MATH1. Part II

Probability and Statistics

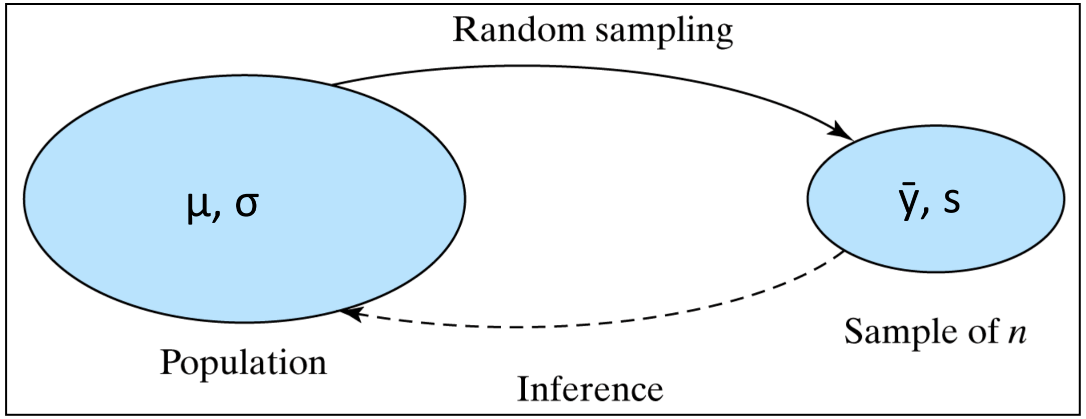
Chapter 6

Confidence Intervals

6.1 Statistical Estimation

Statistical inference

- Statistical estimation is a form of statistical inference in which we use the data to
  1) determine an estimate of some feature of the population and
  2) assess the precision of the estimate.
- Most of time, μ and σ are <u>unknow</u>.
  1) How to estimate μ?
  2) How to assess the reliability or precision of this estimate?



1) $\bar{y}$ is an estimate of μ.
2) s is an estimate of σ.

6.1 Statistical Estimation

Statistical inference

- It is intuitively reasonable that, for a sample of observations on a quantitative variable Y, the **sample** mean ȳ are estimates of the **population** mean μ :
  - As an estimate of μ, the sample mean ȳ is imprecise to the extent that it is affected by <u>sampling error</u>.
  - <u>Magnitude of the sampling error</u> (the amount of discrepancy between ȳ and μ): the standard deviation of the sampling distribution of Ȳ : $\boldsymbol{\sigma_{\bar{Y}} = \sigma/\sqrt{n}}$

- A natural estimate of <u>the magnitude of the sampling error</u> (the standard deviation of the sampling distribution of Ȳ), $\sigma_{\bar{Y}} = \sigma/\sqrt{n}$, would be $\boldsymbol{s/\sqrt{n}}$.

DEFINITION The **standard error of the mean** is defined as

$$\mathrm{SE}_{\bar{Y}} = \frac{s}{\sqrt{n}}$$

← SE is an estimate of $\sigma_{\bar{Y}}$

6.2 Standard Error of the Mean

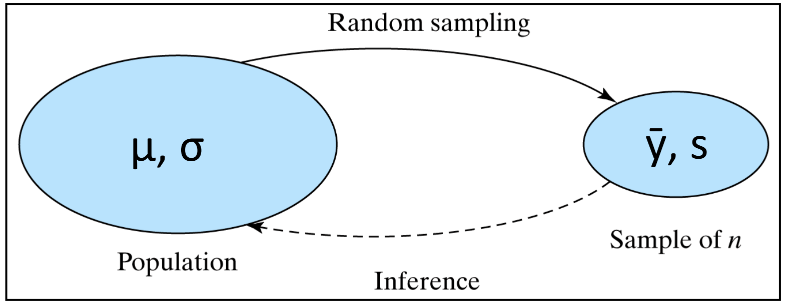
Standard error of the mean

DEFINITION The **standard error of the mean** is defined as

$$\mathrm{SE}_{\bar{Y}} = \frac{s}{\sqrt{n}}$$

← SE is an estimate of $\sigma_{\bar{Y}}$

- SE incorporates the two factors that affect reliability:
  1) the inherent variability of the observations (expressed through s),
  2) the sample size (n).



- SE can be interpreted in terms of the expected sampling error.

- Thus, the SE is a measure of the reliability or precision of $\bar{y}$ as an estimate of μ; the smaller the SE, the more precise the estimate.

6.2 Standard Error of the Mean

Standard error of the mean

Example 6.2.1 Butterfly Wings

- Researchers are studying the wing areas of male monarch butterfly.
- The data are given in Table 6.1.1

Table 6.1.1 Wing areas of male monarch butterflies

Wing area (cm^2)				
33.9	33.0	30.6	36.6	36.5
34.0	36.1	32.0	28.0	32.0
32.2	32.2	32.3	30.0	

- What is the sample mean and sample standard deviation?

- What is the standard error of the mean? (n=14)

6.2 Standard Error of the Mean

Standard error of the mean

Example 6.2.1 Butterfly Wings

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32.2	32.2	32.3	30.0	

- What is the sample mean and sample standard deviation?
  - $\bar{y}$ = 32.8143 ≈ 32.81 cm^2 and s = $\sqrt{\sum_{i=1}^{n}(yi-\bar{y})^2/(n-1)}$ = 2.4757 ≈ 2.48 cm^2 .
- What is the standard error of the mean? (n=14)
  - $SE_{\bar{Y}} = s/\sqrt{n}$ = 2.4757/ $\sqrt{14}$ = 0.6617 cm^2

6.2 Standard Error of the Mean

Standard error (SE) vs. Standard deviation (SD)

- **Standard error** (SE) describes the unreliability (due to sampling error) in the *mean* of the sample as an estimate of the mean of the population.
- **Standard deviation** (s, or SD) describes the dispersion of the data.

Example 6.2.2 Lamb birthweights

- A geneticist weighed 28 female lambs at birth. The birthweights are shown in Table 6.2.1.

Table 6.2.1 Birthweights of 28 Rambouillet lambs

Birthweight (kg)						
4.3	5.2	6.2	6.7	5.3	4.9	4.7
5.5	5.3	4.0	4.9	5.2	4.9	5.3
5.4	5.5	3.6	5.8	5.6	5.0	5.2
5.8	6.1	4.9	4.5	4.8	5.4	4.7

- What is SD? Explain the meaning of SD.
- What is SE? Explain the meaning of SE.

6.2 Standard Error of the Mean

Standard error (SE) vs. Standard deviation (SD)

Example 6.2.2 Lamb birthweights

- A geneticist weighed 28 female lambs at birth. The birthweights are shown in Table 6.2.1.
- What is SD? Explain the meaning of SD.
  - $s = 0.65$ kg
  - SD, s, describes the variability of birth weights among the lambs in the sample.
  - SD is indicated as a deviation from $\bar{y}$ (Fig).
- What is SE? Explain the meaning of SE.
  - SE = 0.12 kg.
  - SE indicates the variability associated with the sample mean (5.17 kg), viewed as an estimate of the population mean birthweight.
  - SE is indicated as variability associated with $\bar{y}$ Itself (Fig).

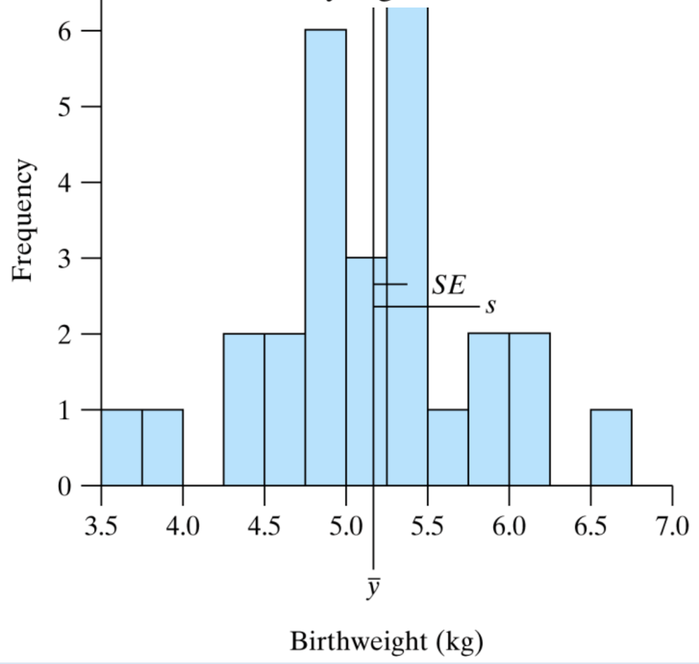


Figure 6.2.1 Birthweights of twenty-eight lambs

6.2 Standard Error of the Mean

Standard error (SE) vs. Standard deviation (SD)

- Another way to highlight the contrast between SE and SD is to consider samples of various **sizes**.

Example 6.2.2 Lamb birthweights (continued)

- A geneticist weighed 28 female lambs at birth. The birthweights are shown in Table 6.2.1.

	$n = 28$	$n = 280$	$n = 2{,}800$	$n \to \infty$
$\bar{y}$	5.17	5.19	5.14	$\bar{y} \to \mu$
s	0.65	0.67	0.65	$s \to \sigma$
SE	0.12	0.040	0.012	$\mathrm{SE} \to 0$

Sample distribution

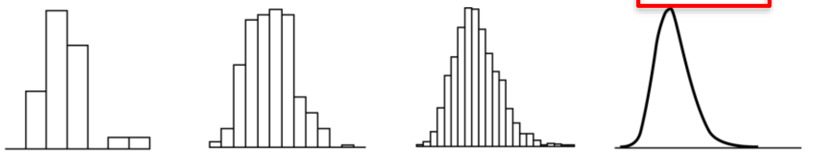


Figure 6.2.2 Samples of various sizes from the lamb birthweight population

– As the sample size increases, the sample mean and SD tend to approach more closely the population mean and SD.

– SE, by contrast, tends to decrease as n increases; when n is very large, the SE is very small and so the sample mean is a very precise estimate of the population mean.

6.3 Confidence Interval for μ

Standard error of the mean (SE) and Standard deviation (SD) (section 6.2)

- $\boldsymbol{SE_{\bar{Y}} = s/\sqrt{n}}$
- measures how far $\bar{y}$ is likely to be from the population mean μ.

Confidence interval for μ: Basic idea

Invisible man analogy:

- Figure 6.3.1 is a drawing of an invisible man walking his dog.
- The dog is within 2 standard errors of the man 95% of the time.
- We can take the interval "dog ± 2 x SE" as an interval that typically would include the man.
- Indeed, we could say that we are 95% confident that the man is in this interval.

Figure 6.3.1 Invisible man walking his dog

6.3 Confidence Interval for μ

Standard error of the mean (SE) and Standard deviation (SD) (section 6.2)

- $SE_{\bar{Y}} = s/\sqrt{n}$
- measures how far $\bar{y}$ is likely to be from the population mean μ.

Confidence interval for μ: Basic idea

The basic idea of a confidence interval:

- **population mean μ** - man, we cannot see it directly
- **sample mean $\bar{y}$** - dog, we can see it.
- Use $\bar{y}$ and SE to construct an interval.
- We hope this interval can include what we cannot see, the population mean μ.

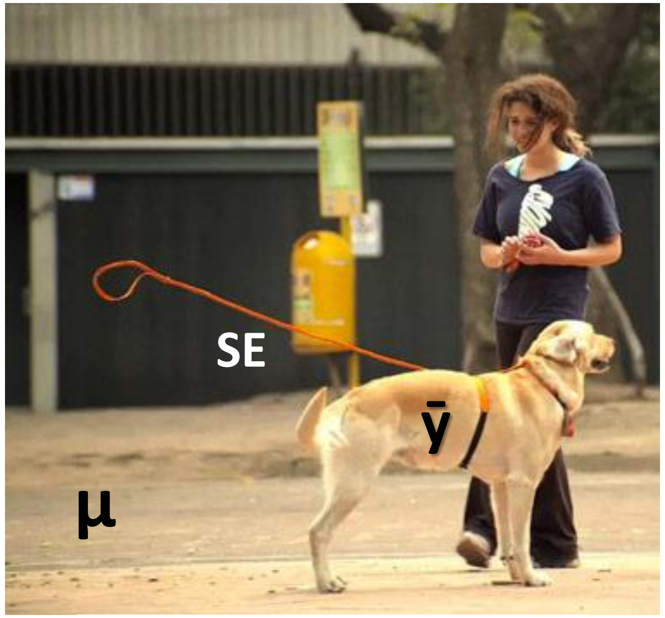


Figure 6.3.1 Invisible man walking his dog

6.3 Confidence Interval for μ

Confidence interval for μ: Mathematics

- From Theorem 5.2.1, if the population distribution of Y is normal, then the sampling distribution of $\bar{Y}$ is normal, regardless of the sample size n.

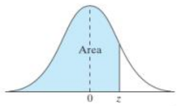
- If random variable Y has a normal distribution, then $Z = (\bar{Y} - \mu) / (\sigma/\sqrt{n})$ has a standard normal (Z) distribution.

- From **Table 3**,
  $\Pr\{-1.96 < (\bar{Y} - \mu) / (\sigma/\sqrt{n}) < 1.96\} = 0.95$
  So, $\Pr\{\bar{Y} - 1.96 \times \sigma/\sqrt{n} < \mu < \bar{Y} + 1.96 \times \sigma/\sqrt{n}\} = 0.95$

- Thus, the interval $\bar{Y} \pm 1.96 \times \sigma/\sqrt{n}$ will contain μ for 95% of all samples.

- **A 95% confidence interval for μ**: $\bar{Y} \pm 1.96 \times \sigma/\sqrt{n}$

TABLE 3 Areas Under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
–3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
–3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
–3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

6.3 Confidence Interval for μ

Confidence interval for μ: Mathematics

- **A 95% confidence interval for μ**: $\bar{Y} \pm 1.96 \times \sigma/\sqrt{n}$
  - This interval cannot be used for data analysis because it contains a quantity, standard deviation of population, **σ** , which cannot be determined from the data.
  - “Student” (British scientist named W. S. Gosset) discovered that, if the data come from a **normal population** and if we replace **σ** in the interval by the sample SD, **s**, then the 95% interpretation can be preserved if the **multiplier** of $\sigma/\sqrt{n}$ (i.e., **1.96**) is replaced by a suitable quantity;
  - the new quantity is denoted $t_{0.025}$ and is related to a distribution known as Student’s *t* distribution.

Confidence interval for μ : Method

- **A 95% confidence interval for μ**: $\bar{y} \pm t_{0.025} \times s/\sqrt{n}$

6.3 Confidence Interval for μ

Confidence interval for μ : Method

- **A 95% confidence interval for μ**: $\bar{y} \pm t_{0.025} \times s/\sqrt{n}$
  - **s** is sample SD; n is sample size
  - **$t_{0.025}$** is determined from **Student's t distribution** with df = n-1
    - $t_{0.025}$ is called the "<u>two-tailed 5% critical value</u>" of Student's *t* distribution
    - Defined as: the interval between **$-t_{0.025}$** and **$+t_{0.025}$** contains 95% of the area under the t curve, as shown in Figure 6.3.3.
    - **Table 4** show other critical values of Student's t distribution

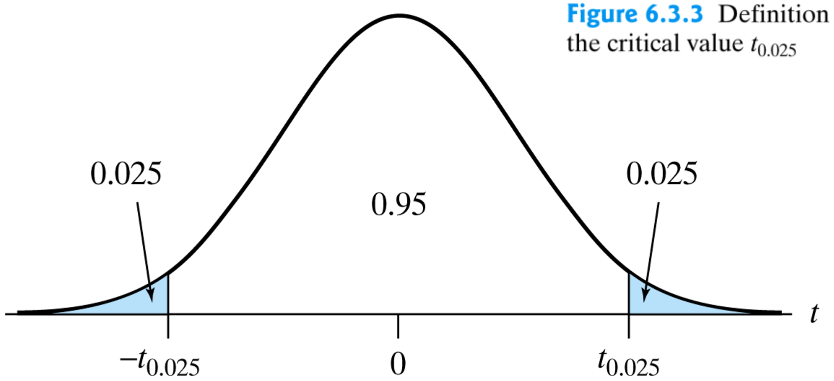
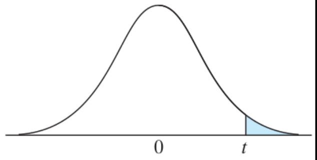


Figure 6.3.3 Definition of the critical value $t_{0.025}$

TABLE 4 Critical Values of Student's *t* Distribution



	UPPER TAIL PROBABILITY									
df	0.20	0.10	0.05	0.04	0.03	0.025	0.02	0.01	0.005	0.0005
1	1.376	3.078	6.314	7.916	10.579	12.706	15.895	31.821	63.657	636.619
2	1.061	1.886	2.920	3.320	3.896	4.303	4.849	6.965	9.925	31.599
3	0.978	1.638	2.353	2.605	2.951	3.182	3.482	4.541	5.841	12.924

6.3 Confidence Interval for μ

Student's t distribution

- **Table 4** show other critical values of Student’s t distribution

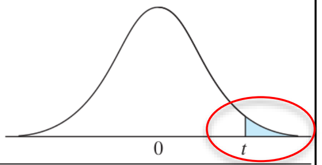
Example 6.3.1 Butterfly Wings

- data came from a normal population
- $n = 14$, $\bar{y} = 32.8143$ cm^2 and $s = 2.4757$ cm^2
- What is the **95%** confidence interval for μ ?
  - $df = n-1 = 14-1 = 13$; $\mathbf{t}_{0.025} = 2.160$ (Table 4)
  - A 95% confidence interval for μ:

$$\bar{y} \pm \mathbf{t}_{0.025} \times s/\sqrt{n}$$
$$= 32.8143 \pm 2.160 \times 2.4757/\sqrt{14}$$
$$= 32.8143 \pm 1.43 \text{ or } (31.4, 34.2)$$

  - The population mean wing area is between 31.4 cm ^{2} and 34.2 cm ^{2} with 95% confidence.

TABLE 4 Critical Values of Student’s *t* Distribution



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4	0.941	1.533	2.132	2.333	2.601	2.776	2.999	3.747	4.604	8.610
5	0.920	1.476	2.015	2.191	2.422	2.571	2.757	3.365	4.032	6.869
6	0.906	1.440	1.943	2.104	2.313	2.447	2.612	3.143	3.707	5.959
7	0.896	1.415	1.895	2.046	2.241	2.365	2.517	2.998	3.499	5.408
8	0.889	1.397	1.860	2.004	2.189	2.306	2.449	2.896	3.355	5.041
9	0.883	1.383	1.833	1.973	2.150	2.262	2.398	2.821	3.250	4.781
10	0.879	1.372	1.812	1.948	2.120	2.228	2.359	2.764	3.169	4.587
11	0.876	1.363	1.796	1.928	2.096	2.201	2.328	2.718	3.106	4.437
12	0.873	1.356	1.782	1.912	2.076	2.179	2.303	2.681	3.055	4.318
13	0.870	1.350	1.771	1.899	2.060	2.160	2.282	2.650	3.012	4.221
14	0.868	1.345	1.761	1.888	2.046	2.145	2.264	2.624	2.977	4.140
15	0.866	1.341	1.753	1.878	2.034	2.131	2.249	2.602	2.947	4.073
16	0.865	1.337	1.746	1.869	2.024	2.120	2.235	2.583	2.921	4.015
17	0.863	1.333	1.740	1.862	2.015	2.110	2.224	2.567	2.898	3.965
18	0.862	1.330	1.734	1.855	2.007	2.101	2.214	2.552	2.878	3.922
19	0.861	1.328	1.729	1.850	2.000	2.093	2.205	2.539	2.861	3.883
20	0.860	1.325	1.725	1.844	1.994	2.086	2.197	2.528	2.845	3.850
21	0.859	1.323	1.721	1.840	1.988	2.080	2.189	2.518	2.831	3.819
22	0.858	1.321	1.717	1.835	1.983	2.074	2.183	2.508	2.819	3.792
23	0.858	1.319	1.714	1.832	1.978	2.069	2.177	2.500	2.807	3.768
24	0.857	1.318	1.711	1.828	1.974	2.064	2.172	2.492	2.797	3.745
25	0.856	1.316	1.708	1.825	1.970	2.060	2.167	2.485	2.787	3.725
26	0.856	1.315	1.706	1.822	1.967	2.056	2.162	2.479	2.779	3.707
27	0.855	1.314	1.703	1.819	1.963	2.052	2.158	2.473	2.771	3.690
28	0.855	1.313	1.701	1.817	1.960	2.048	2.154	2.467	2.763	3.674
29	0.854	1.311	1.699	1.814	1.957	2.045	2.150	2.462	2.756	3.659
30	0.854	1.310	1.697	1.812	1.955	2.042	2.147	2.457	2.750	3.646
40	0.851	1.303	1.684	1.796	1.936	2.021	2.123	2.423	2.704	3.551
50	0.849	1.299	1.676	1.787	1.924	2.009	2.109	2.403	2.678	3.496
60	0.848	1.296	1.671	1.781	1.917	2.000	2.099	2.390	2.660	3.460
70	0.847	1.294	1.667	1.776	1.912	1.994	2.093	2.381	2.648	3.435
80	0.846	1.292	1.664	1.773	1.908	1.990	2.088	2.374	2.639	3.416
100	0.845	1.290	1.660	1.769	1.902	1.984	2.081	2.364	2.626	3.390
140	0.844	1.288	1.656	1.763	1.896	1.977	2.073	2.353	2.611	3.361
1000	0.842	1.282	1.646	1.752	1.883	1.962	2.056	2.330	2.581	3.300
∞	0.842	1.282	1.645	1.751	1.881	1.960	2.054	2.326	2.576	3.291
	60%	80%	90%	92%	94%	95%	96%	98%	99%	99.9%
	CRITICAL VALUE FOR CONFIDENCE LEVEL									

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6.3 Confidence Interval for μ

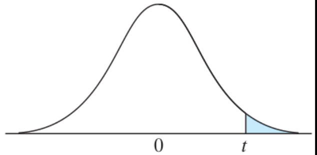
Student's t distribution

- **Table 4** show other critical values of Student’s t distribution

Example 6.3.1 Butterfly Wings

- data came from a normal population
- $n = 14$, $\bar{y} = 32.8143$ cm^2 and $s = 2.4757$ cm^2 .
- What is the **90%** confidence interval for μ ?

TABLE 4 Critical Values of Student’s *t* Distribution



df	UPPER TAIL PROBABILITY 0.20	0.10	0.05	0.04	0.03	0.025	0.02	0.01	0.005	0.0005
1	1.376	3.078	6.314	7.916	10.579	12.706	15.895	31.821	63.657	636.619
2	1.061	1.886	2.920	3.320	3.896	4.303	4.849	6.965	9.925	31.599
3	0.978	1.638	2.353	2.605	2.951	3.182	3.482	4.541	5.841	12.924
4	0.941	1.533	2.132	2.333	2.601	2.776	2.999	3.747	4.604	8.610
5	0.920	1.476	2.015	2.191	2.422	2.571	2.757	3.365	4.032	6.869
6	0.906	1.440	1.943	2.104	2.313	2.447	2.612	3.143	3.707	5.959
7	0.896	1.415	1.895	2.046	2.241	2.365	2.517	2.998	3.499	5.408
8	0.889	1.397	1.860	2.004	2.189	2.306	2.449	2.896	3.355	5.041
9	0.883	1.383	1.833	1.973	2.150	2.262	2.398	2.821	3.250	4.781
10	0.879	1.372	1.812	1.948	2.120	2.228	2.359	2.764	3.169	4.587
11	0.876	1.363	1.796	1.928	2.096	2.201	2.328	2.718	3.106	4.437
12	0.873	1.356	1.782	1.912	2.076	2.179	2.303	2.681	3.055	4.318
13	0.870	1.350	1.771	1.899	2.060	2.160	2.282	2.650	3.012	4.221
14	0.868	1.345	1.761	1.888	2.046	2.145	2.264	2.624	2.977	4.140
15	0.866	1.341	1.753	1.878	2.034	2.131	2.249	2.602	2.947	4.073
16	0.865	1.337	1.746	1.869	2.024	2.120	2.235	2.583	2.921	4.015
17	0.863	1.333	1.740	1.862	2.015	2.110	2.224	2.567	2.898	3.965
18	0.862	1.330	1.734	1.855	2.007	2.101	2.214	2.552	2.878	3.922
19	0.861	1.328	1.729	1.850	2.000	2.093	2.205	2.539	2.861	3.883
20	0.860	1.325	1.725	1.844	1.994	2.086	2.197	2.528	2.845	3.850
21	0.859	1.323	1.721	1.840	1.988	2.080	2.189	2.518	2.831	3.819
22	0.858	1.321	1.717	1.835	1.983	2.074	2.183	2.508	2.819	3.792
23	0.858	1.319	1.714	1.832	1.978	2.069	2.177	2.500	2.807	3.768
24	0.857	1.318	1.711	1.828	1.974	2.064	2.172	2.492	2.797	3.745
25	0.856	1.316	1.708	1.825	1.970	2.060	2.167	2.485	2.787	3.725
26	0.856	1.315	1.706	1.822	1.967	2.056	2.162	2.479	2.779	3.707
27	0.855	1.314	1.703	1.819	1.963	2.052	2.158	2.473	2.771	3.690
28	0.855	1.313	1.701	1.817	1.960	2.048	2.154	2.467	2.763	3.674
29	0.854	1.311	1.699	1.814	1.957	2.045	2.150	2.462	2.756	3.659
30	0.854	1.310	1.697	1.812	1.955	2.042	2.147	2.457	2.750	3.646
40	0.851	1.303	1.684	1.796	1.936	2.021	2.123	2.423	2.704	3.551
50	0.849	1.299	1.676	1.787	1.924	2.009	2.109	2.403	2.678	3.496
60	0.848	1.296	1.671	1.781	1.917	2.000	2.099	2.390	2.660	3.460
70	0.847	1.294	1.667	1.776	1.912	1.994	2.093	2.381	2.648	3.435
80	0.846	1.292	1.664	1.773	1.908	1.990	2.088	2.374	2.639	3.416
100	0.845	1.290	1.660	1.769	1.902	1.984	2.081	2.364	2.626	3.390
140	0.844	1.288	1.656	1.763	1.896	1.977	2.073	2.353	2.611	3.361
1000	0.842	1.282	1.646	1.752	1.883	1.962	2.056	2.330	2.581	3.300
∞	0.842	1.282	1.645	1.751	1.881	1.960	2.054	2.326	2.576	3.291
CRITICAL VALUE FOR CONFIDENCE LEVEL	60%	80%	90%	92%	94%	95%	96%	98%	99%	99.9%

6.3 Confidence Interval for µ

Student's t distribution

- **Table 4** show other critical values of Student's t distribution

Example 6.3.1 Butterfly Wings

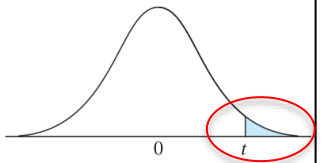
- data came from a normal population
- $n = 14$, $\bar{y} = 32.8143$ cm^2 and $s = 2.4757$ cm^2 .
- What is the **90%** confidence interval for µ ?
  - df = n-1 = 14-1 = 13; $t_{0.05} = 1.771$ (Table 4)
  - A 90% confidence interval for µ:

    $\bar{y} \pm t_{0.05} \times s/\sqrt{n}$

    $= 32.8143 \pm 1.771 \times 2.4757/\sqrt{14}$

    $= 32.8143 \pm 1.1718$ or (31.6, 34.0)
  - The population mean wing area is between 31.6 cm^2 and 34.0 cm^2 with 90% confidence.

TABLE 4 Critical Values of Student's *t* Distribution



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19	0.861	1.328	1.729	1.850	2.000	2.093	2.205	2.539	2.861	3.883
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60	0.848	1.296	1.671	1.781	1.917	2.000	2.099	2.390	2.660	3.460
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∞	0.842	1.282	1.645	1.751	1.881	1.960	2.054	2.326	2.576	3.291
	60%	80%	90%	92%	94%	95%	96%	98%	99%	99.9%
	CRITICAL VALUE FOR CONFIDENCE LEVEL									

6.3 Confidence Interval for μ

Relationship to Sampling Distribution of $\bar{Y}$

- Figure 6.3.7 shows a particular sample mean ($\bar{y}$) and its associated 95% confidence interval for μ, superimposed on the sampling distribution of $\bar{Y}$.
- Notice that the particular confidence interval does contain μ; this will happen for 95% of samples.

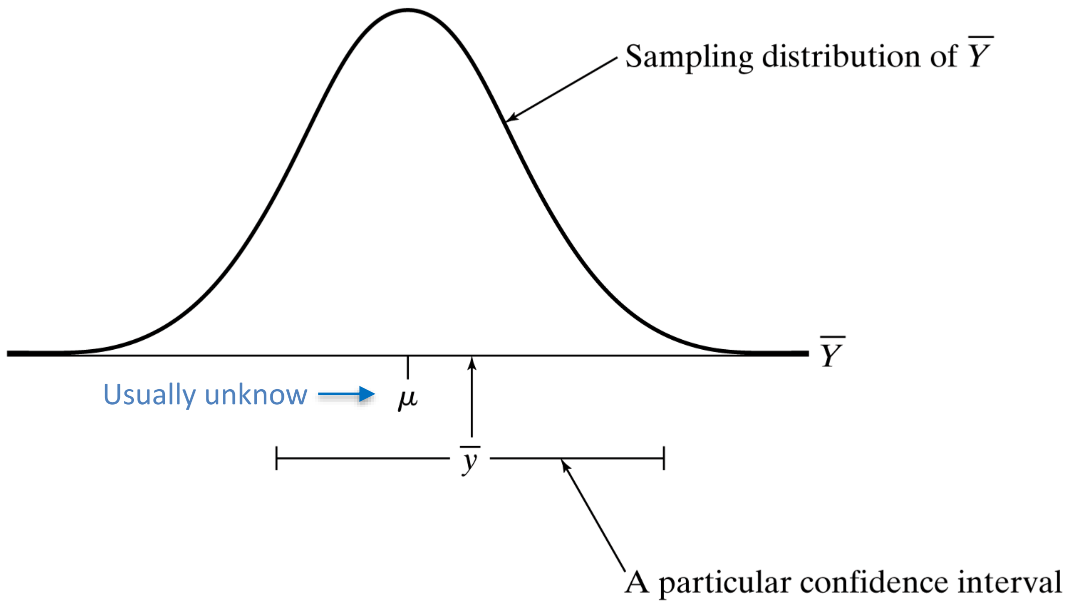


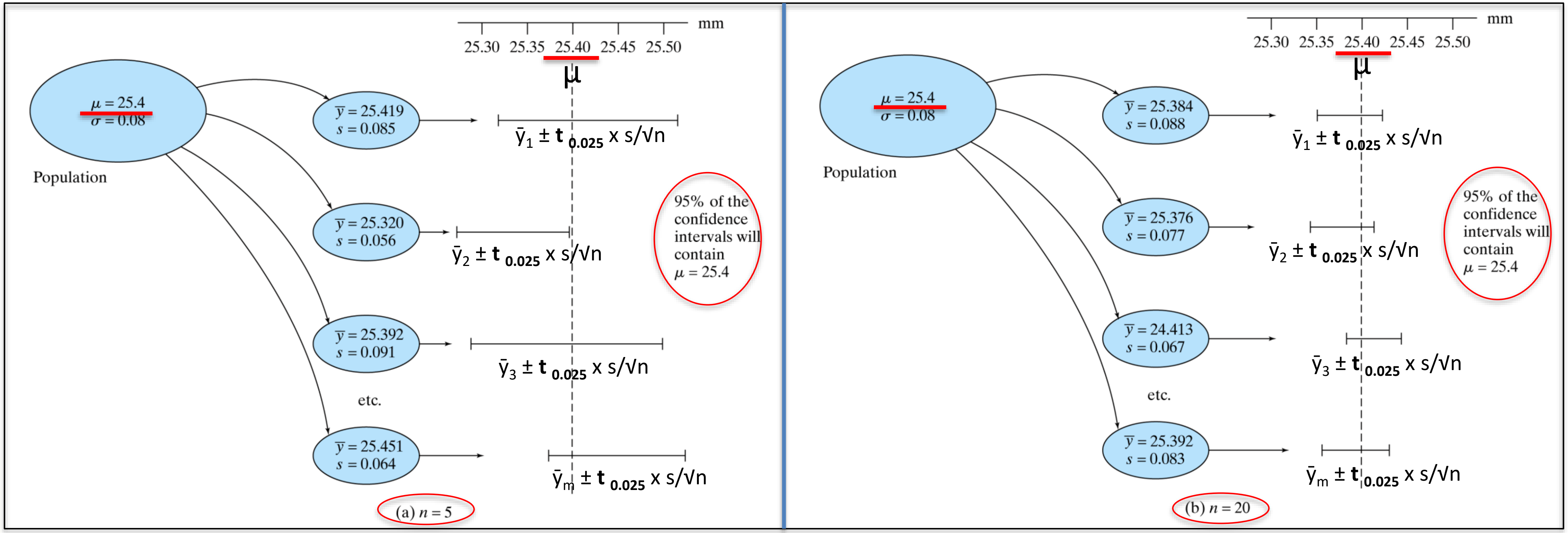
Figure 6.3.7 Relationship between a particular confidence interval for μ and the sampling distribution of $\bar{Y}$

6.3 Confidence Interval for μ

Confidence intervals and randomness

Meta-study interpretation of a confidence level

Figure 6.3.5 Confidence intervals for mean bill length



- The population mean, μ, is between $\bar{y} \pm t_{0.025} \times s/\sqrt{n}$ with 95% confidence.
- The **larger** samples tend to produce **narrower** confidence intervals.

6.3 Confidence Interval for μ

Interpretation of a Confidence Interval

Example 6.3.4 Bone Mineral Density (normal distribution)

- 94 women. The average bone mineral density was 0.878 g/cm^2 , with a standard deviation of 0.126 g/cm^2
- What is the 95% confidence interval for μ? What is the meaning of this interval?

Example 6.3.5 Seeds per Fruit (normal distribution)

- 12 fruit. The average number of seeds was 320, with a standard deviation of 125.
- What is the 90% confidence interval for μ? What is the meaning of this interval?

6.3 Confidence Interval for μ

Interpretation of a Confidence Interval

Example 6.3.4 Bone Mineral Density (normal distribution)

- 94 women. The average density was 0.878 g/cm^2 , with a standard deviation of 0.126 g/cm^2
- What is the 95% confidence interval for μ? What is the meaning of this interval?
  - t multiplier is $t_{0.025} = 1.984 \rightarrow 0.878 \pm 1.984(0.126/\sqrt{94}) \rightarrow (0.852, 0.904)$
  - Thus, we are 95% confident that the mean hip bone mineral density of all women age 45 to 64 who take CEE for 36 months is between 0.852 g/cm^2 and 0.904 g/cm^2 .

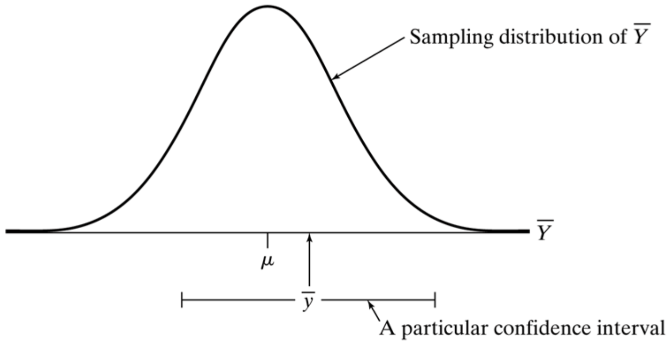
Example 6.3.5 Seeds per Fruit (normal distribution)

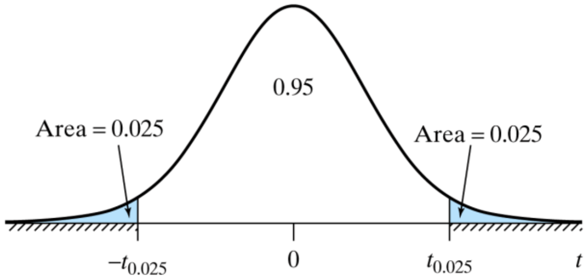
- 12 fruit. The average number of seeds was 320, with a standard deviation of 125.
- What is the 90% confidence interval for μ? What is the meaning of this interval?
  - t multiplier is $t_{0.05} = 1.796 \rightarrow 320 \pm 1.796\ (125/\sqrt{12}) \rightarrow (255, 385)$
  - Thus, we are 90% confident that the (population) mean number of seeds per fruit for Vallisneria americana is between 255 and 385.

6.3 Confidence Interval for μ

One-sided Confidence Intervals

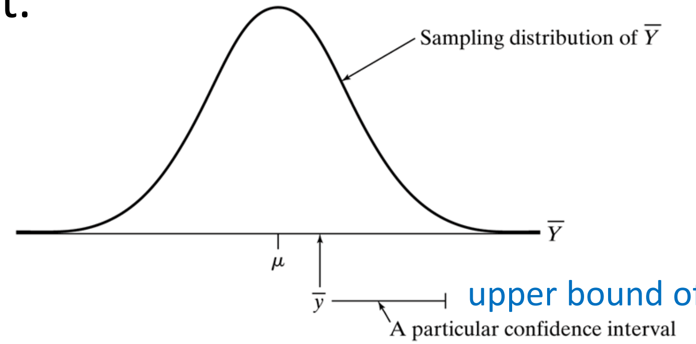
- **Two-sided confidence intervals**: most confidence intervals are of the form "estimate ± margin of error"; these are known as two-sided intervals.



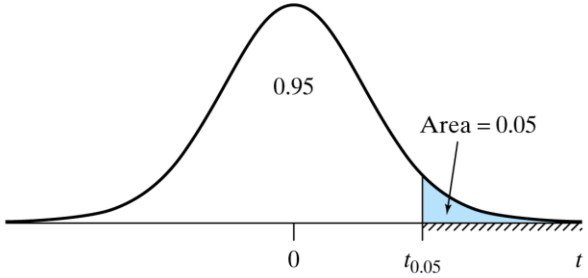


- $t_{0.025}$ is called the "two-tailed 5% critical value" of Student's t distribution

- **One-sided confidence intervals**: when only a lower bound, or only an upper bound, is of interest.



upper bound of μ

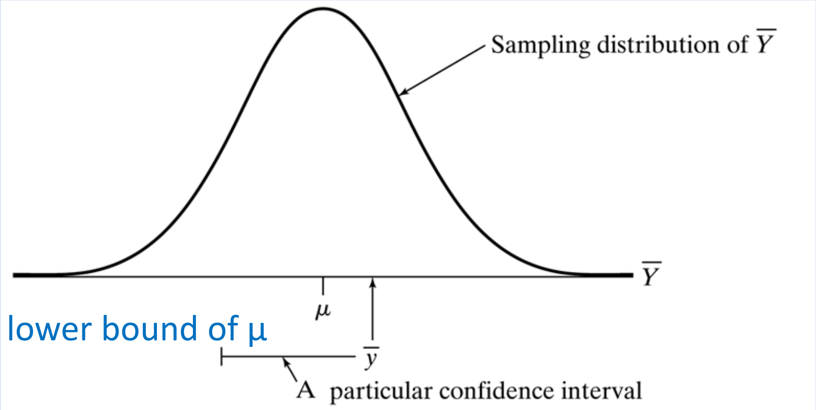


6.3 Confidence Interval for μ

One-sided Confidence Intervals

Example 6.3.6 Seeds per Fruit - One-sided

- data came from a normal population
- 12 fruit. The average number of seeds was 320, with a standard deviation of 125.
- we want a lower bound on μ, the population mean, but we are not concerned with how large μ might be.
- Construct a 95% confidence interval for μ with lower bound.



6.3 Confidence Interval for μ

One-sided Confidence Intervals

Example 6.3.6 Seeds per Fruit - One-sided

- data came from a normal population
- 12 fruit. The average number of seeds was 320, with a standard deviation of 125.
- Construct a 95% confidence interval for μ with lower bound.

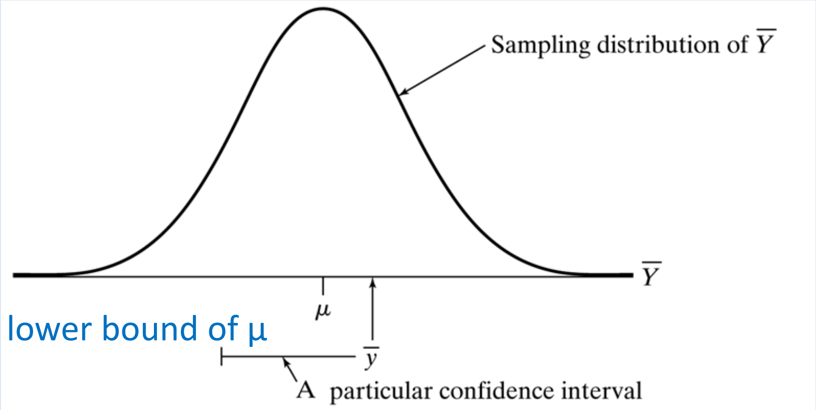
  — a one-sided 95% (lower) confidence interval uses the fact that $\Pr(-t_{0.05} < t < \infty) = 0.95$

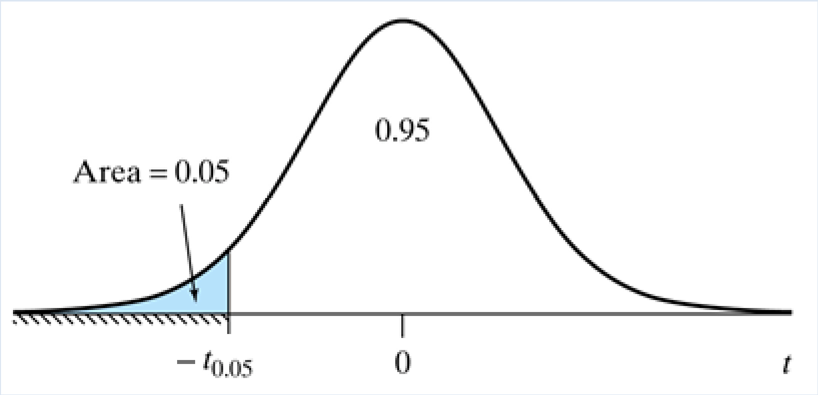
  — $\Pr\{ \bar{Y} - t_{0.05} \times s/\sqrt{n} < \mu < \infty\} = 0.95$

  — multiplier is $\mathbf{t_{11,\ 0.05} = 1.796}$, and we get

  $320 - 1.796(36) = 320 - 65 = 255 \rightarrow (255, \infty)$

  — Thus, we are 95% confident that the (population) mean number of seeds per fruit for *Vallisneria americana* is at least 255.





To be continued ….

Tomorrow

6.4 Planning a Study to Estimate μ

Two-sided Confidence Intervals

- A **95%** confidence interval for μ: $\bar{y} \pm \mathbf{t}_{0.025} \times \mathbf{s}/\sqrt{\mathbf{n}}$
- The precision with which a population mean can be estimated is determined by two factors:
  - (1) the variability of the observed variable Y, **s**.
    - In some situations the variability of Y cannot, and perhaps should not, be reduced
    - For some other cases, the variability of Y can be reduced by holding *extraneous* conditions as constant as possible
  - (2) the sample size, **n**.
    - *What sample size will be sufficient to achieve a desired degree of precision in estimation of the population mean?*

Example 6.4.1 Butterfly Wings

- n = 14, $\bar{y}$ = 32.81 cm^2 and s = 2.48 cm^2. SE = 0.66 cm^2
- If we perform a new study and want SE be no more than 0.4 cm^2, how many sample do we needed?

6.4 Planning a Study to Estimate μ

Example 6.4.1 Butterfly Wings

- n = 14, $\bar{y}$ = 32.81 cm^2 and s = 2.48 cm^2. SE = 0.66 cm^2
- If we perform a new study and want SE be no more than 0.4 cm^2, how many sample do we needed?
  - $SE_{\bar{Y}} = \frac{s}{\sqrt{n}} \leq 0.4$
  - As a preliminary guess of the SD, we use the value from the old study, s = 2.48 cm^2
  - $SE_{\bar{Y}} = \frac{Guessed\ s}{\sqrt{n}} = \frac{2.48}{\sqrt{n}} \leq$ 0.4 = Desired SE → n ≥ 38.4
  - New study should include at least 39 butterflies.

- A **95%** confidence interval for μ: $\bar{y}$ ± **t** $_{0.025}$ x **s**/√**n**
- ± **t** $_{0.025}$ x Desired SE:
  - How much error one is willing to tolerate in the estimate of μ.
  - **margin of error for 95% confidence** is **t** $_{0.025}$ x SE

6.5 Conditions for Validity of Estimation Methods

Summary of Conditions

Student's t method of constructing a confidence interval for μ is appropriate if the following conditions hold:

1. Conditions on the design of the study

- (a) It must be reasonable to regard the data as a **random sample** from a **large** population.
- (b) The observations in the sample must be **independent** of each other.

2. Conditions on the form of the population distribution

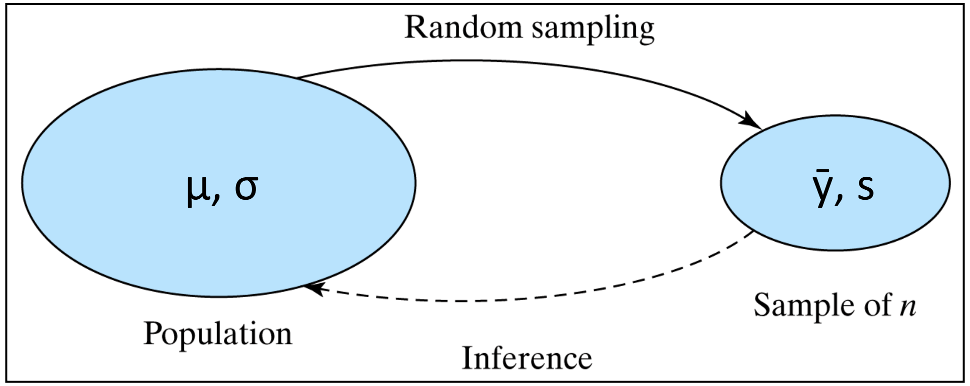
- (a) If n is small, the population distribution must be approximately normal.
- (b) If n is large, the population distribution need not be approximately normal.

* The requirement that the data are a random sample is the most important condition.

6.5 Conditions for Validity of Estimation Methods

Conditions for Validity of the SE Formula

- 1. Data be viewed "as if" they had been generated by **random sampling**



- 2. The use of the standard error formula $SE = s/\sqrt{n}$ requires two further conditions:
  - 1) The **population size must be large** compared to the sample size (see explanation in textbook).
    - This requirement is rarely a problem in the life sciences;
    - The sample can be as much as 5% of the population without seriously invalidating the SE formula.*
  - 2) The **observations must be independent** of each other.
    - This requirement means that the n observations actually give n <u>independent</u> pieces of information about the population.

6.5 Conditions for Validity of Estimation Methods

Conditions for Validity of the SE Formula

- 2. The use of the standard error formula SE = s /√n requires two further conditions:
  - 2) The **observations must be independent** of each other.
    - This requirement means that the n observations actually give n independent pieces of information about the population.
    - Data often <u>fail to meet the independence</u> requirement if the experiment or sampling regime has a **hierarchical structure**, in which observational units are "**nested**" within sampling units.

Figure 6.5.1 Hierarchical data structure of Example 6.5.2

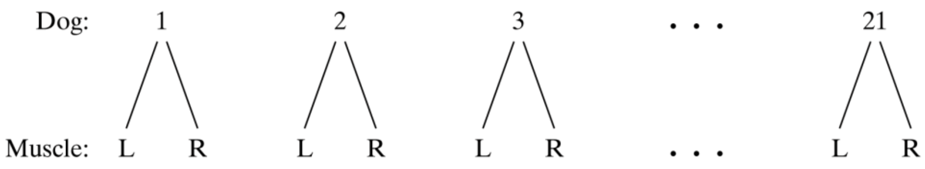
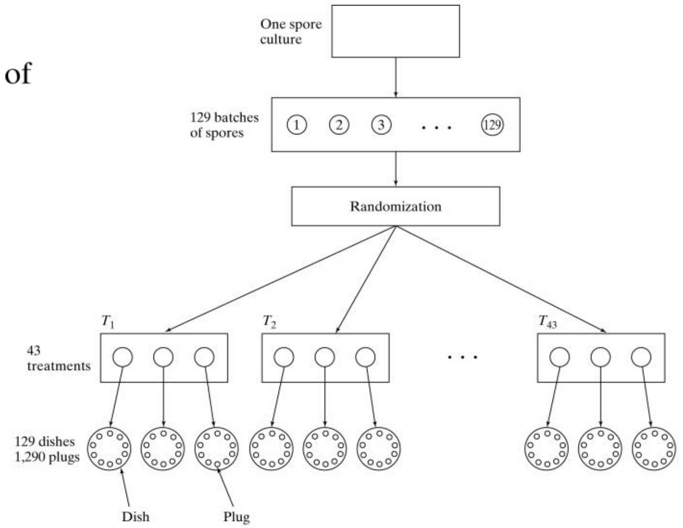


Figure 6.5.2 Design of spore germination experiment

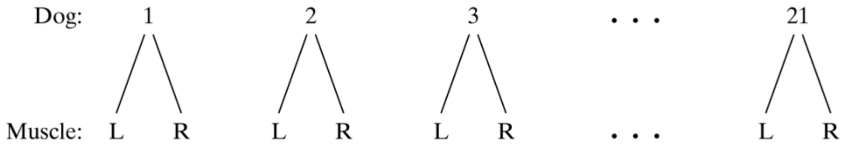


6.5 Conditions for Validity of Estimation Methods

Conditions for Validity of the SE Formula

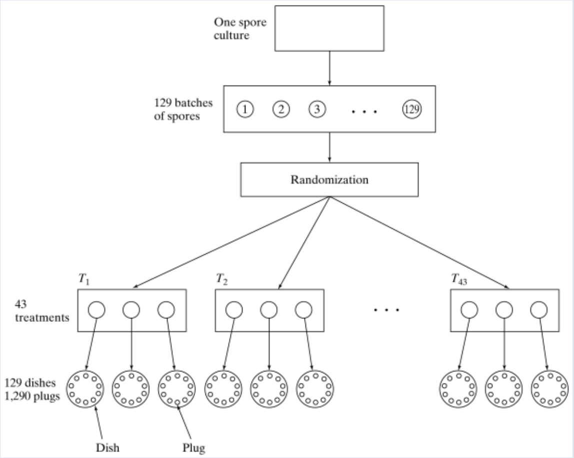
Example 6.5.2 Canine Anatomy

- The weigh of coccygeus muscle.
- The left side and the right side of the tail muscle were weighed for each of 21 female dogs.
- What is the sample size n? 21 or 42?



Example 6.5.3 Germination of spores

- There were 129 batches of spores, which were randomly allocated to the 43 treatments,
- Each batch of spores resulted in one petri dish, and each petri dish resulted in 10 plugs.
- Comparing T1 vs. other treatments.
- What is the sample size n for T1? 30 or 3?



6.5 Conditions for Validity of Estimation Methods

Conditions for Validity of the SE Formula

Example 6.5.2 Canine Anatomy

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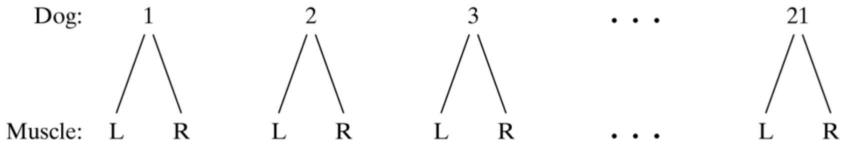
- The left side and the right side of the tail muscle were weighed for each of 21 female dogs.
- What is the sample size n? 21 or 42?



— Because of the symmetry of the coccygeus, n = 21

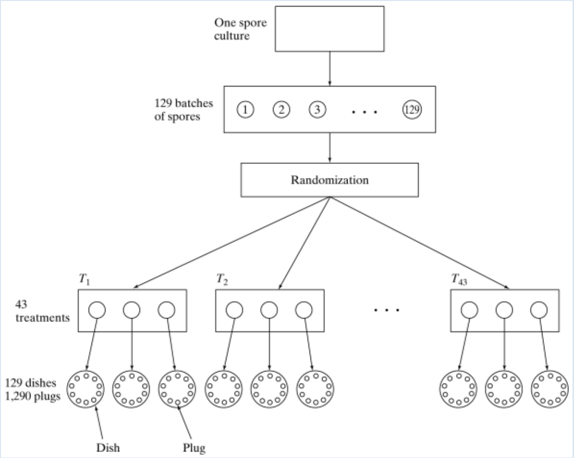
— Many other similar examples: 90 nerve cells that come from only 3 different cats; 60 young mice who come from only 10 litters...

Example 6.5.3 Germination of spores

- There were 129 batches of spores, which were randomly allocated to the 43 treatments,
- Each batch of spores resulted in one petri dish, and each petri dish resulted in 10 plugs.
- Comparing T1 vs. other treatments.
- What is the sample size n for T1? 30 or 3?

— n = 3

— In the language of experimental design, plugs are **nested** within petri dishes.



6.5 Conditions for Validity of Estimation Methods

Conditions for Validity of a Confidence interval for μ

The validity of **Student's t method** for constructing confidence intervals also depends on the form of the population distribution of the observed variable Y.

- If Y follows a normal distribution in the population, then Student's t method is exactly valid.
- If Y do NOT follow a normal distribution in the population, how large must the sample be in order for the confidence interval to be approximately valid?
  - depends on the degree of nonnormality of the population distribution: If the population is only moderately nonnormal, then n need not be very large.

(Review) Theorem 5.2.1 - 3 shape: The Sampling Distribution of $\bar{Y}$

a) If the population distribution of Y is normal, then the sampling distribution of $\bar{Y}$ is normal, regardless of the sample size n.

b) **Central Limit Theorem**: If n is large, then the sampling distribution of $\bar{Y}$ is approximately normal, even if the population distribution of Y is not normal.

6.5 Conditions for Validity of Estimation Methods

Verification of Conditions

- In practice, the preceding “conditions” are often “assumptions” rather than known facts. However, it is always important to check whether the conditions are reasonable in a given case.
  - — random sampling model vs. biases in the choice of experimental material
  - — independence vs. nonindependence of the observations
  - — If distribution is approximately normal
    - normal quantile plot

Example 6.5.4 Sediment Yield

- Thus, we are 95% confident that the **mean natural logarithm** of sediment yield for the Black River is between 2.20 and 4.22.

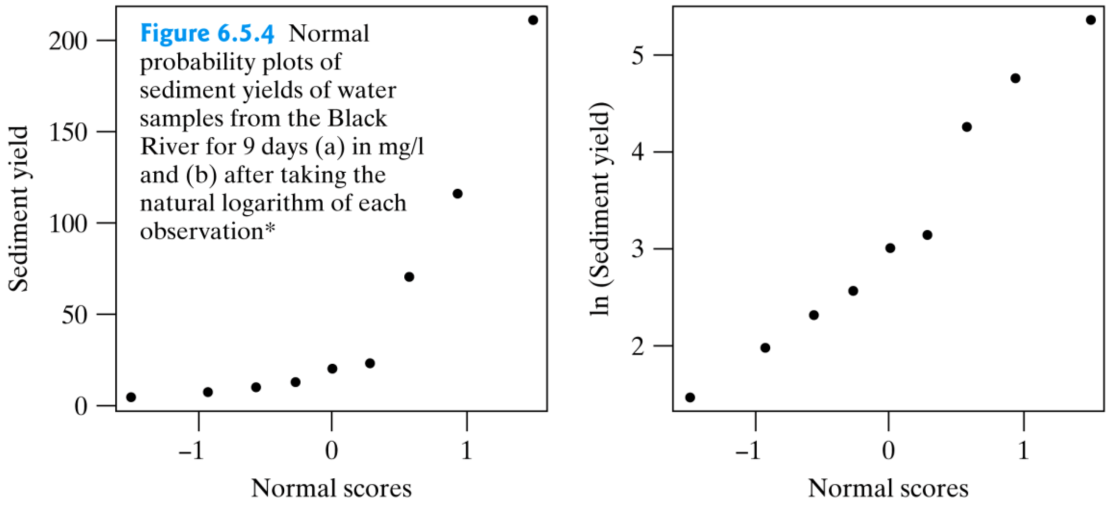


Figure 6.5.4 Normal probability plots of sediment yields of water samples from the Black River for 9 days (a) in mg/l and (b) after taking the natural logarithm of each observation*

6.6 Comparing Two Means

Notation

- Comparison of two or more samples from **different** populations
- Two populations: μ1, σ1; μ2, σ2.
- The notation is exactly parallel to our earlier notation, but now a subscript (1 or 2) is used to differentiate between the two samples.

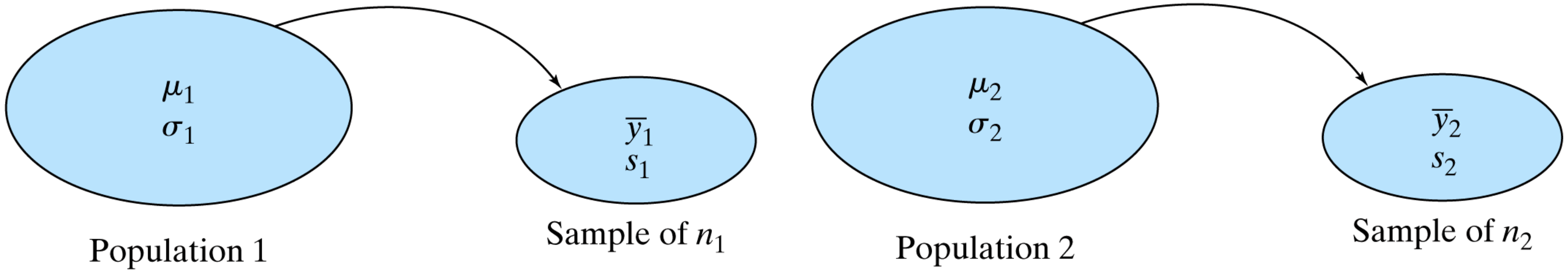


Figure 6.6.1 Notation for comparison of two samples

6.6 Comparing Two Means

Standard Error of ($\bar{Y}_1$ -$\bar{Y}_2$)

- The difference between two sample means ($\bar{Y}1$ -$\bar{Y}2$)
  - ($\bar{Y}1$ -$\bar{Y}2$) is an estimate of the quantity (μ1 - μ2).

DEFINITION The **standard error of $\bar{Y}_1 - \bar{Y}_2$** is defined as

$$\mathrm{SE}_{(\bar{Y}_1 - \bar{Y}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

  - The standard error of ($\bar{Y}1$ -$\bar{Y}2$) tells us how much precision to attach to this difference between $\bar{Y}1$ and $\bar{Y}2$

6.6 Comparing Two Means

Standard Error of ($\bar{Y}_1$ - $\bar{Y}_2$)

- The following alternative form of the formula shows how the SE of the difference is related to the individual SEs of the means:

DEFINITION The **standard error of $\bar{Y}_1 - \bar{Y}_2$** is defined as

$$\mathrm{SE}_{(\bar{Y}_1-\bar{Y}_2)} = \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = \sqrt{SE_1{}^2 + SE_2{}^2}$$

  – Where $\boldsymbol{SE}_{\bar{Y}} = \boldsymbol{s}/\sqrt{\boldsymbol{n}}$: $SE_1 = \boldsymbol{SE}_{\bar{Y}1} = \boldsymbol{s_1}/\sqrt{\boldsymbol{n_1}}$, $SE_2 = \boldsymbol{SE}_{\bar{Y}2} = \boldsymbol{s_2}/\sqrt{\boldsymbol{n_2}}$

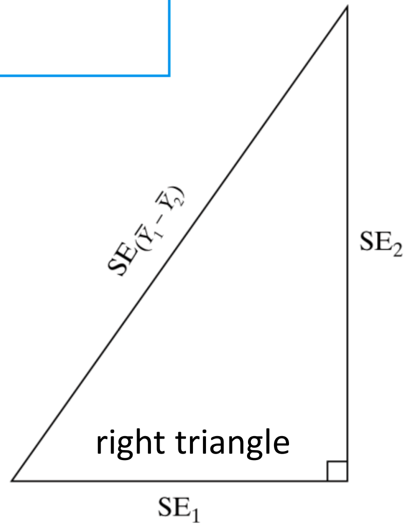


Figure 6.6.2 SE for a difference

6.6 Comparing Two Means

Standard Error of ($\bar{Y}_1$ -$\bar{Y}_2$)

- $$\mathrm{SE}_{(\bar{Y}_1-\bar{Y}_2)} = \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = \sqrt{SE_1{}^2+SE_2{}^2}$$
  - Where $\boldsymbol{SE}_{\bar{Y}} = \boldsymbol{s}/\sqrt{\boldsymbol{n}}$: $SE_1 = \boldsymbol{SE}_{\bar{Y}\mathbf{1}} = \boldsymbol{s_1}/\sqrt{\boldsymbol{n_1}}$, $SE_2 = \boldsymbol{SE}_{\bar{Y}\mathbf{2}} = \boldsymbol{s_2}/\sqrt{\boldsymbol{n}_2}$

Example 6.6.2 Vital capacity

- For the vital capacity data, preliminary computations yield the results in Table 6.6.2.
- What is the Standard Error of ($\bar{Y}1$ -$\bar{Y}2$)?

Table 6.6.2

	Brass player	Control
s^2	0.1892	0.1232
n	7	5
SE	0.164	0.157

Example 6.6.3 Tonsillectomy

- Table 6.6.3 pain score that each child reported, on a scale of 0–10, four days after surgery.
- What is the Standard Error of ($\bar{Y}1$ -$\bar{Y}2$)?

Table 6.6.3 Pain score

	Type of surgery	
	Conventional	Coblation
Mean	4.3	1.9
SD	2.8	1.8
n	49	52

6.6 Comparing Two Means

Standard Error of ($\bar{Y}_1$ - $\bar{Y}_2$)

- $$\mathrm{SE}_{(\bar{Y}_1 - \bar{Y}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{SE_1{}^2 + SE_2{}^2}$$
  - Where $\boldsymbol{SE_{\bar{Y}} = s/\sqrt{n}}$: SE_1 = $\boldsymbol{SE_{\bar{Y}1} = s_1/\sqrt{n_1}}$, $SE_2 = \boldsymbol{SE_{\bar{Y}2} = s_2/\sqrt{n_2}}$

Example 6.6.2 Vital capacity

- What is the Standard Error of ($\bar{Y}1$ - $\bar{Y}2$)?

$$\boldsymbol{SE}_{(\bar{Y}1 - \bar{Y}2)} = \sqrt{0.1892/7 + 0.1232/5} = 0.227$$

Table 6.6.2

	Brass player	Control
s^2	0.1892	0.1232
n	7	5
SE	0.164	0.157

Example 6.6.3 Tonsillectomy

- What is the Standard Error of ($\bar{Y}1$ - $\bar{Y}2$)?

$$\boldsymbol{SE}_{(\bar{Y}1 - \bar{Y}2)} = \sqrt{2.8^2/49 + 1.8^2/52} = 0.47$$

Table 6.6.3 Pain score

	Type of surgery	
	Conventional	Coblation
Mean	4.3	1.9
SD	2.8	1.8
n	49	52

The standard error of ($\bar{Y}1$ -$\bar{Y}2$) tells us how much precision to attach to this difference between $\bar{Y}1$ and $\bar{Y}2$

6.7 Confidence Interval for (μ1 - μ2)

- **95% Confidence interval for μ :**

$$\bar{y} \pm \mathbf{t}_{0.025} \times \mathbf{SE}_{\bar{Y}}$$

- Analogously, **95% Confidence interval for μ1 - μ2 :**

$$(\bar{y}_1 - \bar{y}_2) \pm \mathbf{t}_{0.025} \times \mathbf{SE}_{(\bar{Y}_1 - \bar{Y}_2)}$$

- The critical value $\mathbf{t}_{0.025}$ is determined from Student's t distribution using degrees of freedom* given as

$$\mathrm{df} = \frac{(\mathrm{SE}_1^2 + \mathrm{SE}_2^2)^2}{\mathrm{SE}_1^4/(n_1 - 1) + \mathrm{SE}_2^4/(n_2 - 1)}$$

  - Where $SE_1 = \boldsymbol{SE}_{\bar{Y}1} = \boldsymbol{s_1}/\sqrt{\boldsymbol{n_1}}$, $SE_2 = \boldsymbol{SE}_{\bar{Y}2} = \boldsymbol{s_2}/\sqrt{\boldsymbol{n}_2}$

- approximate degrees of freedom is to use the smaller of ($\boldsymbol{n_1}$ - 1) and ($\boldsymbol{n_2}$ - 1).
  - somewhat conservative, in the sense that the true confidence level is a bit larger than 95% when $t_{0.025}$ is used.
- approximate the degrees of freedom as $\boldsymbol{n_1} + \boldsymbol{n_2} - 2$
  - somewhat liberal, in the sense that the true confidence level is a bit smaller than 95% when $t_{0.025}$ is used.

6.7 Confidence Interval for (μ1 - μ2)

- **The standard error of (Ȳ1 -Ȳ2)**

Example 6.7.1 Fast Plants

- Height distributions often follow a normal curve.
- The standard error of (Ȳ1 -Ȳ2) ?
- 95% confidence interval for (μ1 - μ2) ?
- What is the meaning of this interval?

Table 6.7.1 Fourteen-day height of control and of ancy plants (cm)

	Control (Group 1)	Ancy (Group 2)
	10.0	13.2
	13.2	19.5
	19.8	11.0
	19.3	5.8
	21.2	12.8
	13.9	7.1
	20.3	7.7
	9.6	
n	8	7
$\bar{y}$	15.9	11.0
s	4.8	4.7
SE	1.7	1.8

6.7 Confidence Interval for (μ1 - μ2)

- **The standard error of (Ȳ1 -Ȳ2)**

Example 6.7.1 Fast Plants

- Height distributions often follow a normal curve.
- The standard error of (Ȳ1 -Ȳ2) ?
- 95% confidence interval for (μ1 - μ2) ?
- What is the meaning of this interval?
  - $\boldsymbol{SE}_{(\bar{Y}1\,-\,\bar{Y}2)} = \sqrt{4.8^2/8 + 4.7^2/7} = 2.46$
  - df = $(1.7^2 + 1.8^2)^2 / (1.7^4 / 7 + 1.8^4 / 6) = 12.8$
  - $t_{12.8,\ 0.025} = 2.164$
  - df ~ 12, in which case the t multiplier is 2.179. Or we could approximate $t_{0.025} \approx 2$.)
  - The confidence interval formula gives (15.9 - 11.0) $\pm$ (2.164)(2.46)
  - **The 95% confidence interval** for (μ1 - μ2) is (-0.42, 10.22)

Table 6.7.1 Fourteen-day height of control and of ancy plants (cm)

	Control (Group 1)	Ancy (Group 2)
	10.0	13.2
	13.2	19.5
	19.8	11.0
	19.3	5.8
	21.2	12.8
	13.9	7.1
	20.3	7.7
	9.6	
n	8	7
$\bar{y}$	15.9	11.0
s	4.8	4.7
SE	1.7	1.8

6.7 Confidence Interval for (µ1 - µ2)

- **The standard error of ($\bar{Y}1$ -$\bar{Y}2$)**

Example 6.7.1 Fast Plants

- Height distributions often follow a normal curve.
- The standard error of ($\bar{Y}1$ -$\bar{Y}2$) ?
- 95% confidence interval for (µ1 - µ2) ?
- What is the meaning of this interval?
  - **The 95% confidence interval** for (µ1 - µ2) is (-0.42, 10.22)
  - Thus, we are 95% confident that the population average 14-day height of fast plants when water is used (µ1) is between 0.4 cm lower and 10.2 cm higher than the average 14-day height of fast plants when ancy is used (µ2).

Table 6.7.1 Fourteen-day height of control and of ancy plants (cm)

	Control (Group 1)	Ancy (Group 2)
	10.0	13.2
	13.2	19.5
	19.8	11.0
	19.3	5.8
	21.2	12.8
	13.9	7.1
	20.3	7.7
	9.6	
n	8	7
$\bar{y}$	15.9	11.0
s	4.8	4.7
SE	1.7	1.8

6.7 Confidence Interval for (μ1 - μ2)

- **95% Confidence interval for μ :**

$$\bar{y} \pm t_{0.025} \times SE_{\bar{Y}}$$

- Analogously, **95% Confidence interval for μ1 - μ2 :**

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.025} \times SE_{(\bar{Y}_1 - \bar{Y}_2)}$$

  – The critical value $t_{0.025}$ is determined from Student's t distribution

Conditions for validity

– In Section 6.5 we stated the conditions that make a confidence interval for a mean valid: We require that the data can be thought of as (1) a random sample from (2) a normal population.

– Likewise, when comparing two means, we require two **independent**, **random samples** from normal populations.

– If the sample sizes are large, then the condition of normality is not crucial (due to the Central Limit Theorem).

6.8 Perspective and Summary

Choice of Confidence level

- Often chosen a confidence level equal to 95%; however, there is nothing wrong with an 80% confidence interval, for example.

Summary of Estimation Methods

STANDARD ERROR OF THE MEAN

$$\mathrm{SE}_{\overline{Y}} = \frac{s}{\sqrt{n}}$$

CONFIDENCE INTERVAL FOR μ

95% confidence interval: $\bar{y} \pm t_{0.025}\mathrm{SE}_{\overline{Y}}$

Critical value $t_{0.025}$ from Student's t distribution with df $= n - 1$.

Intervals with other confidence levels (e.g., 90%, 99%) are constructed analogously (using $t_{0.05}$, $t_{0.005}$, etc.).

The confidence interval formula is valid if (1) the data can be regarded as a random sample from a large population, (2) the observations are independent, and (3) the population is normal. If n is large then condition (3) is less important.

6.8 Perspective and Summary

STANDARD ERROR OF $\bar{y}_1 - \bar{y}_2$

$$\mathrm{SE}_{(\bar{Y}_1 - \bar{Y}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\mathrm{SE}_1^2 + \mathrm{SE}_2^2}$$

CONFIDENCE INTERVAL FOR $\mu_1 - \mu_2$

95% confidence interval:

$$(\bar{y}_1 - \bar{y}_2) \pm t_{0.025}\mathrm{SE}_{(\bar{Y}_1 - \bar{Y}_2)}$$

Critical value $t_{0.025}$ from Student's t distribution with

$$\mathrm{df} = \frac{(\mathrm{SE}_1^2 + \mathrm{SE}_2^2)^2}{\mathrm{SE}_1^4/(n_1 - 1) + \mathrm{SE}_2^4/(n_2 - 1)}$$

where $\mathrm{SE}_1 = s_1/\sqrt{n_1}$ and $\mathrm{SE}_2 = s_2/\sqrt{n_2}$.

Confidence intervals with other confidence levels (90%, 99%, etc.) are constructed analogously (using $t_{0.05}$, $t_{0.005}$, etc.).

The confidence interval formula is valid if (1) the data can be regarded as coming from two independently chosen random samples, (2) the observations are independent within each sample, and (3) each of the populations is normally distributed. If n_1 and n_2 are large, condition (3) is less important.

Summary of Estimation Methods (continued)

Summary

Chapter 6 - Confidence Intervals

- 6.1 Statistical Estimation
- 6.2 Standard Error of the Mean
- 6.3 Confidence Interval for μ
- 6.4 Planning a Study to Estimate μ
- 6.5 Conditions for Validity of Estimation Methods
- 6.6 Comparing Two Means
- 6.7 Confidence Interval for (μ1 - μ2)
- 6.8 Perspective and Summary

Homework

Chapter 6

- **6.2.3 ; 6.2.7 ;**
- **6.3.5 ; 6.3.14 ; 6.3.19 ; 6.3.20 ;**
- **6.4.1 ; 6.4.3 ; 6.4.4 ;**
- **6.5.5 ;**
- **6.6.2 ; 6.6.6 ; 6.6.9 ;**
- **6.7.2 ; 6.7.4 ; 6.7.5 ; 6.7.9 .**