

Let $W_T = h_T^{\frac{1}{1-\eta}}$

$\frac{\partial W_T}{\partial h_T} = \frac{g W_T}{h_T} = g h_T^{\frac{1}{1-\eta}} \cdot \frac{\partial h_T}{\partial h_T} \cdot h_T = W_T \cdot g$



$\frac{\partial W_T}{\partial \tau} = \frac{\partial h_T}{\partial \tau} \cdot \frac{1}{h_T} = \frac{1}{\eta g}$

$S_T = \frac{\mu \phi}{\mu \phi + \frac{1}{g} - \eta}$

$e_T = \eta \tau \cdot g W_T = \eta g \cdot \tau \cdot h_T^{\frac{1}{1-\eta}}$

$h_T = \eta^{\frac{1}{1-\eta}} \cdot \tau^{\frac{\eta}{1-\eta}} \cdot (g h_T^{\frac{1}{1-\eta}})^{\frac{1}{1-\eta}} \cdot a_T^{\frac{1}{1-\eta}} \cdot S_T^{\frac{\phi}{1-\eta}} \cdot \left(\frac{2\tilde{u}_T}{M}\right)^{\frac{\sigma}{1-\eta}}$

$1 - \frac{\eta}{1-\eta}(g-1) = \frac{1-\eta-\eta g+\eta}{1-\eta} = \frac{1-\eta g}{1-\eta}$

$h_T^{\frac{1-\eta g}{1-\eta}} = \eta^{\frac{1}{1-\eta}} \cdot \tau^{\frac{\eta}{1-\eta}} \cdot a_T^{\frac{1}{1-\eta}} \cdot S_T^{\frac{\phi}{1-\eta}} \cdot \left(\frac{2\tilde{u}_T}{M}\right)^{\frac{\sigma}{1-\eta}}$

$h_T = \left(\eta^{\frac{1}{1-\eta}} \cdot \tau^{\frac{\eta}{1-\eta}} \cdot a_T^{\frac{1}{1-\eta}} \cdot S_T^{\frac{\phi}{1-\eta}} \cdot \left(\frac{2\tilde{u}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \right)^{\frac{1}{1-\eta g}}$

$e_T = g \left(\eta^{\frac{1}{1-\eta}} \cdot \tau^{\frac{\eta}{1-\eta}} \cdot a_T^{\frac{1}{1-\eta}} \cdot S_T^{\frac{\phi}{1-\eta}} \cdot \left(\frac{2\tilde{u}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \right)^{\frac{1}{1-\eta g}}$

$1 + \frac{\eta g}{1-\eta g} = \frac{1}{1-\eta g}$

$\left(\frac{1-S_T}{1-S_0}\right)^{\frac{1-\eta}{M}} \cdot \frac{\tau_T}{\tau_0} \cdot \left(\frac{a_T}{a_0}\right)^{\alpha} \cdot \left(\frac{S_T}{S_0}\right)^{\phi} \cdot \left(\frac{1+e_T}{1+e_0}\right)^{1-\eta} \cdot \frac{g h_T^{\frac{1}{1-\eta}}}{A_0} \cdot \left(\frac{1-\eta}{g-1}\right)^{1-\eta} = 1$

$\left(\frac{1-S_T}{1-S_0}\right)^{\frac{1-\eta}{M}} \cdot \frac{\tau_T}{\tau_0} \cdot \left(\frac{a_T}{a_0}\right)^{\alpha} \cdot \left(\frac{S_T}{S_0}\right)^{\phi} \cdot \left(\frac{1+e_T}{1+e_0}\right)^{1-\eta} \cdot \left(\frac{1-\eta}{g-1}\right)^{1-\eta} \cdot \frac{g}{A_0} \cdot \left(\eta^{\frac{1}{1-\eta}} \cdot \tau^{\frac{\eta}{1-\eta}} \cdot a_T^{\frac{1}{1-\eta}} \cdot S_T^{\frac{\phi}{1-\eta}} \cdot \left(\frac{2\tilde{u}_T}{M}\right)^{\frac{\sigma}{1-\eta}}\right)^{\frac{1}{1-\eta g}} = 1$

$\alpha + \alpha \frac{g-1}{1-\eta g} = \alpha \left(\frac{1-\eta g + g-1}{1-\eta g} \right) = \alpha g \frac{1-\eta}{1-\eta g}$

$1 + \frac{\eta(g-1)}{1-\eta g} = \frac{1-\eta}{1-\eta g}$

$\left(\frac{1-S_T}{1-S_0}\right)^{\frac{1-\eta}{M}} \cdot \frac{\tau_T}{\tau_0} \cdot \left(\frac{a_T}{a_0}\right)^{\alpha g \frac{1-\eta}{1-\eta g}} \cdot \left(\frac{S_T}{S_0}\right)^{\phi} \cdot \left(\frac{1+e_T}{1+e_0}\right)^{1-\eta} \cdot \left(\frac{1-\eta}{g-1}\right)^{1-\eta} \cdot \frac{g}{A_0} \cdot \eta^{\frac{1}{1-\eta g}} \cdot \left(\frac{2\tilde{u}_T}{M}\right)^{\frac{\sigma(1-\eta)}{1-\eta g}} = 1$

$\tilde{H}_T^* = \eta^{\frac{\eta \beta}{\sigma(1-\eta g)}} \cdot \left(\frac{2\tilde{u}_T}{M}\right)^{\frac{\beta}{1-\eta g}} \sum_g \tau_{T,g} \int a_{T,g}^{\frac{\alpha \beta}{\sigma(1-\eta g)}} \cdot S_{T,g}^{\frac{\phi \beta}{\sigma(1-\eta g)}} f_{T,g}(a) da$

$\tilde{H}_T^* = \eta^{\frac{\eta \beta}{\sigma(1-\eta g-\beta)}} \cdot \left(\frac{2}{M}\right)^{\frac{\beta}{1-\eta g-\beta}} \left(\sum_g \tau_{T,g} \int a_{T,g}^{\frac{\alpha \beta}{\sigma(1-\eta g)}} \cdot S_{T,g}^{\frac{\phi \beta}{\sigma(1-\eta g)}} f_{T,g}(a) da \right)^{\frac{1-\eta g}{1-\eta g-\beta}}$