MATH1001 Worksheet III-3 (solution)

6.3.3-4

As part of a study of the development of the thymus gland, researchers weighed the glands of five chick embryos after 14 days of incubation. The thymus weights (mg) were as follows:

29.6   21.5   28.0   34.6   44.9

For these data, the mean is 31.7 and the standard deviation is 8.7.

(a) Calculate the standard error of the mean.

(b) Construct a 90% confidence interval for the population mean.

(c) Construct a 95% confidence interval for the population mean.

(d) Interpret the confidence interval you found in part (a). That is, explain what the numbers in the interval mean. (See Examples 6.3.4 and 6.3.5.)

(a) $\bar{y} = 31.720$ mg; $s = 8.729$ mg; $n = 5$.

The standard error of the mean is

$SE_{\bar{y}} = s/\sqrt{n} = 8.729/\sqrt{5} = 3.89 \approx 3.9$ mg.

(b) The degrees of freedom are $df = n - 1 = 5 - 1 = 4$. The critical value is $t_{0.05} = 2.132$.

The 90% confidence interval for μ is

$\bar{y} \pm t_{0.05} \times s/\sqrt{n}$

$31.7 \pm 2.132\ (8.729/\sqrt{5})$

$(23.4, 40.0)$ or $23.4 < \mu < 40.0$ mg.

(c) The degrees of freedom are $df = n - 1 = 5 - 1 = 4$. The critical value is $t_{0.025} = 2.776$.

The 95% confidence interval for μ is

$\bar{y} \pm t_{0.025} \times s/\sqrt{n}$

$31.7 \pm 2.776\ (8.729/\sqrt{5})$

$(20.9, 42.5)$ or $20.9 < \mu < 42.5$ mg.

(d) We are 95% confident that the mean thymus gland weight in the population of chick embryos is between 20.9 and 42.5 mg.

6.4.2

A medical researcher proposes to estimate the mean serum cholesterol level of a certain population of middle-aged men, based on a random sample of the population. He asks a statistician for advice. The ensuing discussion reveals that the researcher wants to estimate the population mean to within ±6 mg/dl or less, with 95% confidence. Thus, the standard error of the mean should be 3 mg/dl or less. Also, the researcher believes that the standard deviation of serum cholesterol in the population is probably about 40 mg/dl. How large a sample does the researcher need to take?

We use the inequality: Guessed SD / $\sqrt{n} \leqslant$ Desired SE.

In this case, the desired SE is 3 mg/dl and the guessed SD is 40 mg/dl. Thus, the inequality is

$40/\sqrt{n} \leqslant 3$ or $40/3 \leqslant \sqrt{n}$ which means that $n \geqslant 177.8$, so a sample of n = 178 men is needed.

6.7.8

In a field study of mating behavior in the Mormon cricket (Anabrus simplex), a biologist noted that some females mated successfully while others were rejected by the males before coupling was complete. The question arose whether some aspect of body size might play a role in mating success. The accompanying table summarizes measurements of head width (mm) in the two groups of females.

	Successful	Unsuccessful
n	22	17
$\bar{y}$	8.498	8.440
s	0.283	0.262

(a) Construct a 95% confidence interval for the difference in population means. [Note: Formula (6.7.1) yields 35.7 degrees of freedom for these data.]

(b) Interpret the confidence interval from part (a) in the context of this setting.

(c) Using your interval computed in (a) to support your answer, is there strong evidence that the population mean head width is indeed larger for successful maters than unsuccessful maters?

(a) Let 1 denote successful and let 2 denote unsuccessful.

$SE_{(\bar{y}_1 - \bar{y}_2)} = \sqrt{(0.283^2/22 + 0.262^2/17)} = 0.08763.$

$(8.498 - 8.440) \pm (2.021)(0.08763)$ (using df = 40)

$(-0.12, 0.24)$ or $-0.12 < \mu_1 - \mu_2 < 0.24$ mm.

(b) We are 95% confident that the population mean head width of all females who mate successfully (μ_1) is smaller than that for rejected females (μ_2) by an amount that might be as much as 0.12 mm or is larger than that for rejected females (μ_2) by an amount that might be as large as 0.24 mm.

(c) There is no compelling evidence that the mean head width for the successful and unsuccessful maters differs, as zero is contained in the confidence interval computed in part (a). That is, a difference in mean head width of zero (no difference) is consistent with this data.

7.2.10

In a study of the development of the thymus gland, researchers weighed the glands of 10 chick embryos. Five of the embryos had been incubated 14 days, and five had been incubated 15 days. The thymus weights were as shown in the table. 11 [Note: Formula (6.7.1) yields 7.7 df.]

(a) Use a t test to compare the means at $\alpha = 0.10$.

(b) Note that the chicks that were incubated longer had a smaller mean thymus weight. Is this "backward" result surprising, or could it easily be attributed to chance? Explain.

	Thymus weight (MG)	
	14 Days	15 Days
	29.6	32.7
	21.5	40.3
	28.0	23.7
	34.6	25.2
	44.9	24.2
n	5	5
$\bar{y}$	31.72	29.22
s	8.73	7.19

(a) H_0: mean thymus weight is the same at 14 and 15 days ($\mu_1 = \mu_2$)

H_A: mean thymus weight is not the same at 14 and 15 days ($\mu_1 \neq \mu_2$)

$$SE_{(\bar{Y}1 - \bar{Y}2)} = \sqrt{\frac{8.73^2}{5} + \frac{7.19^2}{5}} = 5.06$$

$t_s = (31.72 - 29.22)/5.06 = 0.49$. df = $n_1 + n_2 - 2 = 8$. (Formula (6.7.1) gives df = 7.7.) Table 4 gives $t_{0.20} = 0.889$; thus P –value > 0.40, so we do not reject H_0.

There is insufficient evidence ($P > 0.40$) to conclude that mean thymus weight is different at 14 and 15 days.

(b) According to the P-value found in part (a), the fact that $\bar{y}_1$ is greater than $\bar{y}2$ could easily be attributed to chance.

8.2.1

In an agronomic field experiment, blocks of land were subdivided into two plots of 346 square feet each. Each block provided two paired observations: one for each of the varieties of wheat. The plot yields (lb) of wheat are given in the table.

(a) Calculate the standard error of the mean difference between the varieties.

(b) Test for a difference between the varieties using a paired t test at a = 0.05. Use a nondirectional alternative.

(c) Test for a difference between the varieties the wrong way, using an independent-samples test. Compare with the result of part (b).

	Variety		
Block	1	2	Difference
1	32.1	34.5	−2.4
2	30.6	32.6	−2.0
3	33.7	34.6	−0.9
4	29.7	31.0	−1.3
Mean	31.53	33.18	−1.65
SD	1.76	1.72	0.68

(a) The standard deviation of the four sample differences is given as 0.68. The standard error is

$$SE_{\bar{D}} = \frac{s_D}{\sqrt{n_D}} = \frac{0.68}{\sqrt{4}} = 0.34.$$

(b) H_0: The mean yields of the two varieties are the same ($\mu_D = 0$)
H_A: The mean yields of the two varieties are different ($\mu_D \neq 0$)

$t_s = -1.65/0.34 = -4.85$. With df = 3, Table 4 gives $t_{0.01} = 4.541$ and $t_{0.005} = 5.841$;
thus, $0.01 < \text{P-value} < 0.02$. At significance level $\alpha = 0.05$, we reject H_0 if $P < 0.05$.
Since $0.01 < P < 0.02$, we reject H_0.
There is sufficient evidence ($0.01 < P < 0.02$) to conclude that Variety 2 has a higher mean yield than Variety 1.

(c) H_0: The mean yields of the two varieties are the same ($\mu_1 = \mu_2$)
H_A: The mean yields of the two varieties are different ($\mu_1 \neq \mu_2$)

$$SE_{(\bar{Y}1 - \bar{Y}2)} = \sqrt{\frac{1.76^2}{4} + \frac{1.72^2}{4}} = 1.230.$$

$t_s = -1.65/1.230 = -1.34$. With df = 6, Table 4 gives $t_{0.20} = .906$ and $t_{0.10} = 1.440$.
Thus, $0.20 < \text{P-value} < 0.40$ and we do not reject H_0.
There is insufficient evidence ($0.20 < P < 0.40$) to conclude that the mean yields of the two varieties are different. (By contrast, the correct test, in part (b), resulted in rejection of H_0.)