The Allocation of Teaching Talent and Human Capital Accumulation

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Model

Endowments, Preferences

- Each period, a measure M of agents is born and lives for two periods ("young" and "old")
- Individuals born with occupation-specific abilities drawn from a joint bivariate Fréchet distribution with c.d.f.

$$F(a^T, a^O) = \exp\left[-a_T^{-\theta} - a_O^{-\theta}\right],$$

- Individuals have $\log$ preferences over leisure and consumption (no discounting)

Model

Technologies

- Children ("young") make occupation-specific educational investments (in units of time and output)
- Adults work as teachers or production workers.
- Technologies are occupation-specific:

Human capital production (teaching) depends on teacher's h^T, child's ability a, and child's educational investments (time s and goods e) according to:

$$h'(a) = \left(h^T\right)^{\beta} a^{\alpha} s\,(a)^{\phi}\, e(a)^{\eta}$$

Final output production depends on adult worker's human capital h^O and exogenous productivity A^O:

$$y = A^O h^O$$

Model

Values

$$V^g(a^T, a^O, \widetilde{H}^T) = \max_{\{s^O, s^T, e^O, e^T\}} \left\{ V^{O,g}(a^O, \widetilde{H}^T), V^{T,g}(a^T, \widetilde{H}^T) \right\}$$

where

$$\begin{aligned}
V^{O,g}(a^O, \widetilde{H}^T) &= \ln\left(1 - s^O\left(a^O, \widetilde{H}^T\right)\right) \\
&+ \mu \ln \left[h'^O A'^O (1 - \tau'^{O,g,w}) \color{red}{(1-t)} \right. \\
&\left. - e^O(a^O, \widetilde{H}^T)(1 + \tau^{O,g,e}) \right], \\
V^{T,g}(a^T, \widetilde{H}^T) &= \ln\left(1 - s^T\left(a^T, \widetilde{H}^T\right)\right) \\
&+ \mu \ln \left[\omega'(h'^T, \widetilde{H'}^T)(1 - \tau'^{T,g,w}) \color{red}{(1-t)} \right. \\
&\left. - e^T(a^T, \widetilde{H}^T)(1 + \tau^{T,g,e}) \right]
\end{aligned}$$

Model (cont'd)

Constraints, Laws of Motion

$$T = \int_0^\infty \omega\left(h^{T'}(a)\right) f'^{T}(a)da$$

$$f^T(a) = \int_0^{\bar{a}^{-1}(a)} f\left(a,b\right)db$$

$$f^O(b) = \int_0^{\bar{a}(b)} f\left(a,b\right)da$$

Aggregate laws of motion for $\widetilde{H}^T$ and H^O:

$$\widetilde{H'}^T = \int_0^\infty \left(\left(\frac{2\widetilde{H}^T}{M}\right)^\sigma a^\alpha s^T\left(a,\widetilde{H}^T\right)^\phi e^T(a,\widetilde{H}^T)^\eta\right)^{\frac{\beta}{\sigma}} f^T(a)da$$

$$H'^O = \int_0^\infty \left(\frac{2\widetilde{H}^T}{M}\right)^\sigma a^\alpha s^O\left(a,\widetilde{H}^T\right)^\phi e^O(a,\widetilde{H}^T)^\eta f^O(a)da$$

Model (cont'd)

Occupational Threshold

$$a^{T*}(a^O) = \bar{a}\big(a^O, \widetilde{H}^T\big)$$

such that

$$V^O(a^O, \widetilde{H}^T) = V^T\left(a^{T*}(a^O), \widetilde{H}^T\right), \text{ for all } a^O \in (0, \infty)$$

Model (cont'd)

- Assignment of students to teachers is random
  $\Rightarrow$ distribution of students' skill identical across classrooms

- Teachers with different h^T vary with respect to class *size*

- $\omega(\cdot,\cdot)$ is proportional to the *number of students* in a teacher's class and to :

$$
\begin{aligned}
\omega(h^T, \widetilde{H}^T) &= \lambda N(h^T, \widetilde{H}^T) \\
&= \frac{H'^O A'^O}{\frac{M}{2}\int_o^\infty f^O(a)da} \underbrace{\overbrace{N(1, \widetilde{H}^T)}^{=\frac{M}{2}\frac{1}{\widetilde{H}^T}} \left(h^T\right)^{\frac{\beta}{\sigma}}}_{=N(h^T, \widetilde{H}^T)} \\
&= \frac{H'^O A'^O}{\int_o^\infty f^O(a)da} \frac{\left(h^T\right)^{\frac{\beta}{\sigma}}}{\widetilde{H}^T}
\end{aligned}
$$

Optimal Investments for Prospective Teachers

$$s^T = \frac{\mu\phi}{\mu\phi + \frac{\beta}{\sigma} - \eta}$$

$$e^T = \left(\frac{(1-t)(1-\tau'^{T,w}) \left(\frac{2}{M}\right)^{\beta} A'^{O} (s^T)^{\frac{\phi\beta}{\sigma}} \left(\frac{\eta\beta}{\sigma}\right) \left(\widetilde{H}^T\right)^{\beta} (a^T)^{\frac{\alpha\beta}{\sigma}}}{1+\tau^{T,e}} \right)^{\frac{1}{1-\frac{\eta\beta}{\sigma}}}$$

$$\times \left(\frac{H'^{O}}{\int_o^{\infty} f^O(a)da} \left(\widetilde{H'}^{T}\right)^{-1} \right)^{\frac{1}{1-\frac{\eta\beta}{\sigma}}}$$

Aggregate Laws of Motion

$$\widetilde{H'}^{T} = \Bigg[\left(\frac{(1-\tau'^{T,w}) \left(\frac{\eta\beta}{\sigma}\right)}{1+\tau^{T,e}} \right)^{\eta} \left(\frac{(1-\tau'^{O,w})\eta}{1+\tau^{O,e}} \right)^{\frac{\eta}{1-\eta}\eta} \left((1-t)A'^{O} \right)^{\frac{\eta}{1-\eta}}$$

$$\times \left(\frac{2}{M} \right)^{\frac{\sigma}{1-\eta}} (s^{T})^{\phi} (s^{O})^{\frac{\eta}{1-\eta}\phi} \left(\frac{\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^{O}(a)da}{\int_0^\infty f^{O}(a)da} \right)^{\eta}$$

$$\times \left(\int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^{T}(a)da \right)^{\frac{\sigma-\eta\beta}{\beta}} \left(\widetilde{H}^{T} \right)^{\frac{\sigma}{1-\eta}} \Bigg]^{\frac{\beta}{\sigma}}$$

$$H'^{O} = \left(\frac{(1-t)(1-\tau'^{O,w})A'^{O}\eta}{1+\tau^{O,e}} \right)^{\frac{\eta}{1-\eta}} \left(\frac{2}{M} \right)^{\frac{\sigma}{1-\eta}} (s^{O})^{\frac{\phi}{1-\eta}}$$

$$\times \left(\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^{O}(a)da \right) \left(\widetilde{H}^{T} \right)^{\frac{\sigma}{1-\eta}}$$

Optimal Investments for Prospective Teachers

$$s^T = \frac{\mu\phi}{\mu\phi + \frac{\beta}{\sigma} - \eta}$$

$$\begin{aligned} e^T = &\left((1-t)\left(\tfrac{2}{M}\right)^{\sigma} A'^{O} \eta (s^O)^{\phi} \left(\widetilde{H}^T\right)^{\sigma} \left(a^T\right)^{\alpha}\right)^{\frac{1}{1-\eta}} \\ &\times \frac{(1-\tau'^{T,w})(1-\tau'^{O,w})^{\frac{\eta}{1-\eta}}}{(1+\tau^{T,e})(1+\tau^{O,e})^{\frac{\eta}{1-\eta}}} \cdot \frac{\beta}{\sigma} \\ &\times \left(\frac{\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^O(a)da}{\int_0^\infty f^O(a)da}\right) \left(\int_0^\infty a^{\frac{\alpha}{\frac{\sigma}{\beta}-\eta}} f^T(a)da\right)^{-1} \end{aligned}$$

Optimal Investments for Prospective "Other" Workers

$$s^O = \frac{\mu\phi}{\mu\phi + 1 - \eta}$$

$$e^O = \left(\frac{(1-t)(1-\tau'^{O,w}) \left(\frac{2}{M}\right)^{\sigma} A'^{O} (s^O)^{\phi} \eta \left(\widetilde{H}^T\right)^{\sigma} \left(a^O\right)^{\alpha}}{1+\tau^{O,e}} \right)^{\frac{1}{1-\eta}}$$

Some Parameter Restrictions

- $\beta < 1 - \eta$ to guarantee existence of stable $\widetilde{H^T} = \widetilde{H^T}' > 0$
- $\frac{\sigma}{\beta} > \eta$ and $\mu\phi > 0$ for $s^{T*} \in (0, 1)$
- $1 > \eta$ and $\mu\phi > 0$ for $s^{O*} \in (0, 1)$

Occupational Choice Boundary...

... does not depend on aggregate state $\widetilde{H^T}$

$$\left(\frac{1-\tau'^{O,w}}{1-\tau'^{T,w}}\right)\left(\frac{1-\eta}{1-\frac{\eta\beta}{\sigma}}\right)\left(\frac{1-s^O}{1-s^T}\right)^{\frac{1}{\mu}}\left(\frac{(a^O)^{\frac{\alpha}{1-\eta}}}{\left(\bar{a}^T(a^O)\right)^{\frac{\alpha\beta}{\sigma-\eta\beta}}}\right)$$
$$=\left(\frac{\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^O(a)da}{\int_0^\infty f^O(a)da}\right)\left(\int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^T(a)da\right)^{-1}$$

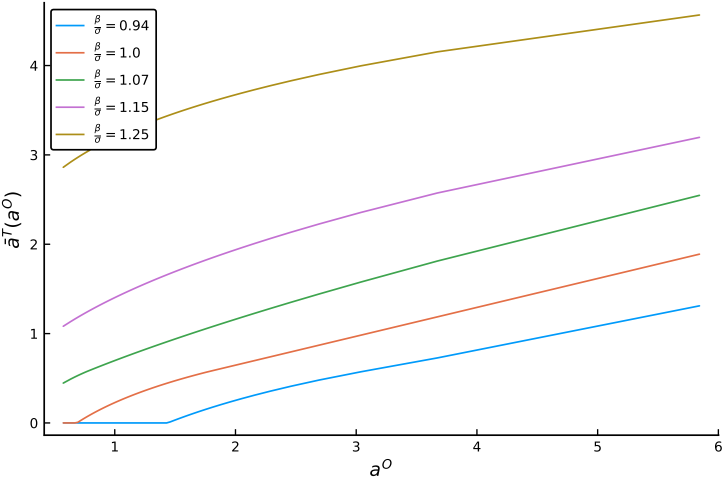
Equilibrium

The equilibrium is characterized by:

1. the optimal investment for prospective teachers
2. the optimal investment for prospective "other" workers
3. the aggregate laws of motion
4. the occupational choice boundary and the corresponding densities f^T and f^O

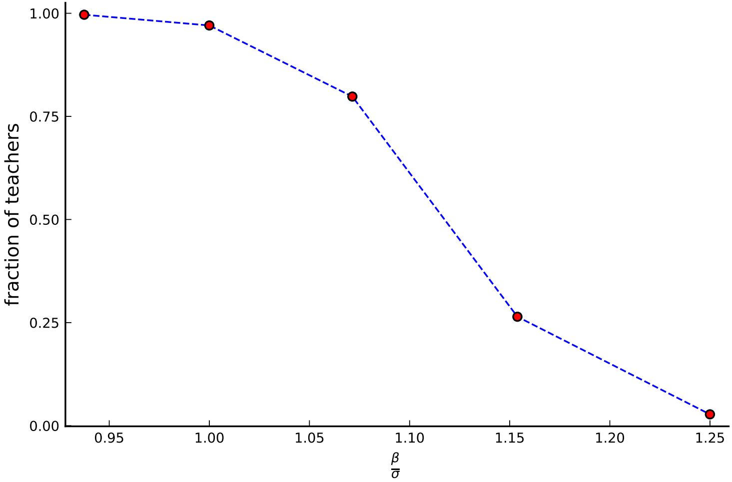
Occupational Threshold

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



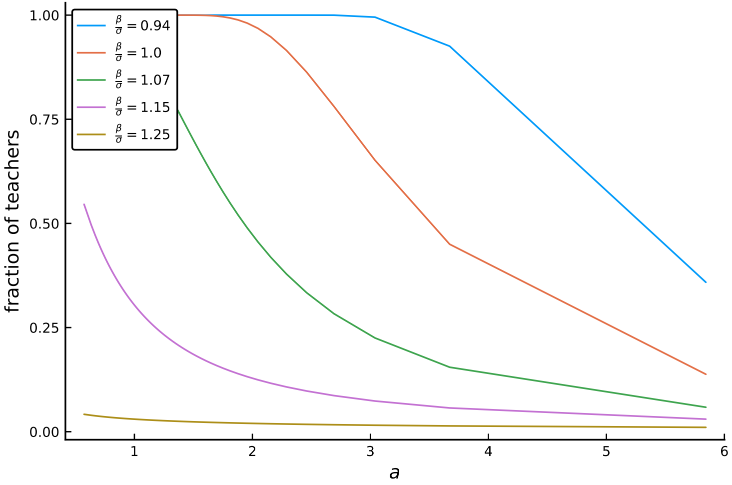
Occupational Choice (Aggregate)

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



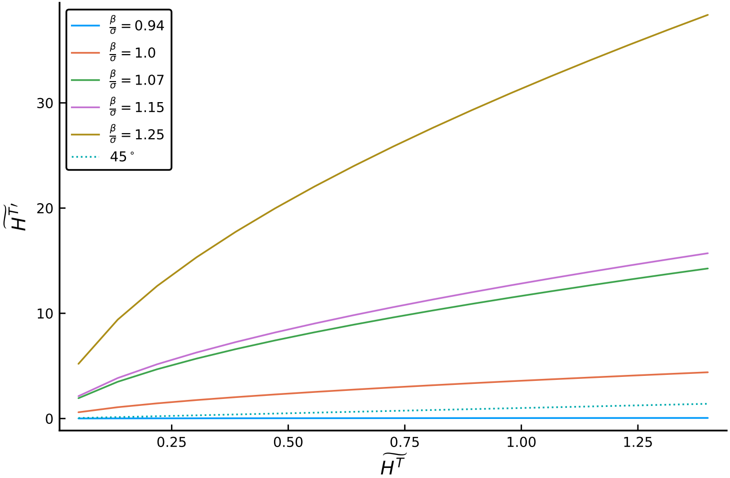
Occupational Split

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



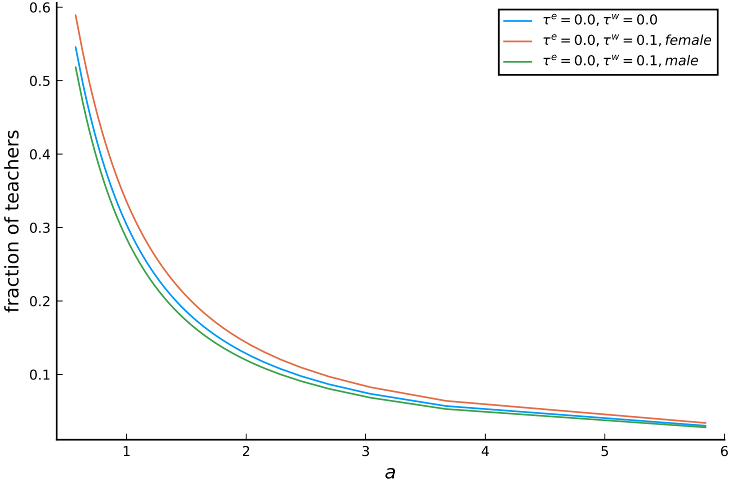
Aggregate Law of Motion for $\widetilde{H^T}$

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



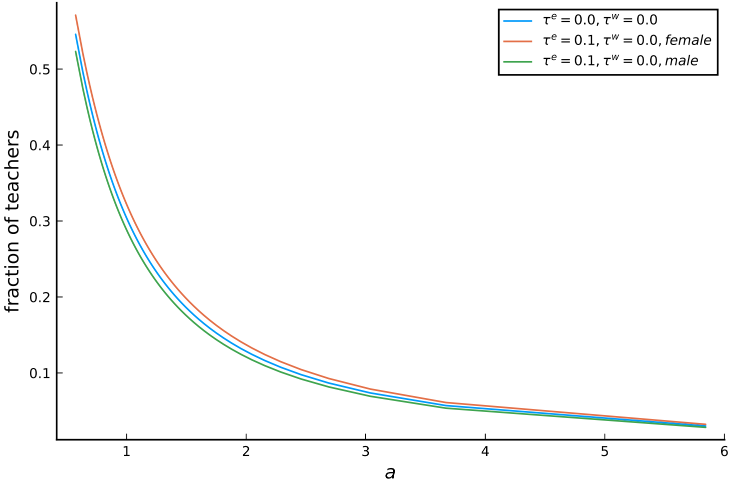
Effect of τ^w on Occupational Choice

$\frac{\beta}{\sigma} = 1.15$



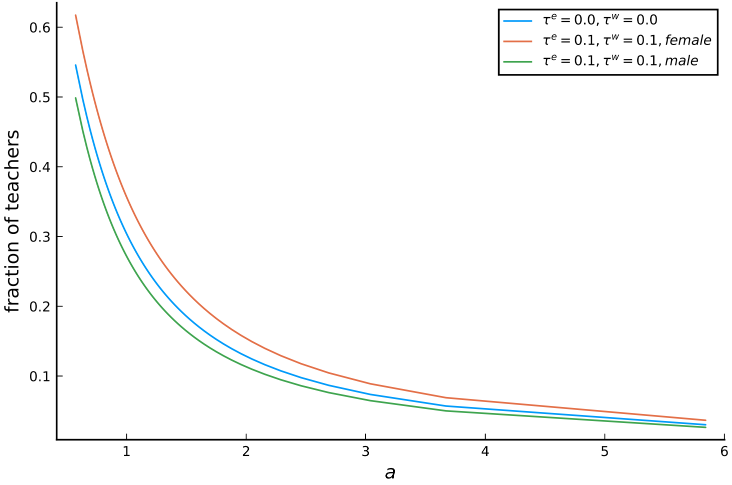
Effect of τ^e on Occupational Choice

$\frac{\beta}{\sigma} = 1.15$



Combined Effect of τ^w and τ^e on Occupational Choice

$\frac{\beta}{\sigma} = 1.15$



Extensions

- Multiple locations:
  - finite number of locations, use Lucas & Moll (2014) and Martellini (2019) to solve
  - teachers' salaries funded by local lump-sum taxes
  - sorting of teachers and children into locations (high wage = high tax)
  - not sure if variation in lump-sum tax is sufficient to prevent everyone from locating in a single location, may need to think of additional congestion forces
  - to start with, solve with two locations ("high vs. low")