1、 chance of having: full-house $C^{13}_1 \cdot C^4_3 \cdot C^{12}_1 \cdot C^4_2 / C^{52}_5 = 3744/2598960$

$= 6/4165 \doteqdot 0.001441$

choose a number and pick 3 of'em

another number pick 2 of'em

Flush $C^4_1 \cdot (C^{13}_5 - 10) / C^{52}_5 = 5108/2598960$

$\doteqdot 0.001965$

pick a suit

select 5 out of 13 and minus 10 combinations of straight flush

prob to get full-house out of 5 cards is approximately 0.00144, it is even rarer then flush's prob of approximately 0.001965.

2. X, Y represents the winning times when one of E and W hits 4 wins, N is the # of games played.

if E wins the series

N	X, Y	P(X, Y)	combination
4	4, 0	p^4	xxxx
5	4, 1	$C^4_1 \cdot p^4 (1-p)$	(3x1y) x
6	4, 2	$C^5_2 \cdot p^4 \cdot (1-p)^2$	(3x2y) x
7	4, 3	$C^6_3 \cdot p^4 \cdot (1-p)^3$	(3x3y) x

combination of x and y but the last game must be x

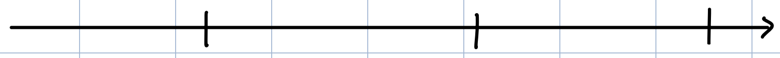
if W wins

N	X, Y	P(X, Y)	combination
4	(0, 4)	$(1-p)^4$	yyyy
5	(1, 4)	$C^4_1 (1-p)^4 \cdot p$	(3y1x) y
6	(2, 4)	$C^5_2 \cdot (1-p)^4 \cdot p^2$	(3y2x) y
7	(3, 4)	$C^6_3 \cdot (1-p)^4 \cdot p^3$	(3y3x) y

combination of x and y but the last game must be y

$$E(N) = 4\cdot[p^4 + (1-p)^4] + 5\cdot C^4_1 [p^4 (1-p) + (1-p)^4 p] + 6\cdot C^5_2 [p^4 (1-p)^2 + (1-p)^4 p^2]$$
$$+ 7\cdot C^6_3 [p^4 (1-p)^3 + (1-p)^4 p^3]$$

$$E(N|p=\tfrac{1}{2}) = 4\cdot(\tfrac{1}{16} + \tfrac{1}{16}) + 5\cdot 4\cdot(\tfrac{1}{32} + \tfrac{1}{32}) + 6\cdot 10\cdot(\tfrac{1}{64} + \tfrac{1}{64}) + 7\cdot 20 + (\tfrac{1}{128} + \tfrac{1}{128})$$
$$= \frac{8}{16} + \frac{40}{32} + \frac{120}{64} + \frac{280}{128} = \frac{93}{16} = 5.8125 \#$$

3. 

Let the arrival time follows uniform distribution

$X \sim U(0,80)$ and the show starts at 80.

If x is between $60\sim80$, I will witness the show.

$P(X \text{ between } 60\sim80) = \frac{1}{4}$ #

4. clerk 1 service time $\equiv X \sim exp(u_1)$

clerk 2 service time $\equiv Y \sim exp(u_2)$ $\perp$

$f_X(x) = \frac{1}{u_1} \cdot e^{-x/u_1}, \; x \geq 0$

$f_Y(y) = \frac{1}{u_2} \cdot e^{-y/u_2}, \; y \geq 0$

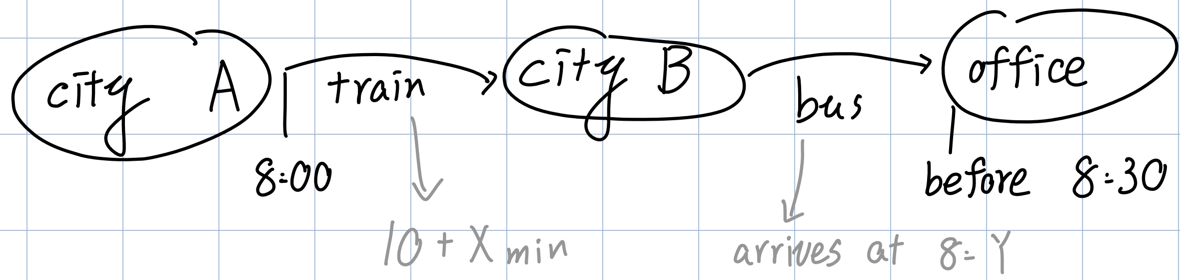
a. $P(X \geq 10) = e^{-10/u_1}$

b. by $P(X > s+t \mid X > s) = P(X > t)$

$$P(X > Y) = \int_0^\infty e^{-y/u_1} \cdot \frac{1}{u_2} e^{-y/u_2} \, dy = \int_0^\infty \frac{1}{u_2} \cdot e^{-y\left(\frac{1}{u_1} + \frac{1}{u_2}\right)} \, dy$$

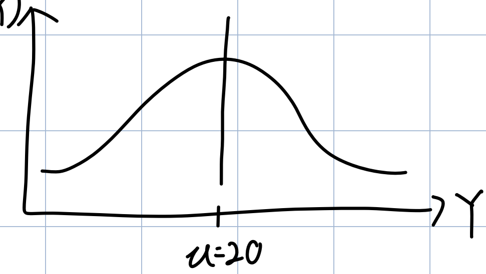
$$= \frac{1}{u_2} \cdot \left(\frac{1}{\frac{1}{u_1} + \frac{1}{u_2}} \right) = \frac{\frac{1}{u_2}}{\frac{1}{u_1} + \frac{1}{u_2}} = \frac{u_1}{u_2 + u_1}$$ #

5、



let the delay time of train $\equiv X$

let the arrival time of bus $\equiv Y \sim N(20, 4)$



X	4	6	8	10	12
P(X)	1/16	1/8	1/2	1/4	1/16

given $X=4$, probability of being late is $P(Y<14)$, $P(Y>20)$

$X=6$, 〃 $P(Y<16)$, $P(Y>20)$

$X=8$, 〃 $P(Y<18)$, $P(Y>20)$

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given $X=10$, $X=12$ will be late no matter what.

can catch up only if Y is exact 20,

and the $P(Y=20)=0$

$$P(\text{late}) = P(X=4)\cdot P(Y<14, Y>20) + P(X=6)\cdot P(Y<16, Y>20)$$
$$+ P(X=8)\cdot P(Y<18, Y>20) + P(X=10) + P(X=12)$$
$$= 1/16\cdot(0.0013+0.5) + 1/8\,(0.0228+0.5) + 1/2\cdot(0.1587+0.5) + 1/4 + 1/16$$
$$= 0.73853125 \#$$

6. Let get cancer $\equiv A$, $P(A)=0.01$

X-ray tested positive $\equiv B$, MRI tested positive $\equiv C$,

sensitivity $=0.99$, $P(B|A)=0.99$, $P(B^c|A)=0.01$

$P(A\cap B)/P(A)=0.99$

$P(A\cap B)=0.99\cdot 0.01=0.0099$

specificity $=0.99$, $P(B^c|A^c)=0.99$, $P(B|A^c)=0.01$

(a) $P(A|B)=P(A\cap B)/P(B) = 0.0099/0.0198 = 1/2$

$P(A\cap B)=0.0099$

$P(B)=P(B|A)\cdot P(A)+P(B|A^c)\cdot P(A^c)$

$= 0.99\cdot 0.01+0.01\cdot 0.99= 0.0198$

(b)

$P(A|B.C)$

sensitivity: 0.999

0.00999

$$= \frac{P(B.C|A)\cdot P(A)}{P(B.C)}$$

$$= \frac{P(B|A)\cdot P(C|A)\cdot P(A)}{0.99\cdot 0.999\cdot 0.01+0.01\cdot 0.001\cdot 0.99} = \frac{0.99\cdot 0.999\cdot 0.01}{0.0099099} \fallingdotseq 0.999 \#$$

7. 25 people

(a) all birthdays are different

$$\frac{365}{365}\cdot\frac{364}{365}\cdots\frac{341}{365} = \prod_{k=0}^{24}\frac{(365-k)}{k} \doteqdot e^{-S}$$

$$S \doteqdot \sum_{k=1}^{25}\frac{k}{365} = (1+25)\cdot 25/2 \cdot \frac{1}{365} \doteqdot 0.445$$

$$e^{-0.445} \doteqdot 0.431 \#$$

(b)

$$1-0.431 = 0.569 \#$$

8.

$H_0: p=0.4$ vs $H_1: p\neq 0.4$

r.v $X \equiv$ numbers of user in sample

$X \sim Bin(n=600, p=0.4)$

$X \approx N(\mu=np=240, \sigma^2=np(1-p)=144)$

$Z = \frac{X-\mu}{\sqrt{\mu}} = \frac{X-240}{12}$, $X=264$, $Z=2$

$X=214$, $Z=-2$

$$P(X>264)+P(X<214) = P(Z>2)+P(Z<-2)$$

$$= 0.0228+0.0228 = 0.0456 \#$$

9. r.v $X \equiv$ lifetime of a lightbulb

$X \sim Exp(\beta = 10)$

by $P(X > a+b \mid X > a) = P(X > b)$

$P(X > 5+a \mid X > a) = P(X > 5)$

$P(X > 5) = e^{-5/10} = e^{-\frac{1}{2}} \fallingdotseq 0.6065$ #

If the distribution is not exponential, memoryless property shall not hold. In the case, the working hour before Jack entered the room of the bulb is required to compute the probability.

10. H_0: grade between OR and Prob are independent

H_1: not independent

Prob \ OR	A	B	C	
A	24 15.75	11 13.5	10 15.75	45
B	7 8.75	13 7.5	5 8.75	25
C	4 10.5	6 9	20 10.5	30
	35	30	35	100

$(24-15.75)^2/15.75 = 4.29$

$(11-13.5)^2/13.5 = 0.46$

$(10-15.75)^2/15.75 = 2.1$

$(7-8.75)^2/8.75 = 0.35$

$(13-7.5)^2/7.5 = 4.03$

$(5-8.75)^2/8.75 = 1.61$

$(4-10.5)^2/10.5 = 4.02$

$(6-9)^2/9 = 1$

$(20-10.5)^2/10.5 = 8.6$

$\chi^2 = 4.29 + 0.46 + \cdots + 1 + 8.6 = 26.46$, $df = (r-1)(c-1) = 4$

$\chi^2_{0.01}(4) = 13.277$, since $\chi^2 = 26.46 > 13.277$

reject H_0.