The Allocation of Teaching Talent and Human Capital Accumulation

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Abstract

The educational landscape in the U.S. has gone through major changes since the end of World War II. Real expenditures per student have risen from approximately $2,100 to more than $10,000 by the turn of the century and more than $12,000 since the Great Recession. At the same time, the student-teacher ratio has fallen from a national average of almost 27 in 1955 to 16 by the 2010s. Yet despite the rise in expenditures and the reduction in class sizes, educational outcomes in the U.S. don't compare very favorably with countries at similar income levels. One aspect of U.S. education that has not garnered a lot of attention until fairly recently is occupational choice. We add an education sector to an otherwise standard Hsieh et al. (2019)-style model to explore the extent to which changes in career opportunities in other occupations affect the selection of workers into teaching careers. In our model, changes in the allocation of teaching talent have implications for the evolution of class size as well as quality of instruction and hence the accumulation of human capital during the workers' formative years. This gives rise to a trade-off between static and dynamic efficiency, which we quantify by way of a structural model. In order to discipline the parameterization of the model, we compare model-generated moments to their empirical counterparts in three longitudinal surveys: Project TALENT, the NLSY79, and NLSY97. Project TALENT surveys 377,000 U.S. high school students in 1960 and follows them for more than a decade. Together with the two NLSY's, it provides a detailed account of, among many others, the educational trajectories and career paths of young American adults over the course of more than four decades.

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1 Introduction

The educational landscape in the U.S. has gone through major changes since the end of World War II. Real expenditure per student has risen from approximately $2,100 to more than $10,000 by the turn of the century and more than $12,000 since the Great Recession. At the same time, the student-teacher ratio has fallen from a national average of almost 27 in 1955 to 16 by the 2010s. Yet despite the rise in expenditures and the reduction in class sizes, educational outcomes in the U.S. don't compare very favorably with countries at similar income levels.

In this paper, we develop a novel unified theory of occupational choice and human capital formation with the aim of reconciling several salient stylized facts. In addition to the change in the student-teacher ratios and per-student-expenditures mentioned above, they are: (1) the increase in the share of women in teaching jobs from 4.6 percent in 1970 to 6.7 percent in 2010; (2) the drop in the share of men who are teachers from 2.9 percent to 2.1 percent during that same time; (3) the sharp rise in the female labor force participation rate; (4) the slight decline in the male labor force participation rate; and (5) the evolution of the skill composition by gender and occupation between 1970 and 2010.[1]

In order to account for these stylized facts, we add an education sector to an otherwise standard Hsieh et al. (2019)-style model to explore the extent to which changes in career opportunities in other occupations – including home production – affect the selection of workers into teaching careers. In our model, teaching is distinct from other occupations in the economy. The teacher's human capital is one of several inputs into an education technology through which students of different abilities accumulate human capital. Human capital, in turn, is the key input for the production of final goods and services. In other words, workers in non-teaching occupations produces final output while teachers contribute to the production of human capital in today's students who, in turn, will either produce units of final output next period or train the next generation of students.

In contrast to a standard Roy (1951)-type model, we allow for the possibility that occupational choice is shaped by comparative as well as absolute advantage. The elasticity of income with respect to the worker's human capital can be different from unity and this implies that two workers with identical *comparative* advantages but different *absolute* advantage may select distinct occupations. This variation gives us additional flexility to account for the evolution of employment shares across occupations separately from changes in the distribution of skills or human capital across jobs.

[1]The secular rise in educational achievement as well and gender- and occupation-specific variations in this trend are additional salient aspects that merit further attention. As will become apparent in sections 2 and 3, the current version of our model is not particularly well-equipped to address this stylized fact. This is an area for future work.

The question of whether low or high-skill workers are more like to enter (or exit, for that matter) the teaching profession is critical for understanding whether static efficiency gains associated with the elimination of labor market discrimination and educational barriers in non-teaching occupations are muted or amplified by dynamic effects that operate through our model's human capital accumulation channel. The previous literature has explored the static gains and losses extensively in the context of linear models. The key contribution of our paper is *(a)* the introduction of non-linear wage functions that emphasize the role of absolute advantage in addition to the prior focus on comparative advantage and *(b)* the intergenerational dynamics of human capital accumulation that mainly operate through the distinct education sector in our model.

In addition to our theoretical contribution, our paper revives and older, but rarely used longitudinal dataset. *Project TALENT* surveyed a representative sample of approximately 377 thousand U.S. high school students in 1959. These same students were interviewed again one, five, and eleven year after they graduated from high school. This dataset was digitized somewhat recently and we use it in combination with the NLSY79 and NLSY97 for our quantitative work. Together, the three surveys paint a rich and detailed picture of American youth spanning more than four decades. Importantly, respondents in all three surveys take aptitude tests that allow us to construct ability profiles, which play a central role in our quantitative work.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 4 reviews the data and we calibrate the model to match salient empirical moments in section 5. Section 6 concludes.

2 Model

The model is populated by a constant measure M of workers who are employed in one of the I sectors of the economy. Each individual is born with an exogenous I-dimensional vector of skills $\vec{a}$, where each element characterizes her innate ability in any one of the $i \in \{1, \dots, I\}$ occupations in the economy, each corresponding to one of the sectors or industries. This vector of abilities is drawn from a known joint distribution $F(\vec{a})$. Moreover, the measure of workers is partitioned into G different groups, which are indexed by $g \in \{1, \dots, G\}$. Workers are born into one of these groups and membership is permanent. Moreover, we assume that the distribution of abilities is identical across groups, i.e., $F^g(\vec{a}) = F^{g'}(\vec{a}) = F(\vec{a})$.

Without loss of generality, occupation 1 is *teaching*. In this economy, teachers are distinct from other occupations since their technology produces human capital rather than the homogenous final good in the remaining $I - 1$ sectors. This kind of distinction is salient since we are assuming that the economy is populated by overlapping generations of workers who accumulate human capital when young, which then pays off when they are of working age (or "old").

2.1 Preferences and Technologies

As in Roy (1951), workers make occupational choices based on their full range of idiosyncratic abilities $\vec{a}$ and the corresponding "payoffs." To this basic mechanism we add Hsieh et al. (2019)-style forces that distort the allocation of skill across occupations and a human capital formation technology with time, good, and teacher inputs.

The timing convention is similar to Hsieh et al. (2019). In the first period, prospective workers – or students – make human capital investments. Since there is no uncertainty, students have perfect foresight and their investment decision will depend on their fully anticipated occupational choice in period 2. They retire at the end of their working period and are replaced by a new cohort of students with measure $\frac{M}{2}$. The environment is stationary in the sense that the members of each cohort draw their ability vectors from the same distribution F_a.

The worker's utility depends on her consumption and time devoted to human capital investment:

$$U = \max_{C'_g, \{s_{i,g}, e_{i,g}\}_{i=1}^{I}} \left\{ \mu \ln C'_g + \ln\left(1 - s_{i,g}\right) \right\}_{i=1}^{I} \tag{1}$$

where

$$C'_g = (1-t')(1-\tau^{\omega'}_{i,g})\omega'_{i,g}(h_{i,g}) - \sum_{i=1}^{I}(1+\tau^{e}_{i,g})e_{i,g} \tag{2}$$

$$h'_{i,g} = \kappa(h_{T,g}, a_i)(s_{i,g})^{\phi}(e_{i,g})^{\eta}\big(N(h_{T,g})\big)^{-\sigma} \tag{3}$$

$$\kappa(h_{T,g}, a_i) = \left(h_{T,g}\right)^{\beta}\left(a_i\right)^{\alpha}. \tag{4}$$

C'_g is consumption of the homogeneous final good when old, $s_{i,g}$ is the time allocated to occupation-specific human capital formation when young, and $e_{i,g}$ are units of the final good invested in occupation-specific human capital formation when workers are still students. These investments and the corresponding consumption in the following period are occupation-specific and indexed by the superscript i.[2] The periodic time endowment is set to unity. $1-s_{i,g}$ is leisure time when young, and labor is supplied inelastically when individuals reach working age.

Labor income is taxed at a constant marginal rate $t \in [0, 1)$ and $\tau^{\omega}_{i,g} < 1$ is a group-g-and-

[2] Nothing prevents students from making investments in human capital in multiple occupations. Since there is no uncertainty in future payoffs, they will, however, select only one in equilibrium.

occupation-i-specific labor market distortion. We allow for the possibility of *negative* distortions. Finally, $\tau^{e}_{i,g}$ is a group and occupation-specific educational barrier that raises the cost of good investments in human capital.

We assume that the draws from $F_a(\cdot)$ are i.i.d. across agents but we allow for the possibility that draws are not independent across occupations.

The production technologies in occupations $2, \ldots, I$ are linear in efficiency units of labor $h_{i,g}$ with productivities $\{A_i\}_{i=2}^{I}$:

$$y_{i,g} = A_i \times h_{i,g}. \tag{5}$$

Since human capital is the only factor of production, we can characterize the income of a worker who is employed in a non-teaching job by:

$$\omega_i\left(h_{i,g}\right) = y_{i,g}. \tag{6}$$

Teaching is distinct from other occupations since it "transforms" students with skill endowment $\vec{a}$ into workers with human capital $\vec{h}$ based on equation (3). The acquisition of human capital depends on the teacher's input $h_{T,g}$, the student's talent $\vec{a}$, the student's time and good investments $s_{i,g}$ and $e_{i,g}$, as well as class size $N(h_{T,g})$, which we allow to vary with the teacher's $h_{T,g}$.

2.2 Occupational Choice

Individuals will choose the occupation that delivers the highest lifetime utility. The workers' choice between non-teaching occupations is typical in the sense that it is based on a conventional Roy (1951)-type model. Teaching, on the other hand, is modeled in a way that deviates from the standard assumptions in two key respects: (1) the teachers' human capital is an input into the *students'* human capital production function rather than the production function for final goods and (2) the teacher's wage $\omega_{T,g}\left(h_{T,g}\right)$ need not be linear in $h_{T,g}$. This implies that the choice of becoming a teacher doesn't just depend on a worker's comparative advantage, it also depends on her *absolute* advantage.

Due to the special nature of teaching, we will discuss the key characteristics based on a version of the model with $I = 2$. This allows us to highlight the novel elements of the model more effectively. In this version, we label the two occupations with subscripts T (for *teaching*) and O (for *other*). Later, and especially when we parameterize the model to match moments in U.S. data, we will re-introduce the standard *Roy* features.

2.2.1 Teachers and Class Size

In equations (3) and (4), the student's ability in occupation i, denoted a_i, and the teacher's human capital h_T are complements. This implies positive sorting between teachers and students. This effect, however, is tempered by the teacher's span-of-control over the students in her class. $N(\cdot)$ denotes the class size and the coefficient $-\sigma$ captures the stylized fact that in larger classes teachers can pay less attention to one individual student. Here, we allow class size to depend on $h_{T,g}$ and we are focusing on $N(h_{T,g})$ such that – for given $e_{i,g}$ and $s_{i,g}$ – students are indifferent between teachers with different levels of human capital, say $h_{T,g}$ and ${h_{T,g}}'$:

$$N({h_{T,g}}') = \left(\frac{{h_{T,g}}'}{h_{T,g}}\right)^{\frac{\beta}{\sigma}} N(h_{T,g}). \tag{7}$$

All $\frac{M}{2}$ students in the economy attend school. Therefore, the following resource constraint must be satisfied:

$$\frac{M}{2} = \int_0^\infty N(h_{T,g}) d\mathcal{F}_T(h_{T,g}) \tag{8}$$

where $\mathcal{F}_T(\cdot)$ is the c.d.f. of human capital in teaching.

Then, the resource constraint (8) anchors the class size distribution.

Proposition 1 (Class size distribution) *For a given teacher with human capital $h_{T,g}$, her class size is characterized by:*

$$\left(\frac{2\widetilde{H}_T}{M}\right)^{\sigma} = h_{T,g}^{\beta} N(h_{T,g})^{-\sigma}, \tag{9}$$

where $\widetilde{H}_T = \sum_{g=1}^{G} \int_0^\infty {h_{T,g}}^{\frac{\beta}{\sigma}} d\mathcal{F}_T(h_{T,g})$.

2.2.2 Student-Teacher Matching

While this guarantees that students are indifferent between different teachers, it may be the case that better teachers prefer to work with higher ability students. This can happen, for instance, if students make human capital investments in terms of goods as a function of the teacher's human capital $h_{T,g}$.

For simplicity, we assume that students make investments as a function of a and occupation i, but not $h_{T,g}$. The joint assumption that $e_{i,g}(a_i)$ is a function of a (but not $h_{T,g}$) and that $N(h_{T,g})$ is a function of $h_{T,g}$ (but not a_i) implies that the student-teacher matching is random, i.e. the human capital production function is modular in $h_{T,g}$ and a_i.[3]

2.3 Values

The structure we impose on class size and student-teacher matching simplifies the characterization of an equilibrium considerably. While idiosyncratic heterogeneity still matters, the aggregate state can be summarized by a single moment of the distribution of human capital among current teachers (which we discuss in more detail below) rather than the entire distribution. Thanks to this feature we can characterize stationary equilibria as well as full transition paths sharply.

The idiosyncratic state consists of the I-dimensional vector of abilities $\vec{a}$. Recall that we assume $I = 2$ for the time being, where T denotes teaching and O lumps together all other occupations.

A student in group g solves a maximization problem which entails a discrete occupational choice as well as continuous human capital investments in terms of time and final goods. Formally, she solves:

$$V_g(a_T, a_O, \widetilde{H}_T) = \max_{\{s_{O,g}, s_{T,g}, e_{O,g}, e_{T,g}\}} \left\{ V_{O,g}(a_O, \widetilde{H}_T), V_{T,g}(a_T, \widetilde{H}_T) \right\} \tag{10}$$

where

$$\begin{aligned} V_{O,g}(a_O, \widetilde{H}_T) &= \ln\left(1 - s_{O,g}\left(a_O, \widetilde{H}_T\right)\right) \\ &\quad + \mu \ln\Big[{h'}_{O,g} A'_O (1-t')(1-\tau^{\omega'}_{O,g}) \\ &\quad - e_{O,g}(a_O, \widetilde{H}_T)(1+\tau^{e}_{O,g})\Big], \end{aligned} \tag{11}$$

$$\begin{aligned} V_{T,g}(a_T, \widetilde{H}_T) &= \ln\left(1 - s_{T,g}\left(a_T, \widetilde{H}_T\right)\right) \\ &\quad + \mu \ln\Big[\omega'(h'_{T,g}, \widetilde{H}'_T)(1-t')(1-\tau^{\omega'}_{T,g}) \\ &\quad - e_{T,g}(a_T, \widetilde{H}_T)(1+\tau^{e'}_{T,g})\Big] \end{aligned} \tag{12}$$

To make further progress, we need to characterize an occupational choice threshold along which a student with abilities (a^T, a^O) is indifferent between teaching and production work. More formally, the threshold is a function

[3] The anecdotal evidence suggests that student-teacher assignments don't involve systematic sorting at the school level. There is, however, evidence in support of sorting between families with highly educated parents and high-ability teachers into better funded schools or school districts. Identifying the empirically relevant extent of this type of sorting is work in progress. Future versions of our model may eventually incorporate it.

$$a^*_{T,g}(a_O) = \bar{a}_g\big(a_O, \widetilde{H}_T\big)$$

such that

$$V_{O,g}(a_O, \widetilde{H}_T) = V_{T,g}\left(a^*_{T,g}(a_O), \widetilde{H}_T\right), \text{ for all } a_O \in (0,\infty) \tag{13}$$

Unless it's necessary, we will omit the aggregate state argument $\widetilde{H}_T$ from $\bar{a}_g(\cdot)$ in order to keep the notation clean and tidy.[4]

The optimization problem is subject to the following constraints:

$$\begin{aligned} t\Bigg[&\sum_{g=1}^{G}\int_0^\infty \omega_T\left(h_{T,g}(a)\right) f_{T,g}(a)da \\ &+\sum_{g=1}^{G}\int_0^\infty \omega_O\left(h_{O,g}(a)\right) f_{O,g}(a)da\Bigg] = \sum_{g=1}^{G}\int_0^\infty \omega_T\left(h_{T,g}(a)\right) f_{T,g}(a)da \end{aligned} \tag{14}$$

where

$$f_{T,g}(a) = \int_0^{\bar{a}_g^{-1}(a)} f\big(a,b\big)db, \tag{15}$$

$$f_{O,g}(b) = \int_0^{\bar{a}_g(b)} f\big(a,b\big)da. \tag{16}$$

$f(a,b)$ is the p.d.f. associated with the c.d.f. $F(\cdot)$ from section 2.1 in the special case with $I=2$, $f_{T,g}(a)$ describes today's distribution of teachers, and $f_{O,g}(a)$ characterizes the distribution of other workers. Workers have perfect foresight and correctly anticipate next period's tax rate t' and future labor market distortions $\left\{\tau^{\omega'}_{O,g}, \tau^{\omega'}_{T,g}\right\}_{g=1}^{G}$ that satisfy (14)-(16) one period from today.[5]

[4] We will show later that the occupational threshold is indeed independent from $\widetilde{H}_T$. At this point, this result is not (yet) obvious.

[5] In the more general case with $I>2$, $i=1$ indexes teachers and the measures of teachers and other workers from equations (15) and (16) are given by:

$$f_{T,g}(a_T) = \sum_{i=2}^{I}\int_0^{\bar{a}_{i,g}^{-1}(a_T)} \Pi_{j\neq i}F\left(\left[\frac{A_i}{A_j}\cdot\frac{1-\tau^{\omega'}_{i,g}}{1-\tau^{\omega'}_{j,g}}\cdot\left(\frac{1+\tau^e_{i,g}}{1+\tau^e_{j,g}}\right)^{-\eta}\right]^{\frac{1}{\alpha}}\cdot a_i\right) f_{i,g}(a)da \text{ and}$$

$$f_{i,g}(a_i) = F(\bar{a}_{i,g}(a_i))\Pi_{j\neq i}F\left(\left[\frac{A_i}{A_j}\cdot\frac{1-\tau^{\omega'}_{i,g}}{1-\tau^{\omega'}_{j,g}}\cdot\left(\frac{1+\tau^e_{i,g}}{1+\tau^e_{j,g}}\right)^{-\eta}\right]^{\frac{1}{\alpha}}\cdot a_i\right), \text{ respectively.}$$

The aggregate laws of motion for human capital in teaching ($\widetilde{H}_T$) and "other" (H_O) are:

$$H'_O = \sum_{g=1}^{G} \int_0^\infty \left(\frac{2\widetilde{H}_T}{M}\right)^\sigma a^\alpha s_{O,g}\left(a, \widetilde{H}_T\right)^\phi e_{O,g}(a, \widetilde{H}_T)^\eta f_{O,g}(a)da \tag{17}$$

$$\widetilde{H}'_T = \sum_{g=1}^{G} \int_0^\infty \left(\left(\frac{2\widetilde{H}_T}{M}\right)^\sigma a^\alpha s_{T,g}\left(a, \widetilde{H}_T\right)^\phi e_{T,g}(a, \widetilde{H}_T)^\eta\right)^{\frac{\beta}{\sigma}} f_{T,g}(a)da \tag{18}$$

Teachers are public sector employees and their compensation is funded by tax revenues. Equation (14) is the government's balanced budget requirement, which is a salient feature in the U.S. context.

Equation (17) is the aggregation over h_O of the current cohort of students, while equation (18) is the $\left(\frac{\beta}{\sigma}\right)^{\text{th}}$ moment of the distribution of h_T of the current cohort of students.

The teachers' wage profile ω_T is exogenous. We assume that ω_T is strictly increasing in the teacher's human capital h_T and continuously differentiable. We allow, in particular, for the possibility that ω_T is not linear in h_T.[6]

3 Equilibrium

Definition 1 *Given next period's tax rate t', the labor market distortions and educational barriers $\left\{\left\{\tau^e_{i,g}, \tau^{\omega'}_{i,g}\right\}_{i=1}^{I}\right\}_{g=1}^{G}$, the occupational choices of today's old agents and aggregate human capital $\widetilde{H}_T$ and H_O, the equilibrium consists of the individual investment decisions of young agents, $\{e_{T,g}, s_{T,g}, e_{O,g}, s_{O,g}\}$, the occupational choice boundary $a^*_{T,g}(a_O)$, the corresponding densities $f_{T,g}(\cdot)$ and $f_{O,g}(\cdot)$, and occupation- and group-specific wage profiles $\{\omega_{T,g}, \omega_{O,g}\}$ such that:*

1. *Given the wage profiles, each individual's choice of goods investment $\{e_{T,g}, e_{O,g}\}$ and time investment $\{s_{T,g}, s_{O,g}\}$ maximizes utility given by (10) subject to the constraints given by (15)-(16).*
2. *The aggregate human capital stocks follow the laws of motion given by (17) and (18).*
3. *The government's budget constraint (14) is satisfied.*

[6]The key implication is that, in contrast to a "standard" Roy model, an agent's occupational choice depends on their comparative as well as absolute advantage.

3.1 Human Capital Investment

Let's first characterize the optimal human capital investment for a prospective teacher with ability a_T from group g.

The F.O.C.s for $s_{T,g}$ and $e_{T,g}$, respectively, after a few steps of algebra are:

$$(1+\tau^e_{T,g})e_{T,g}\left(\frac{\sigma}{\eta\beta}-1\right) = (1-s_{T,g})\,\mu(1-t')(1-\tau^{\omega'}_{T,g})\cdot\frac{\partial\omega'}{\partial h'_{T,g}}\cdot\frac{\partial h'_{T,g}}{\partial s_{T,g}} \tag{19}$$

$$1+\tau^e_{T,g} = (1-t')(1-\tau^{\omega'}_{T,g})\cdot\frac{\partial\omega'}{\partial h'_{T,g}}\cdot\frac{\partial h'_{T,g}}{\partial e_{T,g}}, \tag{20}$$

Let $\tau_{T,g} \equiv \frac{(1-t')\left(1-\tau^{\omega'}_{T,g}\right)}{1+\tau^e_{T,g}}$ be a composite wedge that takes into account the rate at which income is taxed, the extent of labor market discrimination, and educational barriers. Note that $\tau_{T,g} = 1$ when $t' = \tau^{\omega'}_{T,g} = \tau^e_{T,g} = 0$.

The amount of time and goods this individual invests in human capital is then given by:

$$s_{T,g} = \frac{\mu\phi}{\mu\phi+\eta\cdot\tau_{T,g}\cdot\frac{\omega'(h'_{T,g})}{e_{T,g}}-\eta} \tag{21}$$

$$e_{T,g} = \left(\tau_{T,g}\cdot\eta\cdot\left(\frac{2\widetilde{H}_T}{M}\right)^{\sigma}\cdot\left(\frac{\partial\omega'_{T,g}}{\partial h'_{T,g}}\right)\cdot s^{\phi}_{T,g}\cdot a^{\alpha}_T\right)^{\frac{1}{1-\eta}} \tag{22}$$

NOTE THAT A PROSPECTIVE TEACHER'S OPTIMAL INVESTMENT OF RESOURCES DEPENDS ON THE TOTAL HUMAN CAPITAL INVESTMENT OF PROSPECTIVE *production* WORKERS. CLEARLY, IT DEPENDS EXPLICITLY ON $s_{O,g}$, BUT IT ALSO DEPENDS ON $e_{O,g}$ IN EQUATION (36) BELOW, ALBEIT LESS OBVIOUSLY SO. THIS CAPTURES THE FACT THAT TEACHERS ARE COMPENSATED FOR THEIR CONTRIBUTION TO THE ACCUMULATION OF HUMAN CAPITAL THAT CAN BE USED FOR THE PRODUCTION OF THE CONSUMABLE FINAL GOOD.[7]

The F.O.C.s for prospective production workers (i.e., those labeled "other") are analogues of (21) and (22) with the simplification that the wage profile in production (as opposed to teaching) is linear in $h_{O,g}$ and the optimal amount of time and resources invested in human

[7]When the economy has $I > 2$ occupations, the prospective teacher's optimal investment of resources is characterized by:

$$e_{T,g} =$$

capital is given by:

$$s_{O,g} = \frac{\mu\phi}{\mu\phi + 1 - \eta} \tag{23}$$

$$e_{O,g} = \left(\tau_{O,g} \cdot \eta \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\sigma} \cdot A'_O \cdot s_{O,g}^{\phi} \cdot a_O^{\alpha}\right)^{\frac{1}{1-\eta}} \tag{24}$$

where, analogously to the problem of a prospective teacher, we define $\tau_{O,g} \equiv \frac{(1-t')\left(1-\tau_{O,g}^{\omega'}\right)}{1+\tau_{O,g}^{e}}$.

In the special case with $\beta = \sigma$, the model reverts to a standard occupational choice model à la *Roy* and the wage profile is linear in human capital on *both* occupations.

Proposition 2 *If $\beta = \sigma$, then $\frac{d\omega}{dh}$ and $\frac{h^*(h_O)}{h_O}$ are constant, i.e. the occupational cutoff satisfies $h^* = \frac{h_T}{h_O}$ for all h_O.*

3.2 The Aggregate Laws of Motion

The economy is characterized by the following laws of motions: [8]

$$\widetilde{H}'_T = \eta^{\frac{\beta\eta}{\sigma(1-\eta)}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\beta}{1-\eta}} \cdot \sum_{g=1}^{G} \tau_{T,g}^{\frac{\beta\eta}{\sigma(1-\eta)}} \cdot \int_0^{\infty} \left(\frac{\partial\omega_{T,g}}{\partial h_{T,g}}\right)^{\frac{\beta\eta}{\sigma(1-\eta)}} \cdot s_{T,g}^{\frac{\beta\phi}{\sigma(1-\eta)}} \cdot a^{\frac{\beta\alpha}{\sigma(1-\eta)}} f_{T,g}(a)da \tag{25}$$

$$H'_O = \eta^{\frac{\eta}{1-\eta}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \sum_{g=1}^{G} \tau_{O,g}^{\frac{\eta}{1-\eta}} \cdot \int_0^{\infty} {A'}_O^{\frac{\eta}{1-\eta}} \cdot s_{O,g}^{\frac{\phi}{1-\eta}} \cdot a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da \tag{26}$$

3.3 Occupational Choice in Decentralized Competitive Equilibrium with Educational Barriers and Labor Market Discrimination

To highlight the effect of educational barriers and labor market discrimination we add group- and occupation-specific distortions to the model. This is mostly a heuristic exercise since parsimony of our state space enables us to characterize the full transition path from one set of distortions to another. We highlight this feature of the model here since there have arguably been secular changes in educational barriers and labor market distortions for groups other than white males. To fix ideas, however, we simply compare steady states associated with

[8] When the economy has $I > 2$ occupations, the law of motion for the aggregate human capital in teaching is characterized by:

$$\widetilde{H}'_T =$$

.

particular wedges τ indexed by ω (for labor market discrimination), e (for educational barriers), and g for groups (race, gender,...).

We can show that this economy has a steady state in $\widetilde{H}_T$ and we can characterize it in a single equation:[9]

$$\widetilde{H}_T = \eta^{\frac{\beta\eta}{\sigma(1-\eta-\beta)}} \cdot \left(\frac{2}{M}\right)^{\frac{\beta}{1-\eta-\beta}} \cdot \left[\sum_{g=1}^{G} \tau_{T,g}^{\frac{\beta\eta}{\sigma(1-\eta)}} \cdot \int_0^\infty \left(\frac{\partial\omega_{T,g}}{\partial h_{T,g}}\right)^{\frac{\beta\eta}{\sigma(1-\eta)}} \cdot s_{T,g}^{\frac{\beta\phi}{\sigma(1-\eta)}} \cdot a^{\frac{\beta\alpha}{\sigma(1-\eta)}} f_{T,g}(a)da\right]^{\frac{1-\eta}{1-\eta-\beta}} \tag{27}$$

Proposition 3 (Occupational choice boundary) *The occupational choice boundary, denoted $\bar{a}_{T,g}(a_O)$, depends on the magnitude of the educational barriers and labor market frictions, but does not depend on either current or future aggregate human capital in teaching $\widetilde{H}_T$. It is characterized by:*[10]

$$\begin{aligned}
&\frac{a_O^{\frac{\alpha}{1-\eta}}}{a_T^{\frac{\alpha\beta}{\sigma-\eta\beta}}} \cdot \frac{\tau_{O,g}^{\frac{1}{1-\eta}}}{\tau_{T,g}^{\frac{\sigma}{\sigma-\eta\beta}}} \cdot \frac{1-\eta}{1-\frac{\beta\eta}{\sigma}} \cdot \frac{1+\tau_{O,g}^e}{1+\tau_{T,g}^e} \cdot \left(\frac{1-s_{O,g}}{1-s_{T,g}}\right)^{\frac{1}{\mu}} \cdot \frac{s_{O,g}^{\frac{\phi}{1-\eta}}}{s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}}} \\
&= \frac{\sum_{g=1}^G A'^{\frac{1}{1-\eta}}_O \cdot \tau_{O,g}^{\frac{\eta}{1-\eta}} \cdot s_{O,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da}{A'^{\frac{1}{1-\eta}}_O \cdot \sum_{g=1}^G \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f_{T,g}(a)da \cdot \sum_{g=1}^G \int_0^\infty f_{O,g}(a)da}
\end{aligned} \tag{28}$$

The calibration in section 5 will go beyond the simple comparative statics here and match moments along the secular transition from the late 1960s to the more recent past.

4 Data

We parameterize the model with data from longitudinal surveys that cover American youth at three different times after World War II: Project TALENT, NLSY 79, and NLSY 97. While the two NSLY surveys have been used extensively in the literature, Project TALENT is somewhat less known and we present it in some detail below.

[9] When the economy has $I > 2$ occupations, the steady state H_T is characterized by:

$$\widetilde{H}_T =$$

[10] When the economy has $I > 2$ occupations, the occupational choice boundary is characterized by:

4.1 Project TALENT

Project TALENT surveyed approximately 377,000 students enrolled in high school in 1960. The vast majority of these students were in the 14-18 year age range with a few outliers outside this bracket (on both sides). The sample of surveyed students was representative and accounted for 5 percent of the high school population that year. The project was funded by the United States Office of Education (the precursor of the Department of Education) and its goal was to collect high quality data on the achievements, aptitudes, and interests of high school students in the United States, and to examine whether, and how, these data predicted future outcomes in terms of educational attainment, career, and wellbeing.

In the initial wave of surveys, students were subjected to a two-day battery of aptitude, ability, and achievement tests. In addition, students answered questions about dispositional traits, their interests (with respect to occupations and activities), as well as personal and family information. In total, the surveyed students answered more than one thousand questions.

Follow-up surveys were then carried out one, five, and eleven years after the students' expected high school graduation. For instance, students who were in eleventh grade in 1960 were expected to graduate in 1961 and were contacted again in 1962, 1966, and 1972. Ninth graders were contacted in 1963, 1967, and 1973. A lack of federal funding stopped the project during the planning stage for a 17-year follow-up, which never took place.

51 percent of the original participants participated in the 1-year post-graduation surveys. The five and eleven-year follow-ups successfully contacted 35 and 25 percent of the original participants, respectively. One concern with attrition rates of this magnitude is that the survey sample of waves two through five is subject to selection along one or several attributes.

In Table 1 we summarize the descriptive statistics by gender, race, parental education, and parental economic status (all in 1960). The racial composition of the sample exhibits some anomalies. The racial classification in the 1960 base year is problematic since almost 60 percent of the sample are in the "other", "unknown", or "conflicting" category. One conceivable explanation for this unusual pattern as well as the *rise* of black respondents when the sample size is *shrinking* is a retroactive racial re-classification of all the participants in the 5-year post survey.[11]

The rationale for using panel data from Project TALENT and the two NLSYs is that all three surveys allow us to connect scores from aptitude tests to labor market outcomes. Project TALENT contains a number of tests administered in 1960 when the respondents were attending

[11] The classification of respondents into race categories in the base year suffers from inconsistencies (e.g. self-reported vs. pollster's classification). Moreover, the sample proportion of black high school students differs from that reported in the 1960 Census.

	Base Year	1-Year Post	5-Year Post	11-Year Post
Sample Size	368,938	188,138	128,163	92,962
Black (% of sample size)	6,488 (1.8%)	4,003 (2.1%)	4,856 (3.8%)	3,026 (3.3%)
Other / Unknown / Conflicting (% of sample size)	215,786 (58.5%)	73,668 (39.1%)	3,739 (2.9%)	3,758 (4.0%)
Female (% of sample size)	184,912 (50.1%)	98,183 (52.2%)	64,841 (50.6%)	47,417 (51.0%)
Parents: High School Degree (% of sample size)	155,128 (42.05%)	85,804 (45.61%)	60,142 (46.93%)	43,204 (46.47%)
Parents: College Degree (% of sample size)	58.959 (15.98%)	35,065 (18.64%)	25,193 (19.66%)	17,361 (18.68%)
Parents: High Income (% of sample size)	116,628 (33.42%)	65,691 (36.70%)	46,499 (38.10%)	33,097 (37.43%)

Table 1: Project TALENT Demographic Composition

high school, though in different grades.[12] We mainly use composite scores built from a battery of underlying tests measuring verbal and mathematical aptitude. The former includes literature information, vocabulary, and English. The latter contains math information, arithmetic reasoning, introductory high school math, and advanced high school math. Table 2 shows summary statistics for the 88,330 individuals who participated in the 11-year follow-up survey. We focus on the 11-year *post* wave since it contains the bulk of occupational information at a time when the respondents are approximately thirty years old.[13]

ARE WE USING THE PARENTS' EDUCATIONAL ATTAINMENT OR INCOME FOR ANYTHING? IF IT'S IN THE TABLE, WE SHOULD DISCUSS IT SOMEWHERE.

THIS TRANSITION IS ABRUPT. WE START THE DISCUSSION OF TEACHERS IMMEDIATELY AFTER DESCRIBING THE APTITUDE TESTS.

Approximately three quarters of teachers in the sample are female, in the 5-year post as well as the 11-year post surveys. In non-teaching occupations the share of women is 39 percent and women account for 80 percent of respondents who report that they are not in the labor force. On average, teachers have higher mathematics and verbal test scores than respondents in other occupations and those who are not in the labor force. Together, these stylized facts are consistent with the hypothesis that high-ability women are disproportionately choosing

[12] Roughly one quarter of the sample was in each high school grade (i.e. 9 through 12) in the baseline survey.
[13] The 5-year post-graduation survey contains occupational information but we have not made use of it yet.

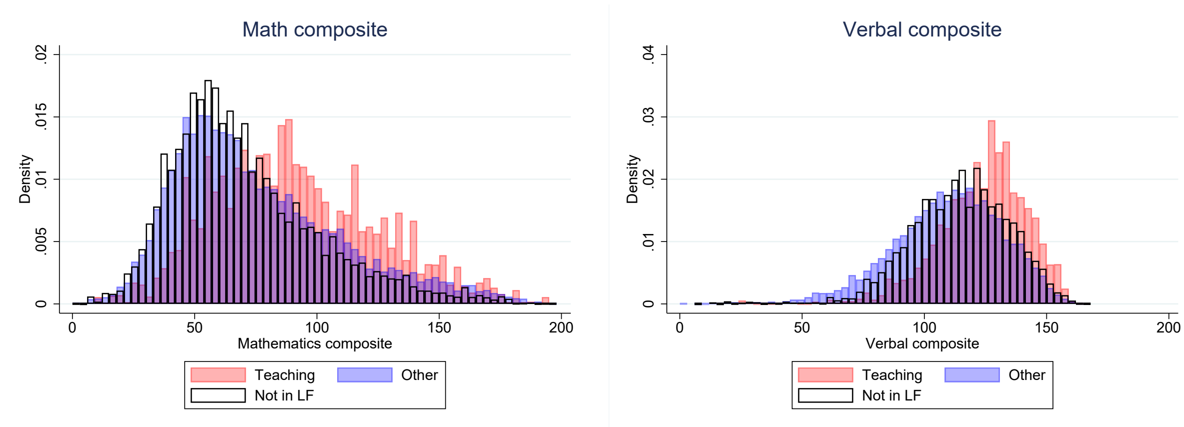
11-Year Post					
	Teachers	Other	Unemployed	Not in Labor Force	Total
Female	3,641	16,063	1,154	23,001	43,859
(% of subsample)	(60.36%)	(30.47%)	(61.84%)	(92.84%)	(51.36%)
Math composite mean	2.964	2.704	2.573	2.477	2.654
(st.dev.)	(0.888)	(1.023)	(1.052)	(0.939)	(1.0)
Verbal composite mean	6.370	5.900	5.909	6.091	5.989
(st.dev.)	(0.809)	(1.031)	(1.145)	(0.929)	(1.0)
Social composite mean	2.915	2.660	2.563	2.753	2.703
(st.dev.)	(0.988)	(1.000)	(0.985)	(0.996)	(1.0)
IQ composite mean	4.330	3.964	3.887	3.978	3.992
(st.dev.)	(0.822)	(1.027)	(1.128)	(0.955)	(1.0)
Observations	6,032	52,726	1,866	24,775	85,399
(% of total sample)	(7.06%)	(61.74%)	(2.19%)	(29.01%)	(100.0%)

Table 2: Project TALENT Test Scores by Occupation

teaching jobs and careers and we will return to this point below.

If the labor market discriminated more against certain groups (gender, race,...) in non-teaching occupations in 1960, we expect these groups to favor teaching over other occupations or to stay out of the labor force altogether. "Not in the labor force" is more a reflection of the absolute level of labor market discrimination against a particular group while the occupational choice between teaching or other occupations reflects differences between group-specific barriers. To the extent that occupations require specific human capital investments we allow for educational barriers to affect occupational choices separately in the model. If both of these barriers declined after 1960, we would expect a narrowing of the educational achievement gap and more talented females should move from teaching into other occupations. This reallocation would lower the average quality of teachers and, therefore, we expect to observe more significant differences in the ability distributions between teachers and other occupations in cohorts who entered the labor market in the 60s and 70s (i.e. our Project TALENT population) compared to younger individuals (i.e. the NLSY79 and NLSY97 respondents).

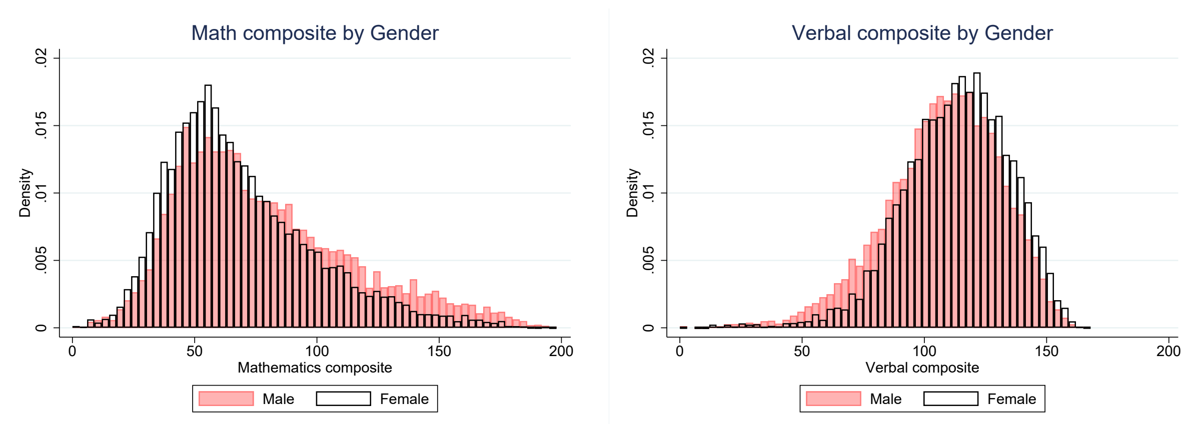
Figure 1 shows the distribution of test scores for math and verbal composite by occupations in Project TALENT. The distribution of math scores in panel 1a is skewed to the right for "other" compared to the distribution of math scores among teachers. Panel 1b shows a similar pattern in terms of relative skewness. The distribution of scores of respondents in non-teaching occupations and those who report not being in the labor force are quite similar. Overall, indi-



(a) Math

(b) Verbal

Figure 1: Composite Scores by Occupation in Project TALENT



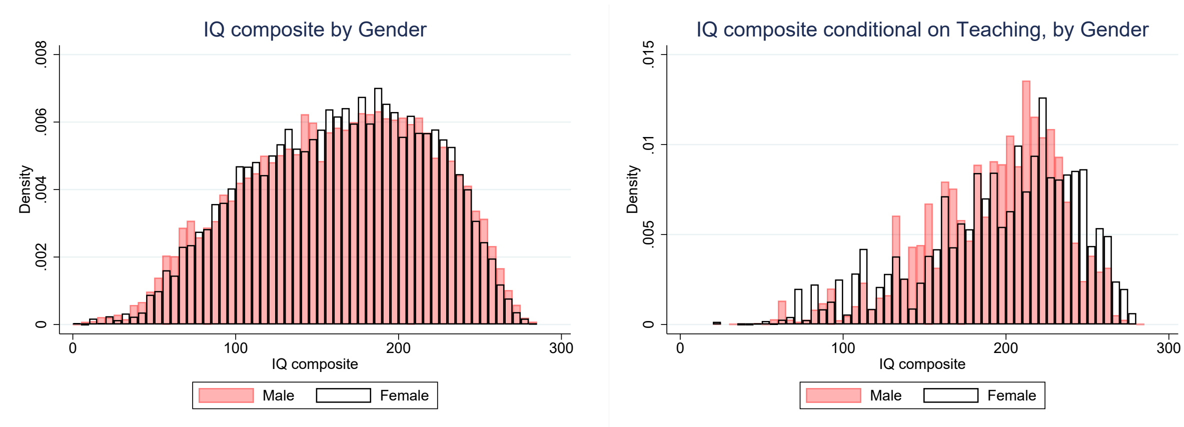
(a) Math composite

(b) Verbal composite

Figure 2: Composite Scores by Gender in Project TALENT

viduals in teaching occupations tend to have higher scores than non-teachers. One possible explanation for these differences is that individuals select into teaching along a non-skill dimension and an obvious selection candidate is gender. Approximately three quarters of all teachers are female (see the first row in the two panels of Figure 2 IS THIS THE CORRECT TABLE/FIGURE REFERENCE?). If women exhibited a distinct distribution of scores this may explain the discrepancy between teaching and other occupations. The data, however, do not suggest that women are innately more or less talented than men. Women's verbal scores slightly dominate the men's while the opposite is true for the math scores (see Figure 2b).

The distributions of composite IQ scores for Project TALENT respondents split by gender are quite similar (see Figure 3). What reconciles Figures 1-3 is the selection of more talented women into teaching, which we plot in Figure 3b. Due to the smaller numbers of individuals



(a) All Occupations

(b) Teaching Only

Figure 3: Composite IQ Scores by Gender in Project TALENT

in Figure 3b compared to the three previous figures (the number of men is particularly low), the plot contains more noise and one should draw any conclusion with appropriate caution. With this in mind, it does appear that teaching attracts higher-ability individuals and that the selection of women along this dimension is somewhat stronger, plausibly due to the lack of career opportunities in other occupations.

The 11-year follow-up survey provides a snapshot of young workers' occupational choices together with the pre-labor market aptitude profile in the early 1970s. For later years, we use data from the two NLSYs and we will give a brief overview in the next section.

4.2 NLSY79 and NLSY97

In contrast to Project TALENT, the two NLSYs initiated in 1979 and 1997 are widely used and many readers are already familiar with the data. Cooksey (2018), among many others, provides a comprehensive overview of the two surveys. For the purposes of our paper, the *Armed Forces Qualification Test* (AFQT) scores are of particular interest since we need proxies to capture differences in the individuals' raw skills, which is the crucial source of exogenous variation in our model. The score consists of four sections of the *Armed Services Vocational Aptitude Battery* (ASVAB): arithmetic reasoning, word knowledge, paragraph comprehension, and numerical operations. The tests took place in 1980 and 1999 for NLSY79 and NLSY97 cohorts, respectively.

Since the tests were administered at different ages (and times) and used different formats, NLS program staff re-normed the scores in 2006, in order to facilitate cross-cohort comparisons.[14] Table 3 shows summary statistics for the individuals in our sample, divided into three categories: (1) teachers, (2) non-teachers/others, (3) not in labor force. Since an individual usually has multiple employment spells, she might be in all these categories over the course of the panel. We define an individual as a teacher if she has at least one spell in teaching that is longer than 9 months (one academic year). An individual is out of labor force if she reported to be out of labor force for 208 consecutive weeks for a reason other than schooling.[15]

If indeed labor market barriers are shrinking over time, we would expect to see some evidence of this in the occupational choice of women in more recent cohorts and hence in the NLSY79 and NLSY97. In these younger cohorts, the share of women among teachers continues to be high. The gender composition of teachers in NLSY 79 is comparable to the one we observe in Project TALENT. In the more recent NLSY 97, the share is a few percentage points lower. In contrast, the share of female workers in other occupations is rising, from approximately 39 percent in Project TALENT to 44 percent in NLSY79, and 45 percent in NLSY97. The gender composition of those who are out of the labor force is heading in the opposite direction. The share of females dropped to 66 percent among NLSY79 respondents and 64 percent among NLSY97 respondents, compared to almost 80 percent in the 11-year post graduation survey from Project TALENT.

One significant difference between the Project TALENT and NLSY respondents are the aptitude scores for those who are not in the labor force. While their average score was higher than the average score of labor force participants in non-teaching occupations in Project TALENT, the ranking is reversed in the two NLSYs. This suggests that the exit rate from "not in the labor force" is correlated with the respondents' ability. High and low ability individuals appear to respond differentially to changes in labor market discrimination and this is further corroborated in a set of plots we present below.

Figure 4 plots the NLSY analogues of the math and verbal scores in Project TALENT shown in Figure 1. Clearly, teaching still attracts higher-ability individuals than "other" occupations or those who are not in the labor force. While the ability distribution of teachers still first-order dominates the scores in "other" occupations in the 1979 survey, there is a remarkable shift in the talent distribution of those who are not in the labor force. It is for more right-skewed than the distribution of workers in "other" occupations. Unfortunately, the 1997 data is considerably more noisy mostly due to the smaller sample size and we're refraining from reading too much into any particular feature of the data. This being said, it is worth highlighting that the

[14] Altonji et al. (2009) also propose a method to enable a comparison of AFQT scores across the two NLSY samples. Note that we cannot construct equivalence scores that would allow as to compare results from aptitude tests in Project TALENT with those from the two NLSYs.

[15] After four years, "not in the labor force" is effectively an absorbing state. The probability of returning to the labor force is less than 1%.

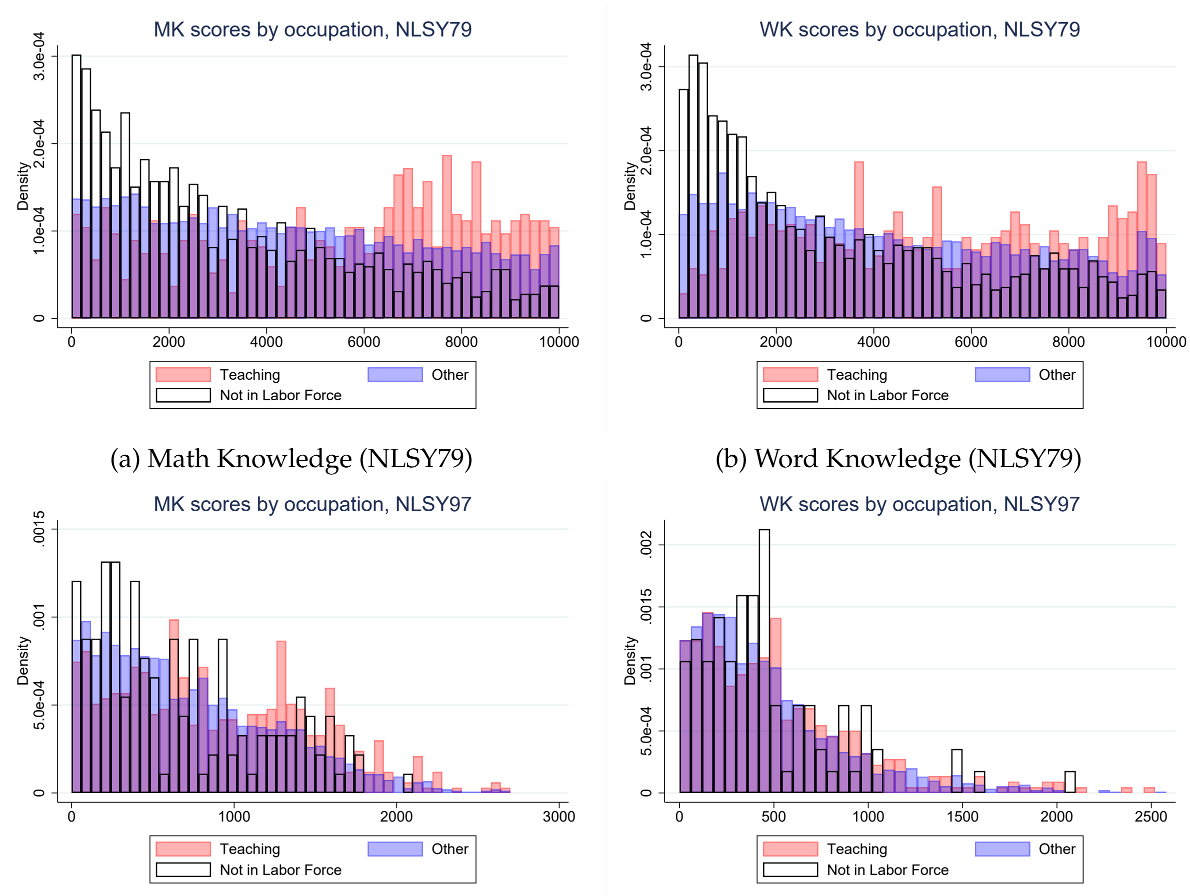
Panel A: NLSY79					
	Teachers	Other	Unemployed	Not in Labor Force	Total
Female	999	2,933	221	1,568	5,721
(% of subsample)	(43.19%)	(44.85%)	(41.39%)	(76.34%)	(50.01%)
Math ability	2.450	2.443	1.919	1.962	2.334
(st.dev.)	(0.932)	(1.018)	(0.935)	(0.912)	(1)
Verbal ability	3.614	3.509	2.920	3.057	3.422
(st.dev.)	(0.864)	(0.971)	(1.088)	(1.074)	(1)
Social ability	3.891	3.824	3.566	3.552	3.777
(st.dev.)	(1.016)	(0.980)	(0.969)	(1.011)	(1)
AFQT	1.780	1.741	1.184	1.270	1.638
(st.dev.)	(0.928)	(1.004)	(0.950)	(0.956)	(1)
Observations	2,313	6,539	534	2,054	11,440
(% of total sample)	(20.22%)	(57.16%)	(4.67%)	(17.95%)	(100.00%)

Panel B: NLSY97					
	Teachers	Other	Unemployed	Not in Labor Force	Total
Female	478	2,184	171	605	3,438
(% of subsample)	(48.53%)	(46.60%)	(48.03%)	(64.71%)	(49.38%)
Math ability	1.309	1.422	1.710	1.780	1.469
(st.dev.)	(0.885)	(0.959)	(1.148)	(1.164)	(1)
Verbal ability	1.475	1.590	1.985	2.001	1.649
(st.dev.)	(0.879)	(0.957)	(1.144)	(1.154)	(1)
Social ability	5.684	5.645	5.478	5.460	5.614
(st.dev.)	(0.904)	(0.967)	(1.108)	(1.158)	(1)
AFQT	1.776	1.665	1.045	1.185	1.585
(st.dev.)	(0.943)	(0.998)	(0.808)	(0.961)	(1)
Observations	985	4,687	356	935	6,963
(% of total sample)	(14.15%)	(67.31%)	(5.11%)	(13.43%)	(100.00%)

Table 3: NLSY Scores by Occupation

definition of "not in the labor force" is *not* identical in the TALENT data on the one hand and the NLSY data on the other.[16] It is, however, suggestive of a trend whereby high-ability in-

[16]The difference stems from the fact that we observe Project TALENT participants relatively infrequently and the respondents' labor market histories are less complete.



(a) Math Knowledge (NLSY79)

(b) Word Knowledge (NLSY79)

(c) Math Knowledge (NLSY97)

(d) Word Knowledge (NLSY97)

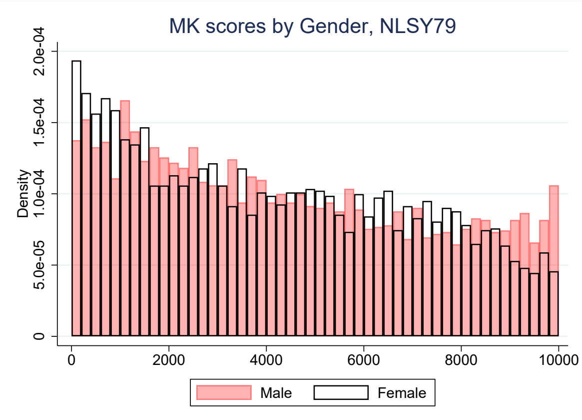
Figure 4: AFQT Scores by Occupations

dividuals are pulled into the labor force more "aggressively" compared to low-skill workers over time.

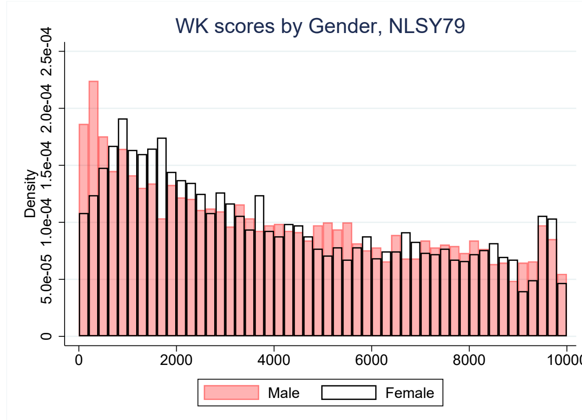
Figures 5-6 are the NLSY analogues of Figures 2-3. WE NEED TO SAY A FEW MORE THINGS ABOUT THIS. IT'S AN ISOLATED STATEMENT RIGHT NOW.

Unfortunately, the composite score distributions aren't comparable between Project TALENT and NLSY cohorts for several reasons. Although the ability tests try to capture math and verbal abilities, the tests themselves are very different. Therefore, it is impossible to compare changes in score distributions over time. Despite this limitation we can still compare the composition (by gender or occupation) *within* cohorts across the three surveys. In fact, the particular *Roy* flavor of our model suggests that this is where the interesting action is.

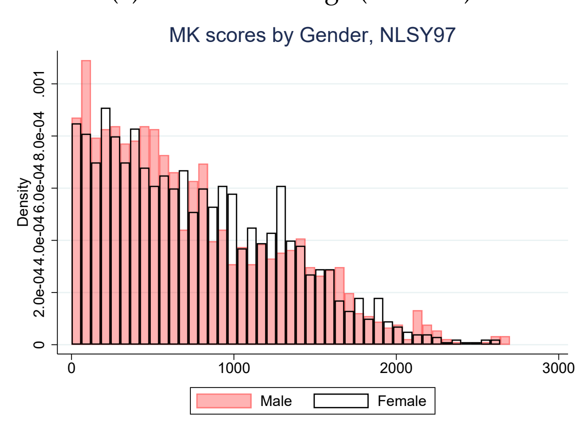
THIS IS A ROUGH TRANSITION. WE JUMP FROM ABILITY TESTS TO EMPLOYMENT AND OCCUPATIONAL HISTORIES SUDDENLY.



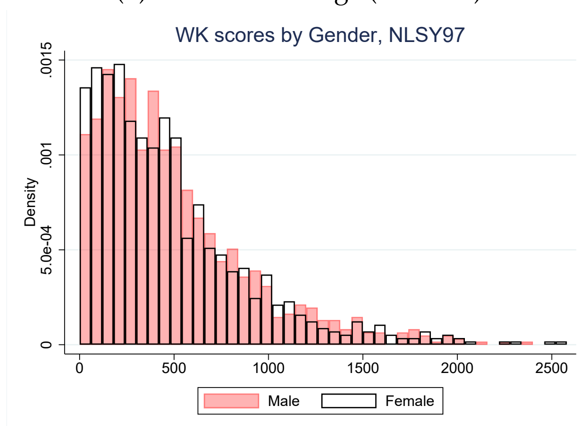
(a) Math Knowledge (NLSY79)



(b) Word Knowledge (NLSY79)

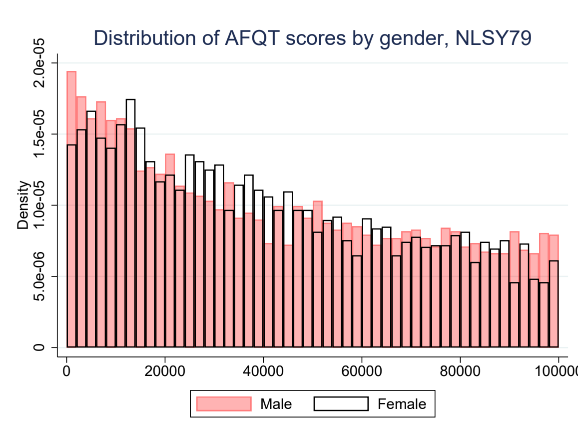


(c) Math Knowledge (NLSY97)

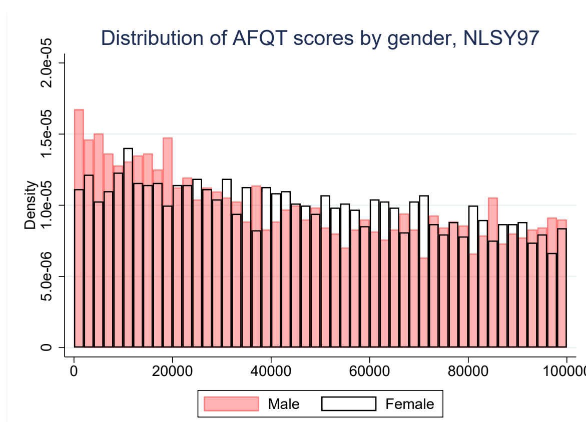


(d) Word Knowledge (NLSY97)

Figure 5: Topic Scores by Gender



(a) AFQT Composite (NLSY79)



(b) AFQT Composite (NLSY97)

Figure 6: AFQT Scores by Gender

Differences in the frequency of the surveys further complicate comparisons between Project TALENT and the NLSY's. The NLSY79 and NLSY97 survey the respondents far more frequently and this enables us to construct more detailed employment histories and occupation choices over the course of panels. While Project TALENT provides data on the respondents' *current* occupation in the 1, 5, and 11-year post waves, it misses some of the higher-frequency churn since it doesn't ask any retrospective career questions. Moreover, the probability of being a teacher one year after high school graduation is vanishingly small (0.08% of the sample). In the figures and tables presented so far we report the occupations reported in the 11-year follow-up study. While certainly less complete than the NLSY data, Project TALENT still offers insights into the career decisions of young adults at a time for which there is only a very limited supply of nationally representative longitudinal survey data.[17]

Project TALENT, NLSY79, and NLSY97 provide us with the data moments that we match to model-generated moments in order to quantify the static and dynamic effects of the implied educational barriers and labor market barriers. While the discussion so far emphasized the contrast between men and women, it is worth emphasizing that our model framework can accommodate more richness in terms of the salient groups of workers. In ongoing work we are incorporating race and ethnicity to more fully capture the salient educational and labor market frictions in the data.[18]

The final element in our calibration element relies on O*Net's characterization of occupation-specific skill requirements. We use a three-dimensional job characterization: mathematics, verbal, and social. WE NEED TO DISCUSS "SOCIAL" EARLIER IN THE SECTION. AFTER ALL, WE ARE USING IT FOR THE CONSTRUCTION OF OCCUPATION-SPECIFIC REQUIREMENTS. We decompose the requirements of each job into an absolute and a comparative dimension in a way that aligns naturally with the notion of absolute and comparative advantage.

We characterize the comparative requirements by a triple of weights based on the dimension-by-dimension skill ranking of each job. To account for the possibility that jobs with similar comparative profiles are characterized by different overall skill intensities, we also create an absolute profile based on the weighted sum of skill-specific rankings. Given the non-linearity of payoffs in our model, the distinction between comparative and absolute advantage is crit-

[17] In ongoing work, we synthetically impose some of the data limitations in Project TALENT on the NLSY in order to get a better sense for underlying changes in labor market conditions and career decisions across cohorts. Evidently we're giving up some valuable details in the process for the benefit of more cross-survey comparability. IS THIS REFERRING TO THE RESTRICTION TO 20-YEAR OLDS IN THE NLSY? IF YES, WE NEED TO BE MORE EXPLICIT IN THE MAIN TEXT.

[18] Since the racial categorization is somewhat problematic in Project TALENT, this extension poses additional challenges.

ical for our calibration strategy[19] I THINK WE NEED TO BE MORE SPECIFIC ABOUT THIS (IF WE ARE EVEN STILL DOING IT)!

Last but not least, we use these skill profiles to project the survey respondents' three-dimensional abilities into an I-dimensional vector of job-specific abilities.

5 Quantitative Analysis

5.1 Calibration Strategy

The aim of the calibration is to parametrize the economy to match aggregate and micro moments that characterize the US economy in 1970, 1990, and 2010 to their model counterparts. We select these years since the respondents from Project TALENT, NLSY79, and NLSY97 are, on average, 29 years old. ThE parameterization will treat these three target years as steady states of the model.

We assume that the idiosyncratic occupation-specific abilities are drawn from a Fréchet distribution and the relative innate talent levels are identical across all groups. Tables 4-6 summarize the elements of a sample parameterization. Table 4 summarizes the assumptions and normalization for our base parameterization of the model. We assume that there are no labor and human capital barriers for men in all occupations. We also assume that labor and human capital barriers in home production are zero for all groups.

Table 5 summarizes the key parameters. Table 6 presents the endogenous variables and their targeted empirical counterparts. Parameter value for $\frac{\beta}{\sigma}$ is a crucial parameter that governs how skill distribution changes with respect to the changes in the barriers. Identification of this parameter allows calibrating the model to the data and conduct quantitative and counterfactual analysis. Another parameter that governs how selection into teaching career responds to the changes in the barriers is η. If $\eta < 1$, a decrease in barriers in other occupations leads to the higher occupational threshold, making teaching career more selective. If $\eta > 1$, a decrease in barriers in other occupations results in the lower occupational threshold.

[19] In this respect, our model differs from a plain-vanilla occupational choice model à la Roy (1951).

Parameter	Definition	Determination	Value
$\tau^w_{i,men}$	Labor market barriers for men	Assumption	0
$\tau^e_{i,men}$	Human capital barriers for men	Assumption	0
$\tau^w_{T,g}$	Labor market barriers for all groups	Assumption	0
$\tau^e_{T,g}$	Human capital barriers for all groups	Assumption	0
$\tau^e_{i,g}$	Human capital barriers for other groups	Normalization	0
A_T	Teachers productivity	Normalization	1
μ_a	Mean ability distribution	Normalization	0
σ_a	St.dev. ability distribution	Normalization	1

Table 4: Assumptions and Normalization

Parameter	Definition	Determination
α	Ability elasticity of human capital	Wage dispersion
η	Goods elasticity of human capital	Aggregate education spending share
ϕ	Time elasticity of human capital	Mincerian return to education for other
A_i	Occupation-specific productivity	Labor market shares for men
$\tau^w_{i,g}$	Labor market barriers for other groups	Group gap by occupation

Table 5: Baseline Parameter Values

Parameter	Definition	Empirical Targets
β	Teacher elasticity of human capital	Skill composition by occupation and group
σ	Class size elasticity of human capital	Normalization to 1
μ	Trade-off between consumption and time spent accumulating human capital	Schooling of teachers relative to schooling of others
λ_m	Scale for male labor market wedges	Share of teachers among men
λ_f	Scale for other group labor market wedges	Share of teachers among other group

Table 6: Benchmark Calibration

6 Conclusion

This paper develops a novel occupational choice model that captures the distinct nature of teaching and learning – and hence human capital accumulation. In addition to the static gains and losses associated with labor market distortions and educational barriers, we can characterize the dynamic effects that are driven by the number and ability of workers who are drawn to the teaching profession. This occupational choice, in turn, shapes the incentives to invest in human capital when the agents are young, i.e. when they are students.

The model also introduces wage profiles that are non-linear in the workers' human capital and this generalization implies that occupational choice is driven by *comparative* as well as *absolute* advantages across the various occupations. This is a significant departure from the canonical occupational choice models in the spirit of Roy (1951). At this point, the extent of this non-linearity is not pinned down precisely and this is an area of ongoing work.

The model reconciles several salient stylized facts. In addition to the change in the student-teacher ratios and per-student-expenditures mentioned above, they are: (1) the increase in the share of women in teaching jobs from 4.6 percent in 1970 to 6.7 percent in 2010; (2) the drop in the share of men who are teachers from 2.9 percent to 2.1 percent during that same time; (3) the sharp rise in the female labor force participation rate; (4) the slight decline in the male labor force participation rate; and (5) the evolution of the skill composition by gender and occupation between 1970 and 2010.

The somewhat loose discussion of our calibration strategy in section 5.1 reflects the work-in-progress nature of our quantitative exercise. The model is otherwise well identified and in ongoing work we are calibrating the remaining parameters to match the salient moments at three different points in time: 1970, 1990, and 2010. We select these years since the respondents from all three longitudinal surveys are, on average, 29 years old. This parameterization will treat these three target years as steady states of the model. Future work will explore the human capital transition dynamics between steady states.

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A Special Case: Teacher's Are Paid their Marginal Product

The model in section 2 assumes that teachers are paid according to an exogenous schedule ω_T and each teacher's level of pay is a function of her human capital h_T. Here we consider a special case wher

A.1 Human Capital Investment

Let's first characterize the optimal human capital investment for a prospective teacher with ability a from group g.

The F.O.C.s for $s_{T,g}$ and $e_{T,g}$, respectively, after a few steps of algebra are:

$$(1+\tau^{e}_{T,g})e_{T,g}\left(\frac{\sigma}{\eta\beta}-1\right) = (1-s_{T,g})\,\mu(1-t')(1-\tau^{\omega'}_{T,g})\frac{\partial\omega'}{\partial s_{T,g}} \tag{29}$$

$$(1-t')(1-\tau^{\omega'}_{T,g})\frac{\partial\omega'}{\partial e_{T,g}} = 1+\tau^{e}_{T,g}, \tag{30}$$

where

$$\frac{\partial\omega'}{\partial s_{T,g}} = \frac{\phi\beta}{\sigma}\lambda'\left(\frac{M}{2\widetilde{H}_T}\right)^{-\beta}\frac{M}{2\widetilde{H}'_T}\left[a^{\alpha}(s_{T,g})^{\phi}(e_{T,g})^{\eta}\right]^{\frac{\beta}{\sigma}}\frac{1}{s_{T,g}} \tag{31}$$

$$\frac{\partial\omega'}{\partial e_{T,g}} = \frac{\eta\beta}{\sigma}\lambda'\left(\frac{M}{2\widetilde{H}_T}\right)^{-\beta}\frac{M}{2\widetilde{H}'_T}\left[a^{\alpha}(s_{T,g})^{\phi}(e_{T,g})^{\eta}\right]^{\frac{\beta}{\sigma}}\frac{1}{e_{T,g}} \tag{32}$$

Let $\tau_{T,g} \equiv \frac{(1-t')\left(1-\tau^{\omega'}_{T,g}\right)}{1+\tau^{e}_{T,g}}$ be a composite wedge that takes into account the rate at which income is taxed, the extent of labor market discrimination, and educational barriers. Note that $\tau_{T,g} = 1$ when $t' = \tau^{\omega'}_{T,g} = \tau^{e}_{T,g} = 0$.

The amount of time and goods this individual invests in human capital is then given by:

$$s_{T,g} = \frac{\mu\phi}{\mu\phi + \frac{\beta}{\sigma} - \eta} \tag{33}$$

$$\begin{aligned} e_{T,g} = \frac{\beta}{\sigma} \cdot \eta^{\frac{1}{1-\eta}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \frac{\sum_{g=1}^{G} {A'_O}^{\frac{1}{1-\eta}} \cdot \tau_{O,g}^{\frac{\eta}{1-\eta}} \cdot s_{O,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da}{\sum_{g=1}^{G} f_{O,g}(a)da} \\ \times \frac{\tau_{T,g}^{\frac{\sigma}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot a_T^{\frac{\alpha\beta}{\sigma-\eta\beta}}}{\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da} \end{aligned} \tag{34}$$

Note that a prospective teacher's optimal investment of resources depends on the total human capital investment of prospective *production* workers. Clearly, it depends explicitly on $s_{O,g}$, but it also depends on $e_{O,g}$ in equation (36) below, albeit less obviously so. This captures the fact that teachers are compensated for their contribution to the accumulation of human capital that can be used for the production of the consumable final good.[20]

The F.O.C.s for prospective production workers (i.e., those labeled "other") are analogues of (33) and (34) with the simplification that the wage profile in production (as opposed to teaching) is linear in $h_{O,g}$ and the optimal amount of time and resources invested in human capital is given by:

$$s_{O,g} = \frac{\mu\phi}{\mu\phi + 1 - \eta} \tag{35}$$

$$e_{O,g} = \left(\tau_{O,g} \cdot \eta \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\sigma} \cdot {A'_O}^{\eta} \cdot s_{O,g}^{\phi} \cdot a_O^{\alpha}\right)^{\frac{1}{1-\eta}} \tag{36}$$

where, analogously to the problem of a prospective teacher, we define $\tau_{O,g} \equiv \frac{(1-t')\left(1-\tau^{\omega'}_{O,g}\right)}{1+\tau^{e}_{O,g}}$.

[20]When the economy has $I > 2$ occupations, the prospective teacher's optimal investment of resources is characterized by:

$$\begin{aligned} e_{T,g} = \frac{\beta}{\sigma} \cdot \eta^{\frac{1}{1-\eta}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \frac{\sum_{i=2}^{I}\sum_{g=1}^{G} {A'_i}^{\frac{1}{1-\eta}} \cdot \tau_{i,g}^{\frac{\eta}{1-\eta}} \cdot s_{i,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{i,g}(a)da}{\sum_{i=2}^{I}\sum_{g=1}^{G} f_{i,g}(a)da} \\ \times \frac{\tau_{T,g}^{\frac{\sigma}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot a_T^{\frac{\alpha\beta}{\sigma-\eta\beta}}}{\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da}. \end{aligned}$$

In the special case with $\beta = \sigma$, the model reverts to a standard occupational choice model à la *Roy* and the wage profile is linear in human capital on *both* occupations.

Proposition 4 *If $\beta = \sigma$, then $\frac{d\omega}{dh}$ and $\frac{h^*(h_O)}{h_O}$ are constant, i.e. the occupational cutoff satisfies $h^* = \frac{h_T}{h_O}$ for all h_O.*

A.2 The Aggregate Laws of Motion

The economy is characterized by the following laws of motions: [21]

$$\widetilde{H}'_T = \left[\left(\frac{\beta}{\sigma}\right)^\eta \cdot \eta^{\frac{1}{1-\eta}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \left(\frac{\sum_{i=2}^{I}\sum_{g=1}^{G} A_i'^{\frac{1}{1-\eta}} \cdot \tau_{i,g}^{\frac{\eta}{1-\eta}} \cdot s_{i,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{i,g}(a)da}{\sum_{i=2}^{I}\sum_{g=1}^{G} f_{i,g}(a)da} \right)^\eta \right.$$
$$\left. \times \left(\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da \right)^{\frac{\sigma-\eta\beta}{\beta}} \right]^{\frac{\beta}{\sigma}} \tag{37}$$

$$H'_O = \sum_{g=1}^{G} \left(\tau_{O,g}^\eta \cdot \eta^\eta \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot A_O'^\eta \cdot s_{T,g}^\phi \right)^{\frac{1}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da \tag{38}$$

A.3 Occupational Wages in Decentralized Competitive Equilibrium

In order to highlight the mechanisms that govern the individuals' occupational choice, we first solve for a decentralized equilibrium that implements the optimal allocation of adults to "teaching" and "other". Put differently, we solve the model for a wage profile $\omega^*(h_{T,g})$ that pays teachers with human capital $h_{T,g}$ their marginal product.[22]

How can we characterize the teacher's marginal product? Recall that only workers produce the final (consumption) good using a linear production function with productivity A_O. Thanks to stable random matching of students to teachers, the ability distribution of students is identical across class rooms and a teacher's marginal value only varies with the number

[21] When the economy has $I > 2$ occupations, the law of motion for the aggregate human capital in teaching is characterized by:

$$\widetilde{H}'_T = \left[\left(\frac{\beta}{\sigma}\right)^\eta \cdot \eta^{\frac{1}{1-\eta}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \left(\frac{\sum_{g=1}^{G} A_O'^{\frac{1}{1-\eta}} \cdot \tau_{O,g}^{\frac{\eta}{1-\eta}} \cdot s_{O,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da}{\sum_{g=1}^{G} f_{O,g}(a)da} \right)^\eta \right.$$
$$\left. \times \left(\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da \right)^{\frac{\sigma-\eta\beta}{\beta}} \right]^{\frac{\beta}{\sigma}}$$

.

[22] In section 2 we specified the model for an arbitrary profile ω. Here we impose further restrictions on this function.

of students in her class room, denoted by $N(h_{T,g})$. Clearly, then, the wage profile is linear in $N(\cdot)$. Since N is not linear in $h_{T,g}$ in general this implies that the wage profile is not linear in $h_{T,g}$ either.

Since teachers are compensated for the incremental human capital in "other", we can characterize the teacher's wage profile.

Proposition 5 (Teacher's wage profile) *For a given teacher with human capital $h_{T,g}$ and aggregate human capital in teaching $\widetilde{H}_T$, the wage in teaching is proportional to the number of students in the teacher's class and is characterized by:*

$$\omega(h_{T,g}, \widetilde{H}_T) = \frac{\overbrace{H'_O A'_O}^{\text{total output (next period)}}}{\underbrace{\int_0^\infty f_{O,g}(a)da}_{\substack{\text{fraction of prospective} \\ \text{production workers in class}}}} \underbrace{\frac{(h_{T,g})^{\frac{\beta}{\sigma}}}{\widetilde{H}_T}}_{\text{fraction of students taught}} \tag{39}$$

where

$$\widetilde{H}'_T = \sum_{g=1}^{G} \int_0^\infty h_{T,g}{}^{\frac{\beta}{\sigma}} f_{T,g}(a)da$$

is the $\left(\frac{\beta}{\sigma}\right)^{th}$ moment of the distribution of human capital among current teachers.

The second term in equation (39) captures the idea that a teacher's wage depends on class size, which is a non-linear function of her human capital $h_{T,g}$. Recall from equations (31) and (32) that a prospective teacher's optimal $s_{T,g}$ and $e_{T,g}$ depends on the quality of today's teachers (i.e. their human capital) and on the slope of the wage profile next period and hence on both today's $\widetilde{H}_T$ as well as tomorrow's $\widetilde{H}'_T$. [23]

$$\omega = \eta^{\frac{\eta}{1-\eta}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \frac{\sum_{g=1}^{G} A_O'^{\frac{1}{1-\eta}} \cdot \tau_{O,g}^{\frac{\eta}{1-\eta}} \cdot s_{O,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da}{\sum_{g=1}^{G} f_{O,g}(a)da}$$
$$\times \frac{\tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot a_T^{\frac{\alpha\beta}{\sigma-\eta\beta}}}{\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da} \tag{40}$$

A.4 Occupational Choice in Decentralized Competitive Equilibrium with Educational Barriers and Labor Market Discrimination

To highlight the effect of educational barriers and labor market discrimination we add group- and occupation-specific distortions to the model. This is mostly a heuristic exercise since parsimony of our state space enables us to characterize the full transition path from one set of distortions to another. We highlight this feature of the model here since there have arguably been secular changes in educational barriers and labor market distortions for groups other than white males. To fix ideas, however, we simply compare steady states associated with particular wedges τ indexed by ω (for labor market discrimination), e (for educational barriers), and g for groups (race, gender,...).

We can show that this economy has a steady state in H_T and we can characterize it in a single

[23]When the economy has $I > 2$ occupations, the teachers' wage profile is characterized by:

$$\omega = \eta^{\frac{\eta}{1-\eta}} \cdot \left(\frac{2\widetilde{H}_T}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \frac{\sum_{i=2}^{I}\sum_{g=1}^{G} A_i'^{\frac{1}{1-\eta}} \cdot \tau_{i,g}^{\frac{\eta}{1-\eta}} \cdot s_{i,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{i,g}(a)da}{\sum_{i=2}^{I}\sum_{g=1}^{G} f_{i,g}(a)da}$$
$$\times \frac{\tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot a_T^{\frac{\alpha\beta}{\sigma-\eta\beta}}}{\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da}$$

equation:[24]

$$\widetilde{H}_T = \Bigg[\left(\frac{\beta}{\sigma}\right)^{\eta} \cdot \eta^{\frac{1}{1-\eta}} \cdot \left(\frac{2}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \left(\frac{\sum_{g=1}^{G} A_i'^{\frac{1}{1-\eta}} \cdot \tau_{O,g}^{\frac{\eta}{1-\eta}} \cdot s_{O,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da}{\sum_{g=1}^{G} f_{O,g}(a)da} \right)^{\eta}$$
$$\times \left(\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da \right)^{\frac{\sigma-\eta\beta}{\beta}} \Bigg]^{\frac{\beta}{\sigma}\cdot\frac{1-\eta}{1-\eta-\beta}} \tag{41}$$

Proposition 6 (Occupational choice boundary) *The occupational choice boundary, denoted $\bar{a}_{T,g}(a_O)$, depends on the magnitude of the educational barriers and labor market frictions, but does not depend on either current or future aggregate human capital in teaching $\widetilde{H_T}$. It is characterized by:*[25]

$$\frac{a_O^{\frac{\alpha}{1-\eta}}}{a_T^{\frac{\alpha\beta}{\sigma-\eta\beta}}} \cdot \frac{\tau_{O,g}^{\frac{1}{1-\eta}}}{\tau_{T,g}^{\frac{\sigma}{\sigma-\eta\beta}}} \cdot \frac{1-\eta}{1-\frac{\beta\eta}{\sigma}} \cdot \frac{1+\tau_{O,g}^{e}}{1+\tau_{T,g}^{e}} \cdot \left(\frac{1-s_{O,g}}{1-s_{T,g}}\right)^{\frac{1}{\mu}} \cdot \frac{s_{O,g}^{\frac{\phi}{1-\eta}}}{s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}}}$$
$$= \frac{\sum_{g=1}^{G} A_O'^{\frac{1}{1-\eta}} \cdot \tau_{O,g}^{\frac{\eta}{1-\eta}} \cdot s_{O,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{O,g}(a)da}{A_O'^{\frac{1}{1-\eta}} \cdot \sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f_{T,g}(a)da \cdot \sum_{g=1}^{G} \int_0^\infty f_{O,g}(a)da} \tag{42}$$

[24]When the economy has $I > 2$ occupations, the steady state H_T is characterized by:

$$\widetilde{H}_T = \Bigg[\left(\frac{\beta}{\sigma}\right)^{\eta} \cdot \eta^{\frac{1}{1-\eta}} \cdot \left(\frac{2}{M}\right)^{\frac{\sigma}{1-\eta}} \cdot \left(\frac{\sum_{i=2}^{I}\sum_{g=1}^{G} A_i'^{\frac{1}{1-\eta}} \cdot \tau_{i,g}^{\frac{\eta}{1-\eta}} \cdot s_{i,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{i,g}(a)da}{\sum_{i=2}^{I}\sum_{g=1}^{G} f_{i,g}(a)da} \right)^{\eta}$$
$$\times \left(\sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{g,T}(a)da \right)^{\frac{\sigma-\eta\beta}{\beta}} \Bigg]^{\frac{\beta}{\sigma}\cdot\frac{1-\eta}{1-\eta-\beta}}$$

[25]When the economy has $I > 2$ occupations, the occupational choice boundary is characterized by:

$$\frac{a_i^{\frac{\alpha}{1-\eta}}}{a_T^{\frac{\alpha\beta}{\sigma-\eta\beta}}} \cdot \frac{\tau_{i,g}^{\frac{1}{1-\eta}}}{\tau_{T,g}^{\frac{\sigma}{\sigma-\eta\beta}}} \cdot \frac{1-\eta}{1-\frac{\beta\eta}{\sigma}} \cdot \frac{1+\tau_{i,g}^{e}}{1+\tau_{T,g}^{e}} \cdot \left(\frac{1-s_{i,g}}{1-s_{T,g}}\right)^{\frac{1}{\mu}} \cdot \frac{s_{i_g}^{\frac{\phi}{1-\eta}}}{s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}}}$$
$$= \frac{\sum_{i=2}^{I}\sum_{g=1}^{G} A_i'^{\frac{1}{1-\eta}} \cdot \tau_{i,g}^{\frac{\eta}{1-\eta}} \cdot s_{i,g}^{\frac{\phi}{1-\eta}} \cdot \int_0^\infty a^{\frac{\alpha}{1-\eta}} f_{i,g}(a)da}{A_i'^{\frac{1}{1-\eta}} \cdot \sum_{g=1}^{G} \tau_{T,g}^{\frac{\eta\beta}{\sigma-\eta\beta}} \cdot s_{T,g}^{\frac{\phi\beta}{\sigma-\eta\beta}} \cdot \int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f_{T,g}(a)da \cdot \sum_{i=2}^{I}\sum_{g=1}^{G} \int_0^\infty f_{i,g}(a)da}$$