MATH1001 Worksheet III-2 (solution)

4.3.9

The serum cholesterol levels of 12- to 14-year-olds follow a normal distribution with mean 155 mg/dl and standard deviation 27 mg/dl. What percentage of 12 to 14-year-olds have serum cholesterol values

(a) 164 or more?

(b) 137 or less?

(c) 186 or less?

(d) 100 or more?

(e) between 159 and 186?

(f) between 100 and 132?

(g) between 132 and 159?

(a) 1 - 0.6293 = 0.3707 or 37.07%

(b) 0.2514 or 25.14%

(c) 0.8749 or 87.49%

(d) 1 - 0.0207 = 0.9793 or 97.93%

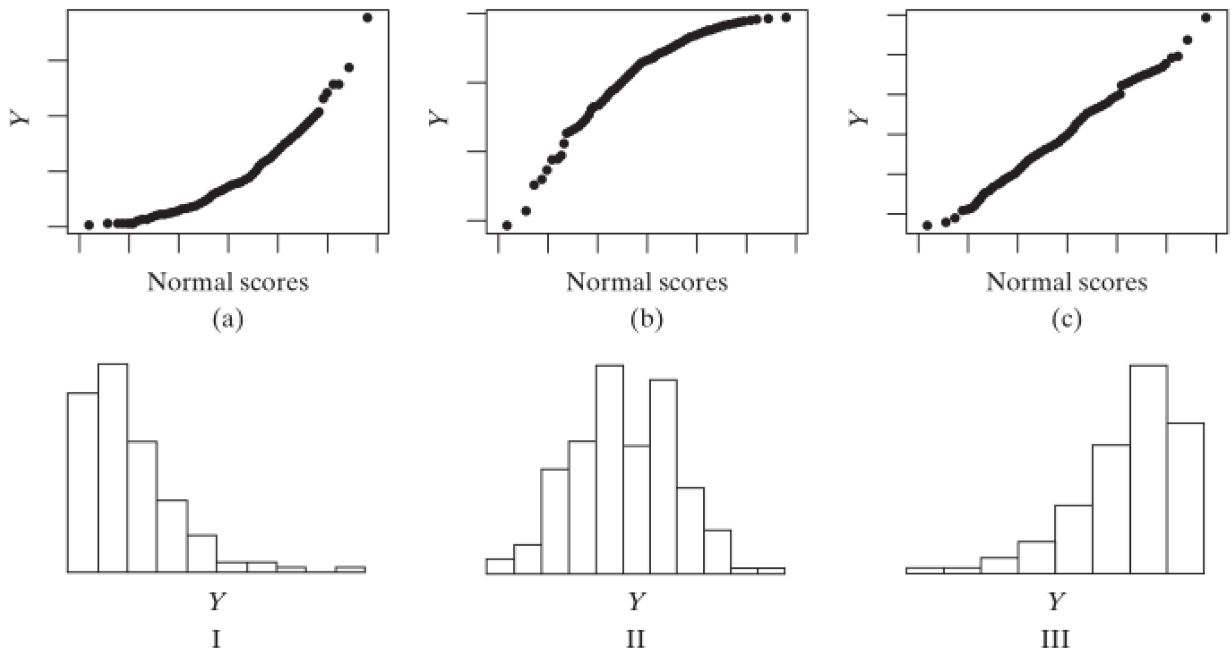
(e) 0.8749 - 0.5596 = 0.3153 or 31.53%

(f) 0.1977 - 0.0207 = 0.1770 or 17.7%

(g) 0.5596 - 0.1977 = 0.3619 or 36.19%

4.4.2

The following three normal probability plots, (a), (b), and (c), were generated from the distributions shown by histograms I, II, and III. Which normal probability plot goes with which histogram? How do you know?



Normal probability plot (a) goes with histogram I, since that histogram is skewed to the right and plot (a) shows larger Y values than would be expected for a symmetric, bell-shaped distribution;

plot (b) goes with histogram III, because that histogram is skewed to the left;

and plot (c) goes with histogram II, which is reasonably symmetric and bell-shaped, which corresponds to a linear normal probability plot.

5.1.2

Consider taking a random sample of size 3 from the knee replacement population of Example 5.1.3. What is the probability that the total cost for those in the sample will be between \$80,000 and \$125,000?

Table 5.1.2 Sampling distribution of total surgery costs for samples of size $n = 3$

Sample total	Sample mean	Probability
0	0.0	1/64
35	11.7	6/64
60	20.0	3/64
70	23.3	12/64
95	31.7	12/64
105	35.0	8/64
120	40.0	3/64
130	43.3	12/64
155	51.7	6/64
180	60.0	1/64

There are three possible total costs in between \$80,000 and \$125,000 when sampling n = 3 patients as listed in Table 5.1.2: \$95,000, \$105,000, and \$120,000. Thus,

Pr{total cost > \$125,000} = Pr{total cost = \$95,000} + Pr{total cost = \$105,000} + Pr{total cost = \$120,000} = 12/64 + 8/64 + 3/64 = 23/64 = 0.3594

5.2.10

In a certain population of fish, the lengths of the individual fish follow approximately a normal distribution with mean 54.0 mm and standard deviation 4.5 mm. We saw in Example 4.3.1 that in this situation, 65.68% of the fish are between 51 and 60 mm long. Suppose a random sample of four fish is chosen from the population. Find the probability that

(a) all four fish are between 51 and 60 mm long.

(b) the mean length of the four fish is between 51 and 60 mm.

(a) In the population, 65.68% of the fish are between 51 and 60 mm long. To find the probability that four randomly chosen fish are all between 51 and 60 mm long, we let "success" be "between 51 and 60 mm long" and use the binomial distribution with n = 4 and p = 0.6568, as follows:

Pr{all 4 are between 51 and 60} = ${}_4C_4\, p^4 (1 - p)^0 = (1)\, 0.6568^4 (1) = 0.1861$.

(b) The mean length of four randomly chosen fish is $\bar{Y}$. Thus, we are concerned with the sampling distribution of $\bar{Y}$ for a sample of size n = 4 from a population with μ = 54 and σ = 4.5. From Theorem 5.2.1, the mean of the sampling distribution of $\bar{Y}$ is

$\mu_{\bar{Y}} = \mu = 54$,

the standard deviation is

$\sigma_{\bar{Y}} = \frac{\sigma}{\sqrt{n}} = \frac{4.5}{\sqrt{4}} = 2.25$,

and the shape of the distribution is normal because the population distribution is normal (part 3a of Theorem 5.2.1).

For $\bar{y} = 60$,

$z = \frac{\bar{y} - \mu_{\bar{Y}}}{\sigma_{\bar{Y}}} = (60 - 54)/2.25 = 2.67$.

From Table 3, the area below 2.67 is 0.9962.

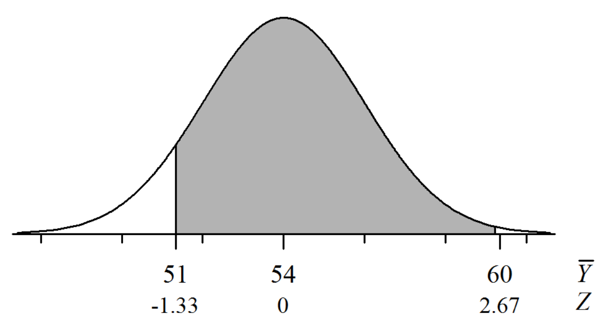
For $\bar{y} = 51$,

$z = \frac{\bar{y} - \mu_{\bar{Y}}}{\sigma_{\bar{Y}}} = (51 - 54)/2.25 = -1.33$.

From Table 3, the area below -1.33 is 0.0918.

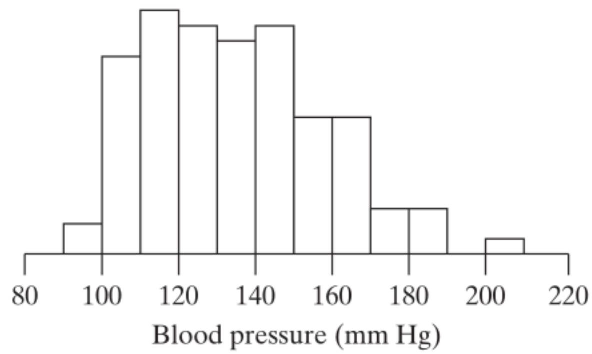
Thus, Pr{$51 \le \bar{Y} \le 60$}

= 0.9962 - 0.0918 = 0.9044.



5.2.18

A medical researcher measured systolic blood pressure in 100 middle-aged men. 5 The results are displayed in the accompanying histogram; note that the distribution is rather skewed. According to the Central Limit Theorem, would we expect the distribution of blood pressure readings to be less skewed (and more bell shaped) if it were based on n = 400 rather than n = 100 men. Explain.



No. The histogram shows the distribution of observations in the sample. Such a distribution would look more like the population distribution for n = 400 than for n = 100, and the population distribution is apparently rather skewed. The Central Limit Theorem applies to the sampling distribution of $\bar{Y}$, which is <u>not</u> what is shown in the histogram.