DETERMINATION OF CLASS SIZE

Indifference condition for students (in terms of class size and teacher's human capital):

$$N(h^T) = N(\hat{h}^T)\left(\frac{h^T}{\hat{h}^T}\right)^{\beta/\sigma}$$

Integration over class size must satisfy:

$$\frac{M}{2} = \int N(h^T)\, dF(h^T),$$ where $F(\cdot)$ is the C.D.F. of human capital in teaching.

Let $\hat{h}^T \equiv \tilde{H}^T$, i.e., let $\hat{h}^T$ be an "average" teacher in the sense that her human capital equals the β/σ th moment of the distribution.

Then,

$$\frac{M}{2} = \int N(\tilde{H}^T)\left(\frac{h^T}{\tilde{H}^T}\right)^{\beta/\sigma} dF(h^T)$$

$$= N(\tilde{H}^T)(\tilde{H}^T)^{-\beta/\sigma}\underbrace{\int (h^T)^{\beta/\sigma} dF(h^T)}_{=\tilde{H}^T}$$

$$= N(\tilde{H}^T)(\tilde{H}^T)^{\frac{\sigma-\beta}{\sigma}}$$

$$\Longleftrightarrow N(\tilde{H}^T) = \frac{M}{2}(\tilde{H}^T)^{\frac{\beta-\sigma}{\sigma}}$$

$$N(h^T) = N(\tilde{H}^T)\left(\frac{h^T}{\tilde{H}^T}\right)^{\beta/\sigma}$$

$$= \frac{M}{2}\left(\tilde{H}^T\right)^{\frac{\beta-\sigma}{\sigma}-\frac{\beta}{\sigma}}\left(h^T\right)^{\beta/\sigma}$$

$$= \frac{M}{2}\left(\tilde{H}^T\right)^{-1}\left(h^T\right)^{\beta/\sigma}$$

$$\frac{2\tilde{H}^T}{M} = \left(h^T\right)^{\beta/\sigma} N\left(h^T\right)^{-1}$$

$$\left(\frac{2\tilde{H}^T}{M}\right)^{\sigma} = \left(h^T\right)^{\beta} N\left(h^T\right)^{-\sigma}$$