An Approximation to Rate-Equalization Fairness with Logarithmic Complexity for QoS

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***Abstract*—Scheduling protocols that provide both rate and fairness guarantees, such as Weighted Fair Queuing, distribute the unused capacity among the flows in proportion to the reserved rate of the flows. Thus, flows whose reserved rate is the largest will receive a larger share of the unused capacity. In earlier work, we have presented a scheduling algorithm that first distributes unused capacity to those flows whose reserved rate is the least. However, the per-packet complexity of this algorithm, known as rate-equalization (REQ) fairness, is linear in the number of flows. In this paper, we present an algorithm that approximates the behavior of REQ fairness but with only logarithmic complexity per packet. Simulation results show that in practical situations the behavior of the approximation algorithm is quite similar to that of pure REQ fairness.**

***Keywords:* Quality of Service Scheduling, Real-Time Traffic, Fairness in Bandwidth Allocation.**

I. Introduction

While the best effort service provided by the Internet suffices for traditional applications, such as email and web browsing, real-time applications such as interactive audio and video require Quality of Service (QoS) guarantees from the network. In particular, they require the network to reserve bandwidth for the application and to provide a bounded end-to-end delay. To provide these QoS guarantees, a broad array of packet scheduling protocols have been developed, such as Virtual Clock (VC) [24], [26] and Weighted Fair Queuing (WFQ) [18]. These and many others belong to a large family of scheduling protocols, known as rate-guaranteed schedulers [6], [11], [13] which provide a guaranteed end-to-end delay bound.

One desirable property of a rate-guaranteed scheduler is fairness. That is, a flow should not be "punished" (temporarily denied service) if it exceeds its reserved rate to take advantage of unused bandwidth in the channel. Some protocols, such as Virtual Clock, are unfair, while others, like WFQ, are fair.

The fairness property is desirable because it may be normal for some flows to violate their reserved packet rate. Examples of such flows are file transfers and multi-resolution video [17]. The sources of these flows will likely reserve from the network the smallest packet rate necessary to receive a minimum quality of service. If the source detects that additional bandwidth is available, it generates packets at a rate higher than its reserved rate. If the source detects that no additional bandwidth is available, it reduces its sending rate. There are several techniques by which a source can detect if additional bandwidth is available, see for example [16] and [20]. Thus, because some flows may be of adjustable rate, the unused bandwidth of a channel should be shared in a fair manner among all flows.

Most scheduling protocols that provide both rate and fairness guarantees, such as WFQ and its variants [18] [1] [7] [12] [2], distribute the unused capacity among the flows in proportion to the reserved rate of the flows. Thus, if flow f reserved twice the rate as flow g, then f's share of the unused bandwidth is twice that of g.

In this paper, we present an alternative approach for fairness in rate-guaranteed schedulers. Our scheduling algorithm first distributes unused capacity to those flows whose reserved rate is the least. This allocation continues until the flows with least reserved rate and the flows with the next-to-least reserved rate are given the same capacity. This continues, level by level, until, if enough unused capacity is available, all flows will receive the same capacity, and the scheduler will simply behave like Fair Queuing.

II. QoS Model

Our QoS model is based on the models of [6] and [11]. A *network* is a set of computers connected via point-to-point communication channels. A *flow* is a sequence of packets, all of which originate at the same source computer and are destined to the same computer. All packets of a flow must traverse the network along a fixed path.

Each flow is characterized by its data rate and an upper bound on its end-to-end delay. Before a source introduces a new flow to the network, enough network resources are reserved to ensure the flow's delay bound is not violated. If enough resources are not available, the flow is rejected.

Each output channel of a computer is equipped with a packet scheduler (see Figure 1). A scheduler receives packets from flows whose path traverses its output channel. Whenever its output channel becomes idle, the scheduler chooses a received packet and forwards it over the output channel. A packet *exits* a scheduler when its last bit packet is transmitted by the output channel. We adopt the following notation for each flow f.

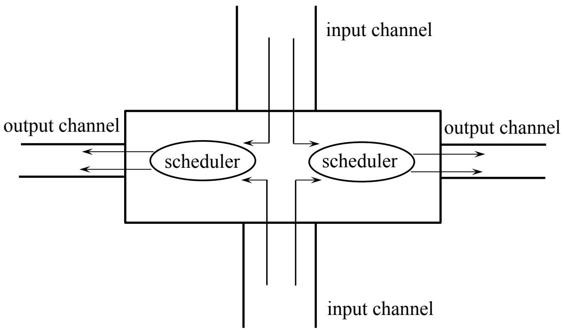


Fig. 1. Schedulers and Channels

CR Server

let $B(t)$ be the set of backlogged flows at time t;
for every flow f,
    if $f \in B(t)$, then
        $\psi_{f,CR}(t) = R_f$
    else
        $\psi_{f,CR}(t) = 0$

Fig. 2. Constant-rate fluid server

R_f	data rate reserved for flow f.
$p_{f,i}$	i^{th} packet of f, $i \geq 1$.
$L_{f,i}$	length of packet $p_{f,i}$.
L_f^{max}	maximum packet length of f.
L^{max}	maximum packet length of all flows.
$A_{f,i}$	arrival time of $p_{f,i}$.
$E_{f,i}$	exit time of $p_{f,i}$.
C	bandwidth of the output channel.

Consider a constant-rate (CR) fluid server[1] whose input is a set of flows, among them f. Let this server forward the bits of each input flow f at exactly its reserved rate R_f. Such a fluid server is defined in Fig. 2, where $B(t)$ are the *backlogged flows* at time t, i.e., flows with bits remaining to be forwarded, and $\psi_{f,CR}(t)$ is the bit rate given to flow f at time t.

We define $S_{f,i,CR}$ as the *start-time* of packet $p_{f,i}$ at the constant-rate server and $F_{f,i,CR}$ as the *finishing time*. They can be computed recursively as follows, where $S_{f,1,CR} = A_{f,1}$.

$$\begin{aligned} S_{f,i,CR} &= \max(A_{f,i},\ F_{f,i-1,CR}) \quad i > 1 \\ F_{f,i,CR} &= S_{f,i,CR} + \frac{L_{f,i}}{R_f} \quad i \geq 1 \end{aligned} \tag{1}$$

Note that $F_{f,i,CR}$ is the value used by the Virtual Clock scheduling protocol to assign priorities to its packets [24], [26].

Assume that a packet scheduler s forwards the packets of an input flow f at a rate of at least R_f. Then, each packet $p_{f,i}$ exits scheduler s not much later than its finishing time at CR, i.e., $F_{f,i,CR}$. We refer to these packet schedulers as *rate-guaranteed schedulers* [6], [11], [13]. More formally, a scheduler s is a rate-guaranteed scheduler iff, for every input flow f of s and every i, $i \geq 1$,

$$E_{f,i,s} \leq F_{f,i,CR} + \delta_{f,s}$$

for some constant $\delta_{f,s}$. For many protocols, such as Virtual Clock [24], [26] and Weighted Fair Queuing [18] (and their many variations),

$$\delta_{f,s} = L_s^{max}/C_s$$

I.e., packets exit by their finishing time in CR plus at most the time to transmit the largest packet size. In this case, admission control is simple[2], the sum of the reserved rates of all the flows through s must be at most C.

III. Rate-Equalization Fairness

We next overview the motivation for rate-equalizing fairness, which we introduced in [10].

On occasions, the bandwidth of a channel is not fully utilized. This occurs either because existing flows are transmitting at a rate that is below their reserved rate, or because the sum of the reserved rates of the flows is less than the output channel's bandwidth. Under these conditions, it is possible for a flow to temporarily exceed its reserved rate, in an attempt to take advantage of bandwidth that is not being used by other flows. The source of the flow can determine if there is unused bandwidth by receiving implicit feedback from the network, in particular, that its packets are experiencing an end-to-end delay that is smaller than expected.

The manner in which unallocated bandwidth is distributed among flows varies from one scheduling protocol to another. We refer to this distribution of unallocated bandwidth as the *fairness method* of the protocol, and it is the main focus of this paper.

Some protocols, like Virtual Clock (VC) [26][24], do not address fairness. In VC, each packet $p_{f,i}$ is labeled with its virtual finishing time, $F_{f,i,CR}$ (see (1)), and packets are forwarded by the scheduler in order of increasing labels. A consequence of this is that, if a flow exceeds its reserved rate, it may later be denied service by the scheduler, for a duration proportional to the time the flow exceeded its rate [7]. This is potentially unbounded. The exit bound in Equation (**??**) still holds, however, because VC is a rate-guaranteed scheduler.

Other rate-guaranteed protocols, such as Weighted Fair Queuing (WFQ) [18] and its variants [1] [7] [12] [2], distribute the unallocated bandwidth among flows in proportion to their reserved rate. Specifically, the effective rate ψ_f that is given to flow f (i.e., the rate at which the scheduler actually forwards the packets of flow f) is

$$\psi_f(t) = \frac{C}{\left(\sum_{g \in B(t)} R_g\right)} \cdot R_f \geq R_f \tag{2}$$

[1]Fluid servers can forward, during any time interval, an arbitray number of bits from any subset of its input flows. They are for reference only and cannot be implemented. This is in contrast to packet schedulers which are used in practice and can only forward one packet at a time.

[2]Other protocols have a *rate-independent delay* [11], [27]: where $\delta_{f,s}$ could be negative. This allows for a smaller per-hop delay, but makes the admission control test quite complex. Such protocols are outside the scope of this paper.

where $B(t)$ are the backlogged flows at time t (i.e., those flows with a non-empty queue) and C is the capacity of the output channel of the scheduler.

Consider another flow g with lesser reserved bandwidth, e.g., $R_f = 2 \cdot R_g$. From (2), it can be easily shown that

$$(\psi_f - R_f) = 2 \cdot (\psi_g - R_g)$$

That is, the manner in which unused bandwidth is allocated to backlogged flows is proportional to the reserved rate of the flow. Hence, the fairness method of WFQ favors flows with a higher reserved rate.

The intuition behind WFQ's fairness method is as follows. If a flow has a reserved rate that is greater than that of other flows, it implies that the user who generates the flow is paying a greater price for the network service, and thus, should receive a greater share of the unallocated bandwidth.

In [10], we presented an alternative fairness method in which the overall objective is to give every flow the same effective rate, provided enough unallocated bandwidth is available. This will result in a value for the effective rate ψ_f that is different from the one given in Equation (2). The intuition behind it is the following. Every flow will be guaranteed its reserved rate R_f. However, flows whose applications are rate-adaptive could reserve the minimum rate possible to satisfy their QoS requirements, and thus minimize expense. Any additional bandwidth is given to those flows that need it the most, i.e., those with the least reserved rate.

A more detaied description is as follows. First, at all times, the effective rate of any flow f, ψ_f, is at least its reserved rate, R_f. I.e., $R_f \leq \psi_f$. Next, consider another flow g, where $R_f < R_g$. By definition, $R_g \leq \psi_g$. In our method, if enough unallocated bandwidth is available, ψ_f will increase until it becomes equal to R_g, and thus, ψ_f will become equal to ψ_g. Thus, flows with lower reserved rates will "catch up" to flows with larger reserved rates.

Assume that more unallocated bandwidth remains. In this case, the remaining unallocated bandwidth will be distributed equally between f and g, maintaining the relationship $\psi_f = \psi_g$. If there exists another flow h, where $R_h > R_g$, then ψ_f and ψ_g increase equally until they reach R_h (assuming enough bandwidth remains), and hence, $\psi_f = \psi_g = \psi_h$.

To summarize, our fairness method attempts to give all flows the same effective rate. However, in doing so, the requirement of $R_f \leq \psi_f$ for all f must be preserved at all times.

We next describe our fairness method in a more formal way by introducing a rate equalization fluid server, which wll then be emulated as close as possible by a packet scheduler.

IV. Rate-Equalization Server and Scheduler

Packet scheduling algorithms that provide fairness, such as WFQ and some of its variants, describe their fairness method via a virtual fluid server. The packet scheduler then mimics the fluid server as much as possible. The fluid server and the packet scheduler have the same input flows. Both have an output channel, and both of these channels have equal capacity.

EQ Server

let $B(t)$ be the set of backlogged flows at time t;
for every flow f,
    if $f \notin B(t)$, then
        $\psi_{f,EQ}(t) = 0$
    else
        let $b_1, b_2, \ldots, b_m$ be the flows of $B(t)$
          ordered by increasing R;
        let j, $1 \leq j \leq m$, be the largest index such that
$$R_{b_j} \leq \frac{C - \sum_{k=j+1}^{m} R_{b_k}}{j} < R_{b_{j+1}};$$
        let $R_{EQ} = \dfrac{C - \sum_{k=j+1}^{m} R_{b_k}}{j}$;
        for each k, $1 \leq k \leq j$,
            $\psi_{f,EQ}(t) = R_{EQ}$;
        for each k, $j+1 \leq k \leq m$,
            $\psi_{f,EQ}(t) = R_{b_k}$;

Fig. 3. Rate equalization fluid server

What distinguishes the fluid server from the packet scheduler is the manner in which it forwards bits. Once the packet scheduler begins to transmit a packet, the transmission of the packet cannot be preempted. The fluid server, on the other hand, can concurrently forward an arbitrary number of bits from a group of flows. This, of course, is bounded by the capacity (bits/sec.) of the output channel.

A. Fluid Server

In light of our earlier discussion on fairness, we define the rate-equalization (EQ) fluid server as follows. Let $\psi_{f,EQ}(t)$ be the instantaneous bit rate given to flow f by the fluid server. This value is computed as shown in Figure 3. If $\psi_{f,EQ}(t)$ does not change during an interval $[t_1, t_2]$, then the total number of bits of f forwarded during this interval is $\psi_{f,EQ}(t_1) \cdot (t_2 - t_1)$.

The steps shown in Figure 3 are as follows. First, the set $B(t)$ of backlogged flows (i.e., with bits remaining to be forwarded) is determined. Obviously, a non-backlogged flow receives a rate of zero. All other flows are then arranged in increasing order of their reserved rate. Then, the index j is found such that

$$R_{b_j} \leq \frac{C - \sum_{k=j+1}^{m} R_{b_k}}{j} < R_{b_{j+1}}$$

In this manner, if the higher rate flows $b_{j+1}, \ldots, b_m$ receive exactly their reserved rate, then there is enough remaining bandwidth that can be equally shared among the lower rate flows $b_1, b_2, \ldots, b_j$.

Consider the following example, which is illustrated in Fig. 4. Let $C = 120$ bits/sec.. There are four flows, $f_1 \ldots f_4$, and all packets are 100 bits long. Let $R_{f_1} = 10$ bits/sec., $R_{f_2} = 20$ bits/sec., $R_{f_3} = 30$ bits/sec., and $R_{f_4} = 40$ bits/sec.. Note that $\sum_{i=1}^{4} R_{f_i} = 100$ bits/sec., leaving 20 bits/sec. unallocated.

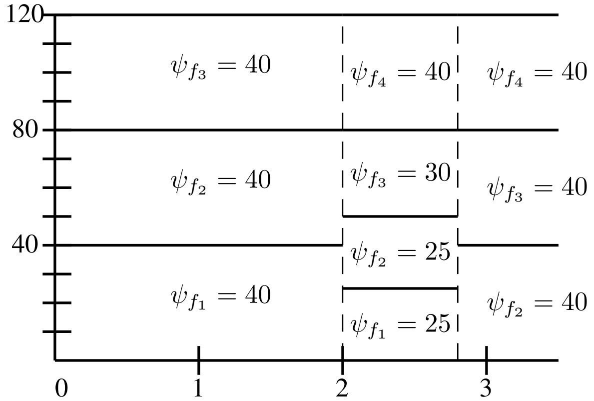


Fig. 4. Fluid server example.

PEQ Scheduler

upon receiving a packet $p_{f,i}$,
    $D_{f,i,PEQ} = \infty$;

if output channel is idle at time t,
    model the behavior of EQ up to time t;
    let $p_{f,i} \in$ Active(t) iff $t \geq S_{f,i,EQ}$;
    for each $p_{f,i} \in$ Active(t),
        $D_{f,i,PEQ} = S_{f,i,EQ} + L_{f,i}/R_f$;
    let $D_{g,j,PEQ} = \min\{D_{f,i,PEQ} \mid p_{f,i} \in \text{Active}(t)\}$;
    forward $p_{g,j}$ to the output channel.

Fig. 5. Packetized rate equalization scheduler

Assume that at time 0, one packet of f_1 arrives, two packets of f_2 arrive, and two packets of f_3 arrive. At time 2 secs., one packet of f_4 arrives.

At time zero, flows f_1, f_2, and f_3 have bits in their queues. Since $C = 120$, there is enough capacity for all three flows to receive the same effective bandwidth of $\psi = 40$ bits/sec.. At time 2, a packet from f_4 arrives. The total reserved rates of the flows is 100 bits/sec., leaving only 20 bits/sec. to equalize the rates among the flows. Equalizing f_1 to f_2 requires 10 bits/sec.. The remaining 10 bits/sec. are distributed equally between f_1 and f_2, but are not enough to increase their effective rates up R_{f_3}. We thus end with $\psi_{f_1} = \psi_{f_2} = 25$ bits/sec. This leaves $\psi_{f_3} = R_{f_3} = 30$ bits/sec., and $\psi_{f_4} = R_{f_4} = 40$ bits sec.

At time $2\frac{4}{5}$, the last bit of the packet of f_1 exits, so only three flows have a non-empty queue. Again, all remaining flows are given an effective bandwidth of 40 bits/sec. We leave it to the reader to determine the remainder of the example.

B. Packet Scheduler

We next overview the packet scheduler for rate-equalization which we presented in [10], where more details can be found.

In general, the purpose of a fluid server is to guide the packet scheduler in the order it chooses to forward packets. Typically, [5][18][21], for every pair of packets, p_1 and p_2, if p_1 finishes service in the fluid server before p_2 finishes service, then the packet scheduler will forward p_1 before p_2. I.e., the packet scheduler tries to emulate the behavior of the fluid server as much as possible. This emulation, of course, is not perfect, because the packet scheduler can only forward one packet at a time, while the fluid server can forward bits of multiple flows (and hence multiple packets) at once.

For most fluid servers [5][18][21], at the moment a packet arrives, the exit time that this packet will have from the fluid server is unknown. This is because the bit rate at which the packet will be served depends not only on the packets currently in the system, but also on packets that are yet to arrive.

In consequence, when a packet $p_{f,i}$ arrives into a packet scheduler, the scheduler assigns to the packet a *virtual exit time* $T_{f,i}$ (see [18] for details on computing this value), such that, for any other packet $p_{g,j}$, $T_{f,i} \leq T_{g,j}$ iff the exit time of $p_{f,i}$ from the fluid server is at most the exit time of $p_{g,j}$. Packets are then forwarded in order of their virtual exit times. Thus, the packet scheduler forwards packets in the same order in which they are forwarded by the fluid server.

A rate-equalizing fluid server, however, does not have this order-preserving property. That is, if two packets $p_{f,i}$ and $p_{g,j}$ are received, not only can't their exit time from the fluid server be determined, but also their relative exit times cannot be determined. I.e., which of $p_{f,i}$ or $p_{g,j}$ exits first depends on the future arrival of packets.

The reason for not having this property is that the relative effective bandwidth, ψ_g/ψ_f, does not remain constant in rate-equalization. Actually, *not* preserving this ratio is one of the objectives of rate-equalization. Hence, when packets $p_{g,j}$ and $p_{f,i}$ arrive, the scheduler is unable to determine which one will exit the fluid server first.

From the above, the rate equalization packet scheduler (PEQ) cannot assign a virtual exit time T to each packet. Instead, we opted in [10] to assign a real-time deadline, $D_{f,i,PEQ}$, to each packet $p_{f,i}$. The deadline is an *upper bound* on the exit time of $p_{f,i}$ from the fluid server. To obtain this upper bound, we take advantage of the fact that both the fluid server and the packet scheduler have the same input flows and the same output channel capacity. This allows the packet scheduler to keep track of the behavior of the fluid server. The upper bound is as follows.

Let $S_{f,i,EQ}$ be the starting time of $p_{f,i}$ in the fluid server, i.e., when its first bit begins service. Note that scheduler PEQ cannot compute this value when $p_{f,i}$ arrives. However, at time t, where $t \geq S_{f,i,EQ}$, PEQ is aware of this value, because it can keep track of the behavior of the server. Thus, the deadline of $p_{f,i}$ is set to

$$D_{f,i,PEQ} = S_{f,i,EQ} + \frac{L_{f,i}}{R_f}.$$

Then, packets are forwarded in order of this deadline.

Note that since $\psi_f \geq R_f$, the above is a true upper bound on the exit time from the fluid server. Furthermore, since scheduler PEQ cannot compute this value until time $S_{f,i,EQ}$,

$p_{f,i}$ is not added to the queue of schedulable packets until this time.

The detailed behavior of the scheduler is shown in Figure 5. More detais can be found in [10], including bounds on the difference between a packet's exit time from the scheduler and from the fluid server.

V. Roadblocks to an Efficient Implementation

Recall that our objective is to find an approximation algorithm that will require $O(\log(n))$ processing for receiving and transmitting a packet, where n is the number of flows in the system. Our scheduling protocol resembles WFQ in the sense that we also have a fluid server, and the packet scheduler attempts to emulate it as close as possible.

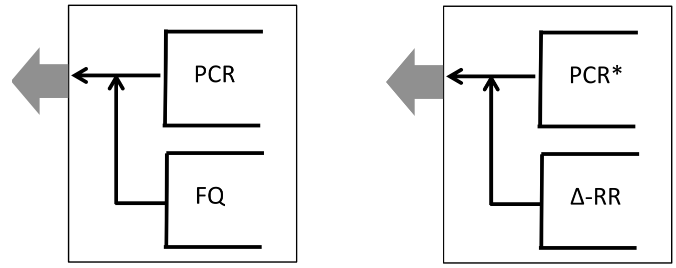
For many years, the best implementation of WFQ had $O(n)$ complexity. This was due to the overhead of computing the virtual time associated with the arrival time of a packet. The virtual time grows inversely proportional to the number of flows backlogged in the fluid server. The $O(n)$ complexity arises because many flows could become not backlogged in a very short period of time.

Several approximations with $O(\log(n))$ complexity, such as Leap-Forward-VC [22], Time-Shift Scheduling [7], and WFQ+ [2], provided a rough approximation of the virtual time. Other approaches reduced the complexity even further to $O(1)$ by sophisticated variations on the classical round-robin algorithm [14][15][19][25][4][28]. All of these provided the same type of fairness as WFQ, i.e., extra bandwidth is allocated in proportional to the reserved rate.

After many years of only having an $O(n)$ implementation, an $O(\log(n))$ implementation of WFQ was presented in [23]. This required the introduction of a complex search structure that organized the "breaking points" in the virtual time into a search tree. Crucial to making this implementation possible is that the ratio of the effective rates, ψ_g/ψ_f for any pair of backlogged flows, remained the same regardless of the arrival or departure of packets from other flows.

However, the ratio mentioned above is not constant in rate-equalization scheduling. In fact, it can vary significantly. To see this, consider again Figure 3. You can consider the set of backlogged flows to always be divided into two subsets: those flows whose effective rate is their reserved rate, and those flows whose effective rates are all the same due to unused bandwidth. The amount of unused bandwidth in turn depends on how many flows are backlogged, which, like in WFQ, can vary signnificantly in a very short period of time. This causes many flows to change from one of these two subsets to another, which in turn drastically changes the ration of effective rates.

For these reasons, a technique similar to the one in [23] is not directly applicable. Although we have not proven a lower bound, we speculate that a precise implementation cannot be done in $O(\log(n))$ time. We thus search for an approximation to the fluid server of rate-equalization that runs in $O(\log(n))$ time. Our approximation, presented below, is quite different from that of earlier works mentioned above due to our significantly different method of defining fairness.



(a) (b)

Fig. 6. Dual-Mode Scheduling

PCR Scheduler

upon receiving a packet $p_{f,i}$,
$\quad D_{f,i,PCR} = F_{f,i,CR}$;

if output channel is idle at time t,
$\quad$ let $p_{f,i} \in \text{Active}(t)$ iff $t \geq S_{f,i,CR}$;
$\quad$ if $\text{Active}(t) \neq \emptyset$ then
$\quad\quad$ let $D_{g,j,PCR} = \min\{D_{f,i,PCR} \mid p_{f,i} \in \text{Active}(t)\}$;
$\quad\quad$ forward $p_{g,j}$ to the output channel.

Fig. 7. Packetized constant-rate scheduler

VI. Dual-Mode Scheduling

From the above discussion, backlogged flows in the fluid server can be cosidered to be in one of two disjoint subsets: *enhanced flows*, whose effective rate is greater than their reserved rate and all members have the same effective rate, and *unenhanced flows*, whose effective rate is simply their reserved rate. This motivates our first packet scheduler design, presented in Section VI-A. Although intuitive, this first attempt is not efficient. We then present our final scheduler design in Section VI-B

A. Flow-Migration Scheduler

Consider Figure 6(a). The service required for an unenhanced flow f is simply a constant rate R_f. This is provided by a packetized constant rate scheduler (PCR), which is described in more detail in Figure 7. It is similat to the Virtual Clock protocol [24], except that it does not allow flows to exceed their reserved rate. Note that this scheduler is non-work-conserving, i.e., there are times when its queues are non-empty yet it does not have any packets considered 'active', so it remains idle. Enhanced flows, on the other hand, have to be served in an equal manner. This is best accomplished by a fair-queuing (FQ) scheduler, also shown in Figure 6(a).

We thus have two schedulers, one for each type of flow. Priority is given to the PCR scheduler. I.e., when the output channel becomes idle, the PCR scheduler is queried for the next packet to be transmitted. Only if the PCR scheduler is unable to provide a packet (due to its queues being empty

A-PEQ Scheduler

upon receiving a packet $p_{f,i}$,
  add $p_{f,i}$ to the queue of f, Q_f;

if output channel is idle at time t,
  model the behavior of PCR^* to dequeue a packet;
  let $p_{f,i}$ be the packet chosen by PCR^*;
  let $\rho_{min} = \min\{\rho_g \mid Q_g \neq \emptyset\}$;
  if $Q_f \neq \emptyset$ then
    dequeue and forward a packet from flow f;
    $\rho_f = \min(\rho_f + 1, \rho_{min} + \Delta)$;
  else
    let g satisfy $\rho_g = \rho_{min}$;
    forward the next packet of flow g;
    $\rho_g = \rho_g + 1$;

Fig. 8. Approximate packetized rate equalization scheduler

or all packets being 'inactive'), then the packet transmitted is chosen from the FQ scheduler. Both of these schedulers can be implemented in $O(\log(n))$ time per packet arrival/departure (FQ using the method of [23]).

The above method should work, provided the membership in the enhanced and unenhanced flow sets remains constant. However, their membership depends on unallocated bandwidth. An increase in unallocated bandwidth enhances more flows, and a decrease unenhances some flows.

Unallocated bandwidth comes from two sources: from bandwidth that is not reserved by any flow, and from flows that have temporarilly stopped creating packets (empty queues). The former is relatively stable, and changes are predictable (when flows are added or removed). In this case, the appropriate movement of flows between the schedulers can be done before a new flow is accepted or removed from the system. The latter, i.e., queues becoming empty, cannot be predicted, and may cause large changes in flow assignments to the two schedulers. This is particularly true if the flows whose queue becomes empty have a large reserved bandwidth. Thus, moving flows from one scheduler to the other is not efficient, which prompts us to present below our final version of the scheduler.

B. Static-Flow-Assignment Scheduler

Our final protocol, *Approximate Packetized rate Equalization* (A-PEQ), is shown in detail in Figure 8, with an abstract view in Figure 6(b). For this implementation, we make the simplifying assumption that all packets of all flows have an equal size, L.[3] There are three major differences from the previous scheduler.

First, *all flows take part in both schedulers*. This solves the problem of having to move a large number of flows between the schedulers in a short period of time.

[3] We will investigate eliminating this restriction in future work.

Second, the scheduler PCR* differs from PCR as follows. PCR* assumes that every flow always has packets available (even if its queue is empty). When it chooses a packet from flow f for transmission, it checks the queue of f. If it is empty, then the packet transmitted is instead a packet chosen by Δ-RR. Eventhough f did not transmit a packet, PCR* updates its state about f as if indeed it had transmitted a packet from f.

Third, instead of FQ, we have a modified round-robin scheduling, which we denote Δ-RR. Each flow f has a round number ρ_f in Δ-RR. When Δ-RR is asked to forward a packet, it chooses it as follows.

- If Δ-RR is called because PCR* is unable to transmit a packet, then Δ-RR chooses a packet from the backlogged flow *with the least round-number*, and increases the flow's round number by one.
- If PCR* is able to transmit a packet from a flow f, then the round-number of f is increased by one, *even though* Δ-RR *did not output a packet*.

The motivation for the above choices is as follows. Consider two flows f and g, where f has a large reserved rate (always unenhanced in the fluid server) and g a small reserved rate (always enhanced in the fluid server). All unenhanced flows, such as f, transmit packets from PCR* at a high rate, so their round numbers in Δ-RR are higher than those of other flows. The slower flows, such as g, are served in round-robin order, and thus receive the same bandwidth.

Consider now two slow flows g and h, with g having a greater reserved rate than h. Note that through their respective packet transmissions at PCR*, the round number of g grows faster than h's. Nonetheless, both flows receive about the same behavior from Δ-RR. This is because the unused bandwidth at PCR* causes Δ-RR to serve the slowest flows, such as h, first, which allows these flows to reach the same round numbers as other flows, such as g.

One final detail remains. Assume the round number of flow f, due to its large reserved rate, grows much larger than that of other flows. Then, assume enough bandwidth becomes available to make f an enhanced flow in the fluid server. However, due its large round number, f will not receive service in Δ-RR for a long time. To avoid this, we place a bound, Δ, on the difference between the round number of any flow and the minimum round number of any backlogged flow, as indicated in Figure 8.

The bound Δ is a tunable parameter of the system. If it is too large, enhanced flows may not receive their due bandwith, and if it is too small, bandwidth may be wasted on unenhanced flows.

VII. Future Work and Concluding Remarks

Several directions for future work are possible. First, it is yet to be determined the tightness of the bound in Theorem **??**. *Also, a smaller value of $\delta_{s,f,i}$ may be possible by using a technique similar to that in [21], as follows. Each packet $p_{f,i}$ does not become "available" until its start time, $S_{s,f,i}$. The scheduler s gives preference to available* *packets, and among these, it forwards the packet with the smallest deadline,* $S_{s,f,i} + L_{f,i}/R_f$*. If no packet is available, then the scheduler chooses packets in a form similar to the virtual rate-equalizing scheduler* $\hat{s}$*. The above will ensure* $\delta_{s,f,i} = L_{f,i}/R_f + L_s^{max}/C_s$*. However, it is not clear how this would affect the bound of Theorem* **??**.

The time complexity of en-queuing or de-queuing a packet in rate-equalization is $O(n)$*, where* n *is the number of flows. For a long time, it was believed that WFQ also had an* $O(n)$ *complexity. However, via sophisticated indexing techniques, a* $O(\log(n))$ *bound was obtained [23]. We will continue to study the complexity of rate-equalization and determine if a lower complexity implementation can be found.*

Finally, scheduling packets over multiple parallel links between neighboring computers has been studied for guaranteed-rate schedulers [9] [8] [3]. We also plan to investigate the impact of rate-equalization over parallel links.

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