New Advances in Quantitative Trade Models

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Motivation

- The analysis and quantification of the effects of economic integration has become increasingly important
- Central question in current trade literature; what are the disaggregate/distributional/welfare effects of trade shocks?
- New quantitative methods are a useful tool to understand the tradeoffs countries, industries, and workers face
- Today: go over these quantitative methods

Agenda

- Simple Trade Model
- Adding Tariffs
- Sectoral Linkages
- Migration and labor market dynamics
- Solution method
- Adding local labor markets (role of geography) and unemployment
- Computation
- References: Eaton and Kortum (2002); Dekle, Eaton, and Kortum (2007); Alvarez and Lucas (2008); Artuc, Chaudhuri, McLaren (2010); Caliendo and Parro (2015), Caliendo, Parro, Rossi-Hansberg, and Sarte (2018); Caliendo and Parro (2015); Caliendo, Dvorkin, Parro (2019)

Ingredients

- We will start with a simple trade model
- **Gravity**, country $n's$ expenditure on goods from i :

$$\pi_{ni} = \frac{A_i [w_i \kappa_{ni}]^{-\theta}}{\sum_{h=1}^{N} A_h [w_h \kappa_{nh}]^{-\theta}}$$

  where i, n, h index countries, w_i are wages at i, $\kappa_{ni} \geq 1$ are trade costs to ship goods from i to n, and θ is the trade elasticity, and A_i represents technology of county i

- **Price index**, in county n

$$P_n = \gamma \left[\sum_{i=1}^{N} A_i (w_i \kappa_{ni})^{-\theta} \right]^{-1/\theta}$$

  where γ is a constant

General Equilibrium

Given L_n, D_n, A_i and κ_{ni}, an equilibrium is a wage vector $\mathbf{w} \in \mathbf{R}^N_{++}$ and prices $\{P_n\}_{n=1}^N$ such that: markets clear, agents maximize utility and firms maximize profits taking prices as given

1. $$x_n = w_n,$$

2. $$P_n = \gamma \left[\sum_{i=1}^N A_i (x_i \kappa_{ni})^{-\theta} \right]^{-1/\theta},$$

3. $$\pi_{ni} = \frac{A_i [x_i \kappa_{ni}]^{-\theta}}{\sum_{h=1}^N A_h [x_h \kappa_{nh}]^{-\theta}},$$

4. $$\sum_{i=1}^N \pi_{ni} w_n L_n - D_n = \sum_{i=1}^N \pi_{in} w_i L_i,$$

General Equilibrium

- Given (L_n, A_n), there exits a unique solution to the equilibrium of the model

Sketch of the Proof: Define the excess demand system $Z(w)$ with

$$Z_n(w_n) = \frac{1}{w_n}\left[\sum_{i=1}^{N} \frac{A_n [x_n \kappa_{in}]^{-\theta}}{\sum_{h=1}^{N} A_h [x_h \kappa_{ih}]^{-\theta}} w_i L_i + D_n - w_n L_n\right].$$

An equilibrium is a wage vector $w \in \mathbf{R}_{++}^{n}$ such that $Z_n(w_n) = 0$ for all n. The prove of existence follows by showing that $Z(w)$ satisfies all the properties of Proposition 17.C.1. in MGW. Uniqueness follows from showing that $Z(w)$ has the gross substitutes property. For a proof see Alvarez and Lucas (2008).

Adding tariffs

- Consider now the case in which countries need to pay tariffs
- Trade costs are now

$$\kappa_{ni} = (1 + \tau_{ni}) d_{ni}$$

  - Now, d_{ni} "iceberg" trade cost (physical loss of resources)
  - $1 + \tau_{ni}$ ad valorem tariff applied in n to goods from i (impact relative prices of goods)

- Note that **Gravity** and **Price index** are the same as before.
- However, need to consider R_n, (revenue from tariffs):

$$R_n = \sum_{i=1}^{N} \tau_{ni} M_{ni},$$

where

$$M_{ni} = X_n \frac{\pi_{ni}}{1 + \tau_{ni}}$$

are country n's imports of goods from country i in county n, where total expenditure is given by

$$X_n = w_n L_n + R_n + D_n.$$

General Equilibrium

Given L_n, D_n, A_i and κ_{ni}, an equilibrium under tariff structure τ is a wage vector $\mathbf{w} \in \mathbf{R}^N_{++}$ and prices $\{P_n\}_{n=1}^N$ such that: markets clear, agents maximize utility and firms maximize profits taking prices as given

1
$$x_n = w_n,$$

2
$$P_n = \gamma \left[\sum_{i=1}^N A_i (x_i \kappa_{ni})^{-\theta} \right]^{-1/\theta},$$

3
$$\pi_{ni} = \frac{A_i [x_i \kappa_{ni}]^{-\theta}}{\sum_{h=1}^N A_h [x_h \kappa_{nh}]^{-\theta}},$$

4
$$\sum_{i=1}^N \frac{\pi_{ni}}{1+\tau_{ni}} X_n - D_n = \sum_{i=1}^N \frac{\pi_{in}}{1+\tau_{in}} X_i,$$

5
$$X_n = w_n L_n + \sum_{i=1}^N \tau_{ni} X_n \frac{\pi_{ni}}{1+\tau_{ni}} + D_n,$$

Equilibrium - Change in trade policy

- Let $(\mathbf{w}, P)$ be an equilibrium under tariff structure τ and let $(\mathbf{w}', P')$ be an equilibrium under tariff structure τ'
- Define $\left(\hat{\mathbf{w}}, \hat{P}\right)$ as an equilibrium under τ' relative to τ, where a variable with a hat "$\hat{x}$" represents the relative change of the variable, namely $\hat{x} = x'/x$. For instance $\hat{\pi} = \frac{\pi'}{\pi}$
- In this way we can solve the model without knowing all parameters (only need trade elasticities, θ)

General Equilibrium in Changes

- Follow the idea in DEK
- Let the counterfactual changes in trade flows given by

$$\pi'_{ni} = \frac{A_i [x'_i \kappa'_{ni}]^{-\theta}}{\sum_{h=1}^{N} A_h [x'_h \kappa'_{nh}]^{-\theta}}$$

- Now use the factual trade flows

$$\pi_{ni} = \frac{A_i [x_i \kappa_{ni}]^{-\theta}}{\sum_{h=1}^{N} A_h [x_h \kappa_{nh}]^{-\theta}}$$

- Express the system in changes

$$\frac{\pi'_{ni}}{\pi_{ni}} = \frac{\left(A_i [x'_i \kappa'_{ni}]^{-\theta}\right) / \left(A_i [x_i \kappa_{ni}]^{-\theta}\right)}{\left(\sum_{h=1}^{N} A_h [x'_h \kappa'_{nh}]^{-\theta}\right) / \sum_{h=1}^{N} A_h [x_h \kappa_{nh}]^{-\theta}}$$

General Equilibrium in Changes

- Express the system in changes

$$\frac{\pi'_{ni}}{\pi_{ni}} = \frac{\left(A_i[x'_i\kappa'_{ni}]^{-\theta}\right) / \left(A_i[x_i\kappa_{ni}]^{-\theta}\right)}{\left(\sum_{h=1}^{N} A_h[x'_h\kappa'_{nh}]^{-\theta}\right) / \sum_{h=1}^{N} A_h[x_h\kappa_{nh}]^{-\theta}}$$

- Or

$$\frac{\pi'_{ni}}{\pi_{ni}} = \frac{[\hat{x}_i\hat{\kappa}_{ni}]^{-\theta}}{\sum_{h=1}^{N} \frac{A_h[x'_h\kappa'_{nh}]^{-\theta}}{\sum_{h=1}^{N} A_h[x_h\kappa_{nh}]^{-\theta}}},$$

- Now multiply and devide each element of the denominator by $A_h[x_h\kappa_{nh}]^{-\theta}$ to obtain

$$\frac{\pi'_{ni}}{\pi_{ni}} = \frac{[\hat{x}_i\hat{\kappa}_{ni}]^{-\theta}}{\sum_{h=1}^{N} \frac{A_h[x_h\kappa_{nh}]^{-\theta}}{\sum_{h=1}^{N} A_h[x_h\kappa_{nh}]^{-\theta}}[\hat{x}_h\hat{\kappa}_{nh}]^{-\theta}} = \frac{[\hat{x}_i\hat{\kappa}_{ni}]^{-\theta}}{\sum_{h=1}^{N} \pi_{nh}[\hat{x}_h\hat{\kappa}_{nh}]^{-\theta}}.$$

Equilibrium conditions

1

$$\hat{x}_n = \hat{w}_n,$$

2

$$\hat{P}_n = \left[\sum_{i=1}^{N} \pi_{ni} \left(\hat{x}_i \hat{\kappa}_{ni}\right)^{-\theta}\right]^{-1/\theta},$$

3

$$\pi'_{ni} = \frac{\pi_{ni} \left[\hat{x}_i \hat{\kappa}_{ni}\right]^{-\theta}}{\sum_{h=1}^{N} \pi_{nh} \left[\hat{x}_h \hat{\kappa}_{nh}\right]^{-\theta}},$$

4

$$\sum_{i=1}^{N} \frac{\pi'_{ni}}{1+\tau'_{ni}} X'_n - D_n = \sum_{i=1}^{N} \frac{\pi'_{in}}{1+\tau'_{in}} X'_i,$$

5

$$X'_n = \hat{w}_n w_n L_n + \sum_{i=1}^{N} \tau'_{ni} X'_n \frac{\pi'_{ni}}{1+\tau'_{ni}} + D_n,$$

Data - Calibration

- Note that we can now solve the model without knowing all parameters (only need trade elasticities, θ) and data

Need data on:

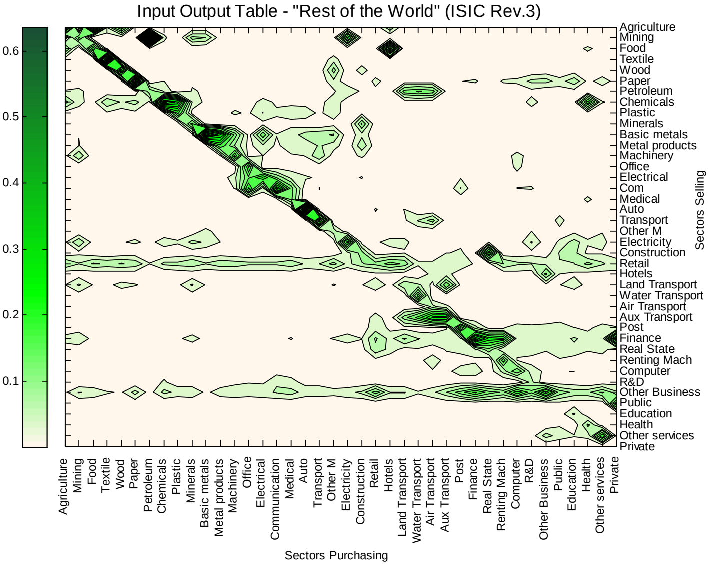
X_{ni}	bilateral trade flows
τ_{ni}	tariffs
$w_n L_n$	value added

Estimate:

θ	dispersion of productivity

Sectors, intermediates and I-O linkages (CP 2015)

- Let $j = 1, .., J$ sectors



Sectors, intermediates and I-O linkages (CP 2015)

- Households problem

$$\max_{\{C_n^j\}} \prod_{j=1}^{J} C_n^{j\ \alpha_n^j} \quad s.t: \sum_j P_n^j C_n^j = I_n,$$

where I_n is households income and α^j is the share of sector j goods in total final demand

$$I_n = w_n L_n + R_n + D_n.$$

- Cost of a unit of the input bundle

$$x_n^j = \Upsilon_n^j w_n^{\gamma_n^j} \underbrace{\prod_{k=1}^{J} P_n^{k\ \gamma_n^{k,j}}}_{\textbf{inputs from all sectors}}$$

where $\sum_{k=1}^{J} \gamma_n^{k,j} = 1 - \gamma_n^j$, and the price of "composite good", "materials", "sectoral good" in country n and sector k : P_n^k

Equilibrium conditions

1

$$x_n^j(\mathbf{w}) = \Upsilon_n^j w_n^{\gamma_n^j} \prod_{k=1}^J \left[P_n^k(\mathbf{w})\right]^{\gamma_n^{k,j}},$$

2

$$P_n^j(\mathbf{w}) = \bar{\gamma}^j \left[\sum_{i=1}^N A_i^j (x_i^j(\mathbf{w})\kappa_{ni}^j)^{-\theta^j}\right]^{-1/\theta^j},$$

3

$$\pi_{ni}^j(\mathbf{w}) = \frac{A_i^j [x_i^j(\mathbf{w})\kappa_{ni}^j]^{-\theta^j}}{\sum_{h=1}^N A_h^j [x_h^j(\mathbf{w})\kappa_{nh}^j]^{-\theta^j}},$$

4

$$\sum_{j=1}^J \sum_{i=1}^N \frac{\pi_{ni}^j(\mathbf{w})}{1+\tau_{ni}^j} X_n^j(\mathbf{w}) - D_n = \sum_{j=1}^J \sum_{i=1}^N \frac{\pi_{in}^j(\mathbf{w})}{1+\tau_{in}^j} X_i^j(\mathbf{w}),$$

5

$$X_n^j(\mathbf{w}) = \sum_{k=1}^J \gamma_n^{j,k} \left(\sum_{i=1}^N \frac{\pi_{in}^k(\mathbf{w})}{1+\tau_{in}^k} X_i^k(\mathbf{w})\right) + \alpha_n^j I_n(\mathbf{w})$$

Equilibrium conditions (relative terms)

1

$$\hat{x}_n^j(\hat{\mathbf{w}}) = \hat{w}_n^{\gamma_n^j} \prod_{k=1}^J \left[\hat{P}_n^k(\hat{\mathbf{w}})\right]^{\gamma_n^{k,j}},$$

2

$$\hat{P}_n^j(\hat{\mathbf{w}}) = \left[\sum_{i=1}^N \pi_{ni}^j [\hat{\kappa}_{ni}^j \hat{x}_i^j(\hat{\mathbf{w}})]^{-\theta^j}\right]^{-1/\theta^j},$$

3

$$\hat{\pi}_{ni}^j(\hat{\mathbf{w}}) = \frac{[\hat{x}_i^j(\hat{\mathbf{w}}) \hat{\kappa}_{ni}^j]^{-\theta^j}}{\sum_{i=1}^N \pi_{ni}^j [\hat{\kappa}_{ni}^j \hat{x}_i^j(\hat{\mathbf{w}})]^{-\theta^j}},$$

4

$$\sum_{j=1}^J \sum_{i=1}^N \frac{\pi_{ni}^{j\prime}(\hat{\mathbf{w}})}{1+\tau_{ni}^{j\prime}} X_n^{j\prime}(\hat{\mathbf{w}}) - D_n = \sum_{j=1}^J \sum_{i=1}^N \frac{\pi_{in}^{j\prime}(\hat{\mathbf{w}})}{1+\tau_{in}^{j\prime}} X_i^{j\prime}(\hat{\mathbf{w}})$$

5

$$X_n^{j'}(\hat{\mathbf{w}}) = \sum_{k=1}^J \gamma_n^{j,k} \left(\sum_{i=1}^N \frac{\pi_{in}^{k'}(\hat{\mathbf{w}})}{1+\tau_{in}^{k'}} X_i^{k'}(\hat{\mathbf{w}})\right) + \alpha_n^j I_n'(\hat{\mathbf{w}})$$

Solving the Model

- Total expenditure in the counterfactual scenario is given by:

$$X_n^{j\prime} = \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} \frac{\pi_{in}^{k\prime}\left(\hat{\mathbf{w}}\right)}{1+\tau_{in}^{k\prime}} X_i^{k\prime} + \alpha_n^j \big(\widehat{w}_n w_n L_n + \sum_{j=1}^{J} X_n^{j\prime} [1\text{-} \sum_i \frac{\pi_{ni}^{j\prime}\left(\hat{\mathbf{w}}\right)}{1+\tau_{ni}^{j\prime}}] + D_n\big)$$

- System of $J \times N$ equations in $J \times N$ total expenditures
- Convenient to re-write the system of equations in matrix form:

$$\Omega(\hat{\mathbf{w}})\,\mathbf{X} = \Delta\left(\hat{\mathbf{w}}\right),$$

  $\mathbf{X}$ is the vector of expenditures for each sector and country and $\Delta\left(\hat{\mathbf{w}}\right)$ is a vector containing the shares of each sector and country in final demand, value added and aggregate trade surplus by country
- $\Omega\left(\hat{\mathbf{w}}\right)$ is a square matrix of dimensions $JN \times JN$.
  - Captures the GE effects of how changes in τ from j and i impact expenditure in all other $j's$ and $i's$

Solving the Model

$$\mathbf{X} = \begin{pmatrix} X_1^{1\prime} \\ \vdots \\ X_1^{J\prime} \\ \vdots \\ X_n^{1\prime} \\ \vdots \\ X_N^{J\prime} \end{pmatrix}_{JN\times 1} \quad ; \quad \Delta\left(\hat{\mathbf{w}}\right) = \begin{pmatrix} \alpha_1^1\left(\hat{w}_1 w_1 L_1 + D_1\right) \\ \vdots \\ \alpha_1^J\left(\hat{w}_1 w_1 L_1 + D_1\right) \\ \vdots \\ \alpha_N^1\left(\hat{w}_N w_N L_N + D_N\right) \\ \vdots \\ \alpha_N^J\left(\hat{w}_N w_N L_N + D_N\right) \end{pmatrix}_{JN\times 1}$$

Solving the Model

- $\Omega(\hat{\mathbf{w}})$ is constructed by adding three square matrices, I, $F(\hat{\mathbf{w}})$ and $\tilde{H}(\hat{\mathbf{w}})$

$$A_n = \begin{pmatrix} \alpha_n^1 \\ \vdots \\ \alpha_n^J \end{pmatrix}_{J\times 1}$$

$$\tilde{F}_n'(\hat{\mathbf{w}}) = \begin{pmatrix} \left(1 - F_n^{1'}(\hat{\mathbf{w}})\right) & \cdots & \left(1 - F_n^{J'}(\hat{\mathbf{w}})\right) \end{pmatrix}_{1\times J}$$

where $F_n^{j'}(\hat{\mathbf{w}})$ are the counterfactual values of $F_n^j(\hat{\mathbf{w}}) = \sum_{\mathbf{i}} \frac{\pi_{\mathbf{ni}}^{\mathbf{j}}}{(\mathbf{1}+\tau_{\mathbf{ni}}^{\mathbf{j}})}$. $F(\hat{\mathbf{w}})$ is defined as

$$F(\hat{\mathbf{w}}) = \begin{pmatrix} A_1 \otimes \tilde{F}_1'(\hat{\mathbf{w}}) & 0_{J\times J} & \cdots & 0_{J\times J} & 0_{J\times J} \\ 0_{J\times J} & A_2 \otimes \tilde{F}_2'(\hat{\mathbf{w}}) & \cdots & \vdots & \vdots \\ 0_{J\times J} & 0_{J\times J} & \ddots & 0_{J\times J} & 0_{J\times J} \\ \vdots & \vdots & \cdots & A_{N-1} \otimes \tilde{F}_{N-1}'(\hat{\mathbf{w}}) & 0_{J\times J} \\ 0_{J\times J} & 0_{J\times J} & \cdots & 0_{J\times J} & A_N \otimes \tilde{F}_N'(\hat{\mathbf{w}}) \end{pmatrix}$$

Solving the Model

- The square matrix $\tilde{H}\left(\hat{\mathbf{w}}\right)$ is given by:

$$\begin{pmatrix}
\gamma_1^{1,1}\tilde{\pi}_{1,1}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_1^{1,J}\tilde{\pi}_{1,1}^{J\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_1^{1,1}\tilde{\pi}_{N,1}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_1^{1,J}\tilde{\pi}_{N,1}^{J\prime}\left(\hat{\mathbf{w}}\right) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\gamma_1^{J,1}\tilde{\pi}_{1,1}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_1^{J,J}\tilde{\pi}_{1,1}^{J\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_1^{J,1}\tilde{\pi}_{N,1}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_1^{J,J}\tilde{\pi}_{N,1}^{J\prime}\left(\hat{\mathbf{w}}\right) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\gamma_N^{1,1}\tilde{\pi}_{1,N}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_N^{1,J}\tilde{\pi}_{1,N}^{J\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_N^{1,1}\tilde{\pi}_{N,N}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_N^{1,J}\tilde{\pi}_{N,N}^{J\prime}\left(\hat{\mathbf{w}}\right) \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\gamma_N^{J,1}\tilde{\pi}_{1,N}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_N^{J,J}\tilde{\pi}_{1,N}^{J\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_N^{J,1}\tilde{\pi}_{N,N}^{1\prime}\left(\hat{\mathbf{w}}\right) & \cdots & \gamma_N^{J,J}\tilde{\pi}_{N,N}^{J\prime}\left(\hat{\mathbf{w}}\right)
\end{pmatrix}$$

  where $\tilde{\pi}_{in}^{k\prime}\left(\hat{\mathbf{w}}\right) = \frac{1}{1+\tau_{in}^{k\prime}}\pi_{in}^{k\prime}\left(\hat{\mathbf{w}}\right)$

- Finally, $\Omega(\hat{\mathbf{w}}) = I - F\left(\hat{\mathbf{w}}\right) - \tilde{H}(\hat{\mathbf{w}})$

Solving the Model

- The interactions presented in $\Omega\left(\hat{\mathbf{w}}\right)$ are the key differences compared to other models
- For example
  - If $\gamma_n^{j,j} = 1$, then $\Omega\left(\hat{\mathbf{w}}\right)$ and expenditures in each i are independent from all other i
  - If $J = 1$ (one sector), $\Omega\left(\hat{\mathbf{w}}\right)$ collapses to a scalar as in Alvarez and Lucas (2007) and Eaton and Kortum (2002)
  - In a two-sector model without tariffs and exogenous sectoral deficit, as in Dekle, Eaton and Kortum (2007), $\Omega\left(\hat{\mathbf{w}}\right)$ depends only on technology and preference parameters, (γ, α).

Solving the Model

- We can proceed to solve for the vector $\mathbf{X}(\hat{\mathbf{w}})$ by inverting the matrix $\Omega(\hat{\mathbf{w}})$

$$\mathbf{X}(\hat{\mathbf{w}}) = \Omega^{-1}(\hat{\mathbf{w}})\,\Delta(\hat{\mathbf{w}})$$

- Substituting $\pi_{in}^{j\prime}(\hat{\mathbf{w}})$, $\mathbf{X}(\hat{\mathbf{w}})$, τ', and D_n into trade balance we obtain:

$$\sum_{j=1}^{J} F_n^{j'}(\hat{\mathbf{w}})\, X_n^{j\prime}(\hat{\mathbf{w}}) - D_n = \sum_{j=1}^{J}\sum_{i=1}^{N} \frac{\pi_{in}^{j\prime}(\hat{\mathbf{w}})}{1+\tau_{in}^{j\prime}} X_i^{j\prime}(\hat{\mathbf{w}})$$

- We reduced the system of equilibrium conditions to a system of N equations and N unknowns (one wage per country).
- Given our initial guess of $\hat{\mathbf{w}}$ we verify if the trade balance holds. If not, we adjust our guess of $\hat{\mathbf{w}}$ until it does

Data - Calibration

- Model with $N = 31$, add "rest of the world" $J = 40$ (20 Tradeable and 20 non-Tradeable)

Need data on:

X_{ni}^{j}	bilateral trade flows
τ_{ni}^{j}	tariffs
γ_{n}^{j}	share of value added in production for each sector j and country n
$\gamma_{n}^{k,j}$	share of sector k goods employed in the production of goods j

Calculate:

α_{n}^{j}	share of final good j in country n consumed

Estimate:

θ^{j}	dispersion of productivity

Data - Calibration

- X_{ni}^{j} COMTRADE. Commodities are defined using the HS 2002 at 6-digits level of aggregation Concordance 2 digit ISIC rev. 3. Merchandise trade
- τ_{ni}^{j} Tariff Data - UNCTAD - TRAINS (Trade Analysis and Information System)
- γ_{n}^{j} from OECD, World Input-Output Database (WIOD), and National Agencies.
- Calibrating α_{n}^{j}

$$\alpha_n^j = \frac{X_n^j - \sum_{k=1}^{J} \gamma_n^{j,k} Y_n^k}{\text{Final absorption}_n},$$

where Y_n^k is gross output

Estimating the Trade Elasticity - CP 2015

- We exploit the multiplicative property of bilateral trade shares
- Consider 3 countries n, h, and i. The cross-product of goods from sector j shipped in one direction between the three countries, from n to h, from h to i, and from i to n is

$$\pi^j_{nh}\pi^j_{hi}\pi^j_{in},$$

  and the cross-product of the same goods shipped in the opposite direction, from h to n, from i to h, and from n to i is

$$\pi^j_{hn}\pi^j_{ih}\pi^j_{ni}.$$

  Take the ratio of both and obtain

$$\frac{X^j_{nh}X^j_{hi}X^j_{in}}{X^j_{hn}X^j_{ih}X^j_{ni}} = \left(\frac{\kappa^j_{ni}}{\kappa^j_{in}}\frac{\kappa^j_{ih}}{\kappa^j_{hi}}\frac{\kappa^j_{hn}}{\kappa^j_{nh}}\right)^{-\theta^j}.$$

Estimating the Trade Elasticity - CP 2015

- From the definition of κ^j_{ni}, trade costs are divided between tariffs (non-symmetric) and iceberg (also non-symmetric) trade costs:

$$\ln \kappa^j_{ni} = \ln \tilde{\tau}^j_{ni} + \ln d^j_{ni}$$

  where $\tilde{\tau}^j_{ni} = (1 + \tau^j_{ni})$
- $\log d^j_{ni}$, can be modeled quite generally as linear functions of cross-country characteristics:

$$\log d^j_{ni} = \nu^j_{ni} + \mu^j_n + \delta^j_i + \varepsilon^j_{ni}$$

- $\nu^j_{ni} = \nu^j_{in}$ captures symmetric bilateral trade costs like distance, language, common border, FTA or not
- μ^j_n captures an importer sectoral fixed effect common to all trading partners of country n
- δ^j_i exporter sectoral fixed effect common to all trading partners of i
- ε^j_{ni} is a random disturbance term, assumed orthogonal to tariffs

Estimating the Trade Elasticity - CP 2015

- We then obtain the following specification

$$\log\left(\frac{X^j_{nh}X^j_{hi}X^j_{in}}{X^j_{hn}X^j_{ih}X^j_{ni}}\right) = -\theta^j \ln\left(\frac{\tilde{\tau}^j_{ni}}{\tilde{\tau}^j_{in}}\frac{\tilde{\tau}^j_{ih}}{\tilde{\tau}^j_{hi}}\frac{\tilde{\tau}^j_{hn}}{\tilde{\tau}^j_{nh}}\right) + \tilde{\varepsilon}^j,$$

  where $\tilde{\varepsilon}^j = \varepsilon^j_{in} - \varepsilon^j_{ni} + \varepsilon^j_{hi} - \varepsilon^j_{ih} + \varepsilon^j_{nh} - \varepsilon^j_{hn}$
- Estimation requires only trade and tariff data (no assumption of bilaterally symmetric trade costs)
- Estimate using maximum number of countries with tariff data (16) for the year 1993
  - 20 Sectors 2 digit ISIC rev. 3

Estimation results

Dispersion-of-productivity parameter							
Sector	$1/\theta^j$	s.e.	N	Sector	$1/\theta^j$	s.e.	N
Agriculture	8.1	(1.8)	496	Basic metals	7.9	(2.5)	388
Mining	15.7	(2.7)	296	Metal products	4.3	(2.1)	404
Food	2.5	(0.6)	496	Machinery n.e.c.	1.5	(1.8)	397
Textile	5.5	(1.1)	437	Office	12.7	(2.1)	306
Wood	10.8	(2.5)	315	Electrical	10.6	(1.3)	343
Paper	9.0	(1.6)	507	Com	7.0	(1.7)	311
Petroleum	51.0	(18.0)	91	Medical	9.9	(1.2)	383
Chemicals	4.7	(1.7)	430	Auto	1.0	(0.8)	237
Plastic	1.6	(1.4)	376	Other Transport	0.3	(1.0)	245
Minerals	2.7	(1.4)	342	Other	5.0	(0.9)	412
Aggregate elasticity	4.55	(0.35)					

Quantifying the trade and welfare effects of NAFTA

- CP 2015 perform two counterfactual exercises

1. NAFTA's Tariff Reductions, the effect of NAFTA's tariff reductions conditional on no other tariff changing
   - How? change tariff structure from 1993 to the year 2005 between NAFTA members and fix the tariff structure for the rest of the world to the levels in 1993
2. *The Effects of NAFTA given World Tariff Changes*, the effects of NAFTA's tariff reductions given observed world tariff reductions
   - How? first introduce observed change in world tariffs from 1993 to 2005 (effects of observed world tariff changes); second recalibrate the model to the year 1993 and introduce change in world tariffs from 1993 to 2005 holding NAFTA 1993 tariffs fixed (effects of observed world tariff changes excluding NAFTA); finally compare the gains between these two exercises, namely the gains from world tariff reductions with and without NAFTA

The effects of NAFTA across different models

	Welfare			
		Multi sector		
Country	One sector	No materials	No I-O	Benchmark
Mexico	0.41%	0.50%	0.66%	1.31%
Canada	-0.08%	-0.03%	-0.04%	-0.06%
U.S.	0.05%	0.03%	0.04%	0.08%

	Imports growth from NAFTA members			
		Multi sector		
Country	One sector	No materials	No I-O	Benchmark
Mexico	60.99%	88.09%	98.96%	118.28%
Canada	5.98%	9.95%	10.14%	11.11%
U.S.	17.34%	26.91%	30.70%	40.52%

Migration, and labor market dynamics

Migration, and labor market dynamics

- Static models, in which workers are assumed to be either instantly costlessly mobile, or perfectly immobile, can say little about important issues
  - How easily the workers displaced from one sector can find employment in other sectors/regions
  - How long will the labor market take to adjust to a shock?
  - Will the steady state feature a lasting differential impact on workers in the import-afflicted sector?
  - What are the lifetime welfare effects on workers in different industries, taking into account moving costs and transitional dynamics?

Ingredients

- We will start with a simple model of migration labor market dynamics
- We model the households' decision of where to supply labor across markets as a dynamic discrete choice/control problem
  - In response to shocks, worker choose whether to remain where she is or to move to another location
  - If the worker moves, she will pay a cost, which has two components:
    - ★ A portion that is the same for all workers making the same move (moving costs, learning costs, etc.)
    - ★ A time-varying idiosyncratic cost or preference (personal situation)

Ingredients

- Idiosyncratic shocks are broadly consistent with observed labor-market behavior:
  - First, gross flows are an order of magnitude larger than net flows, implying large numbers of workers moving in opposite directions at the same time
  - Evidence shows that a significant fraction of workers who change jobs voluntarily move to jobs which pay less than the job the worker left behind
  - These idiosyncratic costs will generate dynamics

Simple model

- N locations indexed by i and n
- The value of a household in location n at time t given by

$$\mathrm{v}_t^n = U(C_t^n) + \max_{\{i\}_{i=1}^N} \left\{ \beta E\left[\mathrm{v}_{t+1}^i\right] - \tau^{n,i} + \nu \epsilon_t^i \right\},$$
$$s.t.\ U(C_t^n) \equiv \log(w_t^n)$$

  - $\beta \in (0,1)$ discount factor
  - $\tau^{n,i}$ additive, *time invariant* migration costs to i from n
  - ϵ_t^i are stochastic *i.i.d idiosyncratic* taste shocks
    - ★ $\epsilon \sim$ Type-I Extreme Value distribution with zero mean
    - ★ $\nu > 0$ is the dispersion of taste shocks

- *Employed* households supply a unit of labor inelastically
  - Receive the competitive market wage w_t^n

Households' problem - Dynamic discrete control

- Denote by $V_t^n \equiv E[\mathrm{v}_t^n]$ to the expected (expectation over ϵ) lifetime utility of a worker in n
- The value of a household in location n at time t given by

$$E\left[\mathrm{v}_t^n\right] = E\left[U(C_t^n) + \max_{\{i\}_{i=1}^N}\left\{\beta E\left[\mathrm{v}_{t+1}^i\right] - \tau^{n,i} + \nu\epsilon_t^i\right\}\right],$$

- We seek to solve for

$$\Phi_t^i = E\left[\max_{\{i\}_{i=1}^N}\left\{\beta E\left[\mathrm{v}_{t+1}^i\right] - \tau^{n,i} + \nu\,\epsilon_t^i\right\}\right]$$

Households' problem - Dynamic discrete control

- Assumption, Type-I Extreme Value

$$F(\epsilon) = \exp\left(-\exp\left(-\epsilon - \bar{\gamma}\right)\right)$$

- Then

$$\Phi_t^i = \nu \log \left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

For the algebra, see the appendix in CDP (2018).

Households' problem - Dynamic discrete choice

- Define $\mu_t^{n,i}$ as the fraction of workers that reallocate from location n to i

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- This fraction is equal to the probability that a given worker moves from n to i at time t. Formally,

$$\mu_t^{n,i} = \Pr\left(\frac{\beta V_{t+1}^i - \tau^{n,i}}{\nu} + \epsilon_t^i \geq \max_{h \neq i}\left\{\frac{\beta V_{t+1}^h - \tau^{n,h}}{\nu} + \epsilon_t^h\right\}\right).$$

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- Fraction of workers that reallocate from location n to i

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^{i} - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}^{h} - \tau^{n,h}\right)^{1/\nu}}$$

For the algebra, see the appendix in CDP (2018).

Households' problem - Dynamic discrete control

Equilibrium conditions:

- The expected (expectation over ϵ) lifetime utility of a worker at n

$$V_t^n = U(C_t^n) + \nu \log \left[\sum_{i=1}^N \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

- Fraction of workers that reallocate from market n to i

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

- Finally, evolution of the distribution of labor across markets

$$L_{t+1}^N = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

- Wages, taken as given $\{w_t^n\}_{t=0}^\infty$

Equilibrium

- State of the economy = distribution of labor $L_t = \{L_t^n\}_{n=1}^N$
  - Fundamentals of the economy given by $\Theta \equiv \left(\tau^{n,i}\right)_{n=1,i=1}^{N,N}$

Definition

Given $\left(L_0, \Theta, \{w_t^n\}_{t=0}^{\infty}\right)$ and (ν), a **sequential competitive equilibrium** is a sequence of $\{L_t,\ \mu_t,\ V_t,\}_{t=0}^{\infty}$ that solves HH dynamic problem.

- With $\mu_t = \{\mu_t^{n,i}\}_{i=1}^N$, and $V_t = \{V_t^n\}_{n=1}^N$

- Note: for estimates of τ Artuc, Chaudhuri and McLaren (2010), and Dix-Carneiro (2014); for estimates of ν see ACM (2010) and CDP (2018)

Welfare

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Welfare

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- Re-writing the value of being in a particular n is given by

$$v_t^n = \underbrace{\log C_t^n}_{\text{current period utility}} + \underbrace{\beta E\left[\mathrm{v}_{t+1}^n\right]}_{\text{value of staying}} + \underbrace{\max_{\{i\}_{i=1}^N}\left\{\beta E\left[\mathrm{v}_{t+1}^i - \mathrm{v}_{t+1}^n\right] - \tau^{n,i} + \nu\,\epsilon_t^i\right\}}_{\text{option value of migration}}$$

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- As before, taking the expected value of this equation, we can write the expected lifetime utility of being in n at time t as

$$V_t^n = \log C_t^n + \beta V_{t+1}^n + \nu \log \left[\sum_{i=1}^N \exp\left(\beta\left(V_{t+1}^i - V_{t+1}^n\right) - \tau^{n,i}\right)^{1/\nu}\right]$$

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- Use

$$\mu_t^{n,n} = \frac{\exp\left(\beta V_{t+1}^n\right)^{1/\nu}}{\sum_{h=1}^N \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

and therefore the option value is given by

$$\nu \log \sum_{h=1}^N \exp\left(\beta\left(V_{t+1}^h - V_{t+1}^n\right) - \tau^{n,h}\right)^{1/\nu} = -\nu \log \mu_t^{n,n}.$$

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Welfare

- Plugging this equation into the value function, we get

$$V_t^n = \log C_t^n + \beta V_{t+1}^n - \nu \log \mu_t^{n,n}$$

- Finally, iterating this equation forward we obtain

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{w_t^n}{(\mu_t^{n,n})^{\nu}}$$

  - $\mu_t^{n,n}$ summarizes the option value of migration

- Crucial difference with static models; a given shock can reduce wages but increase the option value of migration so workers are better off

Trade, migration, and labor market dynamics

Trade and labor market dynamics

- Let's introduce international trade into the model
- We will expand over the production structure
  - Determine wages such that each labor markets clears
  - Given real wages, labor supply determined as before
  - Production structure and international trade will determine labor demand
  - Prices endogenously determined

Households' - Dynamic problem

- As before, equilibrium conditions:
- The expected (expectation over ϵ) lifetime utility of a worker at n

$$V_t^n = U(C_t^n) + \nu \log \left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

but now $\quad U(C_t^n) \equiv \log(w_t^n / P_t^n)$

- Fraction of workers that reallocate from market n to i

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

- Finally, evolution of the distribution of labor across markets

$$L_{t+1}^n = \sum_{i=1}^{N} \mu_t^{i,n} L_t^i$$

Production - Static sub-problem

- **At each t, gravity**
  - Let X_t^n denote the total expenditure on final goods in n
  - Goods market clearing condition is given by

$$X_t^n = w_t^n L_t^n$$

  - The share of total expenditure in market n on goods from i is given by

$$\pi_t^{n,i} = \frac{A_t^i [w_t^i \kappa_t^{n,i}]^{-\theta}}{\sum_{h=1}^N A_t^h [w_t^h \kappa_t^{n,h}]^{-\theta}}$$

- Labor market clearing in n is

$$w_t^n L_t^n = \sum_{i=1}^N \pi_t^{i,n} X_t^i,$$

and we further assume balanced trade (for now).

Production - Static sub-problem

- From the price index,

$$P_t^n = \bar{\gamma} \left[\sum_{i=1}^{N} A_t^i \left[w_t^i \kappa_t^{n,i} \right]^{-\theta} \right]^{-1/\theta}$$

and expenditure shares given by

$$\pi_t^{n,i} = \frac{A_t^i [w_t^i \kappa_t^{n,i}]^{-\theta}}{\sum_{h=1}^{N} A_t^h [w_t^h \kappa_t^{n,h}]^{-\theta}}$$

- Note that real wages are given by

$$\frac{w_t^n}{P_t^n} = \left(\pi_t^{n,n} / T_t^{n,n} \right)^{-1/\theta},$$

where $T_t^{n,i} \equiv \bar{\gamma}^{-\theta} A_t^i \left(\kappa_t^{n,i} \right)^{-\theta}$

Welfare

- Now $\log C_t^n = \log w_t^n / P_t^n$, therefore

$$V_0^n = \sum_{t=0}^{\infty} \beta^t \log \frac{\left(\pi_t^{n,n} / T_t^{n,n}\right)^{-\frac{1}{\theta\gamma}}}{\left(\mu_t^{n,n}\right)^{\nu}} = \frac{\text{gains from trade}}{\text{gains from migration}}$$

- Summarizes welfare equations in static trade models
  - Arkolakis, Costinot and Rodriguez-Clare (2010) and dynamic models with exogenous trade as in Artuc, Chadhuiri and Mclaren (2010)
- Sufficient statistic to measure welfare gains from trade and migration relative to autarky $\pi_t^{n,n} = 1$ and no migration $\mu_t^{n,n} = 1$

Sequential and temporary equilibrium

- Recap: Given real wages, HH dynamic problem solve for the path of labor supply
- Given labor supply at each time t firms decide production and labor demand. Wages clear markets
- General equilibrium: path of employment and path of wages have to be consistent with both the HH dynamic problem and the static sub-problem

Equilibrium conditions

- Let $\tilde{\tau}^{n,i} \equiv e^{\tau^{n,i}}$, $u_t^n \equiv e^{V_t^n}$, then

$$V_t^n = \log(w_t^n / P_t^n) + \nu \log \left[\sum_{i=1}^{N} \exp \left(\beta V_{t+1}^i - \tau^{n,i} \right)^{1/\nu} \right]$$

Equilibrium conditions

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- Can be written as

$$\exp\left(V_t^n\right) = \left(w_t^n / P_t^n\right) \left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]^{\nu}$$

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$$\exp\left(V_t^n\right) = \left(w_t^n / P_t^n\right) \left[\sum_{i=1}^N \exp \left(\beta V_{t+1}^i - \tau^{n,i} \right)^{1/\nu} \right]^\nu$$

- Using $w_t^n / P_t^n = \left(\pi_t^{n,n} / T_t^{n,n}\right)^{-1/\theta}$

$$u_t^n = \left[\sum_{i=1}^N \left(\pi_t^{n,n} / T_t^{n,n}\right)^{-1/\theta\nu} \left(u_{t+1}^i\right)^{\beta/\nu} \left(\tilde{\tau}^{n,i}\right)^{-1/\nu} \right]^\nu$$

Equilibrium conditions - simple system of equations

- Temporary equilibrium, trade

$$\pi_t^{n,i} = \frac{(w_t^i)^{-\theta} T_t^{n,i}}{\sum_{h=1}^{N} (w_t^h)^{-\theta} T_t^{n,h}}$$

$$w_t^n L_t^n = \sum_{i=1}^{N} \pi_t^{i,n} w_t^i L_t^i$$

Equilibrium conditions - simple system of equations

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$$\pi_t^{n,i} = \frac{(w_t^i)^{-\theta} T_t^{n,i}}{\sum_{h=1}^N (w_t^h)^{-\theta} T_t^{n,h}}$$

$$w_t^n L_t^n = \sum_{i=1}^N \pi_t^{i,n} w_t^i L_t^i$$

- Dynamics, migration

$$u_t^n = \left[\sum_{i=1}^N \left(\pi_t^{n,n}/T_t^{n,n}\right)^{-1/\theta\nu} \left(u_{t+1}^i\right)^{\beta/\nu} \left(\tilde{\tau}^{n,i}\right)^{-1/\nu}\right]^\nu$$

$$\mu_t^{n,i} = \frac{\left(u_{t+1}^i\right)^{\beta/\nu} \left(\tilde{\tau}^{n,i}\right)^{-1/\nu}}{\sum_{h=1}^N \left(u_{t+1}^h\right)^{\beta/\nu} \left(\tilde{\tau}^{n,h}\right)^{-1/\nu}}$$

$$L_{t+1}^n = \sum_{i=1}^N \mu_t^{i,n} L_t^i$$

Sequential and temporary equilibrium

- State of the economy = distribution of labor $L_t = \{L_t^n\}_{n=1}^N$
  - Now $\Theta_t \equiv \left(\{A_t^n\}, \{\kappa_t^{n,i}\}, \{\tau^{n,i}\}\right)_{n=1,i=1}^{N,N}$

Definition

Given $\left(L_t, \Theta_t\right)$, a **temporary equilibrium** is a vector of $w_t\left(L_t, \Theta_t\right)$ that satisfies the equilibrium conditions of the static sub-problem

Definition

Given $\left(L_0, \{\Theta_t\}_{t=0}^{\infty}\right)$, a **sequential competitive equilibrium** is a sequence of $\{L_t,\ \mu_t,\ V_t,\ w_t\}_{t=0}^{\infty}$ that solves HH dynamic problem and the temporary equilibrium at each t

Steady state

Definition

A **stationary equilibrium** of the model is a sequential competitive equilibrium such that $\{L_t, \mu_t, V_t, w_t\}_{t=0}^{\infty} = \{\bar{L}, \bar{\mu}, \bar{V}, \bar{w}\}$ are constant for all t.

- At the steady state, $u_t^n = \bar{u}^n$, $\mu_t^{n,i} = \bar{\mu}^{n,i}$, $L_t^n = \bar{L}^n$, $\pi_t^{n,i} = \bar{\pi}^{n,i}$, $w_t^n = \bar{w}^n$, $T_t^{n,i} = \bar{T}^{n,i}$, for all t

Steady state

- Solution to

$$\bar{\pi}^{n,i} = \frac{\left(\bar{w}^{i}\right)^{-\theta} \bar{T}^{n,i}}{\sum_{h=1}^{N} \left(\bar{w}^{h}\right)^{-\theta} \bar{T}^{n,h}}$$

$$\bar{w}^{n} \bar{L}^{n} = \sum_{i=1}^{N} \bar{\pi}^{i,n} \, \bar{w}^{i} \bar{L}^{i}$$

$$\bar{u}^{n} = \left(\bar{\pi}^{n,n} / \bar{T}^{n,n}\right)^{-\frac{1}{\theta(1-\beta)}} \left(\bar{\mu}^{n,n}\right)^{-\frac{\nu}{1-\beta}}$$

$$\bar{\mu}^{n,i} = \frac{\left(\bar{u}^{i}\right)^{\beta/\nu} \left(\tilde{\tau}^{n,i}\right)^{-1/\nu}}{\sum_{h=1}^{N} \left(\bar{u}^{h}\right)^{\beta/\nu} \left(\tilde{\tau}^{n,h}\right)^{-1/\nu}}$$

$$\bar{L}^{n} = \sum_{i=1}^{N} \bar{\mu}^{i,n} \, \bar{L}^{i}$$

Solution Method

Dynamic Hat Algebra (*structural diff and diff*)

Solution method: Dynamic Hat Algebra

- Solving for an equilibrium of the model requires information on Θ
  - Large # of unknowns
- As we increase the dimension of the problem—adding countries, regions, or sectors—the number of parameters grows geometrically
- We solve this problem by computing the equilibrium dynamics of the model in time differences
- Why is this progress?
  - Conditioning on observables one can solve the model without knowing the *levels* of Θ
    - ⋆ Solve for the value function in time differences
- I'll start describing the *Dynamic Hat Algebra* with *constant fundamentals*
  - Then show how to deal with *time varying fundamentals*

Equilibrium conditions

- Expected lifetime utility

$$V_t^n = \log\left(\frac{w_t^n}{P_t^n}\right) + \nu \log \left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu} \right]$$

- Transition matrix (migration flows)

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

Equilibrium conditions

- *Transition* matrix (migration flows) at $t = -1$, **Data**

$$\boldsymbol{\mu}_{\mathbf{-1}}^{\mathbf{n,i}} = \frac{\exp\left(\beta V_0^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_0^h - \tau^{n,h}\right)^{1/\nu}}$$

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- Transition matrix (migration flows) at $t = 0$, Model

$$\mu_0^{n,i} = \frac{\exp\left(\beta V_1^i - \tau^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}$$

Equilibrium conditions

- *Transition* matrix (migration flows) at $t = -1$, **Data**

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- Take the time difference

$$\frac{\mu_0^{n,i}}{\boldsymbol{\mu}_{\mathbf{-1}}^{\mathbf{n,i}}} = \frac{\frac{\exp\left(\beta V_1^i - \tau^{n,i}\right)^{1/\nu}}{\exp\left(\beta V_0^i - \tau^{n,i}\right)^{1/\nu}}}{\sum_{h=1}^{N} \frac{\exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}{\sum_{m=1}^{N} \exp\left(\beta V_0^m - \tau^{n,m}\right)^{1/\nu}}}$$

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- Simplify

$$\frac{\mu_0^{n,i}}{\boldsymbol{\mu_{-1}^{n,i}}} = \frac{\exp\left(V_1^i - V_0^i\right)^{\beta/\nu}}{\sum_{h=1}^N \quad \frac{\exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}{\sum_{m=1}^N \quad \exp(\beta V_0^m - \tau^{n,m})^{1/\nu}}}$$

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- Simplify

$$\frac{\mu_0^{n,i}}{\boldsymbol{\mu_{-1}^{n,i}}} = \frac{\exp\left(V_1^i - V_0^i\right)^{\beta/\nu}}{\sum_{h=1}^{N} \frac{\exp\left(\beta V_1^h - \tau^{n,h}\right)^{1/\nu}}{\sum_{m=1}^{N} \exp\left(\beta V_0^m - \tau^{n,m}\right)^{1/\nu}}}$$

- Use $\boldsymbol{\mu_{-1}^{n,h}}$ once again

$$\mu_0^{n,i} = \frac{\boldsymbol{\mu_{-1}^{n,i}} \exp\left(V_1^i - V_0^i\right)^{\beta/\nu}}{\sum_{h=1}^{N} \boldsymbol{\mu_{-1}^{n,h}} \exp\left(V_1^h - V_0^h\right)^{\beta/\nu}}$$

Equilibrium conditions

- Expected lifetime utility

$$V_t^n = \log\left(\frac{w_t^n}{P_t^n}\right) \; + \nu \log\left[\sum_{i=1}^{N} \exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}\right]$$

- Transition matrix

$$\mu_t^{n,i} \quad = \frac{\exp\left(\beta V_{t+1}^i - \tau^{n,i}\right)^{1/\nu}}{\sum\limits_{h=1}^{N} \quad \exp\left(\beta V_{t+1}^h - \tau^{n,h}\right)^{1/\nu}}$$

Equilibrium conditions - Time differences

- Expected lifetime utility

$$V_{t+1}^n - V_t^n = \log\left(\frac{w_{t+1}^n / w_t^n}{P_{t+1}^n / P_t^n}\right) + \nu \log \left[\sum_{i=1}^{N} \mu_t^{n,i} \, \exp\left(V_{t+2}^i - V_{t+1}^i\right)^{\beta/\nu} \right]$$

- Transition matrix

$$\frac{\mu_{t+1}^{n,i}}{\mu_t^{n,i}} = \frac{\exp\left(V_{t+2}^i - V_{t+1}^i\right)^{\beta/\nu}}{\sum_{h=1}^{N} \mu_t^{n,h} \, \exp\left(V_{t+2}^h - V_{t+1}^h\right)^{\beta/\nu}}$$

where $\frac{w_{t+1}^n / w_t^n}{P_{t+1}^n / P_t^n}$ is the solution to the temporary equilibrium in time differences

Temporary equilibrium conditions

How to solve for the temporary equilibrium in time differences?

- Price index

$$P_t^n(\mathbf{w}_t) = \Gamma^n \left[\sum_{i=1}^{N} A^i [x_t^i(\mathbf{w}_t)\kappa^{n,i}]^{-\theta}\right]^{-1/\theta},$$

- Trade shares

$$\pi_t^{n,i}(\mathbf{w}_t) = \frac{[x_t^i(\mathbf{w}_t)\kappa^{n,i}]^{-\theta} A^i}{\sum_{h=1}^{N} [x_t^h(\mathbf{w}_t)\kappa^{n,h}]^{-\theta} A^h},$$

Temporary equilibrium - Time differences

How to solve for the temporary equilibrium in time differences?

- Price index

$$\dot{P}^n_{t+1} = \left[\sum_{i=1}^{N} \pi^{n,i}_t \left[\dot{w}^i_{t+1}\right]^{-\theta}\right]^{-1/\theta},$$

- Trade shares

$$\pi^{n,i}_{t+1} = \frac{\pi^{n,i}_t \left[\dot{w}^i_{t+1}\right]^{-\theta}}{\sum_{h=1}^{N} \pi^{n,h}_t \left[\dot{w}^h_{t+1}\right]^{-\theta}},$$

- **Notation:** $\dot{P}^n_{t+1} = P^n_{t+1} / P^n_t$, $\dot{w}^i_{t+1} = w^i_{t+1} / w^i_t$,
- Same "dot trick" applies to all equilibrium conditions

Solving the model with constant fundamentals

Proposition

Given $(\mathbf{L_0}, \boldsymbol{\mu_{-1}}, \boldsymbol{\pi_0}, \mathbf{X_0})$, (ν, θ, β), *solving the equilibrium in time differences does not require the level of* Θ, *and solves*

$$\dot{u}^n_{t+1} = (\dot{w}^n_{t+1}/\dot{P}^n_{t+1}) \left(\sum_{i=1}^N \mu^{n,i}_t [\dot{u}^i_{t+2}]^{\beta/\nu} \right)^{1/\nu}, \tag{1}$$

$$\mu^{n,i}_{t+1} = \frac{\mu^{n,i}_t [\dot{u}^i_{t+2}]^{\beta/\nu}}{\sum_{h=1}^N \mu^{n,h}_t [\dot{u}^h_{t+2}]^{\beta/\nu}}, \tag{2}$$

$$L^n_{t+1} = \sum_{i=1}^N \mu^{i,n}_t L^i_t, \tag{3}$$

where $\dot{u}^i_{t+1} \equiv \exp(V^i_{t+1} - V^i_t)$, *and* $\dot{w}^n_{t+1}/\dot{P}^n_{t+1}$ *solves the temporary equilibrium given* $\dot{L}_{t+1}$.

Solution algorithm

Part I: Solving for the sequential competitive equilibrium

The strategy to solve the model given an initial allocation of the economy, $\left(L_0, \pi_0, X_0, \mu_{-1}\right)$, and given an anticipated convergent sequence of changes in fundamentals, $\{\dot{\Theta}_t\}_{t=1}^{\infty}$, is as follows:

1. Initiate the algorithm at $t = 0$ with a guess for the path of $\left\{\dot{u}_{t+1}^{n\,(0)}\right\}_{t=0}^{T}$, where the superscript (0) indicates that it is a guess. The path should converge to $\dot{u}_{T+1}^{n\,(0)} = 1$ for a sufficiently large T. Take as given the set of initial conditions L_0^n, $\mu_{-1}^{n,i}$, $\pi_0^{n,i}$, $w_0^n L_0^n$, $r_0^n H_0^n$.
2. For all $t \geq 0$, use $\left\{\dot{u}_{t+1}^{n\,(0)}\right\}_{t=0}^{T}$ and $\mu_{-1}^{n,i}$ to solve for the path of $\left\{\mu_t^{n,i}\right\}_{t=0}^{T}$ using equation (2).
3. Use the path for $\left\{\mu_t^{n,i}\right\}_{t=0}^{T}$ and L_0^n to get the path for $\left\{L_{t+1}^n\right\}_{t=0}^{T}$ using equation (3)

Solution algorithm

Part I: Solving for the sequential competitive equilibrium

4 Solving for the temporary equilibrium:

   a For each $t \geq 0$, given $\dot{L}^n_{t+1}$, guess a value for $\dot{w}^n_{t+1}$.
   b Obtain $\dot{P}^n_{t+1}$, and $\pi^{n,i}_{t+1}$
   c Use $\pi^{n,i}_{t+1}$, $\dot{w}^n_{t+1}$, and $\dot{L}^n_{t+1}$ to get X^n_{t+1} using labor market clearing
   d Check if the labor market is in equilibrium. If it is not, go back to step (4.a) and adjust the initial guess for $\dot{w}^n_{t+1}$ until labor markets clear.
   e Repeat steps (4.a) through (4.d) for each period t and obtain paths for $\left\{\dot{w}^n_{t+1}, \dot{P}^n_{t+1}\right\}_{t=0}^{T}$

Solution algorithm

Part I: Solving for the sequential competitive equilibrium

5 For each t, use $\mu_t^{n,i}$, $\dot{w}_{t+1}^n$, $\dot{P}_{t+1}^n$, and $\dot{u}_{t+2}^{n\ (0)}$ to solve backwards for $\dot{u}_{t+1}^{n\ (1)}$ using equation (1). This delivers a new path for $\left\{\dot{u}_{t+1}^{n\ (1)}\right\}_{t=0}^{T}$, where the superscript 1 indicates an updated value for u.

6 Take the path for $\left\{\dot{u}_{t+1}^{n\ (1)}\right\}_{t=0}^{T}$ as the new set of initial conditions.

7 Check if $\left\{\dot{u}_{t+1}^{n\ (1)}\right\}_{t=0}^{T} \simeq \left\{\dot{u}_{t+1}^{n\ (0)}\right\}_{t=0}^{T}$. If it is not, go back to step 1 and update the initial guess.

Example

Solving for counterfactuals

- Want to study the effects of changes in fundamentals Θ'/Θ
  - Recall that $\Theta \equiv \left(\{A^n\}, \{\kappa^{n,i}\}, \{\tau^{n,i}\}\right)_{n=1,i=1}^{N,N}$
  - TFP, trade costs, labor migration costs, endowments of local structures, home production
- We can use our solution method to study the effects of changes in Θ
  - One by one or jointly
  - Changes across time and space

Solving for counterfactuals

- Suppose we want to study the effects of a change in $A_t^{\prime i} / A_t^i$
- **Counterfactual I**
  - Economy with $\dot{\Theta}_t' = \dot{A}_t^{\prime i}$ relative to economy with $\dot{\Theta}_t = 1$
    - ★ Pros: requires only data on an initial allocation (for one year)
    - ★ Cons: need to compute the model twice, one under $\dot{\Theta}_t = 1$ "baseline economy", and one under $\dot{\Theta}_t' = \dot{A}_t^{\prime i}$
- **Counterfactual II**
  - Economy with actual change in fundamentals $\dot{\Theta}_t$ relative to an economy with all fundamentals changing except $\dot{A}_t^i$
    - ★ Pros: only requires computing the equilibrium once: "baseline economy" is the **data**
    - ★ Cons: larger data requirements, need data for many t, need to deal with $t = T$?

Equilibrium conditions: Time-varying fundamentals

- Transition matrix (migration flows) $\left\{ {\mu_t}^{n,i} \right\}_{t=0}^{T}$, **Data**

$$ {\mu_t}^{n,i} \quad = \frac{\exp\left(\beta V_{t+1}^{i} - \tau_t^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \quad \exp\left(\beta V_{t+1}^{m} - \tau_t^{n,h}\right)^{1/\nu}} $$

Equilibrium conditions: Time-varying fundamentals

- Transition matrix (migration flows) $\left\{ \mu_t^{n,i} \right\}_{t=0}^{T}$, **Data**

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^{i} - \tau_t^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}^{m} - \tau_t^{n,h}\right)^{1/\nu}}$$

- Transition matrix at t, from Model given fundamentals τ_t'

$$\mu_t'^{n,i} = \frac{\exp\left(\beta V_{t+1}'^{i} - \tau_t'^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}'^{h} - \tau_t'^{n,h}\right)^{1/\nu}}$$

Equilibrium conditions: Time-varying fundamentals

- Transition matrix (migration flows) $\left\{\mu_t^{n,i}\right\}_{t=0}^{T}$, **Data**

$$\mu_t^{n,i} = \frac{\exp\left(\beta V_{t+1}^{i} - \tau_t^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}^{m} - \tau_t^{n,h}\right)^{1/\nu}}$$

- Transition matrix at t, from Model given fundamentals τ_t'

$$\mu_t'^{n,i} = \frac{\exp\left(\beta V_{t+1}'^{i} - \tau_t'^{n,i}\right)^{1/\nu}}{\sum_{h=1}^{N} \exp\left(\beta V_{t+1}'^{h} - \tau_t'^{n,h}\right)^{1/\nu}}$$

- Take the differences at each t, Model relative to Data

$$\mu_t'^{n,i} = \frac{\mu_t^{n,i} \exp\left(V_{t+1}'^{i} - V_{t+1}^{i}\right)^{\beta/\nu} \exp\left(\tau_t'^{n,i} - \tau_t^{n,i}\right)^{-1/\nu}}{\sum_{h=1}^{N} \mu_t^{n,h} \exp\left(V_{t+1}'^{h} - V_{t+1}^{h}\right)^{\beta/\nu} \exp\left(\tau_t'^{n,h} - \tau_t^{n,h}\right)^{-1/\nu}}$$

Equilibrium conditions in "hats"

- Denote by

$$\hat{u}_t^n = \dot{u}_t'^n / \dot{u}_t^n,$$

$$\hat{\tau}_t^{n,i} = \exp\left(\tau_t'^{n,i} - \tau_t^{n,i}\right) / \exp\left(\tau_{t-1}'^{n,i} - \tau_{t-1}^{n,i}\right),$$

$$\dot{\mu}_t^{n,i} = \mu_t^{n,i} / \mu_{t-1}^{n,i},$$

and generically

$$\hat{\Theta}_t = \dot{\Theta}_t' / \dot{\Theta}_t$$

Equilibrium conditions in "hats"

- Denote by

$$\hat{u}_t^n = \dot{u}_t'^n / \dot{u}_t^n,$$

$$\hat{\tau}_t^{n,i} = \exp\left(\tau_t'^{n,i} - \tau_t^{n,i}\right) / \exp\left(\tau_{t-1}'^{n,i} - \tau_{t-1}^{n,i}\right),$$

$$\dot{\mu}_t^{n,i} = \mu_t^{n,i} / \mu_{t-1}^{n,i},$$

and generically

$$\hat{\Theta}_t = \dot{\Theta}_t' / \dot{\Theta}_t$$

- Take the time difference to obtain

$$\mu_t'^{n,i} = \frac{\mu_{t-1}'^{n,i}\left(\hat{\tau}_t^{n,i}\right)^{-1/\nu} \dot{\mu}_t^{n,i}\left(\hat{u}_{t+1}^i\right)^{\beta/\nu}}{\sum_{h=1}^{N} \mu_{t-1}'^{n,h}\left(\hat{\tau}_t^{n,h}\right)^{-1/\nu} \dot{\mu}_t^{n,h}\left(\hat{u}_{t+1}^h\right)^{\beta/\nu}}$$

Solving the model for counterfactuals

Proposition

Given $(L_t, \mu_{\mathbf{t-1}}, \pi_{\mathbf{t}}, X_t)_{t=0}^{\infty}$, (ν, θ, β), *and* $\{\hat{\Theta}_t\}_{t=1}^{\infty}$, *solving the model with the Dynamic Hat-Algebra does not require* Θ_t, *and solves*

$$\hat{u}_t^n = (\hat{w}_t^n / \hat{P}_t^n) \left(\sum_{i=1}^N \mu_t'^{n,i} \, (\hat{\tau}_t^{n,i})^{-1/\nu} \dot{\mu}_t^{n,i} \left(\hat{u}_{t+1}^i \right)^{\beta/\nu} \right)^{\nu} , \tag{4}$$

$$\mu_t'^{n,i} = \frac{\mu_{t-1}'^{n,i} \left(\hat{\tau}_t^{n,i} \right)^{-1/\nu} \dot{\mu}_t^{n,i} \left(\hat{u}_{t+1}^i \right)^{\beta/\nu}}{\sum_{h=1}^N \mu_{t-1}'^{n,h} \left(\hat{\tau}_t^{n,h} \right)^{-1/\nu} \dot{\mu}_t^{n,h} \left(\hat{u}_{t+1}^h \right)^{\beta/\nu}}, \tag{5}$$

$$L_{t+1}'^{n} = \sum_{i=1}^N \mu_t'^{i,n} \, L_t'^{i}, \tag{6}$$

where $\hat{w}_t^n / \hat{P}_t^n$ *solves the temporary equilibrium.*

Solution algorithm

Part II: Solving for counterfactuals

- Timing assumption
  - To compute counterfactuals, we assume that agents at $t = 0$ are not anticipating the change in the path of fundamentals and that at $t = 1$ agents learn about the entire future counterfactual sequence of $\{\Theta'_t\}_{t=1}^{\infty}$.
- Take as given a baseline economy, $\{L_t, \mu_{t-1}, \pi_t, X_t\}_{t=0}^{\infty}$, and a counterfactual convergent sequence of changes in fundamentals, $\{\hat{\Theta}_t\}_{t=1}^{\infty}$.
- To solve for the counterfactual equilibrium, proceed as follows:

Solution algorithm

Part II: Solving for counterfactuals

1. Initiate the algorithm at $t = 0$ with a guess for the path of $\left\{ \hat{u}^{n\,(0)}_{t+1} \right\}_{t=0}^{T}$, where the superscript (0) indicates it is a guess. The path should converge to $\hat{u}^{n\,(0)}_{T+1} = 1$ for a sufficiently large T. Take as given the initial conditions L^n_0, $\mu^{n,i}_{-1}$, $\pi^{n,i}_0$, $w^n_0 L^n_0$, $r^n_0 H^n_0$; the baseline economy, $\{\dot{L}_t, \dot{\mu}_{t-1}, \dot{\pi}_t, \dot{X}_t\}_{t=0}^{\infty}$; and the solution to the sequential competitive equilibrium of the baseline economy.

Solution algorithm

Part II: Solving for counterfactuals

2 For all $t \geq 0$, use $\left\{\hat{u}_{t+1}^{n\,(0)}\right\}_{t=0}^{T}$ and $\{\dot{\mu}_{t-1}\}_{t=0}^{\infty}$ to solve for the path of $\{\mu_t^{\prime n}\}_{t=0}^{T}$ using the following equations:
For $t = 0$,

$$\hat{u}_0^{n\,(0)} = 1,$$

$$\mu_0^{\prime n,i} = \mu_0^{n,i}$$

$$L_1^{\prime n} = L_1^n = \sum_{i=1}^{N} \mu_0^{i,n} L_0^i.$$

For period $t = 1$,

$$\mu_1^{\prime n,i} = \frac{\vartheta_0^{n,i} \left(\hat{u}_2^i\right)^{\beta/\nu}}{\sum_{m=1}^{N} \vartheta_0^{n,m} \left(\hat{u}_2^m\right)^{\beta/\nu}},$$

where

$$\vartheta_0^{n,i\,(0)} = \mu_1^{n,i} \left(\hat{u}_1^{i\,(0)}\right)^{\beta/\nu}.$$

Solution algorithm

For period $t \geq 1$,

$$\mu_t^{\prime n,i} = \frac{\mu_{t-1}^{\prime n,i} \dot{\mu}_t^{n,i} \left(\hat{u}_{t+1}^i\right)^{\beta/\nu}}{\sum_{m=1}^N \mu_{t-1}^{\prime n,m} \dot{\mu}_t^{n,m} \left(\hat{u}_{t+1}^m\right)^{\beta/\nu}}.$$

3 Use the path for $\left\{\mu_t^{\prime n,i}\right\}_{t=0}^T$ and $L_0^{\prime n}$ to get the path for $\left\{L_{t+1}^{\prime n}\right\}_{t=0}^T$ using equation (**??**) in the paper. That is,

$$L_{t+1}^{\prime n} = \sum_{i=1}^N \mu_t^{\prime n,i} L_t^{\prime i}.$$

Solution algorithm

Part II: Solving for counterfactuals

4 Solve for the temporary equilibrium as follows:

a For each $t \geq 0$, given $\hat{L}^n_{t+1}$, guess a value for $\left\{ \hat{w}^n_{t+1} \right\}_{n=1}^{N}$.

b Obtain $\hat{P}^n_{t+1}$, and $\hat{\pi}^{n,i}_{t+1}$ using

$$\hat{P}^n_{t+1} = \left(\sum_{i=1}^{N} \pi'^{n,i}_t \dot{\pi}^{n,i}_{t+1} (\hat{w}^i_{t+1} \hat{\kappa}^{n,i}_{t+1})^{-\theta} (\hat{A}^i_{t+1})^{\theta} \right)^{-1/\theta},$$

and

$$\pi'^{n,i}_{t+1} = \pi'^{n,i}_t \pi^{n,i}_{t+1} \left(\hat{w}^i_{t+1} \hat{\kappa}^{n,i}_{t+1} / \hat{P}^n_{t+1} \right)^{-\theta} (\hat{A}^i_{t+1})^{\theta}.$$

c Solve $X'^n_{t+1} = \hat{w}^n_{t+1} \hat{L}^n_{t+1} w'^n_t L'^n_t \dot{w}^n_{t+1} \dot{L}^n_{t+1}$

d Check if the labor market is in equilibrium using

$$\hat{w}^n_{t+1} \hat{L}^n_{t+1} = \frac{1}{w'^n_t L'^n_t \dot{w}^n_{t+1} \dot{L}^n_{t+1}} \sum_{i=1}^{N} \pi'^{i,n}_{t+1} X'^i_{t+1}.$$

If it is not go back to step (a) and adjust $\left\{ \hat{w}^n_{t+1} \right\}_{n=1}^{N}$ until labor markets clear.

e Repeat steps (a) though (d) for each period t and obtain paths for $\left\{ \hat{w}^n_{t+1}, \hat{P}^n_{t+1} \right\}_{n=1,t=0}^{N,T}$.

Solution algorithm

Part II: Solving for counterfactuals

5 For each t, use $\mu_t'^{n,i}$, $\hat{w}_{t+1}^n$, $\hat{P}_{t+1}^n$, and $\hat{u}_{t+2}^{n\ (0)}$ to solve for backwards $\hat{u}_{t+1}^{n\ (1)}$ using the following equations:
For periods t where $t \geq 2$

$$\hat{u}_t^{n(1)} = \left(\frac{\hat{w}_t^n}{\hat{P}_t^n}\right)\left(\sum_{i=1}^N \mu_{t-1}'^{n,i}\dot{\mu}_t^{n,i}\left(\hat{u}_{t+1}^{i(0)}\right)^{\beta/\nu}\right)^\nu .$$

For period 1:

$$\hat{u}_1^{n\ (1)} = \left(\frac{\hat{w}_1^n}{\hat{P}_1^n}\right)\left(\sum_{i=1}^N \vartheta_0^{n,i\ (0)}\left(\hat{u}_2^i\right)^{\beta/\nu}\right)^\nu .$$

This delivers a new path for $\left\{\hat{u}_{t+1}^{n\ (1)}\right\}$, where the superscript 1 indicates an updated value for $\hat{u}$.

6 Take the path for $\left\{\hat{u}_{t+1}^{n\ (1)}\right\}$ as the new set of initial conditions.

7 Check if $\left\{\hat{u}_{t+1}^{n\ (1)}\right\} \simeq \left\{\hat{u}_{t+1}^{n\ (0)}\right\}$. If it is not, go back to step 1 and update the initial guess.

Adding local labor markets and geography: Full model

Households' problem

- N locations (index n and i) and each has J sectors (index j and k)
- The value of a household in market nj at time t given by

$$\mathrm{v}_t^{nj} = u(c_t^{nj}) + \max_{\{i,k\}_{i=1,k=0}^{N,J}} \left\{ \beta E\left[\mathrm{v}_{t+1}^{ik}\right] - \tau^{nj,ik} + \nu\,\epsilon_t^{ik} \right\},$$

$$s.t.\ u(c_t^{nj}) \equiv \begin{cases} \log(b^n) & if\ \ j=0, \\ \log(w_t^{nj}/P_t^n) & \text{otherwise}, \end{cases}$$

  - $\beta \in (0,1)$ discount factor
  - $\tau^{nj,ik}$ additive, *time invariant* migration costs to *ik* from *nj*
  - ϵ_t^{ik} are stochastic *i.i.d idiosyncratic* shocks
    - $\epsilon \sim$ Type-I Extreme Value distribution with zero mean
    - $\nu > 0$ is the dispersion of shocks

- *Non-employed* HH obtain home production b^n
- *Employed* households supply a unit of labor inelastically
  - Receive the competitive market wage w_t^{nj}
  - Consume $c_t^{nj} = \prod_{k=1}^{J} (c_t^{nj,k})^{\alpha^k}$, where P_t^n is the local price index

Households' problem - Dynamic discrete choice

- Using properties of Type-I Extreme Value distributions one obtains:
- The expected (expectation over ϵ) lifetime utility of a worker at nj

$$V_t^{nj} = u(c_t^{nj}) + \nu \log \left[\sum_{i=1}^N \sum_{k=0}^J \exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu} \right]$$

- Fraction of workers that reallocate from market nj to ik

$$\mu_t^{nj,ik} = \frac{\exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}}{\sum_{m=1}^N \sum_{h=0}^J \exp\left(\beta V_{t+1}^{mh} - \tau^{nj,mh}\right)^{1/\nu}}.$$

- Evolution of the distribution of labor across markets

$$L_{t+1}^{nj} = \sum_{i=1}^N \sum_{k=0}^J \mu_t^{ik,nj} L_t^{ik}$$

Production - Static sub-problem

- Notice that at each t, labor supply across markets is fully determined
  - We can then solve for wages such that labor markets clear, using a very rich static spatial structure (Caliendo et al. 2015 - CPRHS)
- In each nj there is a continuum of intermediate good producers
  - Perfect competition, CRS technology, *idiosyncratic* productivity z^{nj} $\sim$Fréchet$(1, \theta^j)$, deterministic sectoral regional TFP A^{nj}

$$q_t^{nj}(z^{nj}) = z^{nj}\left[A^{nj}\,[l_t^{nj}]^{\xi^n}[h_t^{nj}]^{1-\xi^n}\right]^{\gamma^{nj}}\prod_{k=1}^{J}[M_t^{nj,nk}]^{\gamma^{nj,nk}}$$

- Each n, j produces a final good (for final consumption and materials)
  - CES (elasticity η) aggregator of sector j goods from the lowest cost supplier in the world subject to $\kappa^{nj,ij} \geq 1$ "iceberg" bilateral trade cost

Intermediate goods Final goods

Production - Static sub-problem - Equilibrium conditions

- Sectoral price index,

$$P_t^{nj}(\mathbf{w}_t) = \Gamma^{nj}\left[\sum_{i=1}^{N} A^{ij}\left[x_t^{ij}(\mathbf{w}_t)\kappa^{nj,ij}\right]^{-\theta^j}\right]^{-1/\theta^j}$$

- Let $X_t^{ij}(\mathbf{w}_t)$ be total expenditure. Expenditure shares given by

$$\pi_t^{nj,ij}(\mathbf{w}_t) = \frac{\left[x_t^{ij}(\mathbf{w}_t)\kappa^{nj,ij}\right]^{-\theta^j} A^{ij}}{\sum_{m=1}^{N}\left[x_t^{mj}(\mathbf{w}_t)\kappa^{nj,mj}\right]^{-\theta^j} A^{mj}},$$

where $x_t^{ij}(\mathbf{w}_t)$ is the unit cost of an input bundle

- Labor Market clearing

$$L_t^{nj} = \frac{\gamma^{nj}\left(1-\xi^n\right)}{w_t^{nj}}\sum_{i=1}^{N}\pi_t^{ij,nj}(\mathbf{w}_t)X_t^{ij}(\mathbf{w}_t),$$

where $\gamma^{nj}(1-\xi^n)$ labor share

Input bundle

Sequential and temporary equilibrium

- State of the economy = distribution of labor $L_t = \{L_t^{nj}\}_{n=1,j=0}^{N,J}$
  - Let $\Theta \equiv \left(\{A^{nj}\}, \{\kappa^{nj,ij}\}, \{\tau^{nj,ik}\}, \{H^{nj}\}, \{b^n\}\right)_{n=1,j=0,i=1,k=0}^{N,J,J,N}$

Definition

Given (L_t, Θ), a **temporary equilibrium** is a vector of $w(L_t, \Theta)$ that satisfies the equilibrium conditions of the static sub-problem

Definition

Given (L_0, Θ), a **sequential competitive equilibrium** of the model is a sequence of $\{L_t,\ \mu_t,\ V_t,\ w(L_t, \Theta)\}_{t=0}^{\infty}$ that solves HH dynamic problem and the temporary equilibrium at each t

- With $\mu_t = \{\mu_t^{nj,ik}\}_{n=1,j=0,i=1,k=0}^{N,J,J,N}$, and $V_t = \{V_t^{nj}\}_{n=1,j=0}^{N,J}$

Application: The Rise of China

The rise of China

- U.S. imports from China almost doubled from 2000 to 2007
  - At the same time, manufacturing employment fell while employment in other sectors, such as construction and services, grew
- Several studies document that an important part of the employment loss in manufactures was a consequence of China's trade expansion
  - e.g., Pierce and Schott (2012); Autor, Dorn, and Hanson (2013), Acemoglu, Autor, Dorn, and Hanson (2014)
- We use our model, and apply our method, to quantify and understand the effects of the rise of China's trade expansion, "China shock"
  - Initial period is the year 2000
  - We calculate the sectoral, regional, and aggregate employment and welfare effects of the China shock

Identifying the China "shock"

- Follow Autor, Dorn, and Hanson (2013)
  - We estimate
    $$\Delta M_{USA,j} = a_1 + a_2 \Delta M_{other,j} + u_j,$$
    where j is a NAICS sector, $\Delta M_{USA,j}$ and $\Delta M_{other,j}$ are changes in U.S. and other adv. countries, imports from China from 2000 to 2007
  - Find $a_2 = 1.27$
- Obtain predicted changes in U.S. imports with this specification
- Use the model to solve for the change in China's 12 manufacturing industries TFP $\left\{\hat{A}^{China,j}\right\}_{j=1}^{12}$ such that model's imports match predicted imports from China from 2000 to 2007
  - With model's generated data obtain $a_2 = 1.52$
  - We feed in to our model $\left\{\hat{A}^{China,j}\right\}_{j=1}^{12}$ by quarter from 2000 to 2007 to study the effects of the shock

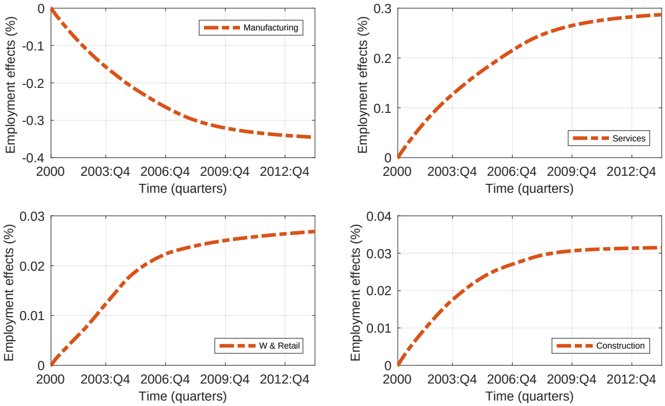
▸ Figure: shock and predicted imports

Taking the model to the data (quarterly)

- Model with 50 U.S. states, 22 sectors + non-empl. and 38 countries
  - More than 1000 labor markets
- Need data for $\left(L_0, \mu_{-1}, \pi_0, VA_0, GO_0\right)$
  - L_0 : PUMS of the U.S. Census for the year 2000
    - Exclude empl. in farming, mining, utilities, and public sect.
  - μ_{-1} : Use CPS to compute intersectoral mobility and ACS to compute interstate mobility
  - π_0 : CFS and WIOD year 2000
  - VA_0 and GO_0 : BEA VA shares and U.S. IO, WIOD for other countries
- Need values for parameters (ν, θ, β)
  - θ : We use Caliendo and Parro (2015)
  - $\beta = 0.99$ Implies approximately a 4% annual interest rate
  - $\upsilon = 5.34$ (implied elasticity of 0.2) Using ACM's data and specification, adapted to our model Estimation
- Need to deal with trade deficits. Do so similar to CPRHS Imbalances

Employment effects

Figure: The effect of the China shock on employment shares



- Chinese competition reduced the share of manufacturing employment by 0.36% in the long run, ∼0.55 million employment loss
  - About 36% of the change not explained by a secular trend

Nonemployment

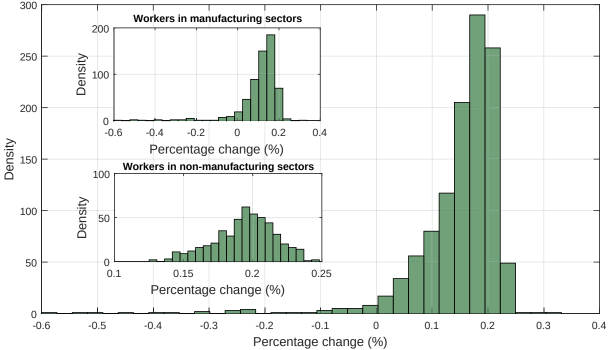
Employment effects: Manufacturing

- Sectors most exposed to Chinese import competition contribute more
  - 1/2 of the decline in manuf. employment originated in the Computer & Electronics and Furniture sectors ▸ Sectoral contributions
    - 1/4 of the total decline comes from the Metal and Textiles sectors
- Unequal regional effects
  - Regions with a larger concentration of sectors that are more exposed to China lose more jobs ▸ Regional contributions
    - California, the region with largest share of employment in Computer & Electronics, contributed to about 9% of the decline

▸ Non-manufacturing

Welfare effects across labor markets

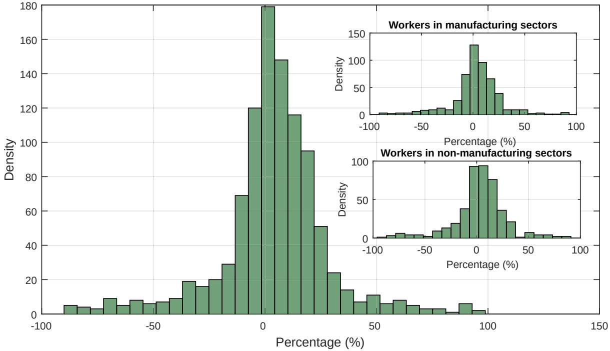
Figure: Welfare effects of the China shock across U.S. labor markets



- Very heterogeneous response to the same aggregate shock welfare
  - Most labor markets gain as a consequence of cheaper imports
  - Unequal regional effects welfare reg real wages reg

Transition cost to the steady state

Figure: Adjustment costs

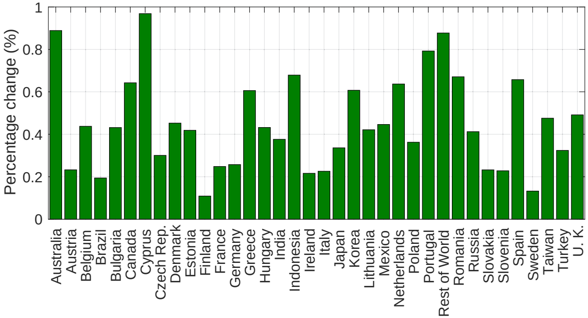


- Adjustment costs reflect the importance of labor market dynamics
  - With free labor mobility AC=0
  - Heterogeneity due to trade/migration frictions and geographic factors

AC

Welfare effects across countries

Figure: Welfare effects across countries



Additional Results - SSDI

- What would have happened if during the China shock, agents expected SSDI benefits and they were not granted?
  - SSDI program amplified the decline in manuf. empl. by about 0.03 percentage points (aprox 50K additional jobs losts due to the program)
    - Also find an increase in the non-employment rate in the long run
- The regions that contribute more to the decline in manuf. empl. due to the SSDI are Mississippi, South Carolina, and Tennessee, each accounting for about 4-5 percent of the total decline

Model with SSDI

Conclusion

- Develop a dynamic and spatial model to quantify the disaggregate effects of aggregate shocks
  - Show how to perform counterfactual analysis in a very rich spatial model without having to estimate a large set of unobservables
- Dynamics and realistic structure matters for capturing very heterogenous effects at the disaggregate level
- Our model can be applied to answer a broader set of questions: changes in productivity or trade costs in any location in the world, commercial policies, and more...

- Where we go from here:
  1- Migration and trade policy in Europe
  2- The Impact of Trade Policy on Firm Location Choice

SSDI

- Federal government finances SSDI by levying taxes τ on labor income of workers across US, N^{US}
- Let L_t^{nD} be the mass of workers in disability in region n at time t and by b^{DI} the SSDI benefit that a worker obtains
- The per-period government budget constraint is given by

$$\sum_{n=1}^{N^{US}} \sum_{k=1}^{J} \tau w_t^{nk} L_t^{nk} + G_t = \sum_{n=1}^{N^{US}} b^{DI} L_t^{nD},$$

  where $\sum_{n=1}^{N^{US}} \sum_{k=1}^{J} \tau \, w_t^{nk} L_t^{nk}$ is aggregate tax revenue from labor income and $\sum_{n=1}^{N^{US}} b^{DI} \, L_t^{nD}$ is the aggregate government expenditure on the SSDI program
- To be eligible for SSDI, households cannot be engaged in a "Substantial Gainful Activity"
  - Non-employed households enter SSDI with probability δ
  - Transition out of SSDI with constant hazard rate $1 - \rho^{DI}$

SSDI

- Hence,

$$V_t^{n0} = \log b^n + \nu(1-\delta)\log\left[\sum_{i=1}^N \sum_{k=0}^J \exp\left(\beta V_{t+1}^{ik} - \tau^{nj,ik}\right)^{1/\nu}\right] + \delta\beta V_{t+1}^{nD},$$

$$V_t^{nD} = \log(b^{DI}/P_t^n) + (1-\rho^{DI})\beta V_{t+1}^{n0} + \rho^{DI}\beta V_{t+1}^{nD}$$

and

$$\begin{aligned}
L_{t+1}^{nD} &= \rho^{DI} L_t^{nD} + \delta L_t^{n0}, \\
L_{t+1}^{n0} &= (1-\rho^{DI})L_t^{nD} + \sum_{i=1}^N \sum_{k\neq 0}^J \mu_t^{ik,n0} L_t^{ik} + \sum_{i=1}^N \mu_t^{i0,n0}(1-\delta)\, L_t^{i0}, \\
L_{t+1}^{nj} &= \sum_{i=1}^N \sum_{k\neq 0}^J \mu_t^{ik,nj} L_t^{ik} + \sum_{i=1}^N \mu_t^{i0,nj}(1-\delta)\, L_t^{i0} \qquad \text{for } j \geq 1
\end{aligned}$$

goods market clearing condition

$$\begin{aligned}
X_t^{jn} &= \sum_{k=1}^J \gamma^{nk,nj} \sum_{i=1}^N \pi_t^{ik,nk} X_t^{ik} \\
&\quad + \alpha^j\left((1-\tau)\sum_{k=1}^J w_t^{nk} L_t^{nk} + b^{DI} L_t^{nD} + \iota^n \chi_t - G_t/N^{US}\right)
\end{aligned}$$

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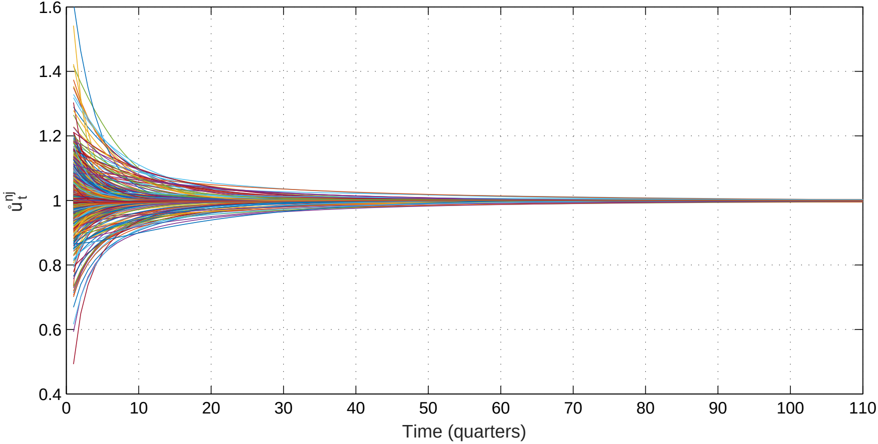
Solving the model (example)

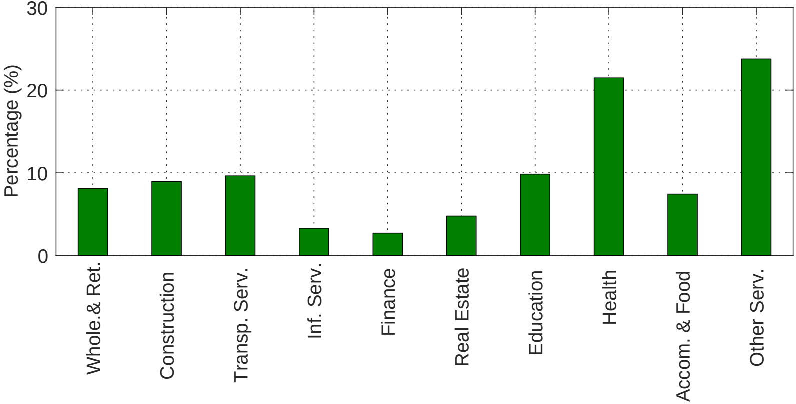
Figure: Equilibrium Value Functions in Time Differences



▸ Proposition

Employment Effects: Non-manufacturing

Figure: Non-manufacturing employment increases (% of total) due to the China trade shock



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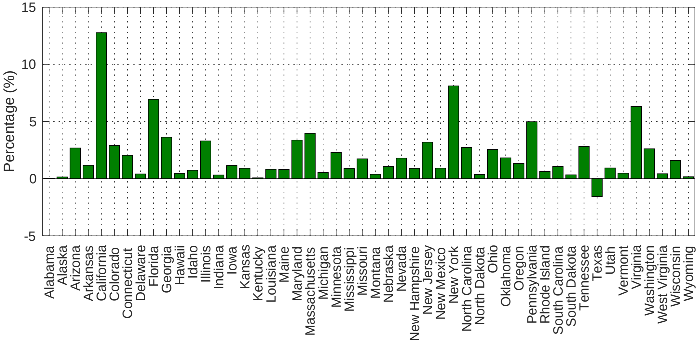
Employment Effects: Non-manufacturing

Figure: Regional contribution to U.S. aggregate non-manufacturing employment increase (%)



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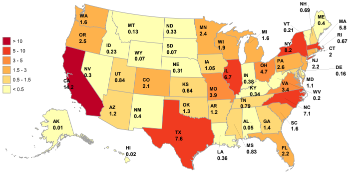
Employment effects: Non-manufacturing

- Non-manufacturing sectors gain employment share
  - More than 50K jobs created in Construction ▸ Sectoral contributions
    - Non-manufacturing sectors benefited from cheaper intermediate goods and influx of workers from the manufacturing sectors
- Unequal regional effects
  - Regional effects are less related to Chinese exposure ▸ Regional contributions

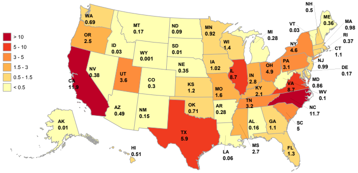
▸ Back ▸ Maps

Regional employment effects

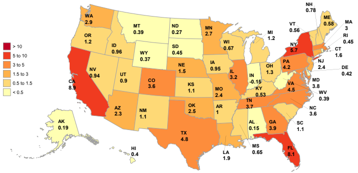
a: Computer and electronics



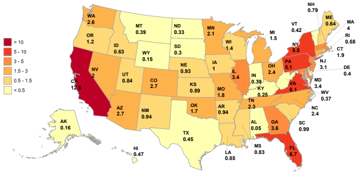
b: Textiles



c: Construction



d: Services



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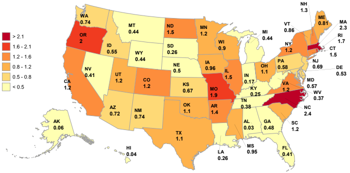
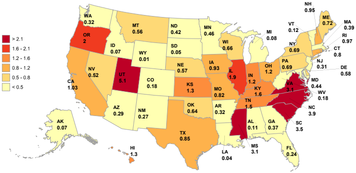
Regional employment effects

Normalized by regional employment share

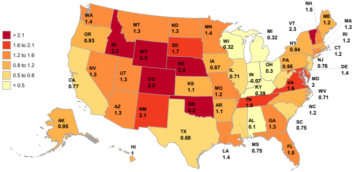
a: Computer and electronics



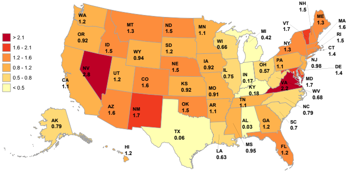
b: Textiles



c: Construction



d: Services



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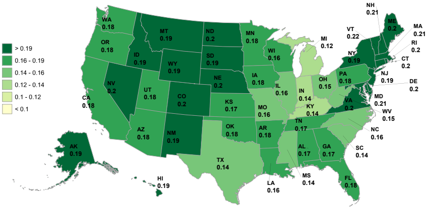
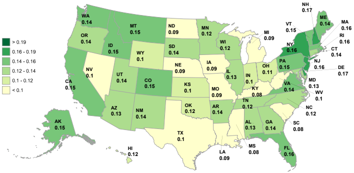
Regional welfare effects

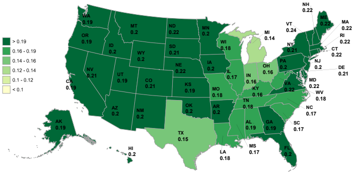
a.1: Regional



a.2: Manufacturing



a.3: Non-manufacturing



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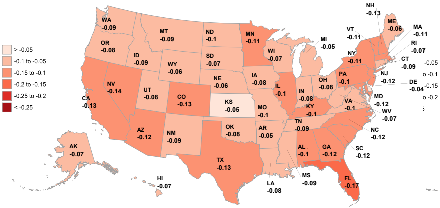
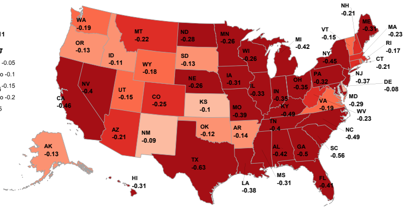
Regional real wage changes in the manuf. sector

a.1: One quarter after 2000



a.2: From 2000 to 2007



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Model - Intermediate goods

- Representative firms in each region n and sector j produce a continuum of intermediate goods with *idiosyncratic* productivities z^{nj}
  - Drawn independently across goods, sectors, and regions from a Fréchet distribution with shape parameter θ^j
  - Productivity of all firms is also determined by a deterministic productivity level A^{nj}
- The production function of a variety with z^{nj} and A^{nj} is given by

$$q_t^{nj}(z^{nj}) = z^{nj}\left[A^{nj}\,[l_t^{nj}]^{\xi^n}[h_t^{nj}]^{1-\xi^n}\right]^{\gamma^{nj}}\prod_{k=1}^{J}[M_t^{nj,nk}]^{\gamma^{nj,nk}},$$

with $\sum_{k=1}^{J}\gamma^{nj,nk} = 1-\gamma^{nj}$

Model - Intermediate good prices

- The cost of the input bundle needed to produce varieties in (nj) is

$$x_t^{nj} = B^{nj} \left[\left(r_t^{nj} \right)^{\xi^n} \left(w_t^{nj} \right)^{1-\xi^n} \right]^{\gamma^{nj}} \prod_{k=1}^{J} [P_t^{nk}]^{\gamma^{nj,nk}}$$

- The unit cost of a good of a variety with draw z^{nj} in (nj) is

$$\frac{x_t^{nj}}{z^{nj}} [A^{nj}]^{-\gamma^{nj}}$$

and so its price under competition is given by

$$p_t^{nj}(z^j) = \min_i \left\{ \frac{\kappa^{nj,ij} \, x_t^{ij}}{z^{ij} [A^{ij}]^{\gamma^{ij}}} \right\},$$

with $\kappa^{nj,ij} \geq 1$ are "iceberg" bilateral trade cost

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Model - Final goods

- The production of final goods is given by

$$Q_t^{nj} = \left[\int_{\mathrm{R}_{++}^N} [\tilde{q}_t^{nj}(z^j)]^{1-1/\eta^{nj}} \phi^j(z^j) dz^j \right]^{\eta^{nj}/(\eta^{nj}-1)},$$

where $z^j = (z^{1j}, z^{2j}, ...z^{Nj})$ denotes the vector of productivity draws for a given variety received by the different n

- The resulting price index in sector j and region n, given our distributional assumptions, is given by

$$P_t^{nj} = \varrho \left[\sum_{i=1}^N [x_t^{ij} \kappa^{nj,ij}]^{-\theta^j} [A^{ij}]^{\theta^j \gamma^{ij}} \right]^{-1/\theta^j},$$

where ϱ is a constant

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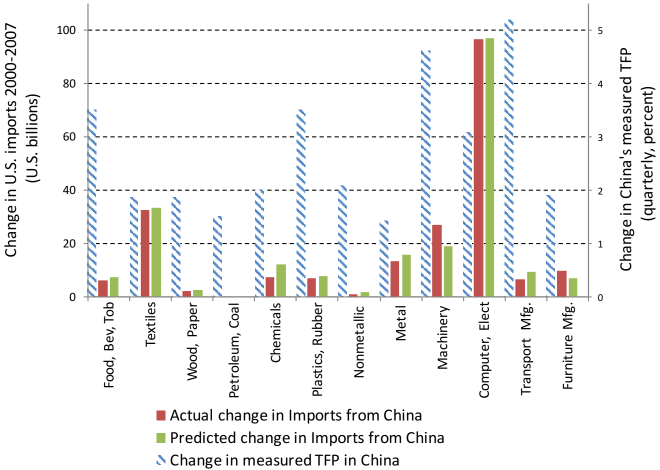
Identifying the China shock

Figure: Predicted change in imports vs. model-based Chinese TFP change



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Taking the model to the data (quarterly)

- $\upsilon = 5.34$ (implied elasticity of 0.2) Using ACM's data and specification, adapted to our model
  - Data: migration flows and real wages for 26 years between 1975-2000, using March CPS
  - We deal with two issues: functional forms, and timing
- Estimating equation

$$\log\left(\frac{\mu_t^{nj,ik}}{\mu_t^{nj,nj}}\right) = \tilde{C} + \frac{\beta}{\nu}\log\left(\frac{w_{t+1}^{ik}}{w_{t+1}^{nj}}\right) + \beta\log\left(\frac{\mu_{t+1}^{nj,ik}}{\mu_{t+1}^{nj,nj}}\right) + \varpi_{t+1},,$$

  - We transform migration flows from five-month to quarterly frequency
  - GMM estimation, past flows and wages used as instruments
  - ACM estimate $\upsilon = 1.88$ (annual), $\upsilon = 2.89$ (five-month frequency)

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Imbalances

- Assume that in each region there is a mass of one of Rentiers
  - Owners of local structures, obtain rents $\sum_{k=1}^{J} r_t^{ik} H^{ik}$
  - Send all their local rents to a global portfolio
  - Receive a constant share ι^i from the global portfolio, with $\sum_{n=1}^{N} \iota^n = 1$
- Imbalances in region i given by

$$\sum_{k=1}^{J} r_t^{ik} H^{ik} - \iota^i \chi_t,$$

where $\chi_t = \sum_{i=1}^{N} \sum_{k=1}^{J} r_t^{ik} H^{ik}$ are the total revenues in the global portfolio

- Rentier uses her income to purchase local goods
  - Same preferences as workers

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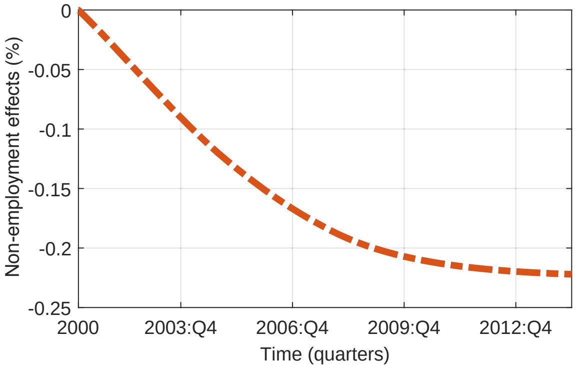
Non-employment shares

Figure: The effect of the China shock on non-employment shares



- Fall mainly due to a decline in flows from non-manuf. to non-empl.
- Flow from manuf. to non-empl. increased in states that are concentrated in the manuf. industries
  - Alabama, Arkansas, Mississippi, Michigan, and Ohio, among others

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Welfare effects from changes in fundamentals

- Let $W_t^{nj}(\hat{\Theta})$ be the welfare effect of change in $\hat{\Theta} = \Theta'/\Theta$

$$W_t^{nj}(\hat{\Theta}) = \sum_{s=t}^{\infty} \beta^s \log \frac{\hat{c}_s^{nj}}{(\hat{\mu}_s^{nj,nj})^{\nu}},$$

  - Note that this is a consumption equivalent measure of welfare
  - $(\hat{\mu}_s^{nj,nj})^{\nu}$ is the change in the option value of migration

- In our model, $\hat{c}_t^{nj} = \hat{w}_t^{nj}/\hat{P}_t^n$ is shaped by several mechanisms,

$$\hat{c}_t^{nj} = \frac{\hat{w}_t^{nj}}{\prod_{k=1}^{J} (\hat{w}_t^{nk})^{\alpha^k}} \prod_{k=1}^{J} \left(\frac{\hat{w}_t^{nk}}{\hat{P}_t^{nk}} \right)^{\alpha^k},$$

  - First component reflects the unequal effects within a region
  - Second component is common to all HH residing in region n, given by

$$\sum_{k=1}^{J} \alpha^k \left(\log(\hat{\pi}_t^{nk,nk})^{-\gamma^{nk}/\theta^k} - \xi^n \log \frac{\hat{L}_t^{nk}}{\hat{H}^{nk}} \right).$$

Welfare effects from changes in fundamentals

- Let $W_t^{nj}(\hat{\Theta})$ be the welfare effect of change in $\hat{\Theta} = \Theta'/\Theta$

$$W_t^{nj}(\hat{\Theta}) = \sum_{s=t}^{\infty} \beta^s \log \frac{\hat{c}_s^{nj}}{(\hat{\mu}_s^{nj,nj})^{\nu}},$$

  - Note that this is a consumption equivalent measure of welfare
  - $(\hat{\mu}_s^{nj,nj})^{\nu}$ is the change in the option value of migration

- In a one sector model with no materials and structures, $\hat{c}_t^n = \hat{w}_t^n / \hat{P}_t^n$

$$W_t^n(\hat{\Theta}) = \sum_{s=t}^{\infty} \beta^s \log \frac{\left(\hat{\pi}_s^{n,n}\right)^{-1/\theta}}{\left(\hat{\mu}_s^{n,n}\right)^{\nu}},$$

- Similar to a ACM (2010) + ACR (2012)

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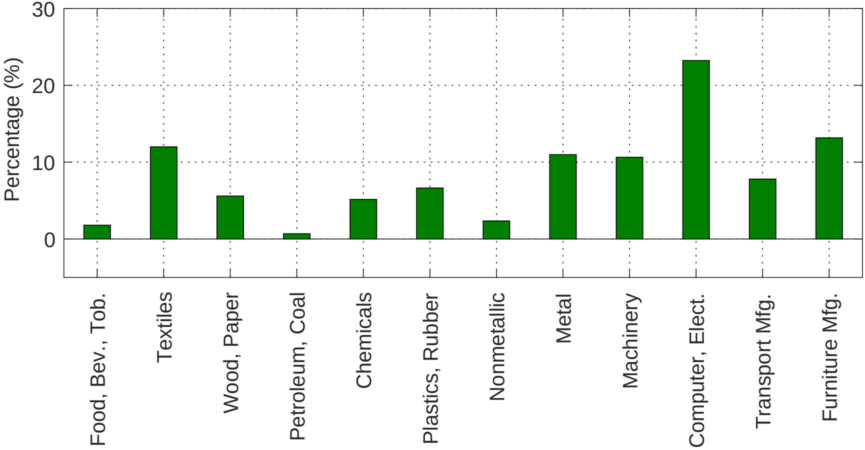
Manufacturing Employment Effects

Figure: Manufacturing employment declines (% of total) due to the China trade shock



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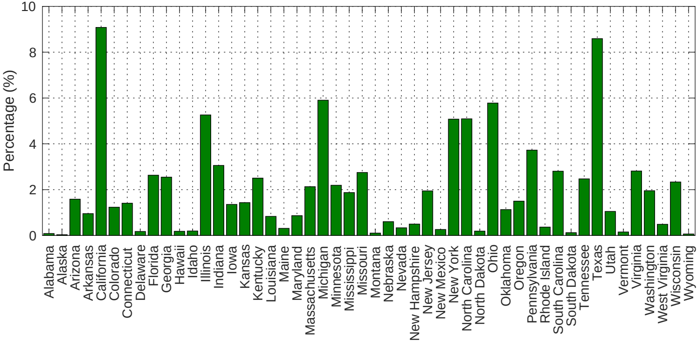
Manufacturing employment effects

Figure: Regional contribution to U.S. aggregate manufacturing employment decline (%)



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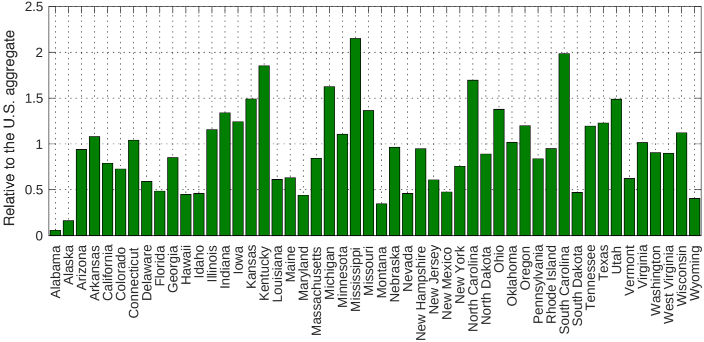
Manufacturing employment effects

Figure: Regional contribution to U.S. agg. mfg. emp. decline normalized by regional emp. share



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Adjustment costs

- We follow Dix-Carneiro (2014)'s measure of adjustment cost
- The steady-state change in the value function due changes in fundamentals is given by $V_{SS}^{nj}(\hat{\Theta}) - V_{SS}^{nj}$
- Therefore, the transition cost for market nj to the new long-run equilibrium, $AC^{nj}(\hat{\Theta})$, is given by

$$AC^{nj}(\hat{\Theta}) = \log\left(\frac{\frac{1}{1-\beta}\left(V_{SS}^{nj}(\hat{\Theta}) - V_{SS}^{nj}\right)}{\sum_{t=0}^{\infty}\beta^{t}\left(V_{t+1}^{nj}(\hat{\Theta}) - V_{t+1}^{nj}\right)}\right),$$

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