Chapter 8

Comparison of Paired Samples

8.1 Introduction

Paired Samples

- In the present chapter we consider the comparison of two samples that are not independent but are paired.
- In a paired design, the observations (Y1 , Y2) occur in pairs;
- the observational units in a pair are linked in some way, so they have more in common with each other than with members of another pair.

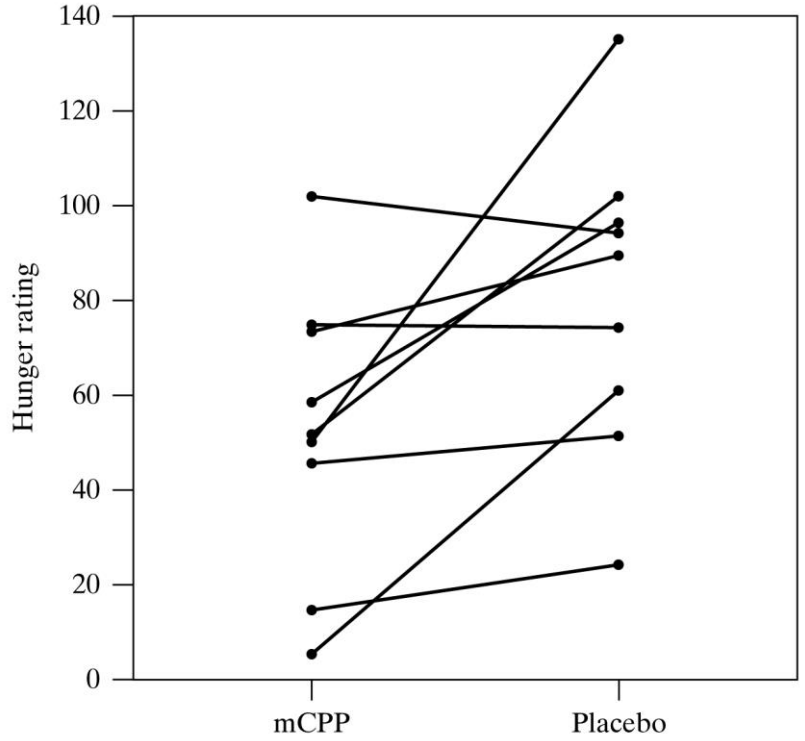

Figure 8.1.1 Dotplots of hunger ratings after mCPP and placebo, with line segments connecting readings on each subject

8.1 Introduction

Paired Samples

Example 8.1.2 Hunger Rating

- each of nine subjects was given drug for 2 weeks; placebo for another 2 weeks,
- the subjects were asked to rate how hungry there were at the end of each 2-week period.
- What are the corresponding hypotheses?

Table 8.1.1 Hunger rating for nine women

	Hunger rating		Difference
Subject	Drug (mCPP)	Placebo	mCPP – Placebo
1	79	78	1
2	48	54	−6
3	52	142	−90
4	15	25	−10
5	61	101	−40
6	107	99	8
7	77	94	−17
8	54	107	−53
9	5	64	−59
Mean	55.3	84.9	−29.6
SD	31.5	34.1	32.8

8.1 Introduction

Paired Samples

Example 8.1.2 Hunger Rating

- What are the corresponding hypotheses?
  - Current experiment uses a paired design (hunger ratings vary from person to person).
  - H_0 : Hunger when taking mCPP is no different from hunger when taking a placebo
  - H_A : Hunger when taking mCPP is different from hunger when taking a placebo

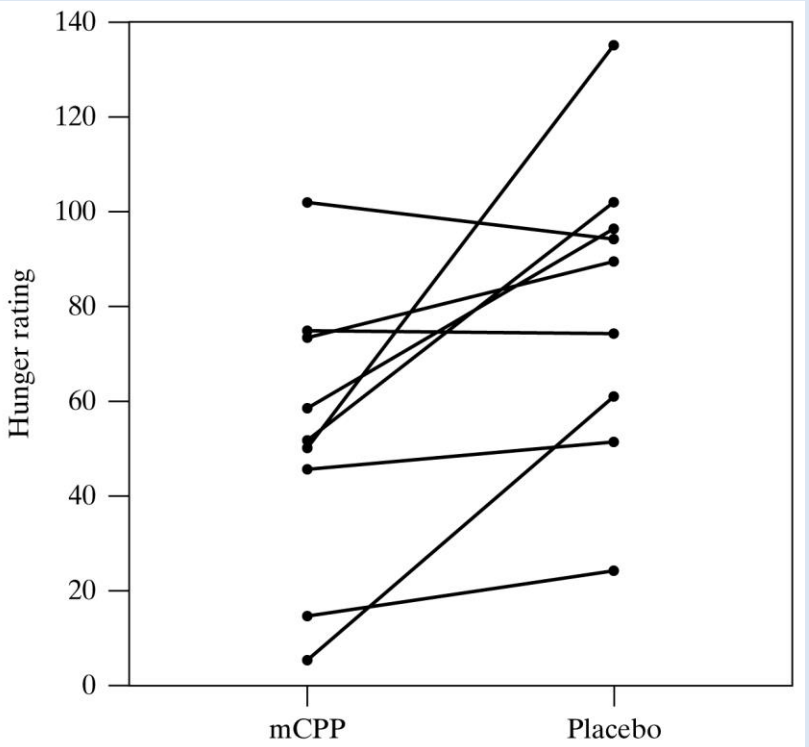

Figure 8.1.1 Dotplots of hunger ratings after mCPP and placebo, with line segments connecting readings on each subject

8.2 The Paired-Sample t Test and Confidence Interval

Analyzing Differences

- In a paired design, the observations (Y1 , Y2) occur in pairs;
- Instead of considering Y1 and Y2 separately, we consider the **difference D**, defined as

$$D = Y1 - Y2$$

- and then $\overline{D} = (\overline{Y}_1 - \overline{Y}_2)$, $\mu_D = \mu_1 - \mu_2$
  - the mean of the difference is equal to the difference of the means.

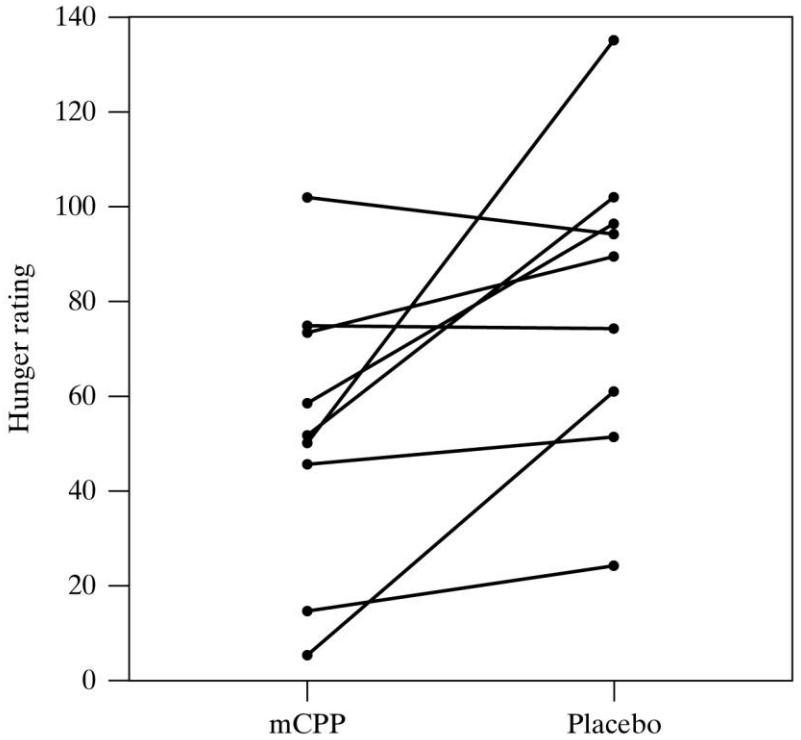

Figure 8.1.1 Dotplots of hunger ratings after mCPP and placebo, with line segments connecting readings on each subject

8.2 The Paired-Sample t Test and Confidence Interval

Summary of Formulas

Standard Error of $\overline{D}$

$$\mathrm{SE}_{\overline{D}} = \frac{s_D}{\sqrt{n_D}}$$

***t* Test**

$$H_0: \mu_D = 0$$

$$t_s = \frac{\overline{d} - 0}{\mathrm{SE}_{\overline{D}}}$$

95% Confidence Interval for μ_d

$$\overline{d} \pm t_{0.025}\mathrm{SE}_{\overline{D}}$$

Intervals with other confidence levels (e.g., 90%, 99%) are constructed analogously (e.g., using $t_{0.05}$, $t_{0.005}$).

8.2 The Paired-Sample t Test and Confidence Interval

Analyzing Differences

Example 8.1.2 Hunger Rating (continued)

- Can we conclude mean hunger rating is reduced more by mCPP than by a placebo? ($\alpha = 0.05$)

Table 8.1.1 Hunger rating for nine women

	Hunger rating		Difference
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8.2 The Paired-Sample t Test and Confidence Interval

Analyzing Differences

Example 8.1.2 Hunger Rating (continued)

- Can we conclude mean hunger rating is reduced more by mCPP than by a placebo? ($\alpha = 0.05$)
  - $H_0 : \mu_D = 0$
  - $H_A : \mu_D \neq 0$
  - The test statistic is
    $t_s = (-29.6 - 0) / (32.8/\sqrt{9}) = -2.71$
  - From Table 4, $t_{8,\,0.02} = 2.449$ and $t_{8,\,0.01} = 2.896$.
  - We reject H_0 and find that there is sufficient evidence ($0.02 < P < 0.04$) to conclude that mean hunger rating is reduced more by mCPP than by a placebo.

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	Hunger rating		Difference
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8.2 The Paired-Sample t Test and Confidence Interval

Analyzing Differences

Example 8.1.2 Hunger Rating (continued)

- Construct a 95% confidence interval for μ_D

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8.2 The Paired-Sample t Test and Confidence Interval

Analyzing Differences

Example 8.1.2 Hunger Rating (continued)

- Construct a 95% confidence interval for μ_D
  - the difference of the means:

$$\bar{d} = 55.3 - 84.9 = -29.6$$

  - the standard error of the mean difference:

$$SE_{\bar{D}} = \frac{s_D}{\sqrt{n_D}} = 32.8/\sqrt{9} = 10.9$$

  - thus, the 95% confidence interval for μ_D is

$$\bar{d} \pm t_{9-1,\,0.025}\, SE_{\bar{D}}$$

$$-29.6 \pm (2.306)\,(32.8/\sqrt{9})$$

Or (-54.8, -4.4)

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8.2 The Paired-Sample t Test and Confidence Interval

Conditions for validity of student's t analysis

The conditions for validity of the paired-sample t test and confidence interval are as follows:

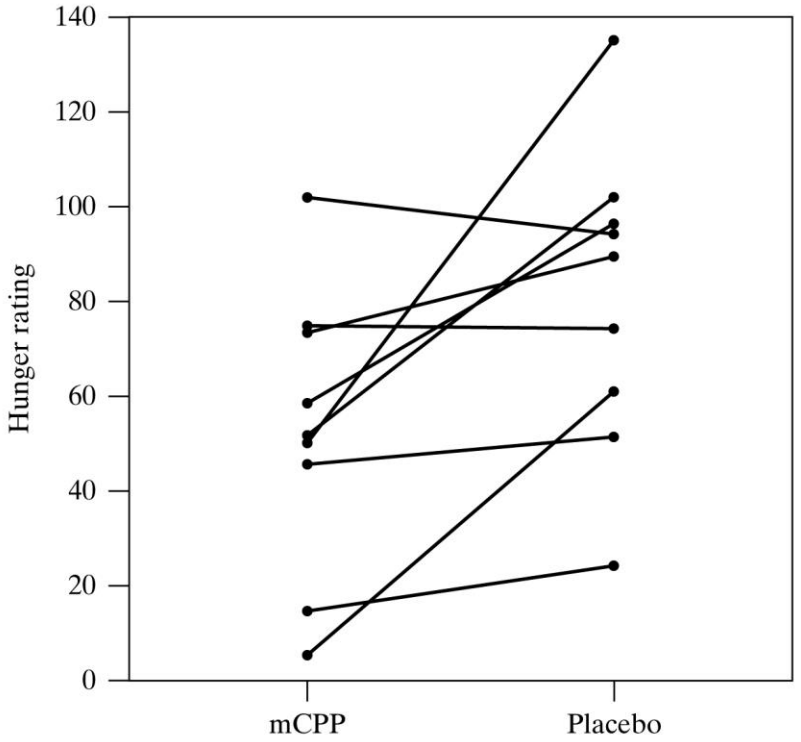
- 1. It must be reasonable to regard the differences (the D's) as a random sample from some large population.

- 2. The population distribution of the D's must be normal. The methods are approximately valid if the population distribution is approximately normal or if the sample size (n_D) is large.

8.3 The Paired Design

Purposes of pairing

- Pairing in an experimental design can serve to reduce bias, to increase precision, or both
- The independent-samples SE formula incorporates all of this variation (expressed through s1 and s2);
- In the paired-sample approach, inter-pair variation has no influence on the calculations because only the D's are used.



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Summary

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- 8.1 Introduction
- 8.2 The Paired-Sample t Test and Confidence Interval
- 8.3 The Paired Design

Homework

Chapter 8

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