

The Allocation of Teaching Talent and Human Capital Accumulation

Simeon Alder¹ Yulia Dudareva¹ Ananth Seshadri¹

¹University of Wisconsin–Madison

Model

Endowments, Preferences

- ▶ Each period, a measure M of agents is born and lives for two periods (“young” and “old”)
- ▶ Individuals born with occupation-specific abilities drawn from a joint bivariate Fréchet distribution with c.d.f.

$$F(a^T, a^O) = \exp \left[-a_T^{-\theta} - a_O^{-\theta} \right],$$

- ▶ Individuals have log preferences over leisure and consumption (no discounting)

Model

Technologies

- ▶ Children (“young”) make occupation-specific educational investments (in units of time and output)
- ▶ Adults work as **teachers** or **production workers**.
- ▶ Technologies are occupation-specific:

Human capital production (teaching) depends on teacher's h^T , child's ability a , and child's educational investments (time s and goods e) according to:

$$h'(a) = (h^T)^\beta a^\alpha s(a)^\phi e(a)^\eta$$

Final output production depends on adult worker's human capital h^O and exogenous productivity A^O :

$$y = A^O h^O$$

Model

Values

$$V^g(a^T, a^O, \tilde{H}^T) = \max_{\{s^O, s^T, e^O, e^T\}} \left\{ V^{O,g}(a^O, \tilde{H}^T), V^{T,g}(a^T, \tilde{H}^T) \right\}$$

where

$$\begin{aligned} V^{O,g}(a^O, \tilde{H}^T) &= \ln \left(1 - s^O(a^O, \tilde{H}^T) \right) \\ &\quad + \mu \ln \left[h'^O A'^O (1 - \tau'^{O,g,w}) (1 - t) \right. \\ &\quad \left. - e^O(a^O, \tilde{H}^T) (1 + \tau^{O,g,e}) \right], \\ V^{T,g}(a^T, \tilde{H}^T) &= \ln \left(1 - s^T(a^T, \tilde{H}^T) \right) \\ &\quad + \mu \ln \left[\omega'(h'^T, \tilde{H}'^T) (1 - \tau'^{T,g,w}) (1 - t) \right. \\ &\quad \left. - e^T(a^T, \tilde{H}^T) (1 + \tau^{T,g,e}) \right] \end{aligned}$$

Model (cont'd)

Constraints, Laws of Motion

$$T = \int_0^\infty \omega \left(h^{T'}(a) \right) f'^T(a) da$$

$$f^T(a) = \int_0^{\bar{a}^{-1}(a)} f(a, b) db$$

$$f^O(b) = \int_0^{\bar{a}(b)} f(a, b) da$$

Aggregate laws of motion for \tilde{H}^T and H^O :

$$\widetilde{H'}^T = \int_0^\infty \left(\left(\frac{2\tilde{H}^T}{M} \right)^\sigma a^\alpha s^T(a, \tilde{H}^T)^\phi e^T(a, \tilde{H}^T)^\eta \right)^{\frac{\beta}{\sigma}} f^T(a) da$$

$$H'^O = \int_0^\infty \left(\frac{2\tilde{H}^T}{M} \right)^\sigma a^\alpha s^O(a, \tilde{H}^T)^\phi e^O(a, \tilde{H}^T)^\eta f^O(a) da$$

Model (cont'd)

Occupational Threshold

$$a^{T*}(a^O) = \bar{a}(a^O, \tilde{H}^T)$$

such that

$$V^O(a^O, \tilde{H}^T) = V^T(a^{T*}(a^O), \tilde{H}^T), \text{ for all } a^O \in (0, \infty)$$

Model (cont'd)

- ▶ Assignment of students to teachers is random
⇒ distribution of students' skill identical across classrooms
- ▶ Teachers with different h^T vary with respect to class size
- ▶ $\omega(\cdot, \cdot)$ is proportional to the *number of students* in a teacher's class and to :

$$\begin{aligned}\omega(h^T, \tilde{H}^T) &= \lambda N(h^T, \tilde{H}^T) \\ &= \frac{H'^O A'^O}{\frac{M}{2} \int_o^\infty f^O(a) da} \underbrace{N(1, \tilde{H}^T)}_{=N(h^T, \tilde{H}^T)} \overbrace{\left(h^T\right)^{\frac{\beta}{\sigma}}}^{=\frac{M}{2} \frac{1}{\tilde{H}^T}} \\ &= \frac{H'^O A'^O}{\int_o^\infty f^O(a) da} \frac{\left(h^T\right)^{\frac{\beta}{\sigma}}}{\tilde{H}^T}\end{aligned}$$

Optimal Investments for Prospective Teachers

$$\begin{aligned}
 s^T &= \frac{\mu\phi}{\mu\phi + \frac{\beta}{\sigma} - \eta} \\
 e^T &= \left(\frac{(1-t)(1-\tau'^{T,w}) \left(\frac{2}{M}\right)^\beta A'^O (s^T)^{\frac{\phi\beta}{\sigma}} \left(\frac{\eta\beta}{\sigma}\right) \left(\tilde{H}^T\right)^\beta (a^T)^{\frac{\alpha\beta}{\sigma}}}{1 + \tau^{T,e}} \right)^{\frac{1}{1-\frac{\eta\beta}{\sigma}}} \\
 &\quad \times \left(\frac{H'^O}{\int_o^\infty f^O(a) da} \left(\widetilde{H'}^T\right)^{-1} \right)^{\frac{1}{1-\frac{\eta\beta}{\sigma}}}
 \end{aligned}$$

Aggregate Laws of Motion

$$\begin{aligned}
 \widetilde{H}'^T &= \left[\left(\frac{(1 - \tau'^{T,w}) \left(\frac{\eta\beta}{\sigma} \right)}{1 + \tau^{T,e}} \right)^\eta \left(\frac{(1 - \tau'^{O,w})\eta}{1 + \tau^{O,e}} \right)^{\frac{\eta}{1-\eta}} \left((1-t)A'^O \right)^{\frac{\eta}{1-\eta}} \right. \\
 &\quad \times \left(\frac{2}{M} \right)^{\frac{\sigma}{1-\eta}} (s^T)^\phi (s^O)^{\frac{\eta}{1-\eta}\phi} \left(\frac{\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^O(a) da}{\int_0^\infty f^O(a) da} \right)^\eta \\
 &\quad \times \left(\int_0^\infty a^{\frac{\alpha\beta}{\sigma-\eta\beta}} f^T(a) da \right)^{\frac{\sigma-\eta\beta}{\beta}} \left(\widetilde{H}^T \right)^{\frac{\sigma}{1-\eta}} \Big]^\beta \\
 H'^O &= \left(\frac{(1-t)(1 - \tau'^{O,w})A'^O\eta}{1 + \tau^{O,e}} \right)^{\frac{\eta}{1-\eta}} \left(\frac{2}{M} \right)^{\frac{\sigma}{1-\eta}} (s^O)^{\frac{\phi}{1-\eta}} \\
 &\quad \times \left(\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^O(a) da \right) \left(\widetilde{H}^T \right)^{\frac{\sigma}{1-\eta}}
 \end{aligned}$$

Optimal Investments for Prospective Teachers

$$\begin{aligned}
 s^T &= \frac{\mu\phi}{\mu\phi + \frac{\beta}{\sigma} - \eta} \\
 e^T &= \left((1-t) \left(\frac{2}{M}\right)^\sigma A'^O \eta (s^O)^\phi \left(\tilde{H}^T\right)^\sigma (a^T)^\alpha \right)^{\frac{1}{1-\eta}} \\
 &\quad \times \frac{(1 - \tau'^{T,w})(1 - \tau'^{O,w})^{\frac{\eta}{1-\eta}}}{(1 + \tau^{T,e})(1 + \tau^{O,e})^{\frac{\eta}{1-\eta}}} \cdot \frac{\beta}{\sigma} \\
 &\quad \times \left(\frac{\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^O(a) da}{\int_0^\infty f^O(a) da} \right) \left(\int_0^\infty a^{\frac{\alpha}{\beta-\eta}} f^T(a) da \right)^{-1}
 \end{aligned}$$

Optimal Investments for Prospective “Other” Workers

$$s^O = \frac{\mu\phi}{\mu\phi + 1 - \eta}$$

$$e^O = \left(\frac{(1-t)(1-\tau'^{O,w}) \left(\frac{2}{M}\right)^\sigma A'^O (s^O)^{\phi\eta} \left(\tilde{H}^T\right)^\sigma (a^O)^\alpha}{1 + \tau^{O,e}} \right)^{\frac{1}{1-\eta}}$$

Some Parameter Restrictions

- ▶ $\beta < 1 - \eta$ to guarantee existence of stable $\widetilde{H^T} = \widetilde{H^{T'}} > 0$
- ▶ $\frac{\sigma}{\beta} > \eta$ and $\mu\phi > 0$ for $s^{T*} \in (0, 1)$
- ▶ $1 > \eta$ and $\mu\phi > 0$ for $s^{O*} \in (0, 1)$

Occupational Choice Boundary...

...does not depend on aggregate state $\widetilde{H^T}$

$$\begin{aligned} & \left(\frac{1 - \tau'^{O,w}}{1 - \tau'^{T,w}} \right) \left(\frac{1 - \eta}{1 - \frac{\eta\beta}{\sigma}} \right) \left(\frac{1 - s^O}{1 - s^T} \right)^{\frac{1}{\mu}} \left(\frac{(a^O)^{\frac{\alpha}{1-\eta}}}{(\bar{a}^T(a^O))^{\frac{\frac{\sigma}{\beta}-\eta}{\beta}}}} \right) \\ &= \left(\frac{\int_0^\infty a^{\frac{\alpha}{1-\eta}} f^O(a) da}{\int_0^\infty f^O(a) da} \right) \left(\int_0^\infty a^{\frac{\frac{\sigma}{\beta}-\eta}{\beta}} f^T(a) da \right)^{-1} \end{aligned}$$

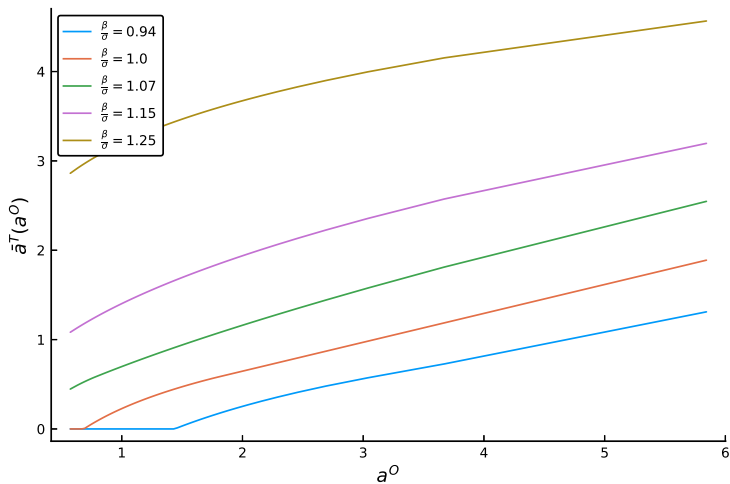
Equilibrium

The equilibrium is characterized by:

1. the optimal investment for prospective teachers
2. the optimal investment for prospective “other” workers
3. the aggregate laws of motion
4. the occupational choice boundary and the corresponding densities f^T and f^O

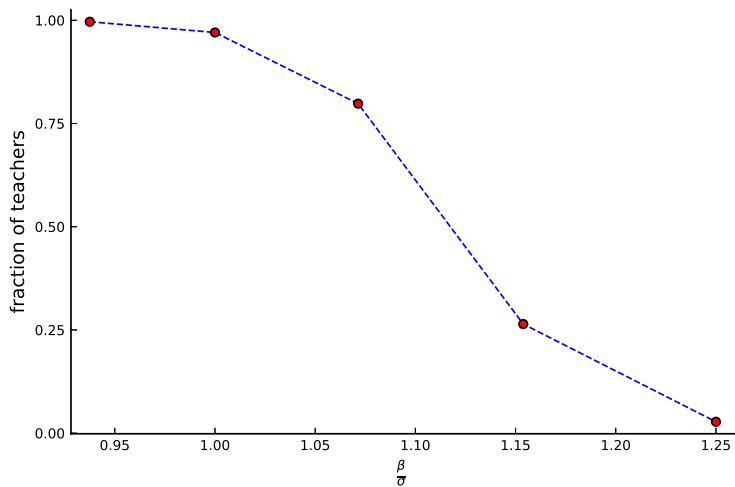
Occupational Threshold

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



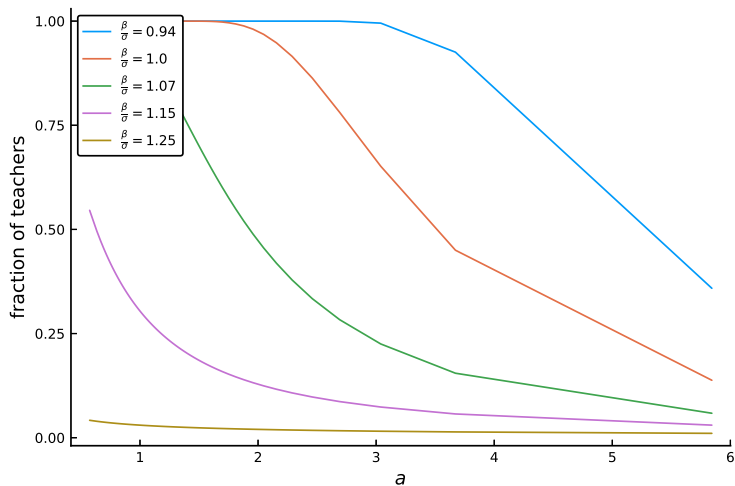
Occupational Choice (Aggregate)

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



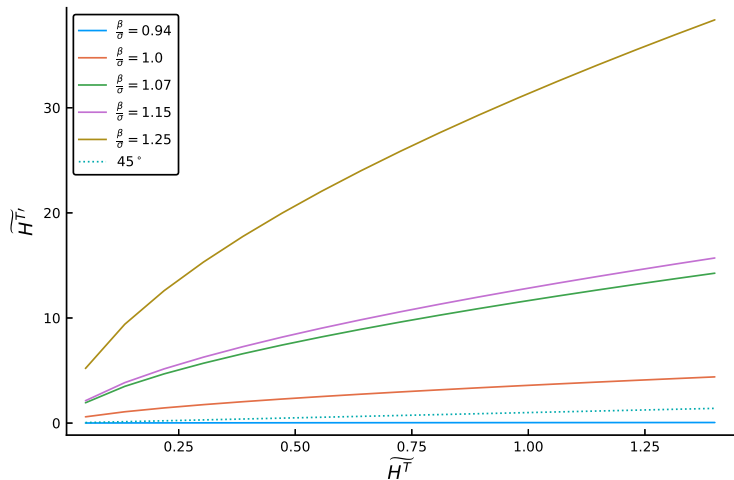
Occupational Split

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



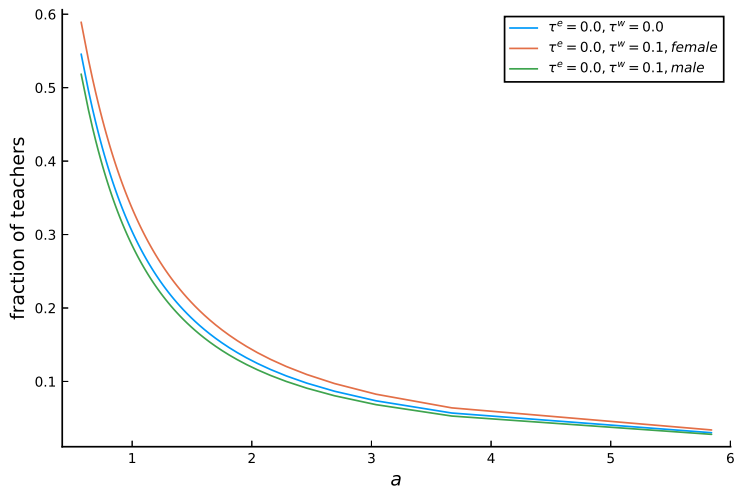
Aggregate Law of Motion for \widetilde{H}^T

Variation in Strength of Superstar Effect in Teaching: $\frac{\beta}{\sigma}$



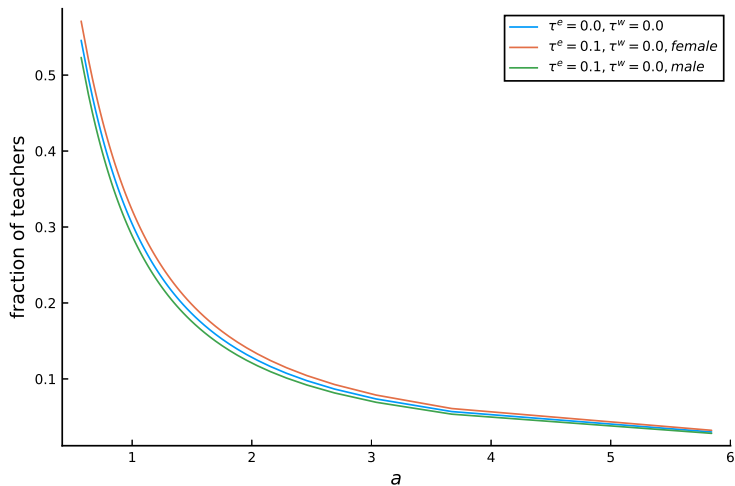
Effect of τ^w on Occupational Choice

$$\frac{\beta}{\sigma} = 1.15$$



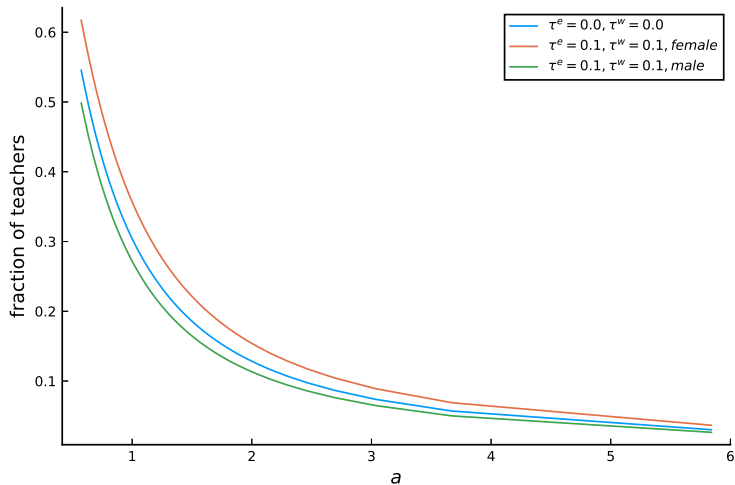
Effect of τ^e on Occupational Choice

$$\frac{\beta}{\sigma} = 1.15$$



Combined Effect of τ^w and τ^e on Occupational Choice

$$\frac{\beta}{\sigma} = 1.15$$



Extensions

- ▶ Multiple locations:
 - ▶ finite number of locations
 - ▶ teachers' salaries funded by combination of local lump-sum (property) and proportional (sales) taxes
 - ▶ one location with $T = 0$, all locations have $t \in (0, 1)$
 - ▶ sorting of teachers and children into locations (high wage = high tax)
 - ▶ sorting breaks global monotonicity between teacher's h^T and class size (still holds locally, with random within-district assignment)
 - ▶ to start with, solve with two locations ("high vs. low")