ADS2 Problem set: Bayesian Inference

ADS2 (based on MI Stefan)

Semester 2, 2023/24

We expect this problem set to take around one hour to complete. But professors are sometimes wrong![citation missing]. If this or future problem sets are too long, please let us know, so we can adjust and plan accordingly.

Please make use of the online discussion board to share your ideas and get help.

A dicy problem - Part 2

How did you tackle the dice problem in the practical?

Here is some of the thinking I did before starting the coding:

We know that the friend threw 7 out of 20 sixes. That means that their die definitely does not have either 0 or 6 sixes. So we are left with five hypotheses:

- H_1: Their die has 1 six
- H_2: Their die has 2 sixes
- ...
- H_5: Their die has 5 sixes

Ultimately, we want to find the most probable hypothesis given the data we have, so we want to find the i that maximises $P_{(H_i|Data)}$.

We can easily (well, quite easily) compute $P_{(Data \mid H_i)}$ for each i, this is just a probability exercise.

We don't have $P_{(Data)}$, but does it matter? Not if all we want to do is find the most probable hypothesis. $P_{(Data)}$ does not depend on the hypothesis chosen, so it's just a constant factor. The maximum of $P_{(H_i|Data)}$ is also the maximum of $P_{(H_i|Data)} \times P_{(Data)}$. So if all we want to do is find the max, we can just go ahead and ignore $P_{(Data)}$.

What about priors though? This is what this exercise is about!

Here are three students with very different approaches to constructing priors.

Aditi: My experience in the world suggests that most playing dice have exactly one 6. In fact, probably all dice I have ever seen had exactly one 6. So $P(H_1)$ would be close to 1. So, I set $P(H_1)$ to 0.99 and $P(H_2) = P(H_3) = P(H_4) = P(H_5) = 0.0025$.

Bo: We know nothing about this die, and dealing with different priors is complicated, so we may as well assume $P(H_i) = 0.2$ for all i $(1 < i < 5)$.

Charlie: I know my friend to be a trickster and I don't trust them. I am 99 % sure the die is not fair, therefore $P(H_1) = 0.01$. I don't know how unfair exactly, so I assume $P(H_2) = P(H_3) = P(H_4) = P(H_5) = 0.2475$.

Does the choice of prior influence the end result?

Also, why do the priors for each person have to add up to 1?

And why does nobody choose priors of exactly 0 or exactly 1? What would happen if they did?
