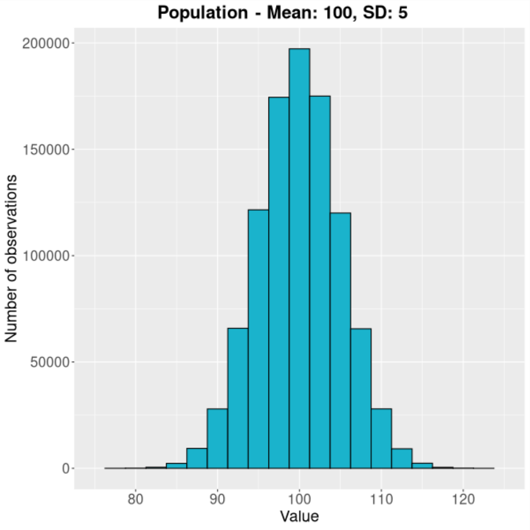
Sampling Distribution & The Central Limit Theorem

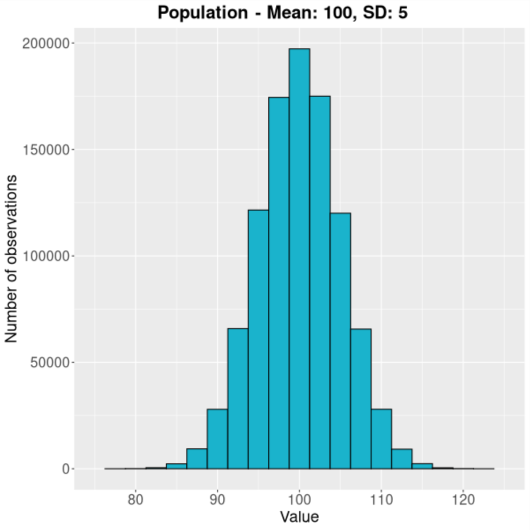
Nicola Romanò - nicola.romano@ed.ac.uk
Based on slides by Duncan McGregor

What’s up with normal distributions?



We talk about normal distributions a lot.

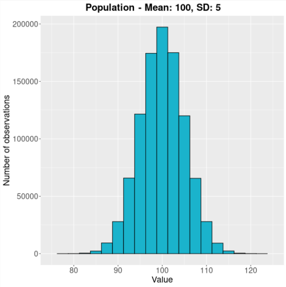
What's up with normal distributions?

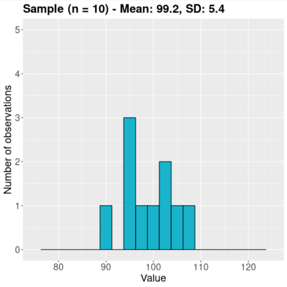


We talk about normal distributions a lot.**But why, actually?**

This lecture is about...

- Properties of sampling distributions.
- The normal distribution...
- ...its properties...
- ...and why it is special.

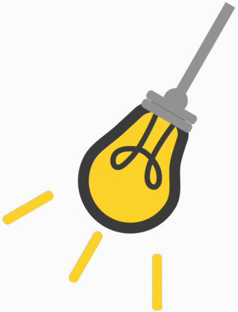




Learning objectives

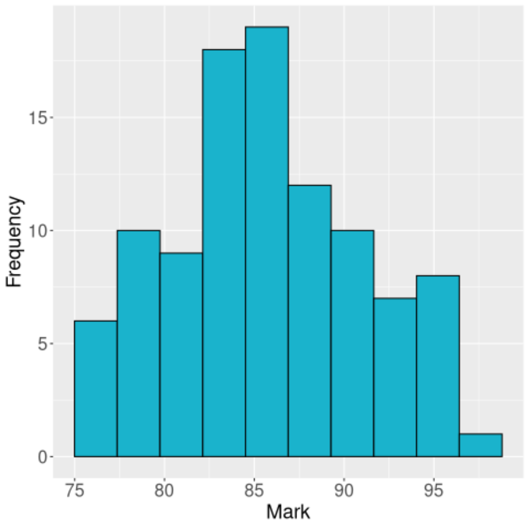
At the end of this lecture, you should be able to:

- Define the standard error of the mean
- Compare sampling distributions and underlying population distributions
- Describe a normal distribution and explain its importance
- Explain the Central Limit Theorem



Examining normal distributions

Create a "virtual class" of 100 exam grades with a mean of 86 and standard deviation of 5.

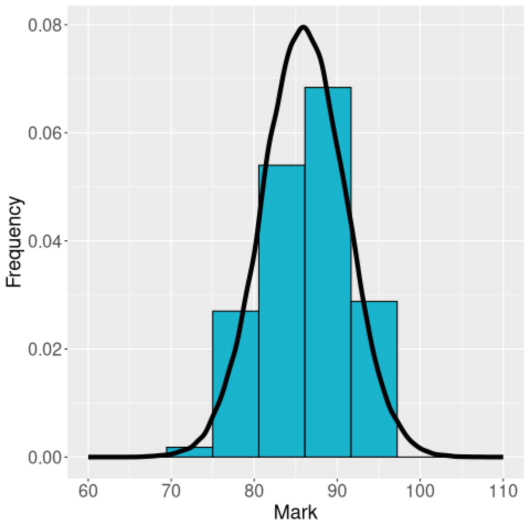


Notation

How would you read this?

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

What are the properties of a normal distribution?



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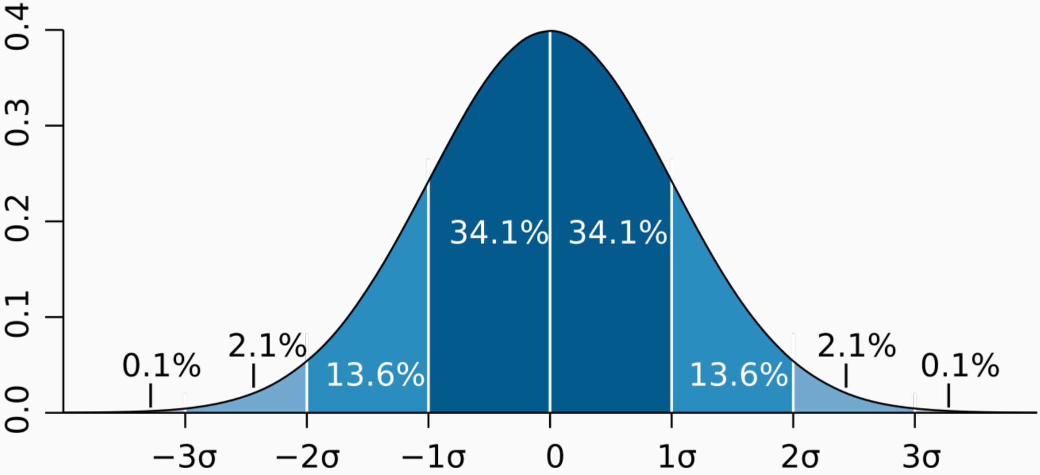
The 68 - 95 - 99.7 Rule



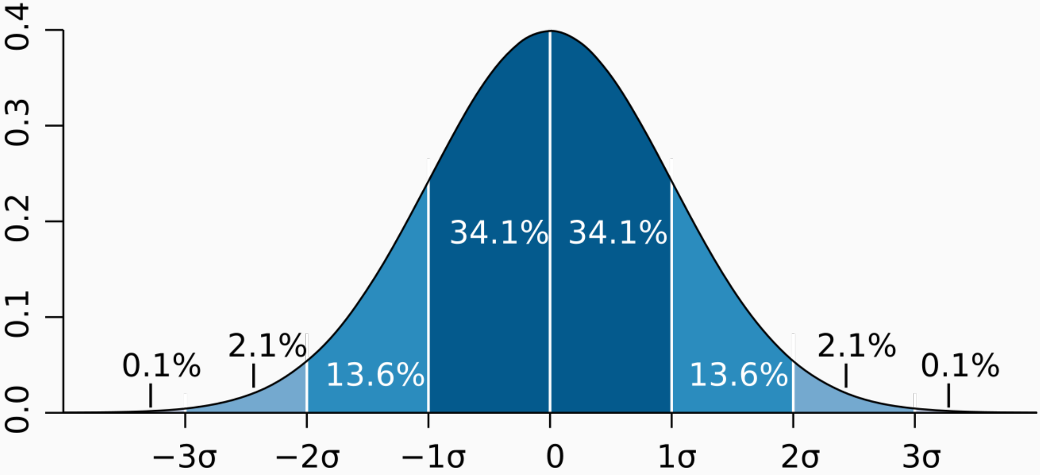
Sampling distributions

From Problem Set 3 (Sampling)

"Take samples of size 5 from a normal distribution, record mean and standard deviation."

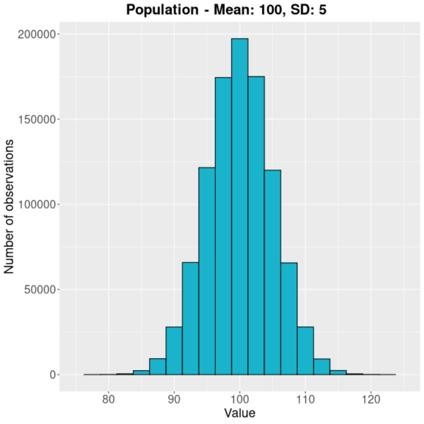
- Is our sample likely to have a higher or lower standard deviation than the population?
- Why?
- How does this relate to sample size?

We are unlikely to sample from the "edges"



From Problem Set 3 (Sampling)

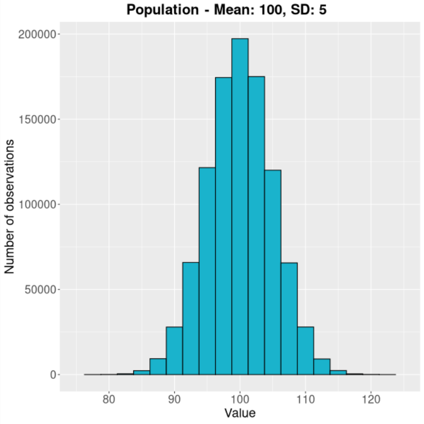
Take samples of size 5 from a normal distribution, record the **mean**



If you do this repeatedly, the distribution of sample means is called the **sampling distribution**.

Where is the sampling distribution centred? How much spread is there?

Population



Sampling distribution (n=5)

?

Fine, but all we (usually) have is a few samples…

How do we know how good a guess our sample mean is for the true population mean?

The **Standard Error of the Mean** (SEM) is a measure of how well your sample mean estimates the true population mean.

$$SEM = \frac{\sigma}{\sqrt{n}}$$

Where

σ is the standard deviation of the population
n is the sample size

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What happens if n increases?
What happens if σ increases?

What is the difference between Standard Error of the Mean and Standard Deviation?

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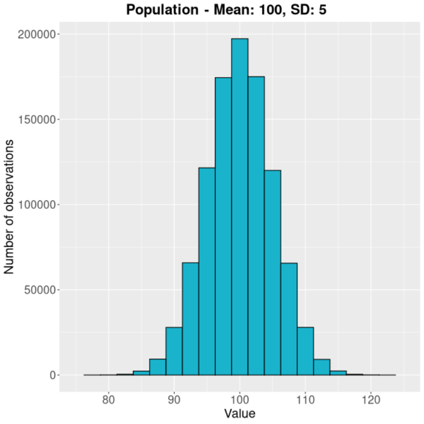
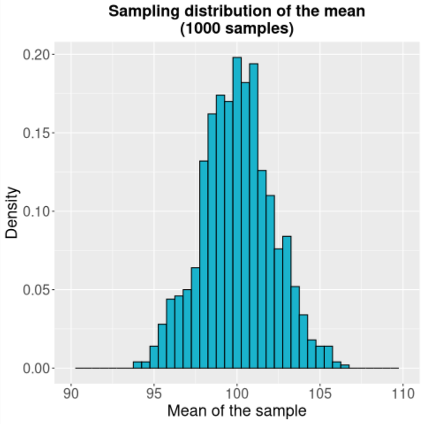
Where do normal distributions come from?

Why do we like normal distributions so much?

Let's recap:





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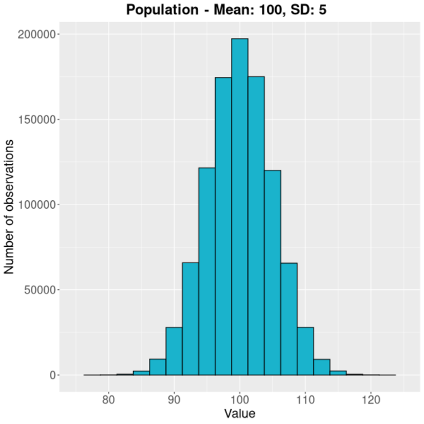

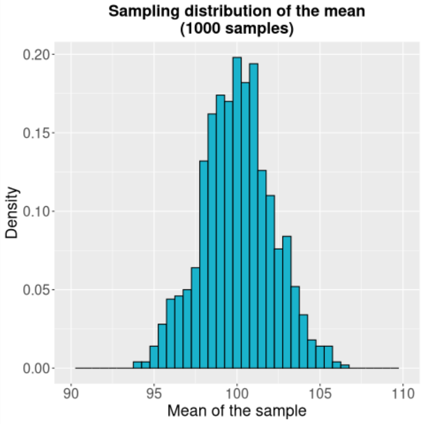

But what happens if we are not sampling from a normal distribution?

The central limit theorem

Source: creaturecast.org

For sample means
Even if a population is not normally distributed, the sampling distribution (for large enough samples) will tend to be normal

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More in general
If we take n independent random variables from any distribution, and take their (normalised) sum, then that sum will tend towards a normal distribution with increasing n.

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Maybe you have seen this in real life before?

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So…why do we love normal distributions so much?

Because it comes up all the time. Even when things are not normally distributed, a normal distribution often “comes out” of parameter combinations, such as taking the mean.

Learning objectives

Now you should be able to:

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- Explain the Central Limit Theorem