from location i to location j at time t, L_{ijt}:

$$\ln L_{ijt} = g(\ln dist_{ij}) + \gamma_{it}^L + \delta_{jt}^L, \tag{2}$$

where gravity implies $\frac{\partial g}{\partial \ln dist_{ij}} < 0$. Figure 8 overlays a non-parametric function $g(\cdot)$ on top of a scatter plot of the relationship between log migration flows and log distance after partitioning out the origin-year and destination-year fixed effects for international migration in both 1960 and 2010; like with the flow of goods, there is a strong negative (and approximately log-linear) relationship in both years. In Figure 9, we impose a linear function $g(\ln dist_{ij}) = \gamma_t \ln dist_{ij}$ and estimate the coefficient γ_t separately for each year of data; again, as with the flow of goods, we find a precisely estimated negative relationship with a coefficient γ_t of between -1 and -2.

Figure 10 shows that the gravity relationship also exists for within country migration and is remarkably stable over the 150 years of data; indeed, as Figure 11 illustrates, the effect of distance on the flow of people is nearly identical within the United States as it is across countries, with a coefficient hovering of about -1.5. (It is interesting to note that unlike for the flow of goods, the gravity coefficient of migration shows no evidence of getting more negative over time).

While the gravity relationship with distance is quite similar for trade and migration, the "push" and "pull" factors (γ_{it}^L and δ_{jt}^L, respectively) are substantially different for migration. Figure 12 shows that there is no systematic relationship between population and either the push or pull factor across countries; within the United States, however, Figure 13 shows the lagged population is strongly correlated with the push factor and the contemporaneous population is strongly correlated with the pull factors.Unlike the flow of goods, there is no systematic correlation between the push and pull factors across countries (Figure 14), but there is a negative correlation between push and pull factors across U.S. states (Figure 15). As we will see below, this is consistent with a theoretical model of migration when the population is not in a steady state and/or migration costs are not symmetric.

3 Gravity: A Simple Framework

We now introduce a simple model that can generate the prevalence of gravity in the data. Suppose there are N locations, where in what follows we define the set $S \equiv \{1, ..., N\}$, each producing a differentiated good. The only factor is labor, and we denote the allocation of labor in location $i \in S$ as L_i and assume the total world labor endowment is $\sum_{i \in S} L_i = \bar{L}$. Given the evidence from the previous section that gravity holds both within and across countries, locations can be interpreted as either regions within a country or countries themselves.

3.1 Demand for Goods: Gravity on Trade Flows

We assume that workers have identical Constant Elasticity of Substitution (CES) preferences over the differentiated varieties produced in each different location. The total welfare in location $i \in S$, W_i, can be written as:

$$W_i = \left(\sum_j q_{ji}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} u_i, \tag{3}$$

where q_{si} is the per-capita quantity of the variety produced in location s and consumed in location i, $\sigma \in (1, \infty)$ is the elasticity of substitution between goods ω, and u_i is the local amenity, discussed below.[2]

Each worker in location i earns a wage w_i and thus the budget constraint is

$$\sum_j p_{ji} q_{ji} = w_i \tag{4}$$

where p_{si} is the price of good from location s in i. Optimization of the worker utility, equation (3), subject to the budget constraint, equation (4), yields the total expenditure in location j on the differentiated variety from location i:

$$X_{ij} = (p_{ij})^{1-\sigma} P_j^{\sigma-1} w_j L_j \quad \text{for all } j \tag{5}$$

where L_j is the total number of workers residing in location j (determined endogenously below) and $P_j \equiv \left(\sum_i (p_{ij})^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is the Dixit-Stiglitz price index.

The production function of each variety is linear in labor and the productivity in location i is denoted by A_i. Thus, the cost of producing variety i is $p_i = w_i / A_i$. Shipping the good from i to final destination j incurs an "iceberg" trade friction, where $\tau_{ij} \geq 1$ units must be shipped in order for one unit to arrive. Thus, the price faced by location j for a factor from i can be written as:

$$p_{ij} = \frac{w_i}{A_i} \tau_{ij}, \tag{6}$$

where τ_{ij} are bilateral trade frictions. Substituting this solution to equation (5) and rearranging we obtain

$$X_{ij} = (\tau_{ij})^{1-\sigma} \left(\frac{w_i}{A_i} \right)^{1-\sigma} P_j^{\sigma-1} w_j L_j. \tag{7}$$

This equation is the modern version of a gravity equation initially derived by Anderson (1979)

[2] While the model attains a non-trivial solution even for $\sigma \in (0, 1)$, we focus on the case where $\sigma > 1$ so that the elasticity of trade flows to trade costs is negative.

and is ubiquitous in modern work in international trade. It is characterized by a bilateral term that is a combination of model parameters, trade costs and the trade elasticity, and origin and destination specific terms which are combinations of endogenous variables and parameters.

More recent work provides a wealth of microfoundations for this structural equation based on comparative advantage, increasing returns, or firm heterogeneity (see for example, Eaton and Kortum (2002); Chaney (2008); Eaton, Kortum, and Kramarz (2011); Arkolakis (2010); Arkolakis, Costinot, and Rodríguez-Clare (2012); Allen and Arkolakis (2014); Redding (2016) among others). Taking log of this equation generates the empirical gravity trade equation (1) presented in Section 2.

To incorporate agglomeration forces in production – which Allen and Arkolakis (2014) show creates an isomorphism with the monopolistic competition models with free entry as in Krugman (1980) – we assume that the productivity of a location is subject to spillovers: $A_i = \bar{A}_i L_i^{\alpha}$ where $\bar{A}_i$ is the exogenous productivity.[3] We focus on the empirically relevant cases of $\alpha \geq 0$, capturing agglomeration externalities due to endogenous entry, scale effects etc.

3.2 Demand for Labor: Gravity on Worker Flows

We next determine the allocation of labor across location. We assume that there is an initial (exogenous) distribution of workers across all locations i denoted by L_i^0, from which all workers choose where to live subject to migration frictions. In particular, the indirect utility function of an owner of one unit of aggregate factor originating from location i and moving to location j is equal to the product of the utility realized in the destination and a bilateral migration disutility ν_{ij} :

$$W_{ij} = \frac{w_j}{P_j} u_j \times \nu_{ij},$$

where $\nu_{ij} = \frac{\left(L_{ij}/L_i^0\right)^{-\beta}}{\mu_{ij}}$ depends both on an (exogenous) iceberg migration friction $\mu_{ij} \geq 1$ and on the (endogenous) number of migrating workers L_{ij}. The parameter $\beta \geq 0$ governs the extent to which migration flows create congestion externalities.

In equilibrium, labor mobility implies that the utility of all agents originating from i is equalized:

$$W_i = \frac{w_j}{P_j} \frac{\bar{u}_j}{\mu_{ij}} \left(L_{ij}/L_i^0\right)^{-\beta}. \tag{8}$$

[3]See Allen and Arkolakis (2014) for a precise discussion of the various isomorphisms to this formulation.

Inverting this expression, we obtain the number of workers that migrate from i to j:

$$L_{ij} = \frac{\left(\frac{w_j}{P_j}\frac{\bar{u}_j}{\mu_{ij}}\right)^{\frac{1}{\beta}}}{W_i^{\frac{1}{\beta}}} L_i^0. \tag{9}$$

Equation (9) is a gravity equation on worker flows as it determines the share of workers in location i as a function of the real wage in location i. By taking logs, it provides a theoretical justification of the empirical gravity specification for the flow of people in equation (2) in Section 2.

We should at this point note that when modeling bilateral migration flows many authors choose alternative, possibly more intuitive, formulations. These formulations yield the same functional form as (8) but capture a variety of microfoundations such as competition for an immobile factor (e.g. land or housing markets) or heterogeneous location preferences across workers. In this latter approach, an agent's utility in location j is the product of the local real wage times a heterogeneous component. This heterogeneity results to different decisions for otherwise identical agents. Assuming a Frechet distribution for this heterogeneity following Eaton and Kortum (2002); Ahlfeldt, Redding, Sturm, and Wolf (2015); Redding (2016) leads to a similar formulation to equation (9), as discussed in Allen and Arkolakis (2014).

3.3 Closing the Model

To close the model we need to satisfy four equilibrium conditions. The first two are associated with the flow of goods. First, the total amount of labor used for the production of goods for all countries equals the labor available in each country i. Written in terms of labor payments, this implies that the total payments accrued to labor in location i must equal to the sales of this location to all the locations in the world, including i,

$$w_i L_i = \sum_j X_{ij}. \tag{10}$$

The second equilibrium condition is that total expenditure equals total labor payments and in turn this equals total payments for goods produced for location i,

$$E_j = w_i L_i = \sum_i X_{ij}. \tag{11}$$

Using equation (7) this expression can be written,

$$w_j L_j = \sum_i (\tau_{ij})^{1-\sigma} \left(\frac{w_i}{A_i}\right)^{1-\sigma} P_i^{\sigma-1} w_j L_j \implies$$

$$P_i^{1-\sigma} = \sum_i (\tau_{ij})^{1-\sigma} \left(\frac{w_i}{A_i}\right)^{1-\sigma}, \tag{12}$$

which is the expression for the Dixit-Stiglitz price index defined above.

The third and fourth equilibrium conditions are associated with the flow of labor. The third condition is that the initial population in location i is equal to the total flows of persons from location i, i.e.:

$$L_i^0 = \sum_j L_{ij} \tag{13}$$

Combined with the migration gravity equation (9) above allows us to write equilibrium welfare of migrants from location i W_i as the CES aggregate of their bilateral utility:

$$L_i^0 = \sum_j \frac{\left(\frac{w_j}{P_j}\frac{\bar{u}_j}{\mu_{ij}}\right)^{\frac{1}{\beta}}}{W_i^{\frac{1}{\beta}}} L_i^0 \iff W_i = \left(\sum_j \left(\frac{w_j}{P_j}\frac{\bar{u}_j}{\mu_{ij}}\right)^{\frac{1}{\beta}}\right)^{\beta}. \tag{14}$$

Substituting this expression for welfare back into the migration gravity equation then allows us to write migration shares of people analogously to expenditure shares on goods:

$$L_{ij}/L_i^0 = \frac{\left(\frac{w_j}{P_j}\frac{\bar{u}_j}{\mu_{ij}}\right)^{\frac{1}{\beta}}}{\sum_j \left(\frac{w_j}{P_j}\frac{\bar{u}_j}{\mu_{ij}}\right)^{\frac{1}{\beta}}}. \tag{15}$$

Finally, the fourth equilibrium condition requires that the in-flow of migrants to location i is equal to its total population:

$$L_j = \sum_{i \in S} L_{ij}. \tag{16}$$

Define the *geography* of the economy as the set of trade costs $\{\tau_{ij}\}$, migration frictions $\{\mu_{ij}\}$, productivities $\{\bar{A}_i\}$, amenities $\{\bar{u}_i\}$, and initial population $\{L_i^0\}$. For any set of elasticities $\{\sigma, \alpha, \beta\}$ and any geography, an equilibrium is defined as a set of wages, labor allocations, price index, and welfare, that satisfy the following four equations:

$$w_i L_i = \sum_j (\tau_{ij})^{1-\sigma} \left(\frac{w_i}{\bar{A}_i L_i^\alpha}\right)^{1-\sigma} P_i^{\sigma-1} w_j L_j \tag{17}$$

$$P_i = \left(\sum_j \left(\frac{\tau_{ji} w_j}{\bar{A}_j L_j^\alpha}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \tag{18}$$

$$L_i = \sum_{j\in S} (\mu_{ji})^{-\frac{1}{\beta}} \left(\frac{w_i}{P_i}\bar{u}_i\right)^{\frac{1}{\beta}} (W_j)^{-\frac{1}{\beta}} L_j^0 \tag{19}$$

$$W_i = \left(\sum_j \left(\frac{w_j}{P_j}\frac{\bar{u}_j}{\mu_{ij}}\right)^{\frac{1}{\beta}}\right)^\beta \tag{20}$$

This yields a system of $4 \times N$ equations and $4 \times N$ unknowns (with one equation being redundant from Walras Law and one price being pinned down by a normalization). Note the symmetry of the labor and goods market clearing conditions: each consists of one market clearing condition and one composite "price index": P_i for goods and W_i for labor.

Precise restrictions that guarantee existence and uniqueness are provided by Allen and Donaldson (2017). Product differentiation implies that there are gains to moving to locations with low population in order to provide labor for the global demand of the local good. In addition agglomeration and dispersion forces act upon this basic mechanism. Intuitively, the agglomeration forces, governed by parameter α, imply increased concentration of economic activity. Dispersion forces, governed by parameter β, imply dispersion of economic activities away from locations with large population. When agglomeration forces are stronger than dispersion forces the possibility of multiple equilibria arises. In these cases agglomeration may act as a self-sustaining force and equilibria where different locations are the ones with the largest population can arise, similar to the spatial models of Krugman (1991); Helpman (1998); Fujita, Krugman, and Venables (1999). In fact, in the version of the model where migration costs are infinite, this happens exactly at $\alpha > \beta$, as discussed in Allen and Arkolakis (2014). Existence of equilibria with positive population is always guaranteed. However, when the agglomeration forces are very strong black hole equilibria with all the activity concentrated in one point may be the only ones that satisfy some refined notion of equilibrium related to stability.[4]

[4]The system of equations has the form of the multi-equation multi-location gravity system analyzed by Allen, Arkolakis, and Li (2014). Using their approach equilibrium existence and uniqueness can be characterized in generalized gravity systems. In addition, their approach provides algorithms to compute the equilibrium of these multi-equation systems efficiently. Refined notions of stability are discussed in Allen and Arkolakis (2014).

Finally, it may be apparent from the above discussion that particular microfoundations of the two key gravity equations do not play a key role in determining many of the properties of the model. A long tradition on modeling gravity in international trade flows is summarized in Arkolakis, Costinot, and Rodríguez-Clare (2012) (see discussion in Proposition 2) by a class of models governed by a single parameter, the elasticity of trade, hereby captured by $1-\sigma$. In addition, more recently, microfoundations for the migration gravity equation have been provided as well. It can be shown that, as far as it concerns the positive properties and the counterfactuals of this expanded class of models, with respect to wages and labor, two composite parameters (in this case functions of the three parameters α, β and σ) determine all its predictions. This class of models that includes geography models with labor mobility, intermediate inputs, non-traded goods and other economic forces is discussed in Allen, Arkolakis, and Takahashi (2014).

4 Model Characterization

We proceed next to characterize this setup by imposing assumptions on the geography of trade costs, τ_{ij}, and the geography of fundamentals, i.e. productivities and amenities, $\bar{A}_i$, $\bar{u}_i$.

4.1 Geography and the distribution of economic activity

We first provide intuition about how geography shapes the spatial distribution of economic activity; these results build upon Allen and Arkolakis (2014), Allen, Arkolakis, and Takahashi (2014) and Allen and Donaldson (2017).

Suppose that trade costs are bilaterally symmetric, i.e. $\tau_{ij} = \tau_{ji}$ for all i and j. Then thegoods gravity equation (7), and two equilibrium conditions, equations (10) and (11), together imply that the origin and destination fixed effects of the *trade* gravity equation are equal up to scale, i.e.:

$$\left(\frac{w_i}{\bar{A}_i L_i^{\alpha}}\right)^{1-\sigma} \propto P_i^{\sigma-1} w_i L_i \;\forall i \in S \tag{21}$$

This accords well with the finding in Section 2.1 that the origin and destination fixed effects of the trade gravity equation as strongly correlated.

What about on the migration side? Suppose that migration costs are bilaterally symmetric, i.e. $\mu_{ij} = \mu_{ji}$ for all i and j. One might wonder if this symmetry, along with the labor gravity equation (9), and the two labor adding up conditions, equations (14) and (16) correspondingly imply that the origin and destination specific terms of the *labor* gravity equation are equal up to scale. It turns out that the answer in general is no, unless the population distribution is in a steady state where $L_i^0 = L_i$. In that case (and only in that case) we have:

$$\left(\frac{w_i}{P_i}\bar{u}_i\right)^{\frac{1}{\beta}} \propto W_i^{-\frac{1}{\beta}} L_i \; \forall i \in S \tag{22}$$

Recall from Section 2.2 that we empirically find virtually no correlation in the origin and destination fixed effects in the migration gravity equation both across countries and across states within the U.S., suggesting that we are either far from a steady state or migration costs are asymmetric (or both). Combining equations (21) and (22), we can express the equilibrium steady-state population and wage of each location solely as a function of that location's productivity, amenity, and geographic location (with the effect of trade costs being summarized by P_i and the effect of migration costs being summarized by W_i):

$$\gamma \ln L_i = \frac{1}{\beta}(\sigma - 1)\ln \bar{A}_i + \frac{\sigma}{\beta}\ln \bar{u}_i + \frac{\sigma}{\beta}\ln W_i - \frac{2\sigma - 1}{\beta}\ln P_i + C_1 \tag{23}$$

$$\sigma\gamma \ln w_i = \sigma(\sigma - 1)\ln \bar{A}_i + (\alpha(\sigma - 1) - 1)\frac{\sigma}{\beta}\ln \bar{u}_i + (\alpha(\sigma - 1) - 1)\frac{\sigma}{\beta}\ln W_i - \sigma\left((\sigma - 1)\left(1 - \frac{\alpha}{\beta}\right) - \frac{1}{\beta}\right) \mathrm{l} \tag{24}$$

where $\gamma \equiv \left(\sigma + \frac{1}{\beta}(1 - \alpha(\sigma - 1))\right)$, C_1 is a constant determined by the size of the aggregate labor market and C_2 is a constant determined by the choice of numeraire. Focusing on the equilibrium distribution of population and assuming $\alpha < \frac{1}{\sigma - 1}$, we can see that more productive places (higher $\bar{A}_i$), higher amenity places (higher $\bar{u}_i$), places with lower migration costs (higher W_i) and places with lower trade costs (lower P_i) all have higher populations, with the responsiveness of the population to the geography governed by the strength of spillovers through the composite term γ.[5]

4.2 Special cases

We now illustrate a number of interesting special cases of this general framework. First, consider the case where workers cannot move from their original location, i.e. where $\mu_{ij} = \infty$ for all $i \neq j$. This is an assumption maintained in gravity trade models such as Anderson (1979); Eaton and Kortum (2002); Chaney (2008); Arkolakis (2010); Arkolakis, Costinot, and Rodríguez-Clare (2012); Costinot and Rodriguez-Clare (2013); Allen, Arkolakis, and Takahashi (2014). Since labor is fixed, we can determine the wage and the price index, w_i and P_i by using equations (17) and (18), equation (19) implies $L_i = L_i^0$, and equation (20)

[5] When $\alpha \geq \frac{1}{\sigma - 1}$, the only type of stable equilibria possible is a black-hole equilibrium where all the population is concentrated in a single location; see Allen and Arkolakis (2014) for an in depth discussion and formal definition of "stability" in this context.

simplifies to $W_i = \frac{w_i}{P_i}\bar{u}_i$.

Second, consider the case where there are no migration frictions, i.e. $\mu_{ij} = 1$ for all $i \neq j$. This is an assumption maintained in economic geography models such as Krugman (1991); Helpman (1998); Redding and Sturm (2008) and in the gravity new economic geography models of Allen and Arkolakis (2014); Redding (2016). In this case, equation (20) implies welfare is equalized across locations,

$$W = W_i = w_i/P_i,$$

Moreover, equation (19) implies that the population residing in location i is

$$\frac{L_i}{\bar{L}} = \frac{\left(\frac{w_i}{P_i}\bar{u}_i\right)^{1/\beta}}{\sum_{i'}\left(\frac{w_{i'}}{P_{i'}}\bar{u}_i\right)^{1/(\beta)}}. \tag{25}$$

A third special case is where there is both free trade and free migration, i.e. $\mu_{ij}, \tau_{ij} = 1$ for all $i \neq j$. This setup is the celebrated Rosen-Roback (1982) model put to use in a number of urban applications (see Glaeser and Gottlieb (2008) and Moretti (2011) for review of applications of this model). With free labor mobility, equation (20) again implies that $W_i = W$ while with free goods mobility, equation (18) implies that the price index equalizes across locations $P_i = P$.[6] Notice that in this special case equations (23) and (24) give an explicit solution of wages and labor in terms of geography of each location. These solutions are intuitive (e.g. under reasonable restrictions labor is increasing in productivities and amenities) and have been heavily exploited by the urban literature.

4.3 Analytical Solutions

In this section, we provide analytical solutions for the equilibrium above for two simple geographies. (Note that the characterization of the equilibrium from equations (23) and (24) provide only partial characterization in the case with positive trade or migration frictions as they are functions of two equilibrium objects, namely the price index and the expected welfare of a location).

[6] In fact, the baseline Rosen-Roback (1982) framework also imposes that elasticity of substitution is infinite across goods, i.e. the goods are perfect substitutes. The assumption is not essential for what is obtained below.

Two countries

In the special case of two countries we assume that the population is fully mobile across locations so that utility equalization holds. Here we do not impose any restriction on the geography of trade costs but we do impose the assumption of symmetry in the geography of productivities and amenities so that $\bar{A}_i = \bar{u}_i = 1$ for all i. Using equation (10) and the gravity equation for trade flowsimplies

$$\begin{aligned} w_i L_i &= w_i^{2-\sigma} P_i^{\sigma-1} L_i + \tau_{ij}^{1-\sigma} w_i^{1-\sigma} w_j L_j P_j^{\sigma-1} \iff \\ L_i &= w_i^{1-\sigma} P_i^{\sigma-1} L_i + \tau_{ij}^{1-\sigma} w_i^{-\sigma} w_j L_j P_j^{\sigma-1} \end{aligned} \tag{26}$$

We next impose utility equalization resulting from the zero migration costs: $\frac{w_i}{P_i} = \frac{w_j}{P_j} = \bar{W}$. This implies that:

$$\frac{P_j^{\sigma-1}}{P_i^{\sigma-1}} = \frac{w_j^{\sigma-1}}{w_i^{\sigma-1}}.$$

Using the definition of the price index we have that

$$\frac{w_i^{1-\sigma}\tau_{ij}^{1-\sigma} + w_j^{1-\sigma}}{w_j^{1-\sigma}\tau_{ji}^{1-\sigma} + w_i^{1-\sigma}} = \frac{w_j^{\sigma-1}}{w_i^{\sigma-1}} \implies$$

$$\left(\frac{w_i}{w_j}\right) = \left(\frac{\tau_{ji}}{\tau_{ij}}\right)^{\frac{1}{2}}. \tag{27}$$

Combining the two results yields:

$$L_i = \bar{W}^{1-\sigma}\left(L_i + \tau_{ij}^{1-\sigma}\left(\frac{w_j}{w_i}\right)^{\sigma} L_j\right).$$

Now take the ratio of the two region's populations:

$$\frac{L_i}{L_j} = \frac{L_i + \tau_{ij}^{1-\sigma}\left(\frac{\tau_{ij}}{\tau_{ji}}\right)^{\frac{\sigma}{2}} L_j}{\tau_{ji}^{1-\sigma}\left(\frac{\tau_{ji}}{\tau_{ij}}\right)^{\frac{\sigma}{2}} L_i + L_j} = \frac{\left(\frac{L_i}{L_j}\right) + \tau_{ij}^{1-\sigma}\left(\frac{\tau_{ij}}{\tau_{ji}}\right)^{\frac{\sigma}{2}}}{\tau_{ji}^{1-\sigma}\left(\frac{\tau_{ji}}{\tau_{ij}}\right)^{\frac{\sigma}{2}}\frac{L_i}{L_j} + 1} \implies$$

$$\frac{L_i}{L_j} = \left(\frac{\tau_{ij}}{\tau_{ji}}\right)^{\frac{1}{2}} \tag{28}$$

In other words, if country j is more open that country i, $\tau_{ij} < \tau_{ji}$, then the population in country i is smaller but wages are higher.

The Line

We now assume that the topography of the world is described by a line, largely drawing from the analysis of Allen and Arkolakis (2014). Let space S be the $[-\pi, \pi]$ interval and suppose that $\alpha = \beta = 0$ and $\bar{A}_i = \bar{u}_i = 1$, i.e. there are no spillovers and all locations have homogeneous exogenous productivities and amenities. Suppose that trade costs are instantaneous and apart from a border b in the middle of the line at location 0; that is, trade costs between locations on the same side of the line are $\tau_{is} = e^{\tilde{\tau}|i-s|}$ and those on different sides are $\tau_{is} = e^{b+\tilde{\tau}|i-s|}$.[7]

Taking logs of equation (23) and differentiating yields the following differential equation:

$$\frac{\partial \ln L_i}{\partial i} = (1 - 2\sigma) \frac{\partial \ln P_i}{\partial i}. \tag{29}$$

It is easy to show that $\frac{\partial \ln P_{-\pi}}{\partial i} = -\tilde{\tau}$ and $\frac{\partial \ln P_{\pi}}{\partial i} = \tilde{\tau}$ in the two edges of the line and $\frac{\partial \ln P_0}{\partial i} = \tilde{\tau} \left(1 - e^{(1-\sigma)b}\right) / \left(1 + e^{(1-\sigma)b}\right)$ in the location of the border which gives us boundary conditions for the value of the differential equation at locations $i = -\pi, 0, \pi$. Intuitively, moving rightward while on the far left of the line reduces the distance to all other locations by τ, thereby reducing the (log) price index by τ. To obtain a closed form solution to equation (29), we differentiate equation (23) twice to show that the equilibrium satisfies the following second order differential equation:

$$\frac{\partial^2}{\partial i^2} L_i^{\tilde{\sigma}} = k_1 L_i^{\tilde{\sigma}} \text{ for } i \in (-\pi, 0) \cup (0, \pi), \tag{30}$$

where $\tilde{\sigma} \equiv (\sigma - 1) / (2\sigma - 1)$ and $k_1 \equiv (1 - \sigma)^2 \tau^2 + 2 (1 - \sigma) \tau W^{1-\sigma}$ can be shown to be negative. Given the boundary conditions above, the equilibrium distribution of labor in both intervals is characterized by the weighted sum of the cosine and sine functions (see example 2.1.2.1 in Polyanin and Zaitsev (2002)):

$$L_i^{\tilde{\sigma}} = k_2 \cos\left(i\sqrt{-k_1}\right) + k_3 \left|\sin\left(i\sqrt{-k_1}\right)\right|.$$

The values of k_1 and the ratio of k_2 to k_3 can be determined using the boundary conditions. Given this ratio, the aggregate labor clearing condition determines their levels.[8] Notice that

[7] This border cost is reminiscent of the one considered in Rossi-Hansberg (2005). As in that model, our model predicts that increases in the border cost will increase trade between locations that are not separated by border and decrease trade between locations separated by the border. Unlike Rossi-Hansberg (2005), however, in our model the border does not affect what good is produced (since each location produces a distinct differentiated variety) nor is there an amplification effect through spillovers (since spillovers are assumed to be local).

[8] More general formulations of the exogenous productivity or amenity functions result to more general

in the case of no border or an infinite border, the solution is the simple cosine function or two cosine functions one in each side of the border, respectively, and $k_3 = 0$, so that the aggregate labor clearing condition directly solves for k_2.[9]

Figure 16 depicts the equilibrium labor allocation in this simple case for different values of the instantaneous trade cost but no border. As the instantaneous trade cost increases, the population concentrates in the middle of the interval where the locations are less economically remote. The lower the trade costs, the less concentrated the population; in the extreme where $\tau = 0$, labor is equally allocated across space. With symmetric exogenous productivities and amenities, wages are lower in the middle of the line to compensate for the lower price index. Figure 17 shows how a border affects the equilibrium population distribution with a positive instantaneous trade cost. As is evident, the larger the border, the more economic activity moves toward the middle of each side in the line; in the limit where crossing the border is infinitely costly, it is as if the two line segments existed in isolation.

Differences in exogenous productivities, amenities and the spillovers also play a key role in determining the equilibrium allocation of labor and wages. The numerical solutions for these cases can be found in Allen and Arkolakis (2014).

5 Taking the model to the data

In this section, we discuss how one can estimate the parameters of the structural model from the previous section and use the model to perform counterfactuals. We conclude the section by illustrating how different assumptions regarding the mobility of goods and labor affects such counterfactual results.

5.1 Estimation

We first discuss how to estimate all the model parameters using readily available data. One particular highlight of the methodology that follows is that it does not require observing all bilateral flows of goods and labor. This is helpful, as oftentimes (especially in sub-national data) such bilateral data is unavailable.

specifications of the second order differential equation illustrated above (see Polyanin and Zaitsev (2002) section 2.1.2 for a number of tractable examples).

[9] Mossay and Picard (2011) obtain a characterization of the population based on the cosine function in a model where there is no trade but agglomeration of population arises due to social interactions that decline linearly with distance. In their case, population density may be zero in some locations while in our case the CES Armington assumption generates a strong dispersion force that guarantees that the equilibrium is regular when agglomeration forces are not too strong, as discussed in Theorem 2.

Estimating Bilateral Frictions

Assume that the log of the trade and migration bilateral frictions (to the power of their respective elasticities, i.e. i.e. $\tau_{ij}^{1-\sigma}$ and $\mu_{ij}^{\frac{1}{\beta}}$) are functions of observables (e.g. distance), i.e. $(1-\sigma)\ln\tau_{ij} = f\left(X_{ij}^{T};\gamma^{T}\right)$ and $\frac{1}{\beta}\ln\mu_{ij} = g\left(X_{ij}^{L};\gamma^{L}\right)$, where $f(\cdot)$ and $g(\cdot)$ are functions known up to a vector of parameters γ^{T} and γ^{L}, respectively. Then taking logs of the goods and labor gravity equations (7) and (9) yield:

$$\ln X_{ij} = f\left(X_{ij}^{T};\gamma^{T}\right) + \gamma_i^{T} + \delta_j^{T} + \varepsilon_{ij}^{T} \tag{31}$$

$$\ln L_{ij} = g\left(X_{ij}^{L};\gamma^{L}\right) + \gamma_i^{L} + \delta_j^{L} + \varepsilon_{ij}^{L}, \tag{32}$$

where γ_i^{T} and γ_i^{L} are the origin fixed effects of the trade and labor gravity equations, δ_j^{T} and δ_j^{L} are the respective destination fixed effects, and we interpret ε_{ij}^{T} and ε_{ij}^{L} as measurement error in the observed bilateral flows. Note that equations (31) and (32) are virtually identical to the empirical gravity equations (1) and (2)presented in Section 2; the key difference is we have now provided a theoretical justification for them.

By far the most common assumption in the literature is that bilateral frictions are increasing in bilateral distance $dist_{ij}$, i.e. $f(\cdot) = \gamma^{T}\ln dist_{ij}$ and $g(\cdot) = \gamma^{L}\ln dist_{ij}$. However, several recent innovations have allowed researchers to go beyond simply using straight-line distances between locations. Donaldson and Hornbeck (2012); Donaldson (forthcoming) uses Dijkstra's algorithm to calculate the least cost route between locations on a graph of the transportation network. Allen and Arkolakis (2014) apply the Fast Marching Method to calculate the least cost route between locations over a continuous geography. Both these methods rely on algorithms originally developed in computer science, leaving $f(\cdot)$ and $g(\cdot)$ implicit functions of the underlying geography. More recently Allen and Arkolakis (2017) derive a closed form solution for $f(\cdot)$ and $g(\cdot)$ as a function of the underlying transportation network assuming that many heterogeneous traders each choose the least cost route over the network.

Regardless of the choice of $f(\cdot)$ and $g(\cdot)$, the fixed effects regression only recovers estimates of the combination of the bilateral frictions and their respective elasticities, i.e. $\tau_{ij}^{1-\sigma}$ and $\mu_{ij}^{-\frac{1}{\beta}}$. However, as we will see, this is all that is needed to recover the remainder of the parameters and conduct counterfactuals. To make this clear, define $T_{ij} \equiv \tau_{ij}^{1-\sigma}$ and $M_{ij} \equiv \mu_{ij}^{-\frac{1}{\beta}}$ for what follows. Moreover, note that estimation can be accomplished even if bilateral flow data is only available for a subset of location pairs, as long as the observables are available for all bilateral pairs. This is helpful, for example, if trade flow data is only available for a subset of countries, but one wishes to construct a model for the entire world.

Recovering Location fundamentals and estimating model elasticities

Given estimates of $T_{ij} \equiv \tau_{ij}^{1-\sigma}$ and $M_{ij} \equiv \mu_{ij}^{-\frac{1}{\beta}}$, we can use the equilibrium structure of the model to recover information about endogenous outcomes in each location, namely the productivities A_i, the amenities u_i, the price index P_i, and the welfare W_i.

To see this, we re-write the equilibrium equations (17)-(20) as follows:

$$p_i^{\sigma-1} = \sum_j T_{ij} \left(\frac{Y_j}{Y_i}\right) P_i^{\sigma-1} \tag{33}$$

$$\left(P_i^{\sigma-1}\right)^{-1} = \sum_j T_{ji} \left(p_j^{\sigma-1}\right)^{-1} \tag{34}$$

$$\left(\omega_i^{\frac{1}{\beta}}\right)^{-1} = \sum_j M_{ji} \left(\frac{L_j^0}{L_i}\right) \left(W_j^{\frac{1}{\beta}}\right)^{-1} \tag{35}$$

$$W_i^{\frac{1}{\beta}} = \sum_j M_{ij} \omega_j^{\frac{1}{\beta}} \tag{36}$$

where $p_i \equiv \frac{w_i}{A_i L_i^{\alpha}}$ is the price of a good produced in location i and $\omega_i \equiv \frac{w_i}{P_i} \bar{u}_i$ is the welfare of residents residing in location i. Since $\{T_{ij}\}$ and $\{M_{ij}\}$ were estimated in the previous step and assuming the income Y_i, initial population L_i^0 and current population L_i are all observed in the data, it can be shown (see Allen and Donaldson (2017)) that there exists a unique (to-scale) set of $\left\{p_i^{\sigma-1}, P_i^{\sigma-1}, \omega_i^{\frac{1}{\beta}}, W_i^{\frac{1}{\beta}}\right\}$ that are consistent with equations (34)-(36). That is, the equilibrium structure of the model allows one to uniquely invert the model to recover these parameters for each location.

There are three important things to note about this inversion procedure. First, the inversion itself does *not* require knowledge of the any model elasticities, i.e. conditional on the bilateral frictions $\{T_{ij}\}$ and $\{M_{ij}\}$, the choice of σ, β, or α does not affect the equilibrium $\left\{p_i^{\sigma-1}, P_i^{\sigma-1}, \omega_i^{\frac{1}{\beta}}, W_i^{\frac{1}{\beta}}\right\}$. Second, if one does know the values of σ, β, and α, one can identify the exogenous productivity and amenity of each location, since:

$$\ln\left(p_i^{\sigma-1}\right) = (\sigma-1)\ln w_i - \alpha(\sigma-1)\ln L_i - (\sigma-1)\ln \bar{A}_i \tag{37}$$

$$\ln\left(\omega_i^{\frac{1}{\beta}}\right) = \frac{1}{\beta}\ln\frac{w_i}{P_i} + \frac{1}{\beta}\ln \bar{u}_i, \tag{38}$$

where the left hand side of both equations ($\ln\left(p_i^{\sigma-1}\right)$ and $\ln\left(\omega_i^{\frac{1}{\beta}}\right)$) are values recovered from the inversion and the right hand side are either observed in the data (the $\ln w_i$ and $\ln L_i$), are recovered from the inversion (the $\ln P_i^{\sigma-1}$) or are the exogenous productivities and amenities (the $\ln \bar{A}_i$ and $\ln \bar{u}_i$).

Third, and perhaps most importantly, note that equations (37) and (38) are estimating equations that allow one to recover the model elasticities σ, α, and β. Specifically, one can regress the $\ln\left(p_i^{\sigma-1}\right)$ recovered from the model inversion on the observed wages $\ln w_i$ and population $\ln L_i$ to recover estimates of σ and α. Given the estimate of σ, one can construct P_i from the model inversion of $P_i^{\sigma-1}$ and then regress $\ln\left(\omega_i^{\frac{1}{\beta}}\right)$ on the observed real wage to recover β. Crucially, because the exogenous productivity and amenity of each location are the residuals of these two equations – and the model tells us that the wage and population will be correlated with the exogenous productivities and amenities (see equations (23) and (24)), ordinary least squares estimation will yield biased estimates. Instead, estimation of the structural parameters requires valid instruments for the wage (both real and nominal) and population that are both correlated with the observed wage and population but uncorrelated with the fundamental productivity and amenity of a location. Two plausible sets of instruments are as follows. First, Allen, Arkolakis, and Takahashi (2014) suggest that the model structure could be used to generate instruments. In particular, the authors calculate the equilibrium wages, price indices, and populations of a hypothetical world where bilateral frictions are a solely a function of observed bilateral distance and local characteristics (e.g. productivity and amenities) are solely a function of known geographic variables (e.g. distance to coast, ruggedness, soil quality, etc.). One could then use these equilibrium variables as instruments for the actual wages, price indices, and populations. This procedure is valid as long as the geographic variables used are also controlled for directly in the regression; indeed, Adao, Arkolakis, and Esposito (2017) show that a two-stage procedure using such a model implied instrument is optimal in the sense that it minimizes the variance of the estimates. An alternative approach would be to use a shock to either trade or migration costs; for example Allen, Morten, and Dobbin (2017) use the construction of wall segments along the border between the U.S. and Mexico as a shock to migration costs. This can be implemented either in a reduced form way (e.g. instrumenting for wages, populations, and the price indices using distance to the border wall, as in Ahlfeldt, Redding, Sturm, and Wolf (2015)) or by using the structure of the model to predict how the trade cost shock changes each of the endogenous variables and using these model-predicted changes in wages, populations, and price indices to estimate the elasticities using the first-differenced versions of equations (37) and (38).

5.2 Conducting Counterfactuals

Given estimates of $T_{ij} \equiv \tau_{ij}^{1-\sigma}$ and $M_{ij} \equiv \mu_{ij}^{-\frac{1}{\beta}}$ and the structural elasticities σ, α, and β from the previous section, it is straightforward to use the equilibrium structure of the model to conduct counterfactuals. Consider any counterfactual change in the bilateral frictions that change $\{T_{ij}, M_{ij}\}$ to $\{T'_{ij}, M'_{ij}\}$. Following Dekle, Eaton, and Kortum (2008), we can express the equilibrium system using the "exact hat algebra" approach where the notation $\hat{x} \equiv \frac{x'}{x}$ is the ratio of the counterfactual value of a variable its current value. We start by writing equilibrium equations (17)-(20) for the future period and using the the identity $x' = \hat{x} \times x$ as follows:

$$\hat{Y}_i = \sum_j \pi_{ij} \hat{T}_{ij} \hat{p}_i^{1-\sigma} \hat{P}_i^{\sigma-1} \tag{39}$$

$$\hat{P}_i^{1-\sigma} = \sum_j \chi_{ji} \hat{T}_{ji} \hat{p}_j^{1-\sigma} \tag{40}$$

$$\hat{L}_i = \sum_{j \in S} \rho_{ji} \hat{M}_{ji} \hat{\omega}_i^{\frac{1}{\beta}} \hat{W}_j^{-\frac{1}{\beta}} \tag{41}$$

$$\hat{W}_i^{\frac{1}{\beta}} = \sum_j \upsilon_{ij} \hat{M}_{ij} \hat{\omega}_j^{\frac{1}{\beta}}, \tag{42}$$

where $\pi_{ij} \equiv \frac{T_{ij} p_i^{1-\sigma} P_j^{\sigma-1} Y_j}{Y_i} = \frac{X_{ij}}{Y_i}$ is the trade export share, $\chi_{ji} \equiv \frac{T_{ji} p_j^{1-\sigma} P_i^{\sigma-1} Y_i}{Y_i} = \frac{X_{ji}}{Y_i}$ as the trade import share, $\rho_{ji} \equiv \frac{M_{ji} W_j^{-\frac{1}{\beta}} L_j^0 \omega_i^{\frac{1}{\beta}}}{L_i} = \frac{L_{ji}}{L_i}$ is the in-migration share, and $\upsilon_{ij} \equiv \frac{M_{ij} W_i^{-\frac{1}{\beta}} L_i^0 \omega_j^{\frac{1}{\beta}}}{L_i^0} = \frac{L_{ij}}{L_i^0}$ is the out migration share. There are several important things to note about the system of equations (39)-(42). First, the kernel of each equation – i.e. π_{ij}, χ_{ji}, ρ_{ji}, and υ_{ij} – are solely a function of observed variables (income Y_i, initial and final populations L_i and L_i^0), estimated variables (the bilateral frictions T_{ij} and M_{ij}) and variables that are recovered from the model inversion (i.e. $\left\{ p_i^{\sigma-1}, P_i^{\sigma-1}, \omega_i^{\frac{1}{\beta}}, W_i^{\frac{1}{\beta}} \right\}$). Moreover, no element of the kernel requires any knowledge of the model elasticities to recover. Hence, even if the bilateral shares are not directly observed, all necessary components of the shares are observed so that the kernels of each equation can be treated as observable. This shows that the "exact hat algebra" approach of Dekle, Eaton, and Kortum (2008) does not require observing bilateral trade or migration flows to implement. Second, the change in bilateral frictions $\hat{M}_{ij}$ and $\hat{T}_{ij}$ depend on the counterfactual of interest, so they are known as well. Hence, equations (39)-(42) comprise $4N$ equations for $6N$ unknowns, namely $\left\{ \hat{Y}_i, \hat{L}_i, \hat{p}_i^{1-\sigma}, \hat{P}_i^{\sigma-1}, \hat{\omega}_i^{\frac{1}{\beta}}, \hat{W}_j^{-\frac{1}{\beta}} \right\}$.

Since the $4N$ equations in the system (39)-(42) do not depend on any of the model elasticities $\{\sigma, \alpha, \beta\}$, one might reasonably ask how these elasticities affect the counterfactuals. The answer is that the choice of model elasticities affects how one can express the changes in incomes $\hat{Y}_i$ and populations $\hat{L}_i$ as a function of the changes in the other endogenous variables $\left\{\hat{p}_i{}^{1-\sigma}, \hat{P}_i^{\sigma-1}, \hat{\omega}_i^{\frac{1}{\beta}}, \hat{W}_j^{-\frac{1}{\beta}}\right\}$. Taking first differences of equations (37) and (37) – using the fact that $\hat{\bar{A}}_i = \hat{\bar{u}}_i = 1$ and $\ln \hat{Y}_i = \ln \hat{L}_i + \ln \hat{w}_i$ – and inverting yields:

$$\ln \hat{Y}_i = \frac{\beta(1+\alpha)}{\alpha} \ln \hat{\omega}_i^{\frac{1}{\beta}} - \frac{1+\alpha}{\alpha(\sigma-1)} \ln \hat{P}_i^{\sigma-1} - \frac{1}{\alpha(\sigma-1)} \ln \hat{p}_i{}^{1-\sigma}$$
$$\ln \hat{L}_i = \frac{\beta}{\alpha} \ln \hat{\omega}_i^{\frac{1}{\beta}} - \frac{1}{\alpha(\sigma-1)} \ln \hat{P}_i^{\sigma-1} - \frac{1-\alpha}{\alpha(1+\alpha)(\sigma-1)} \ln \hat{p}_i{}^{1-\sigma},$$

which allows us to re-write equations the $4N$ equations in the system (39)-(42) as a function of only $4N$ unknowns:

$$\left(\hat{\omega}_i^{\frac{1}{\beta}}\right)^{\frac{\beta(1+\alpha)}{\alpha}} \left(\hat{P}_i^{\sigma-1}\right)^{-\frac{1+\alpha}{\alpha(\sigma-1)}} \left(\hat{p}_i{}^{1-\sigma}\right)^{-\frac{1}{\alpha(\sigma-1)}} = \sum_j \pi_{ij} \hat{T}_{ij} \hat{p}_i{}^{1-\sigma} \hat{P}_i^{\sigma-1} \tag{43}$$

$$\left(\hat{P}_i^{\sigma-1}\right)^{-1} = \sum_j \chi_{ji} \hat{T}_{ji} \hat{p}_j{}^{1-\sigma} \tag{44}$$

$$\left(\hat{\omega}_i^{\frac{1}{\beta}}\right)^{\frac{\beta}{\alpha}} \left(\hat{P}_i^{\sigma-1}\right)^{-\frac{1}{\alpha(\sigma-1)}} \left(\hat{p}_i{}^{1-\sigma}\right)^{-\frac{1-\alpha}{\alpha(1+\alpha)(\sigma-1)}} = \sum_{j \in S} \rho_{ji} \hat{M}_{ji} \hat{\omega}_i^{\frac{1}{\beta}} \hat{W}_j^{-\frac{1}{\beta}} \tag{45}$$

$$\left(\hat{W}_j^{-\frac{1}{\beta}}\right)^{-1} = \sum_j \upsilon_{ij} \hat{M}_{ij} \hat{\omega}_j^{\frac{1}{\beta}}, \tag{46}$$

and a normalization on prices. These four equations can then be solved to determine the effect of the counterfactual on all endogenous variables.

5.3 An Example: The Interstate Highway System

Finally, we illustrate the procedure above with a counterfactual based on Allen and Arkolakis (2014): the effect of the construction of the Interstate Highway System (IHS). In particular, we consider five different versions of that counterfactual with different assumptions regarding the mobility of goods and labor: first, we consider a "trade" model where labor is fixed in each location and the movement of goods is costly; second, we consider the reverse where each location is in autarky (trade costs are infinite) but migration is possible (but costly); third, we consider an "economic geography" model where trade is costly but the movement of labor is costless; fourth, we consider a "labor" model where trade is costless but the movement of labor is costly; and finally we consider the full model where both labor and trade are mobile but subject to bilateral frictions.

For each version of the model, we use data on the wage and population from the 2000 Census as in Allen and Arkolakis (2014) and invert the model using the methodology discussed in Section 5.1 to recover the unique set of variables for each location so that the model exactly matches the data given assumptions on labor and goods mobility.[10] (For models with labor mobility, we assume that the population in the 2000 Census is the steady state population). We use the estimates of the removal of the IHS from Allen and Arkolakis (2014) to construct $\hat{T}_{ij}$ and $\hat{M}_{ij}$ (assuming migration costs and trade costs are affected equally) and apply the counterfactual procedure from Section 5.2 to calculate the change in populations $\hat{L}_i$ and real wages $\frac{\hat{w}_i}{\hat{P}_i}$ in each location. For all simulations, we set $\sigma = 4$, $\alpha = 0.1$, and $\beta = 0.25$, which are (roughly) in line with estimates from the existing literature, see e.g. Simonovska and Waugh (2014) and Rosenthal and Strange (2004).

Figures 18 through 22 present the results; each figure shows the spatial distribution in the change in population and real wage across all U.S. counties by decile, with blue indicating the greatest decline and red indicating the greatest increase. The most important thing to see from the simulations is that the assumptions regarding the mobility of goods and people play a hugely important role in dictating the effects of the removal of the IHS. For example, in the first counterfactual, there is zero correlation between the change in population and the change in real wages – for the simple reason that the population is assumed to be immobile (see Figure 18). Contrast this with the second counterfactual, where there is a perfect correlation between the (log) change in population and the (log) change in real wages – for the simple reason that the change in real wages is determined solely by the change in the location population when each location is in autarky (see Figure 19). This is also true when trade is costly but migration is costless (as is assumed in Allen and Arkolakis (2014)), as welfare will be equalized across all locations (see Figure 20). However, when migration is costly (and trade is possible), the correlation between the change in population and the change in real wages is far from perfect, as Figures 21 and 22 highlight.

Table 1 depicts the pair-wise correlations in predicted changes in real wages and populations across the different model variants. It is somewhat surprising to see how varied the predictions for population changes are: for example, the predictions of model variant #2 (no trade, costly migration) is strongly negatively correlated with the other model variants. Even across the other model variants with labor mobility, the correlation in predicted labor changes is as low as 0.74 (comparing costly trade, free migration to costly trade, costly mi-

[10] To ease computation, we approximate the model variants with "infinitely" costly trade or migration with very large (but finite) trade or migration costs, respectively. Similarly, we approximate model variants with "costless" trade or migration with very small (but positive) trade or migration costs, respectively.

gration). In contrast, the model variants (with the exception of variant #2) largely agree on the change in real wages, with all pair-wise correlations exceeding 0.95.

6 Conclusion

In this primer, we present a review of the simple gravity framework that nests many leading spatial economic models, succinctly incorporates a rich real world geography and matches a number of important empirical patterns in the flow of people and goods. Using a set of newly developed solution techniques and mathematical methods the model can be solved efficiently and be used to conduct relevant policy experiments. Within this environment, the analysis of the economic impact of the interstate highway network highlights the importance of the assumptions on the mobility of labor and goods for evaluating infrastructure policies. We expect that these new tools will further advance the way economists conduct the exploration of space - Economic Science's final frontier.

References

Adao, R., C. Arkolakis, and F. Esposito (2017): "Trade, Agglomeration, and Labor Markets: Theory and EVidence," *Manuscript*.

Ahlfeldt, G. M., S. J. Redding, D. M. Sturm, and N. Wolf (2015): "The economics of density: Evidence from the Berlin Wall," *Econometrica*, 83(6), 2127–2189.

Allen, T., and C. Arkolakis (2014): "Trade and the Topography of the Spatial Economy," *The Quarterly Journal of Economics*.

——— (2017): "The Welfare Effects of Transportation Infrastructure Improvements," *Manuscript*.

Allen, T., C. Arkolakis, and X. Li (2014): "On the existence and uniqueness of trade equilibria," *mimeo, Northwestern and Yale Universities*.

Allen, T., C. Arkolakis, and Y. Takahashi (2014): "Universal Gravity," *NBER Working Paper 20787*.

Allen, T., and D. Donaldson (2017): "The Geography of Path Dependence," *Manuscript*.

Allen, T., M. Morten, and C. d. C. Dobbin (2017): "The Welfare of Walls: Migration and the U.S.-Mexican Border," *Manuscript*.

Anderson, J. E. (1979): "A Theoretical Foundation for the Gravity Equation," *American Economic Review*, 69(1), 106–116.

Anderson, J. E., and E. Van Wincoop (2003): "Gravity with Gravitas: A Solution to the Border Puzzle," *American Economic Review*, 93(1), 170–192.

Arkolakis, C. (2010): "Market Penetration Costs and the New Consumers Margin in International Trade," *Journal of Political Economy*, 118(6), 1151–1199.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New Trade Models, Same Old Gains?," *American Economic Review*, 102(1), 94–130.

Arkolakis, C., S. Demidova, P. J. Klenow, and A. Rodríguez-Clare (2008): "Endogenous Variety and the Gains from Trade," *American Economic Review, Papers and Proceedings*, 98(4), 444–450.

Baldwin, R., and D. Taglioni (2006): "Gravity for dummies and dummies for gravity equations," Discussion paper, National Bureau of Economic Research.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): "Plants and Productivity in International Trade," *American Economic Review*, 93(4), 1268–1290.

Bryan, G., and M. Morten (2015): "Economic development and the spatial allocation of labor: Evidence from Indonesia," *Manuscript, London School of Economics and Stanford University.*

Caliendo, L., M. Dvorkin, and F. Parro (2015): "The impact of trade on labor market dynamics," Discussion paper, National Bureau of Economic Research.

Caliendo, L., and F. Parro (2015): "Estimates of the Trade and Welfare Effects of NAFTA," *The Review of Economic Studies*, 82(1), 1–44.

CFS (2007): "2007 Commodity Flow Survey," *Bureau of Transportation Statistics.*

——— (2012): "2012 Commodity Flow Survey," *Bureau of Transportation Statistics.*

Chaney, T. (2008): "Distorted Gravity: The Intensive and Extensive Margins of International Trade," *American Economic Review*, 98(4), 1707–1721.

Costinot, A., and A. Rodriguez-Clare (2013): "Trade theory with numbers: Quantifying the consequences of globalization," Discussion paper, National Bureau of Economic Research.

Dekle, R., J. Eaton, and S. Kortum (2008): "Global Rebalancing with Gravity: Measuring the Burden of Adjustment," *IMF Staff Papers*, 55(3), 511–540.

Desmet, K., D. K. Nagy, and E. Rossi-Hansberg (2016): "The geography of development," *Princeton University, mimeograph.*

Di Giovanni, J., and A. A. Levchenko (2008): "Putting the Parts Together: Trade, Vertical Linkages, and Business Cycle Comovement," *forthcoming, American Economic Journal: Macroeconomics.*

Donaldson, D. (forthcoming): "Railroads of the Raj," *American Economic Review.*

Donaldson, D., and R. Hornbeck (2012): "Railroads and American Economic Growth: New Data and Theory," Discussion paper.

Eaton, J., and S. Kortum (2002): "Technology, Geography and Trade," *Econometrica*, 70(5), 1741–1779.

Eaton, J., S. Kortum, and F. Kramarz (2011): "An Anatomy of International Trade: Evidence from French Firms," *Econometrica*, 79(5), 1453–1498.

Faber, B., and C. Gaubert (2016): "Tourism and Economic Development: Evidence from Mexico's Coastline," Discussion paper, National Bureau of Economic Research.

Fujita, M., P. Krugman, and A. J. Venables (1999): *The Spatial Economy: Cities, Regions, and International Trade*. MIT Press, Boston, Massachussetts.

Glaeser, E. L., and J. D. Gottlieb (2008): "The economics of place-making policies," Discussion paper.

Head, K., and T. Mayer (2013): *Gravity equations: Workhorse, toolkit, and cookbook*. Centre for Economic Policy Research.

Head, K., T. Mayer, and J. Ries (2010): "The erosion of colonial trade linkages after independence," *Journal of International Economics*, 81(1), 1–14.

Helpman, E. (1998): "The Size of Regions," *Topics in Public Economics. Theoretical and Applied Analysis*, pp. 33–54.

Krugman, P. (1980): "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 70(5), 950–959.

——— (1991): "Increasing returns and economic geography," *Journal of political economy*, 99(3), 483–499.

Melitz, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6), 1695–1725.

Moretti, E. (2011): "Local labor markets," *Handbook of labor economics*, 4, 1237–1313.

Mossay, P., and P. M. Picard (2011): "On spatial equilibria in a social interaction model," *Journal of Economic Theory*, 146(6), 2455–2477.

Polyanin, A., and V. Zaitsev (2002): *Handbook of exact solutions for ordinary differential equations*. Chapman & Hall/CRC.

Redding, S., and D. Sturm (2008): "The Costs of Remoteness: Evidence from German Division and Reunification," *American Economic Review*, 98(5), 1766–1797.

Redding, S. J. (2016): "Goods trade, factor mobility and welfare," *Journal of International Economics*, 101, 148–167.

Redding, S. J., and E. A. Rossi-Hansberg (2017): "Quantitative Spatial Economics," *Annual Review of Economics*, 9(1).

Roback, J. (1982): "Wages, rents, and the quality of life," *The Journal of Political Economy*, pp. 1257–1278.

Rosenthal, S. S., and W. C. Strange (2004): "Evidence on the nature and sources of agglomeration economies," *Handbook of regional and urban economics*, 4, 2119–2171.

Rossi-Hansberg, E. (2005): "A Spatial Theory of Trade," *American Economic Review*, 95(5), 1464–1491.

Ruggles, S., C. A. Fitch, P. Kelly Hall, and M. Sobek (2000): "IPUMS-USA: Integrated Public Use Microdata Series for the United States," *Handbook of international historical microdata for population research. Minneapolis: Minnesota Population Center*, pp. 259–284.

Simonovska, I., and M. E. Waugh (2014): "The elasticity of trade: Estimates and evidence," *Journal of international Economics*, 92(1), 34–50.

Tinbergen, J. (1962): "Shaping the world economy; suggestions for an international economic policy," .

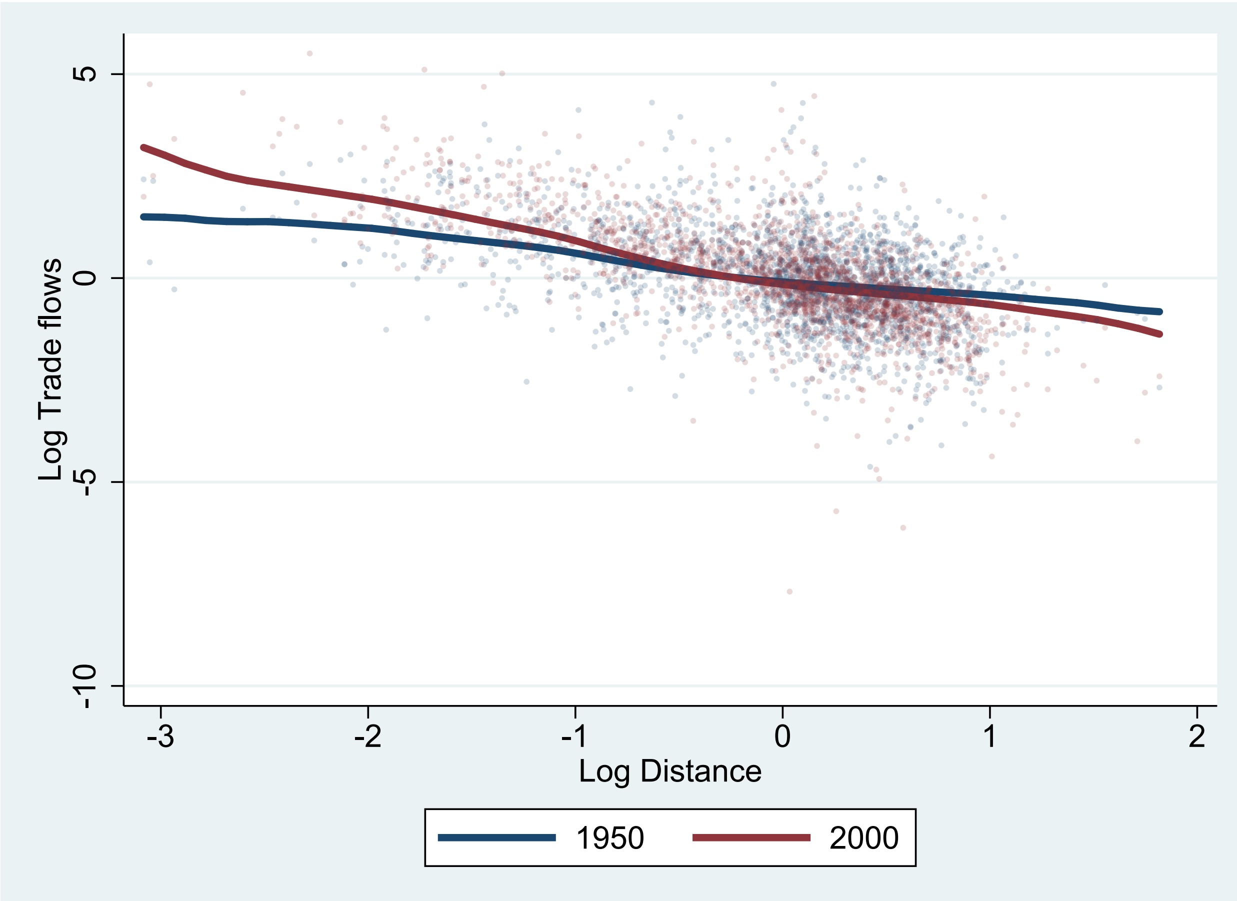
Tombe, T., X. Zhu, et al. (2015): "Trade, migration and productivity: A quantitative analysis of china," *Manuscript, University of Toronto*.

WBG (2011): "2011 World Bank Bilateral Migration Database," *World Bank Group*.

Yeats, A. J. (1998): "Just How Big is Global Production Sharing?," *Policy Research Working Paper 1871, The World Bank*.

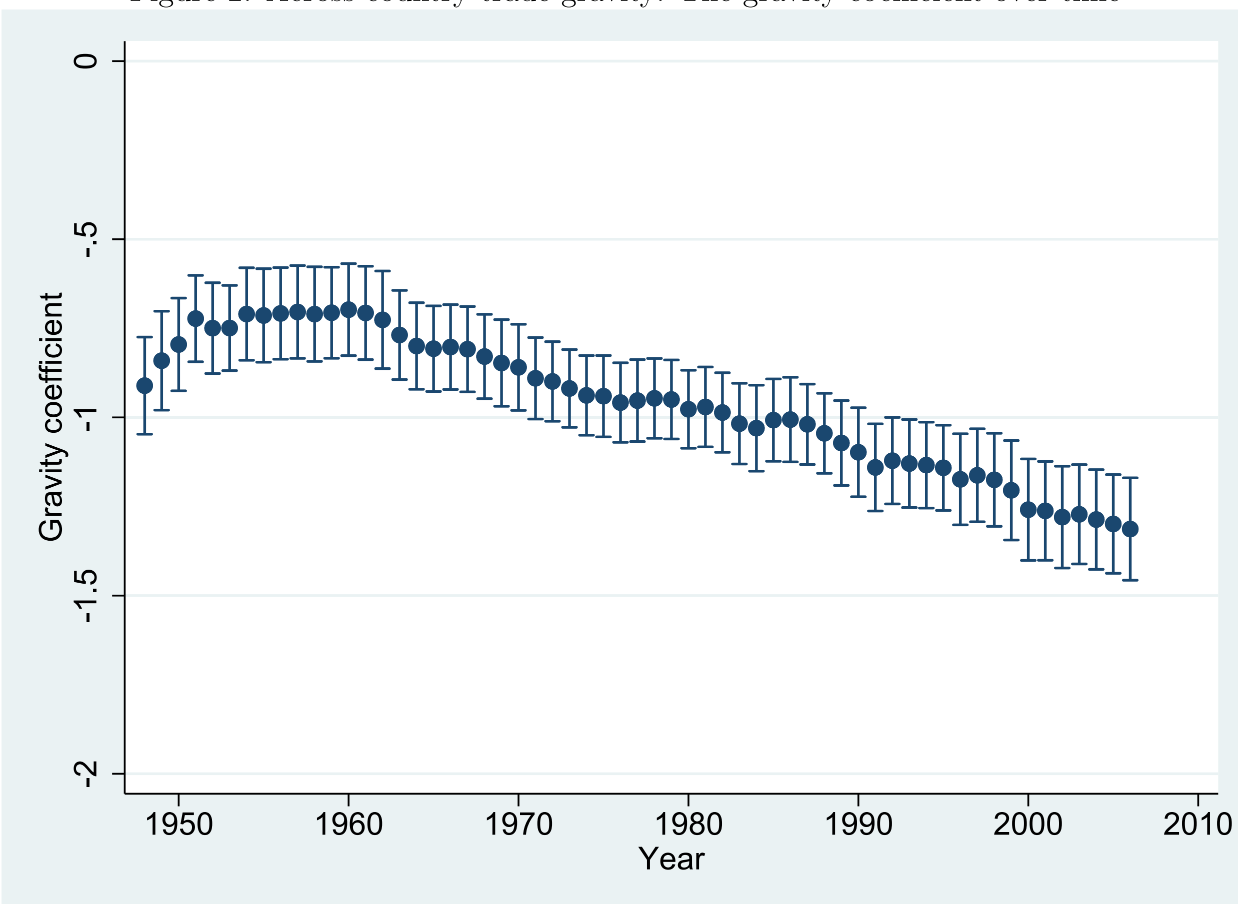
Figures and Tables

Figure 1: Across country gravity: Trade flows between countries over time



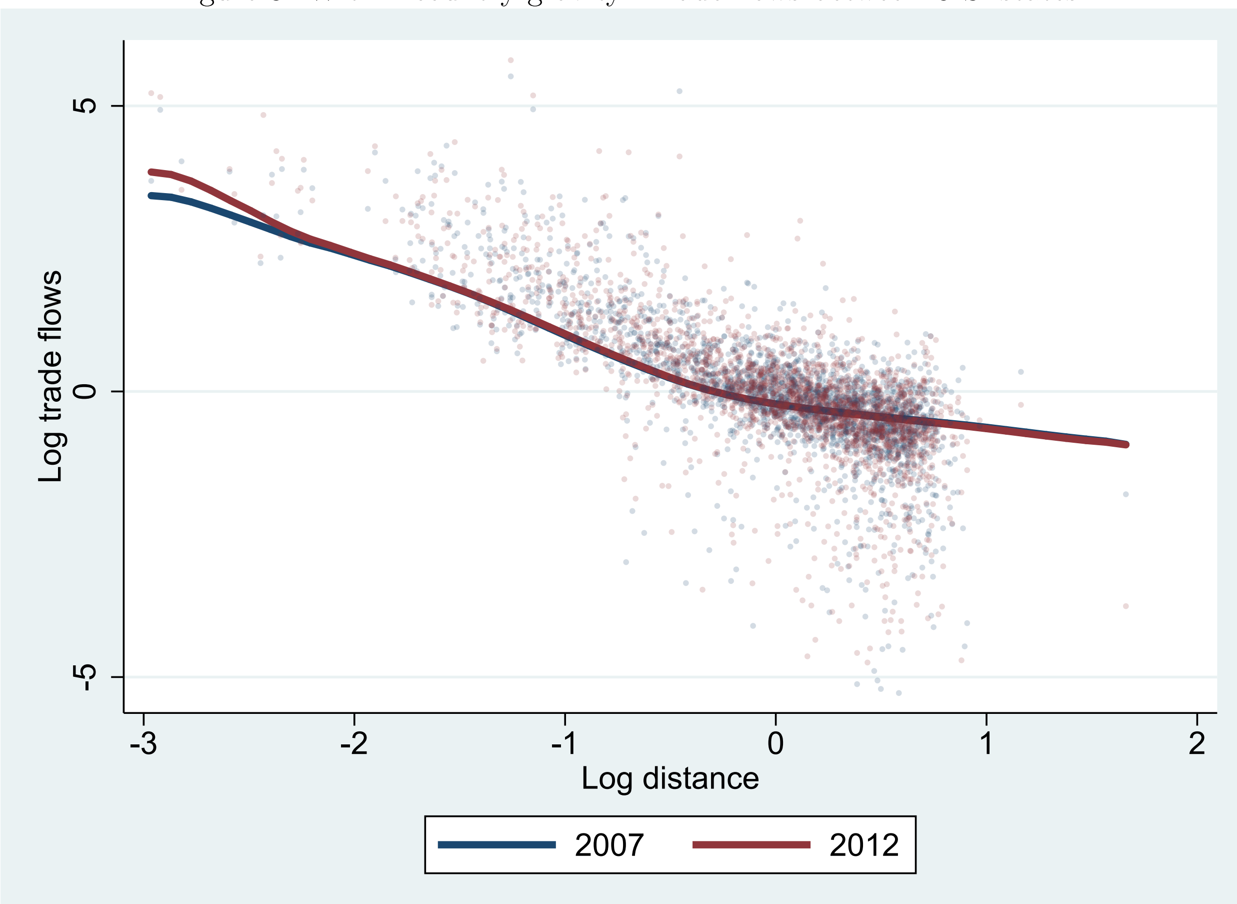
Notes: Data are from Head, Mayer, and Ries (2010). Only bilateral pairs with observed trade flows in both 1950 and 2000 are included. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out the origin-year and destination-year fixed effects.

Figure 2: Across country trade gravity: The gravity coefficient over time



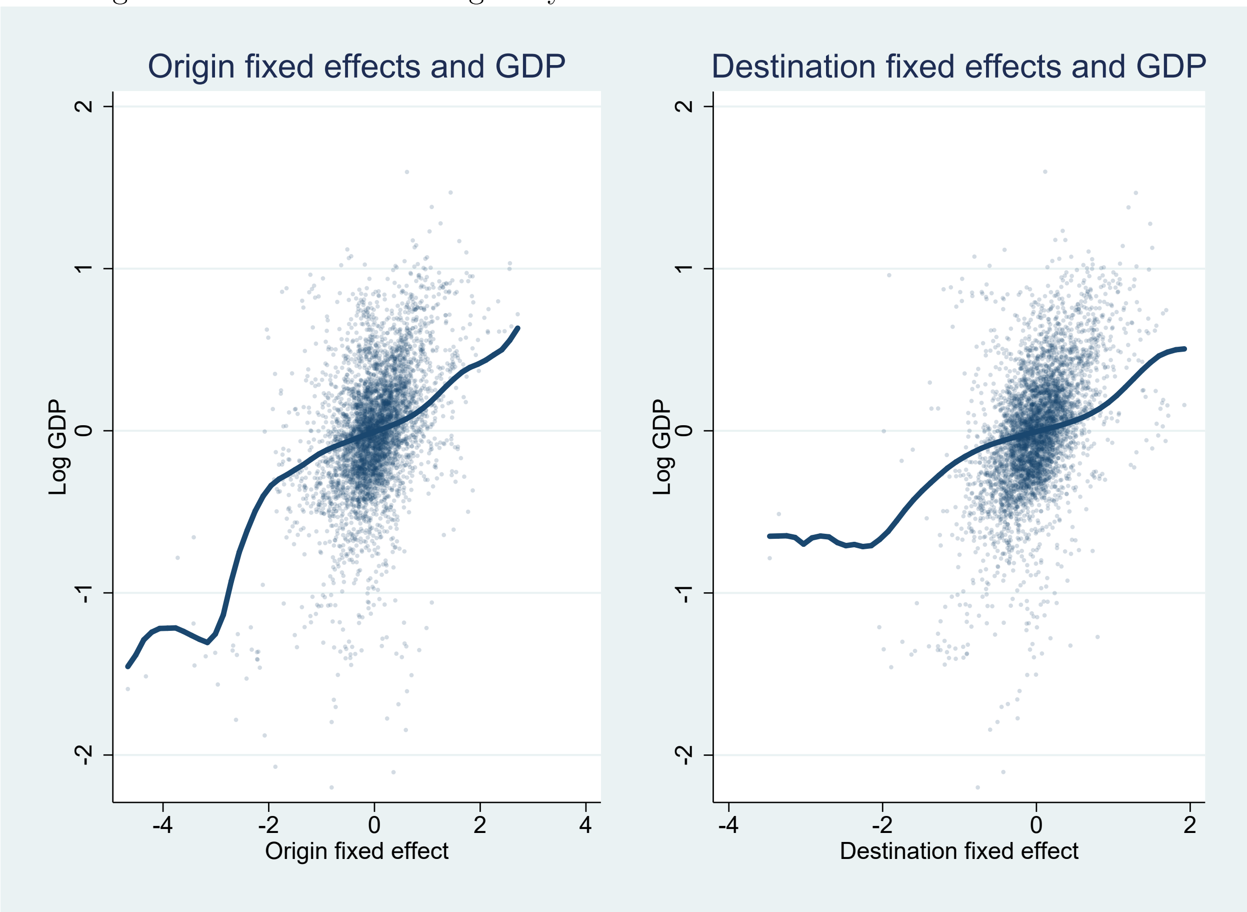
Notes: Data are from Head, Mayer, and Ries (2010). This figure plots the estimated coefficient of distance in a trade gravity regression with origin-year and destination-year fixed effects over time. The bars indicate the 95% confidence interval, where the standard errors are two-way clustered by country of origin and country of destination.

Figure 3: Within country gravity: Trade flows between U.S. states



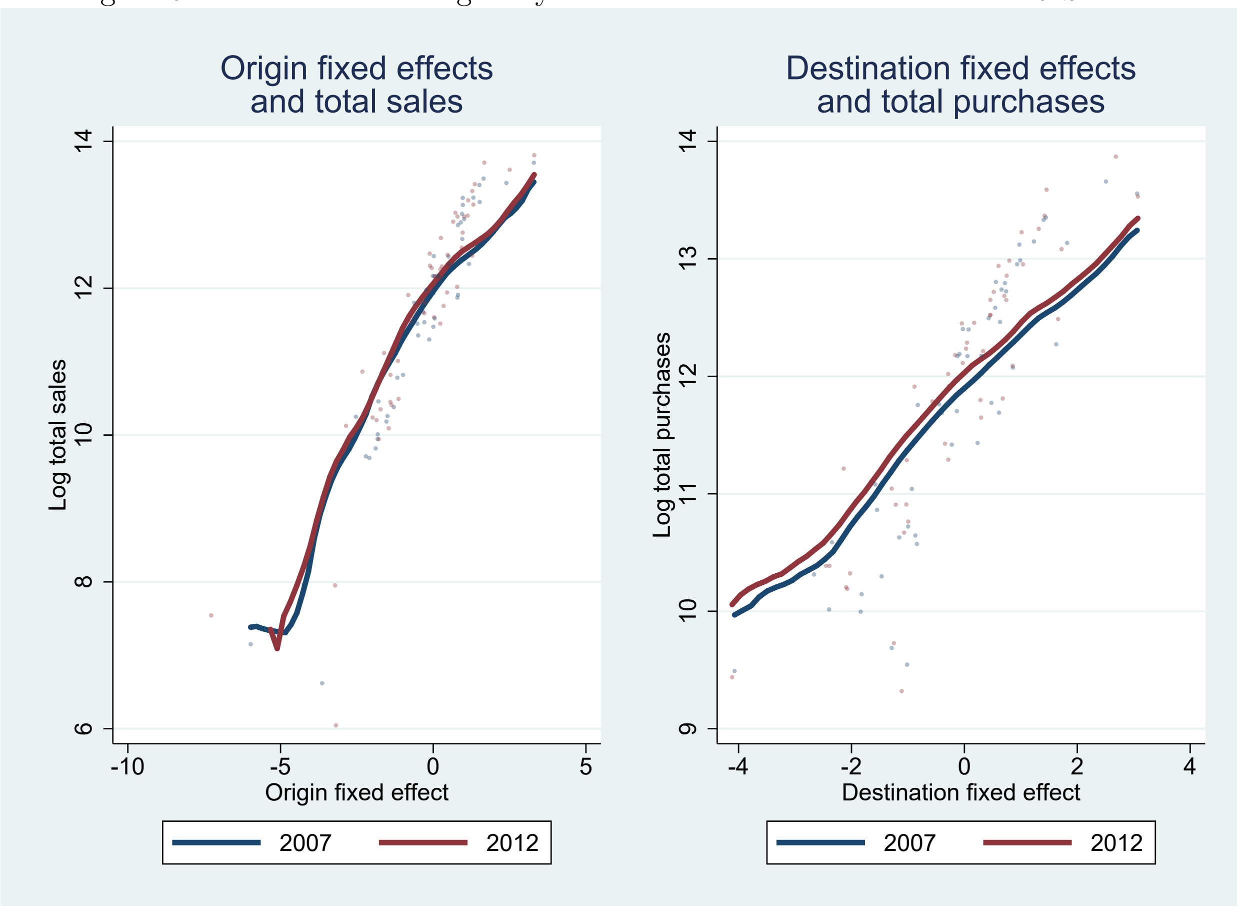
Notes: Data are from 2007 and 2012 Commodity flow surveys (CFS, 2007, 2012). The figure excludes trade flows within each state. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out the origin-year and destination-year fixed effects.

Figure 4: Location size and gravity fixed effects: Trade flows between countries



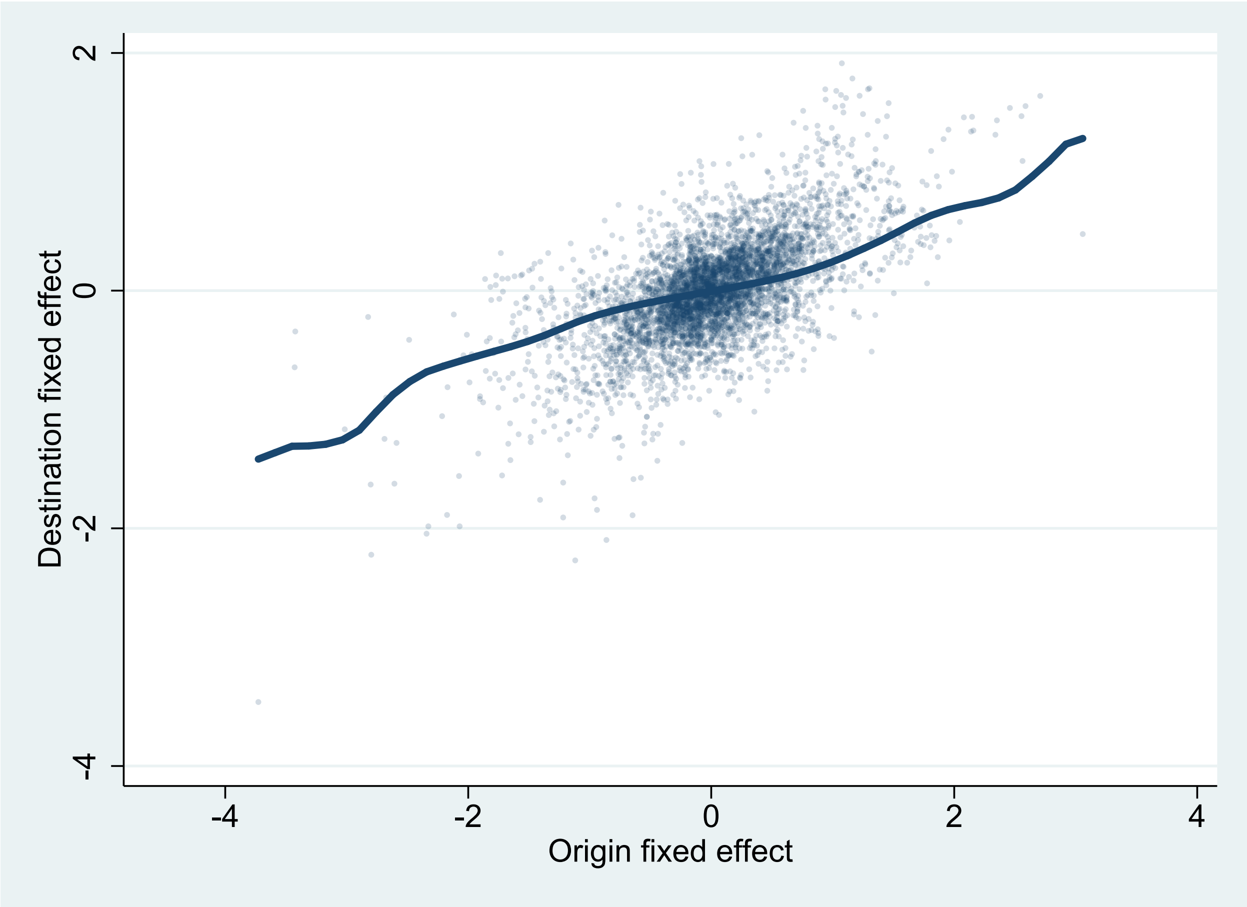
Notes: Data are from Head, Mayer, and Ries (2010). The country-year fixed effects are from a series gravity regression of trade flows on distanceXyear and origin-year and destination-year fixed effects. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out both a country and year fixed effect.

Figure 5: Location size and gravity fixed effects: Trade flows between U.S. states



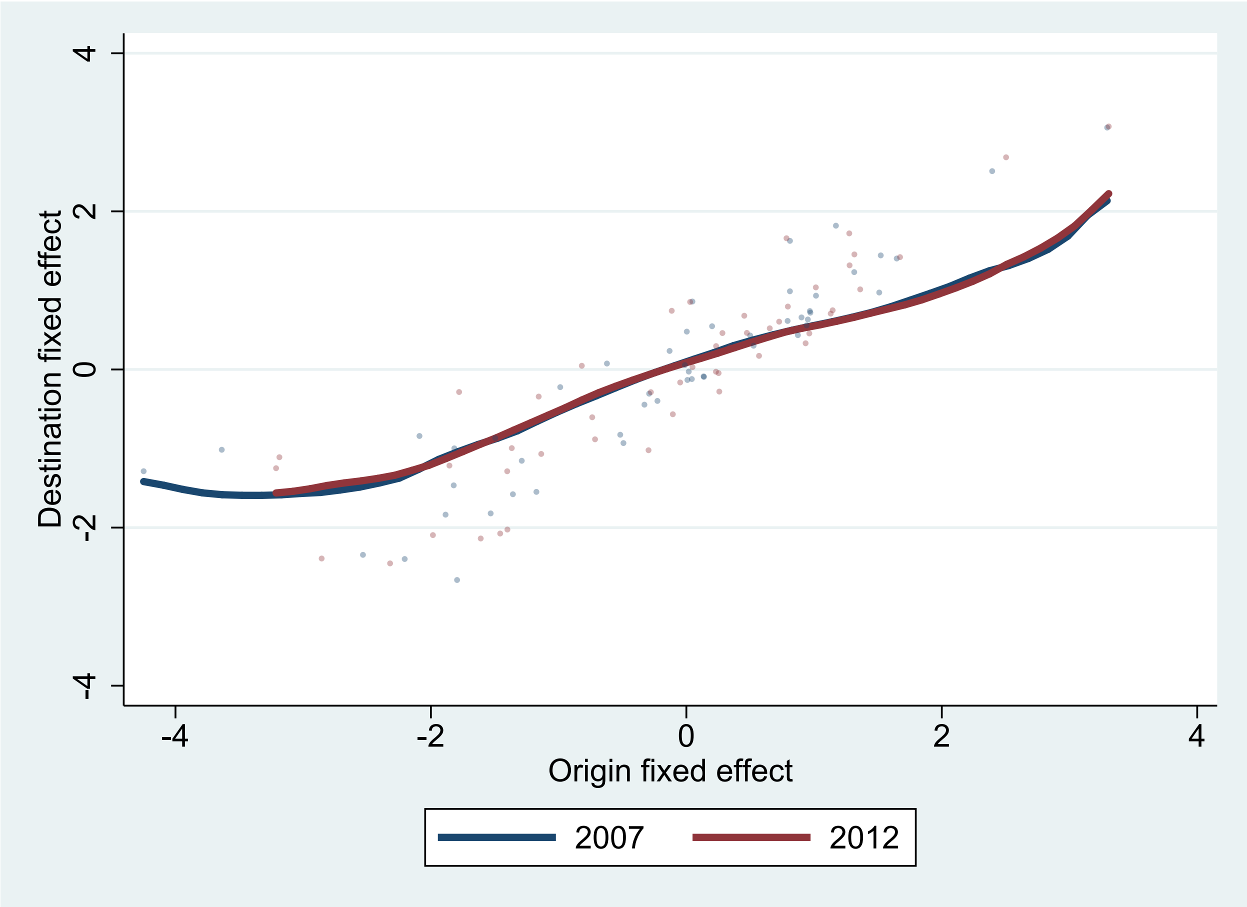
Notes: Data are from 2007 and 2012 Commodity flow surveys (CFS, 2007, 2012). Fixed effects are from a gravity regression of trade flows on distance and origin and destination fixed effects within year. Total sales (purchases) are caculated by summing trade flows across all destinations (origins). The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5.

Figure 6: Origin and destination fixed effects: Trade flows between countries



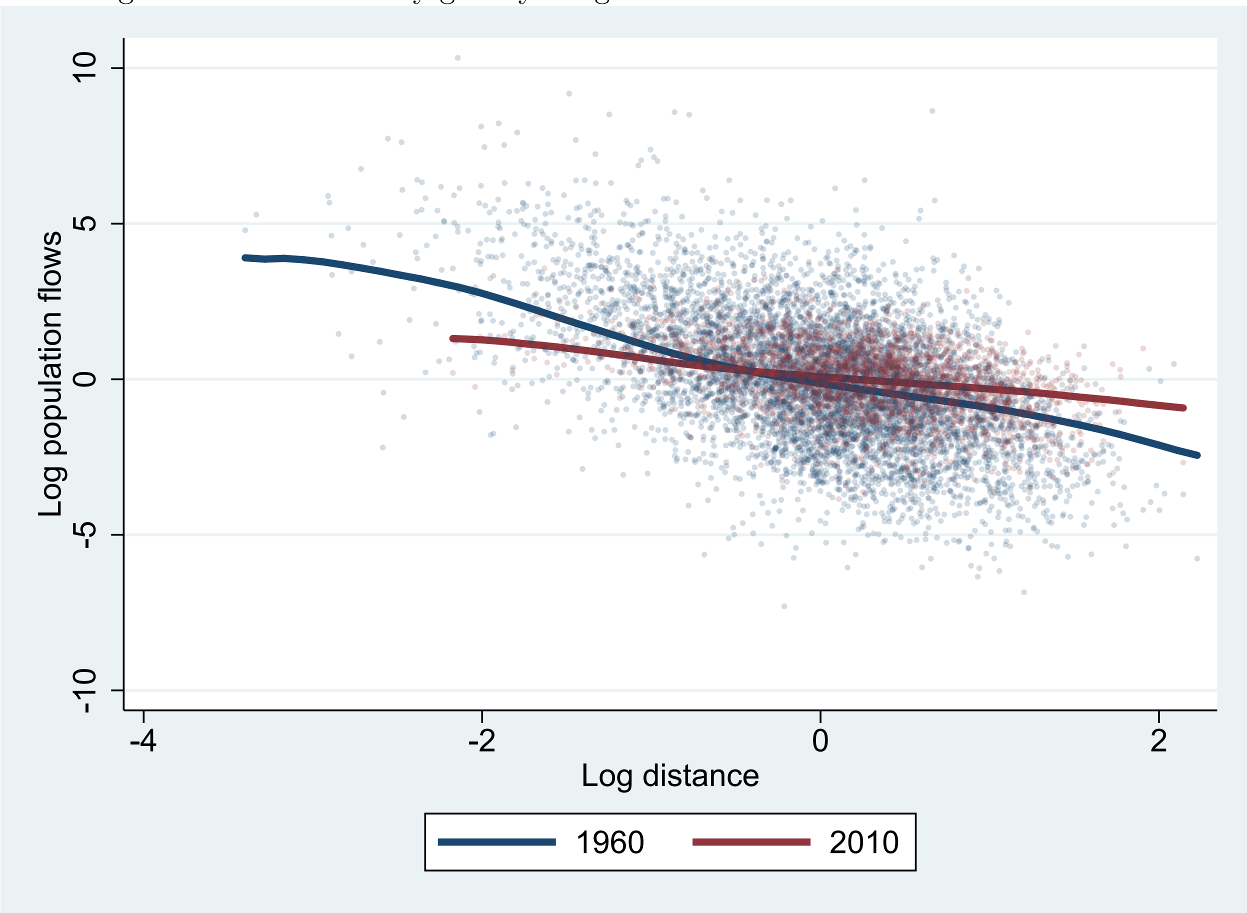
Notes: Data are from Head, Mayer, and Ries (2010). The country-year fixed effects are from a gravity regression of trade flows on distanceXyear and origin-year and destination-year fixed effects for each year. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out both a country and year fixed effect.

Figure 7: Origin and destination fixed effects: Trade flows between U.S. states



Notes: Data are from 2007 and 2012 Commodity flow surveys (CFS, 2007, 2012). Fixed effects are from a gravity regression of trade flows on distance and origin and destination fixed effects within year. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5.

Figure 8: Across country gravity: Migration flows between countries over time

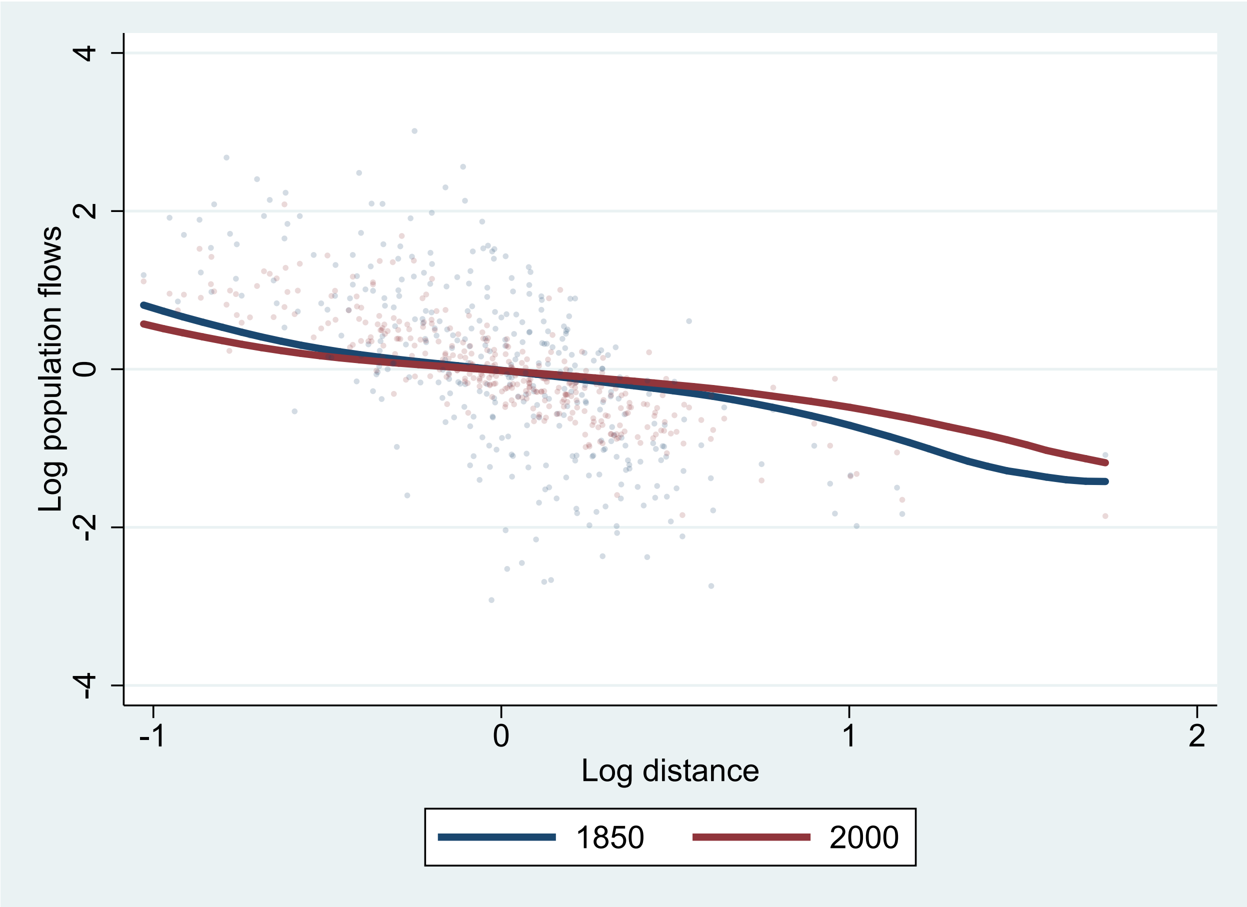


Notes: Data are from Yeats (1998). Excludes own country population shares (i.e. non-migrants). The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out the origin-year and destination-year fixed effects.

Figure 9: Across country gravity: The migration gravity coefficient over time

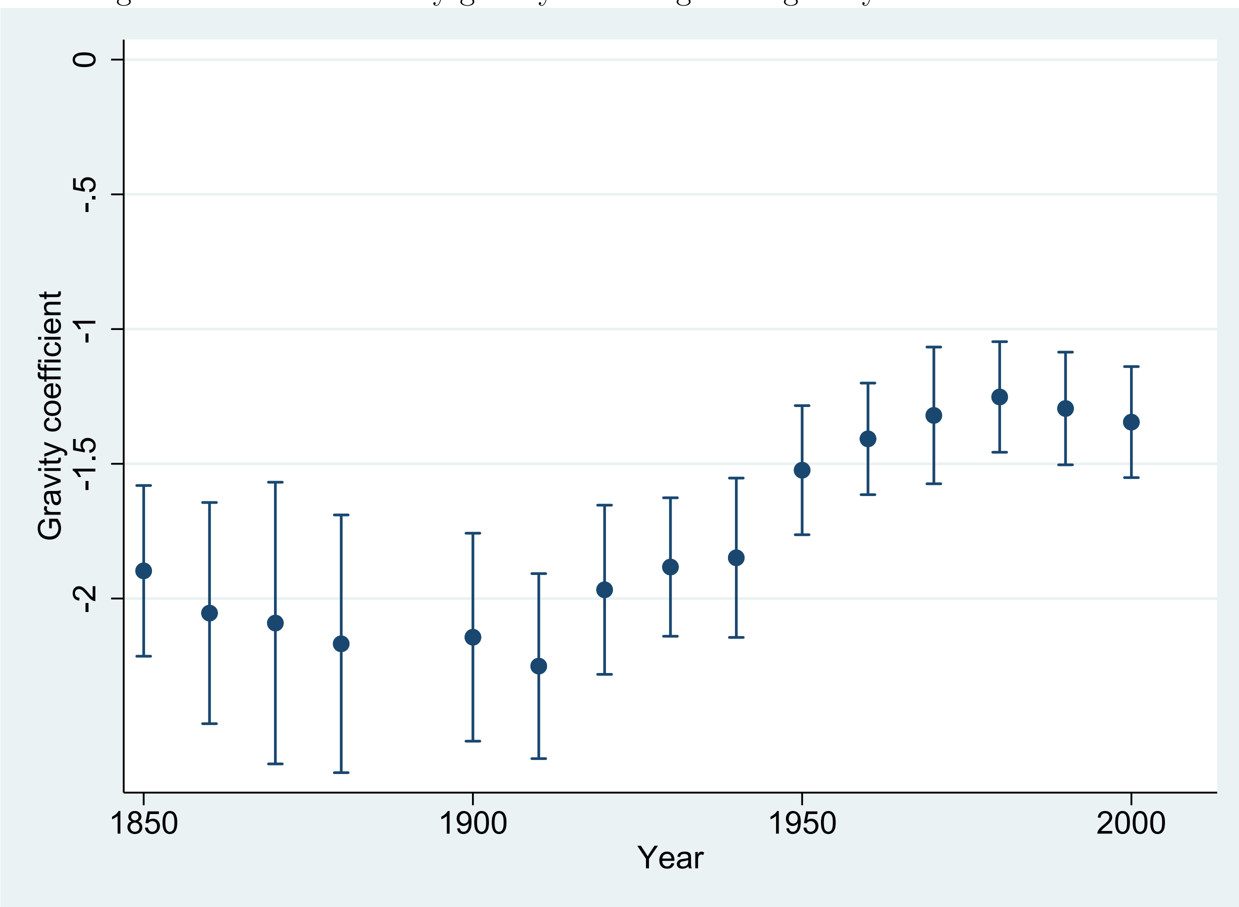
Notes: Data are from Yeats (1998). This figure plots the estimated coefficient of distance in a gravity migration regression with origin-year and destination-year fixed effects over time. The bars indicate the 95% confidence interval, where the standard errors are two-way clustered by country of origin and country of destination.

Figure 10: Within country gravity: Migration flows between U.S. states



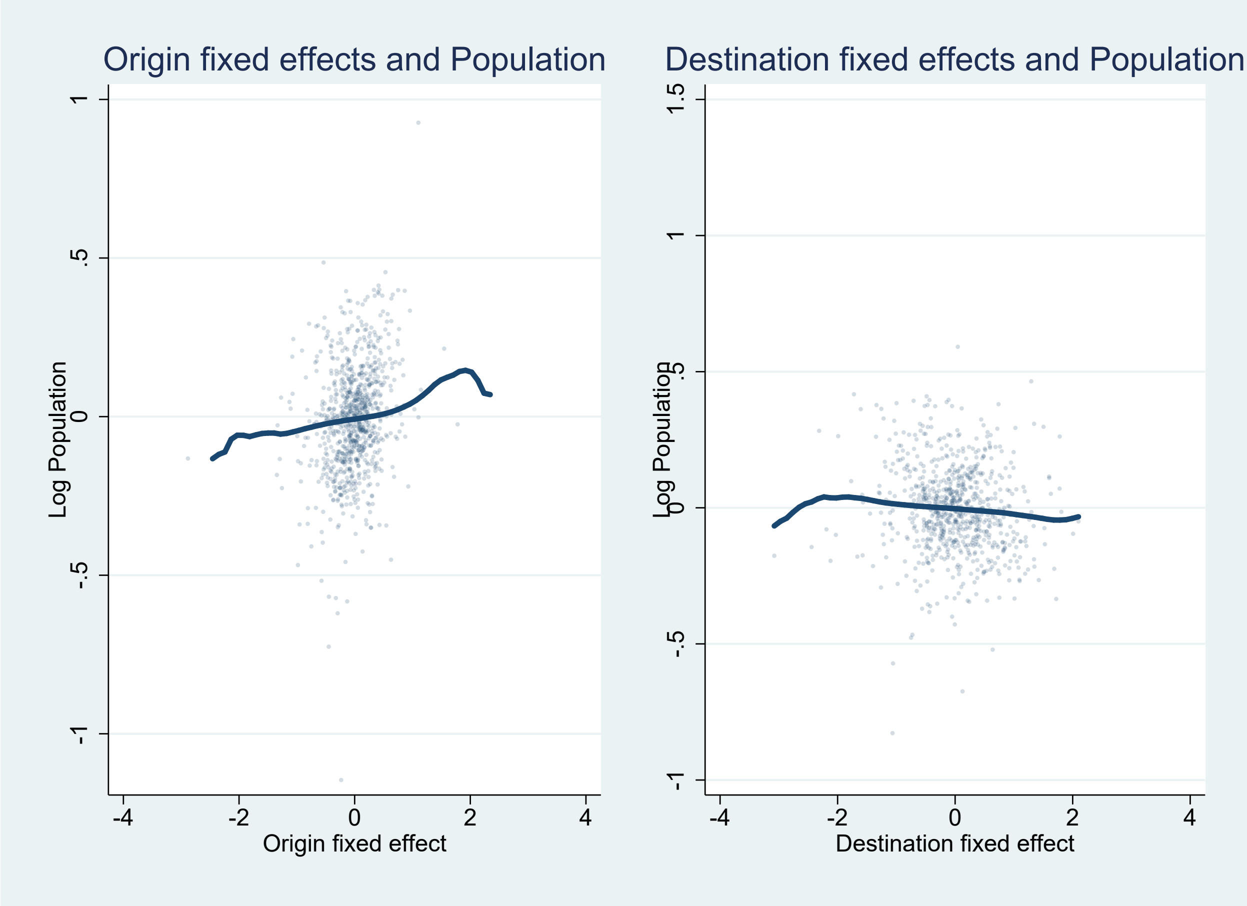
Notes: Data are from the 1850 and 2000 U.S. Censuses Ruggles, Fitch, Kelly Hall, and Sobek (2000), where migration flows are comparing current state of residence of 25-34 year olds to their state of birth. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out the origin-year and destination-year fixed effects.

Figure 11: Within country gravity: The migration gravity coefficient over time



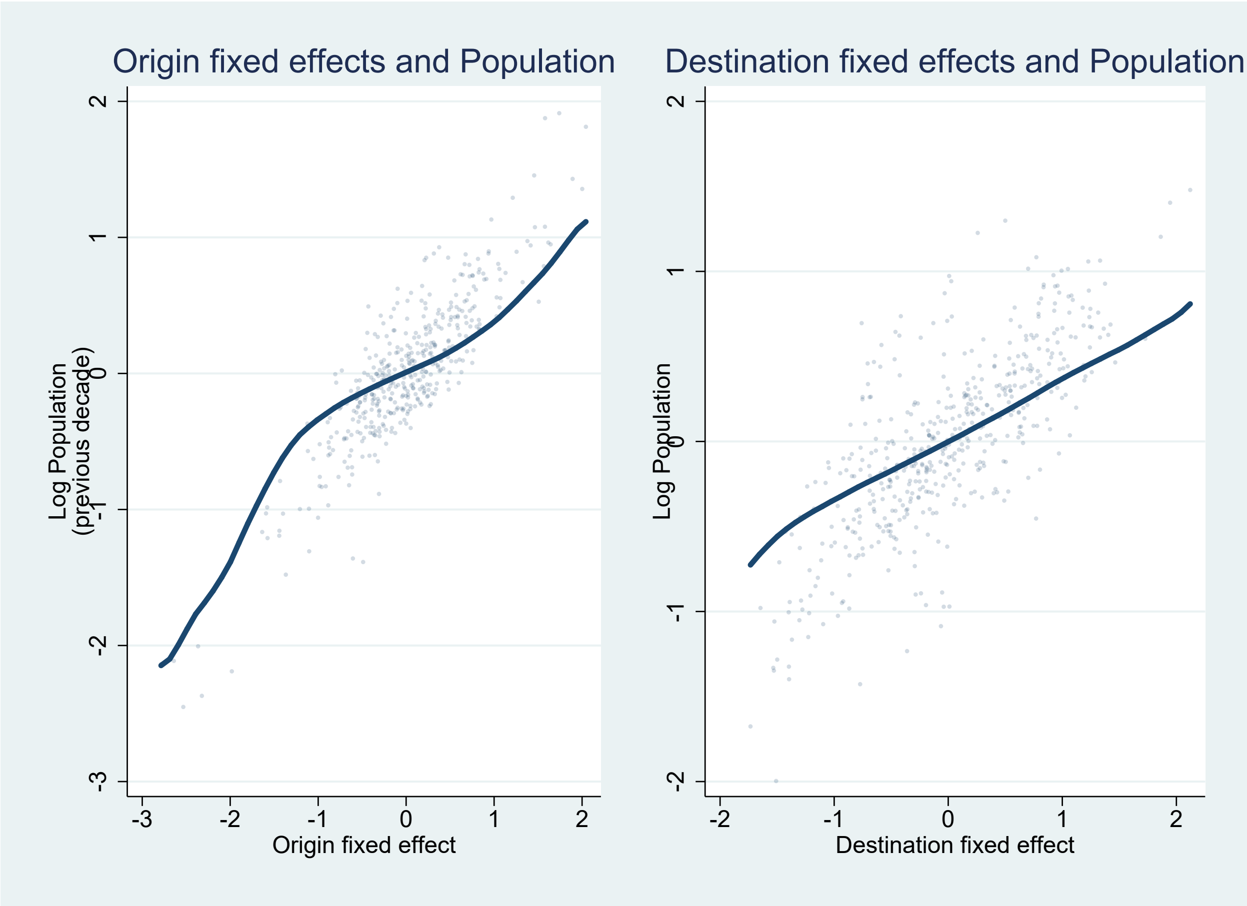
Notes: Data are from the U.S. Censuses from 1850 to 2000 Ruggles, Fitch, Kelly Hall, and Sobek (2000), where migration flows are comparing current state of residence of 25-34 year olds to their state of birth. This figure plots the estimated coefficient of distance in a gravity migration regression with origin-year and destination-year fixed effects over time. The bars indicate the 95% confidence interval, where the standard errors are two-way clustered by state of origin and state of destination.

Figure 12: Location population and gravity fixed effects: Migration flows between countries



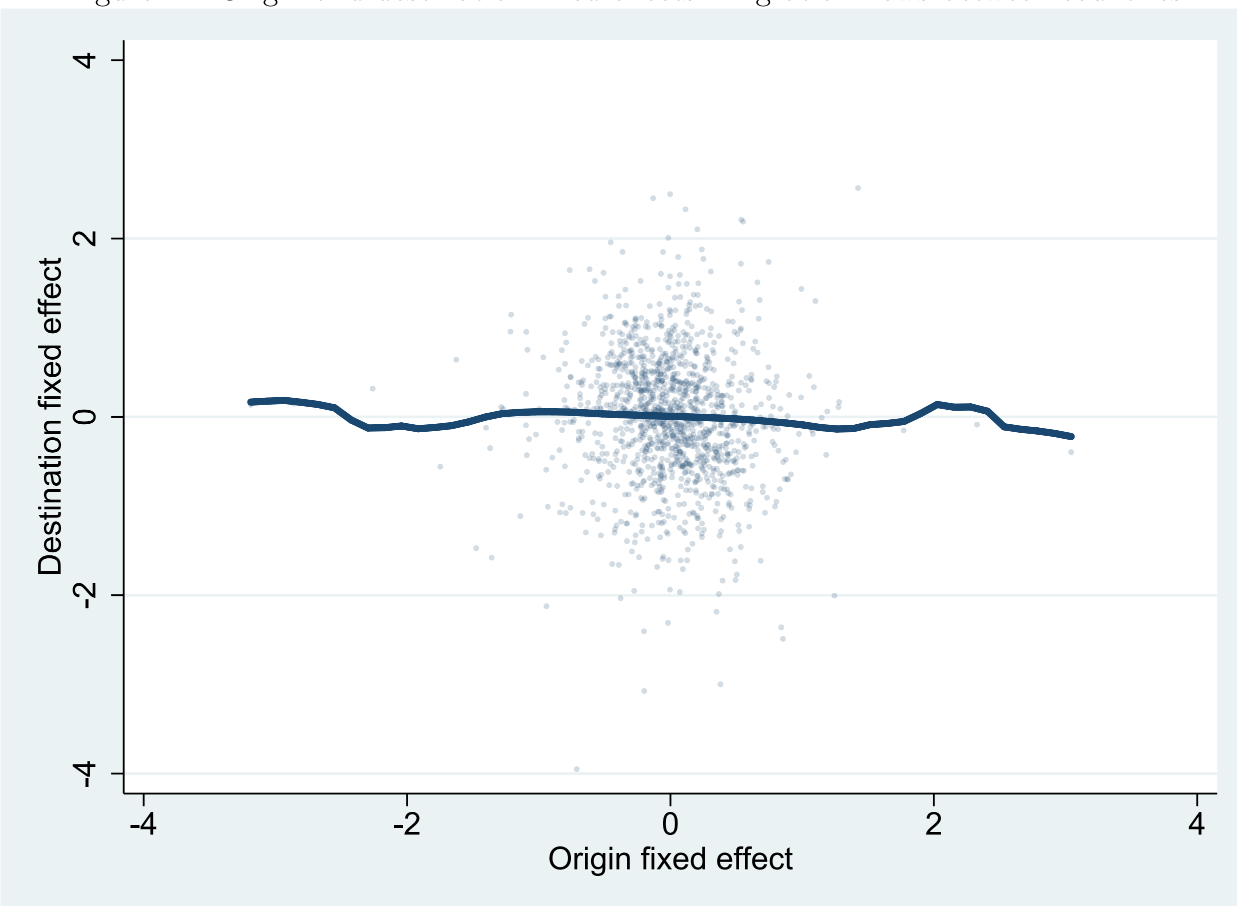
Notes: Data are from Yeats (1998). The country-year fixed effects are from a gravity regression of migration flows on distanceXyear and origin-year and destination-year fixed effects. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out both a country and year fixed effect.

Figure 13: Location size and gravity fixed effects: Migration flows between U.S. states



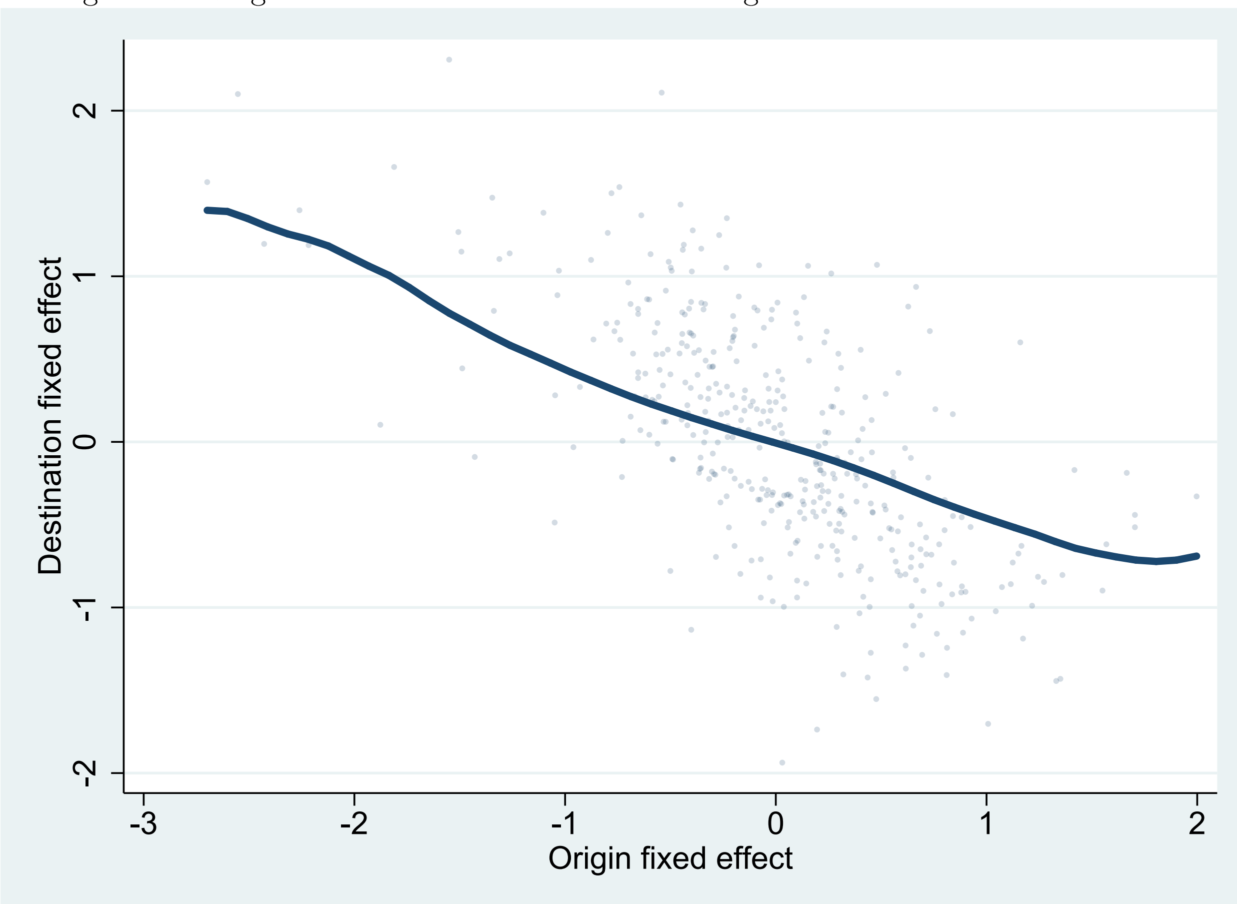
Notes: Data are from the U.S. Censuses from 1850 to 2000 Ruggles, Fitch, Kelly Hall, and Sobek (2000), where migration flows are comparing current state of residence of 25-34 year olds to their state of birth. The state-year fixed effects are from a series gravity regression of migration flows on distanceXyear and origin-year and destination-year fixed effects . The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out both a state and year fixed effect.

Figure 14: Origin and destination fixed effects: Migration flows between countries



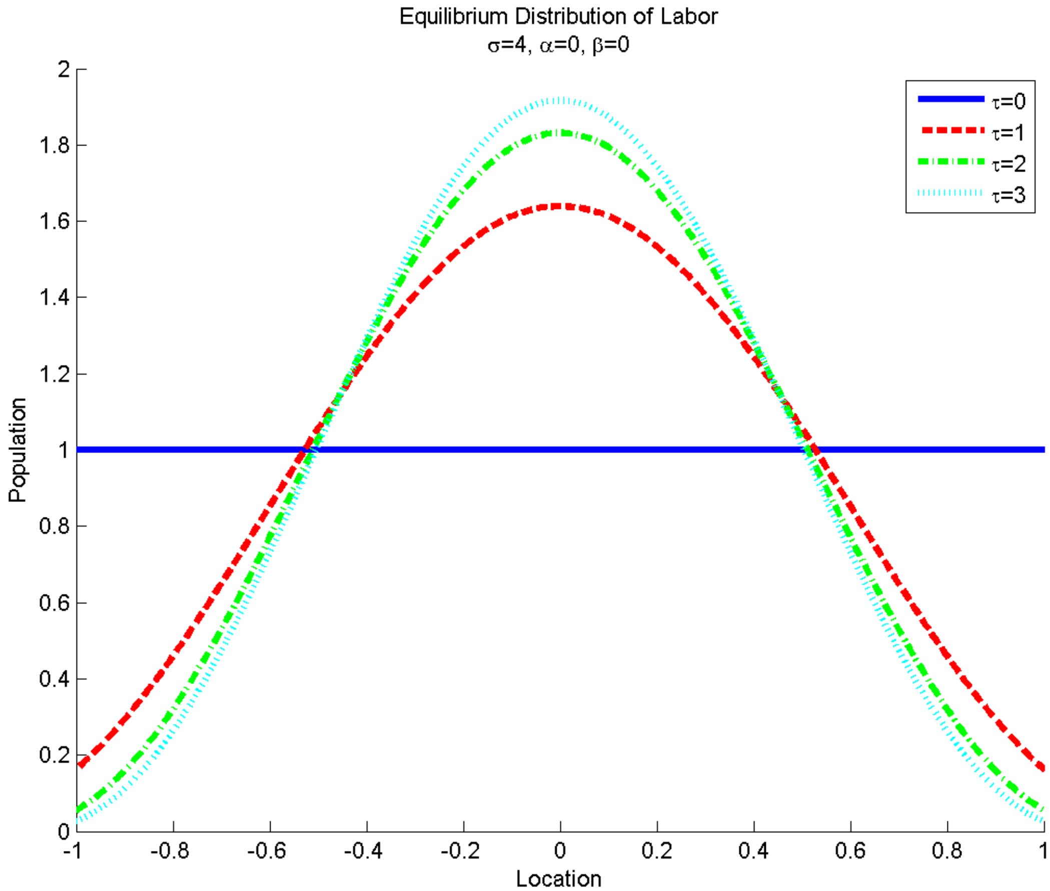
Notes: Data are from Yeats (1998). The country-year fixed effects are from a gravity regression of migration flows on distanceXyear and origin-year and destination-year fixed effects. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out both a country and year fixed effect.

Figure 15: Origin and destination fixed effects: Migration flows between U.S. states



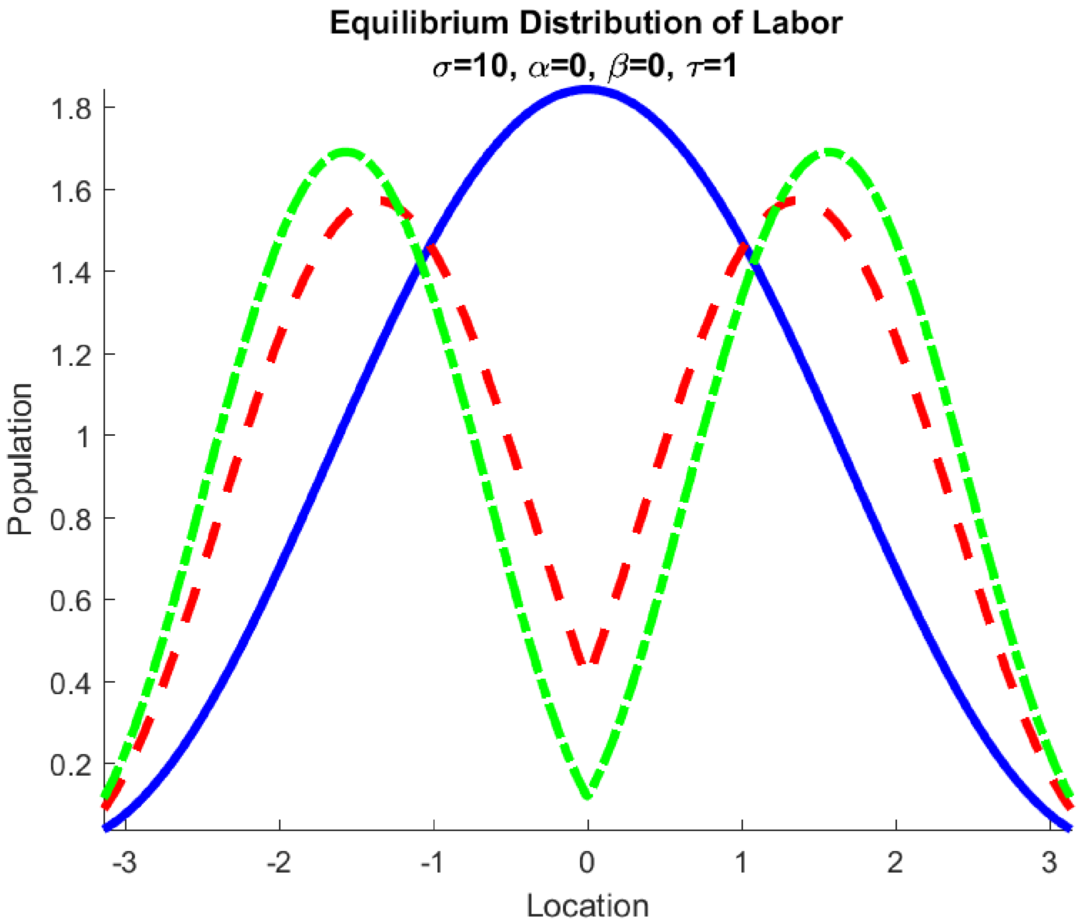
Notes: Data are from the U.S. Censuses from 1850 to 2000 Ruggles, Fitch, Kelly Hall, and Sobek (2000), where migration flows are comparing current state of residence of 25-34 year olds to their state of birth. Fixed effects are from a gravity regression of migration flows on distanceXyear and origin-year and destination-year fixed effects. The thick lines are from a non parametric regression with Epanechnikov kernel and bandwidth of 0.5 after partitioning out both a state and year fixed effect.

Figure 16: Economic activity on a line: Trade costs



Notes: This figure shows how the equilibrium distribution of population along a line is affected by changes in the trade cost. When trade is costless, the population is equal along the entire line. As trade becomes more costly, the population becomes increasingly concentrated in the center of the line where the consumption bundle is cheapest. Source: Allen and Arkolakis (2014).

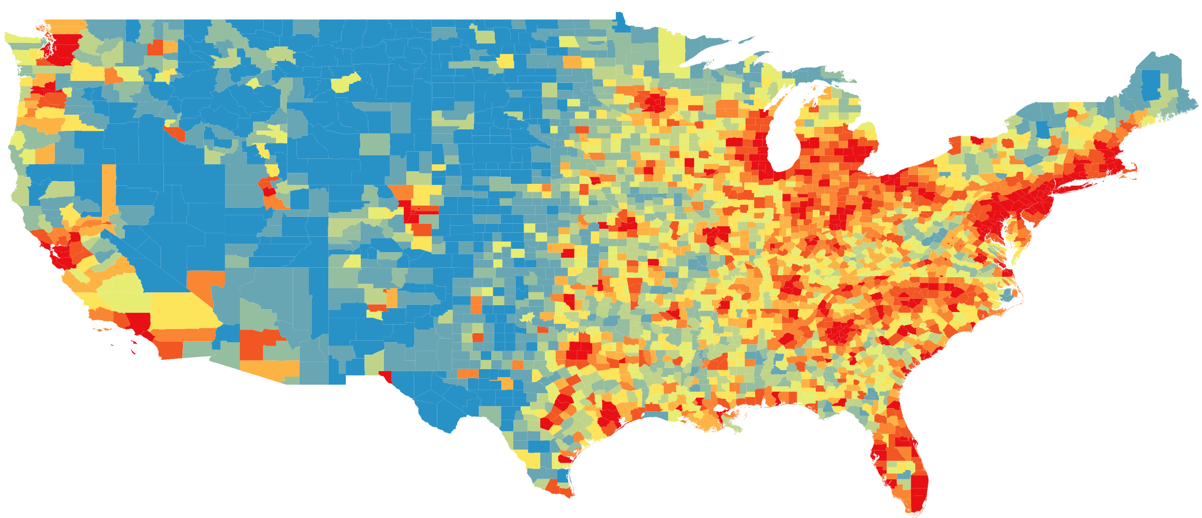
Figure 17: Economic activity on a line: Border costs



Notes: This figure shows how the equilibrium distribution of population along a line is affected by the presence of a border in the center of the line. As crossing the border becomes increasingly costly, the equilibrium distribution of population moves toward the center of each half of the line. Source: Allen and Arkolakis (2014).

Figure 18: Effect of removing the Interstate Highway System: No migration, costly trade

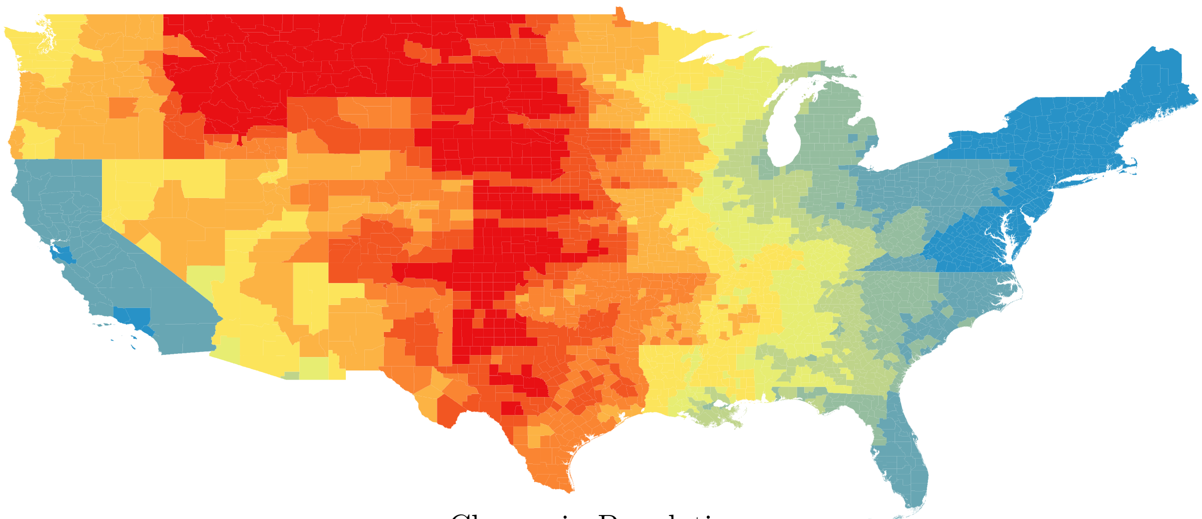
Change in Population



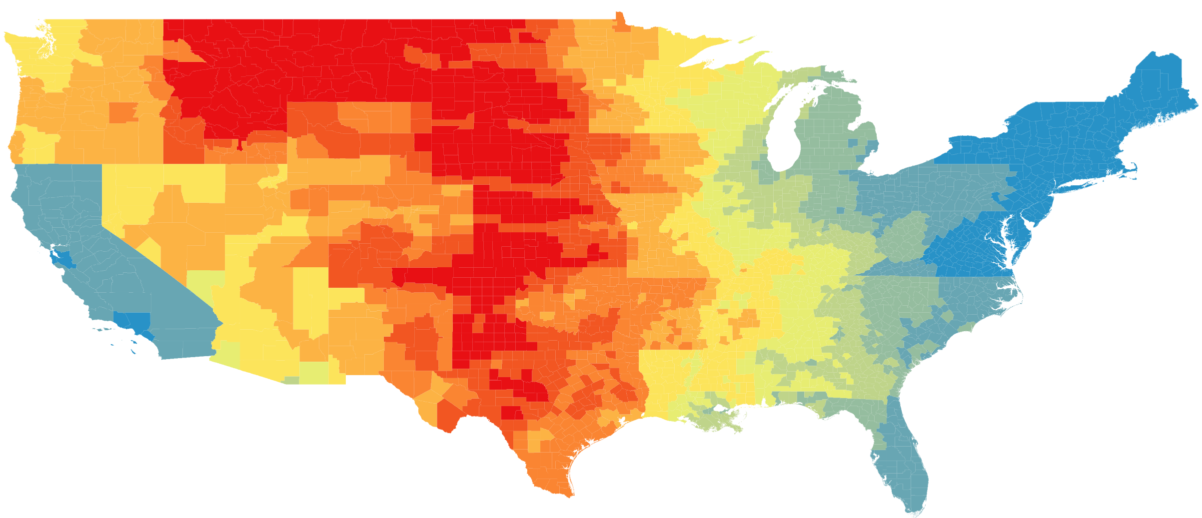
Change in Real Wage

Notes: These maps show the predicted change in population and real wage in every county from removing the Interstate Highway System under the assumption that migration is infinitely costly and trade is costly. The model is calibrated to match the observed population and income in each county from the 2000 Census. The effect of the IHS on trade costs are taken from Allen and Arkolakis (2014). The model parameters assumed are an elasticity of substitution (σ)of 5, a productivity spillover (α) of 0.1 and a disamenity spillover (β) of 0.25, yielding trade and migration elasticities of 4. The color of a county indicates its decile, with blue indicating the greatest decline and red indicating the greatest increase.

Figure 19: Effect of removing the Interstate Highway System: No trade, costly migration



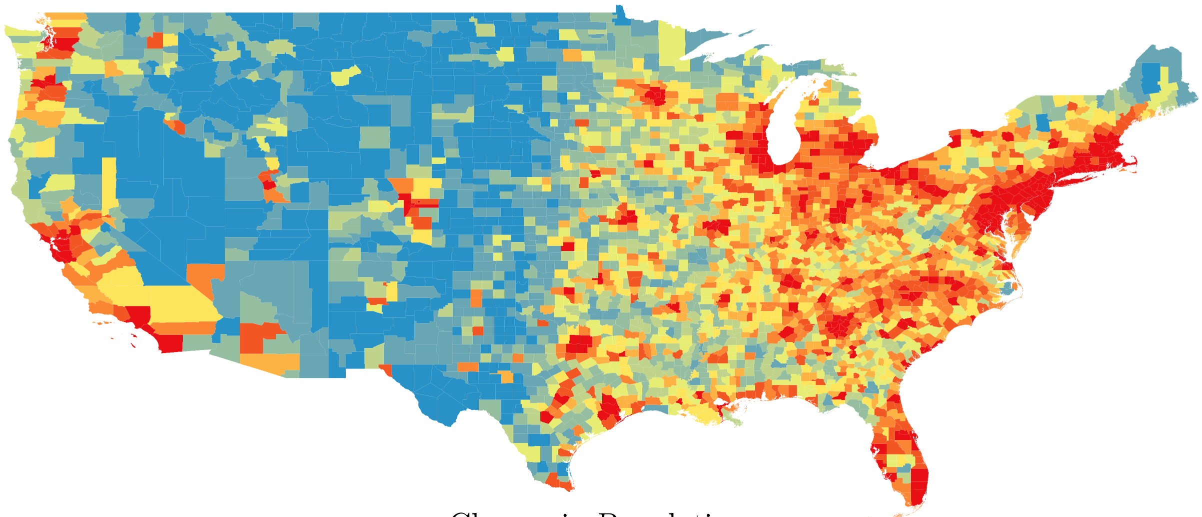
Change in Population



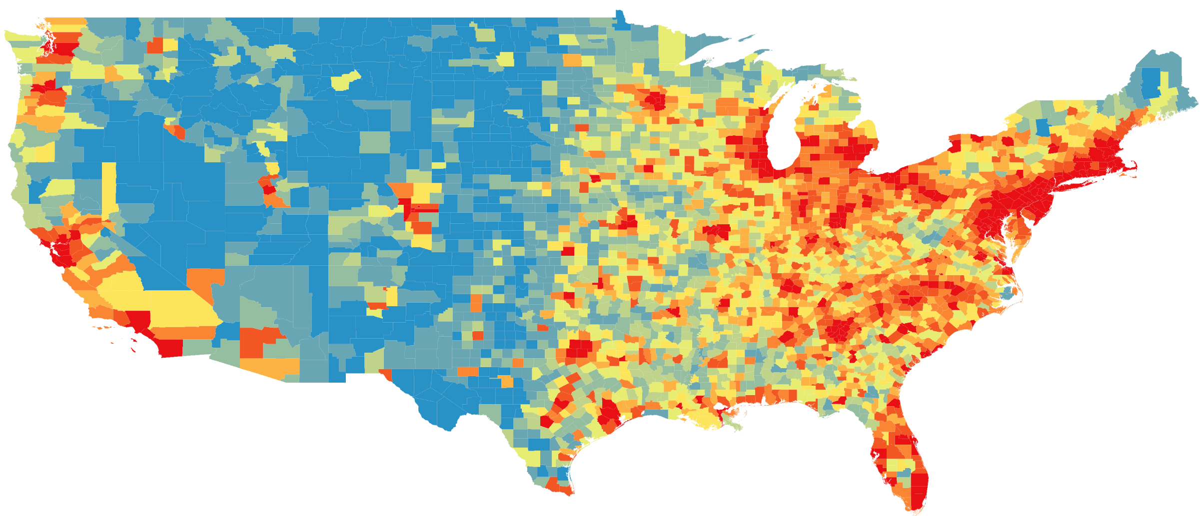
Change in Real Wage

Notes: These maps show the predicted change in population and real wage in every county from removing the Interstate Highway System under the assumption that migration is costly and trade is infinitely costly. The model is calibrated to match the observed population and income in each county from the 2000 Census. The effect of the IHS on migration costs are set equal to the change in trade costs estimated in Allen and Arkolakis (2014). The model parameters assumed are an elasticity of substitution (σ)of 5, a productivity spillover (α) of 0.1 and a disamenity spillover (β) of 0.25, yielding trade and migration elasticities of 4. The color of a county indicates its decile, with blue indicating the greatest decline and red indicating the greatest increase.

Figure 20: Effect of removing the Interstate Highway System: Costly trade, free migration



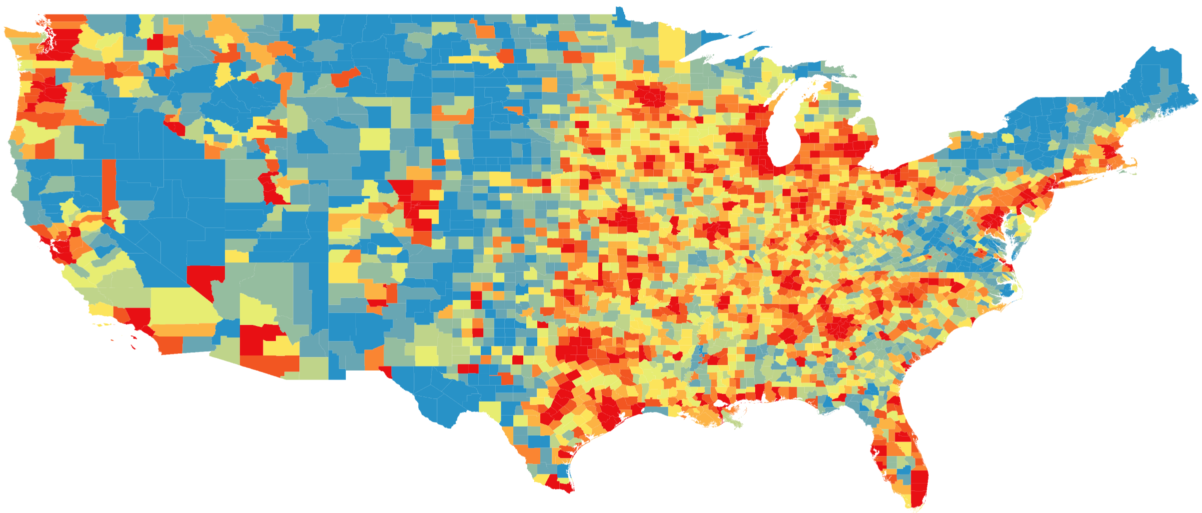
Change in Population



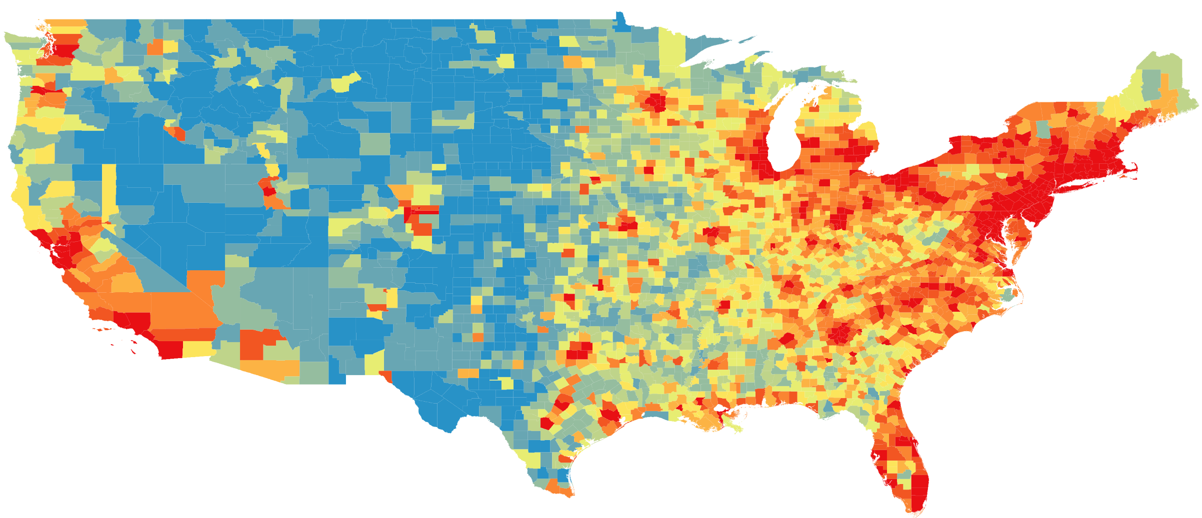
Change in Real Wage

Notes: These maps show the predicted change in population and real wage in every county from removing the Interstate Highway System under the assumption that migration is costless and trade is costly. The model is calibrated to match the observed population and income in each county from the 2000 Census. The effect of the IHS on trade costs are equal to those estimated in Allen and Arkolakis (2014). The model parameters assumed are an elasticity of substitution (σ) of 5, a productivity spillover (α) of 0.1 and a disamenity spillover (β) of 0.25, yielding trade and migration elasticities of 4. The color of a county indicates its decile, with blue indicating the greatest decline and red indicating the greatest increase.

Figure 21: Effect of removing the Interstate Highway System: Free trade, costly migration



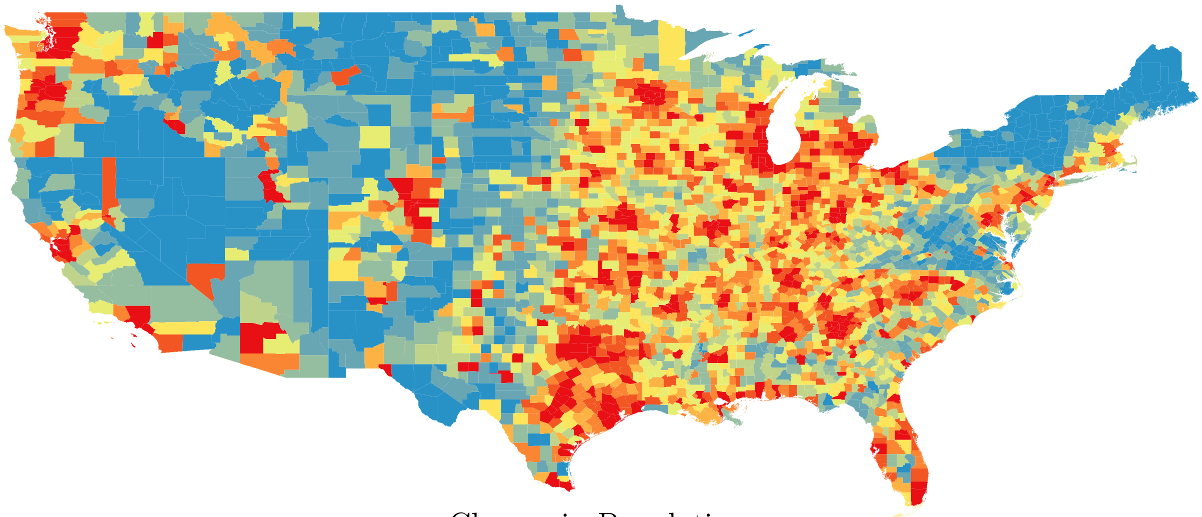
Change in Population



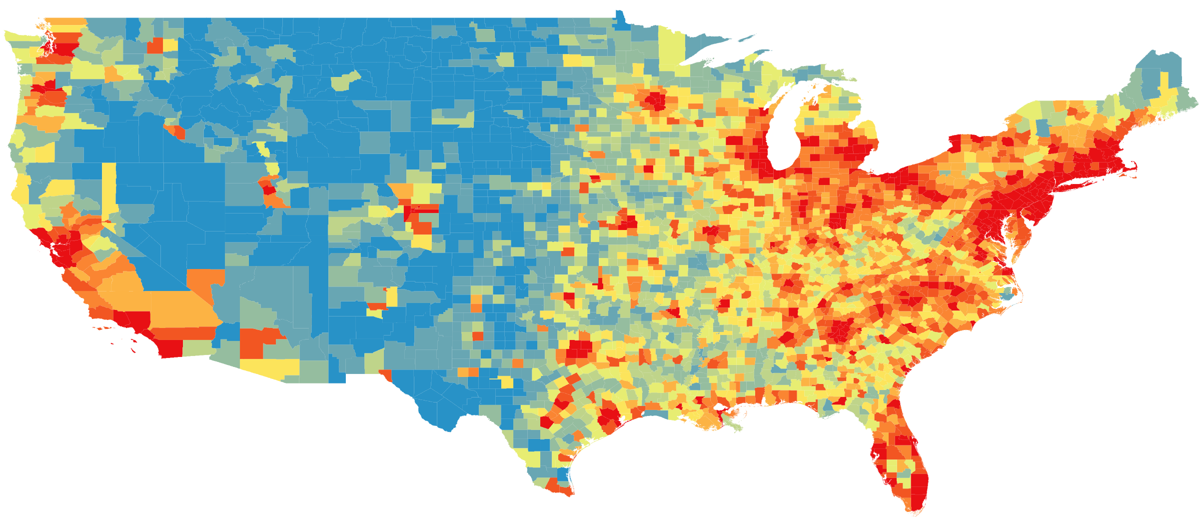
Change in Real Wage

Notes: These maps show the predicted change in population and real wage in every county from removing the Interstate Highway System under the assumption that migration is costly and trade is costless. The model is calibrated to match the observed population and income in each county from the 2000 Census. The effect of the IHS on migration costs are equal to those estimated for the change in trade costs in Allen and Arkolakis (2014). The model parameters assumed are an elasticity of substitution (σ)of 5, a productivity spillover (α) of 0.1 and a disamenity spillover (β) of 0.25, yielding trade and migration elasticities of 4. The color of a county indicates its decile, with blue indicating the greatest decline and red indicating the greatest increase.

Figure 22: Effect of removing the Interstate Highway System: Costly trade, costly migration



Change in Population



Change in Real Wage

Notes: These maps show the predicted change in population and real wage in every county from removing the Interstate Highway System under the assumption that both migration and trade are costly. The model is calibrated to match the observed population and income in each county from the 2000 Census. The effect of the IHS on both migration and trade costs are equal to those estimated for the change in trade costs in Allen and Arkolakis (2014). The model parameters assumed are an elasticity of substitution (σ)of 5, a productivity spillover (α) of 0.1 and a disamenity spillover (β) of 0.25, yielding trade and migration elasticities of 4. The color of a county indicates its decile, with blue indicating the greatest decline and red indicating the greatest increase.

Table 1: Correlation of counterfactual predictions across different model specifications

Population

	Costly trade, No migration	No trade, costly migration	Costly trade, free migration	Free trade, costly migration	Costly trade and migration
Costly trade, No migration	N/A				
No trade, costly migration	N/A	1			
Costly trade, free migration	N/A	-0.6104	1		
Free trade, costly migration	N/A	-0.0945	0.8216	1	
Costly trade and migration	N/A	0.0244	0.742	0.9852	1

Real wage

	Costly trade, No migration	No trade, costly migration	Costly trade, free migration	Free trade, costly migration	Costly trade and migration
Costly trade, No migration	1				
No trade, costly migration	-0.5083	1			
Costly trade, free migration	0.9949	-0.5886	1		
Free trade, costly migration	0.9555	-0.7301	0.9787	1	
Costly trade and migration	0.9841	-0.6506	0.996	0.99	1

Notes: This table shows the pair wise correlations in changes in real income (top) and population (bottom) across the five different model specifications. The shock considered is the removal of the Interstate Highway System (IHS). The models differ only in their assumptions regarding the mobility of goods and labor; when trade and/or migration is ”costly”, the change in the costs due to the destruction of the Interstate Highway System are those estimated for trade costs in Allen and Arkolakis (2014). Each model is calibrated to exactly match the county level population and income data from the 2000 Census given assumed bilateral trade and migration costs. The model parameters assumed are an elasticity of substitution (σ) of 5, a productivity spillover (α) of 0.1 and a disamenity spillover (β) of 0.25, yielding trade and migration elasticities of 4.