浙江大学爱丁堡大学联合学院
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The *t*-test: practical applications and variants

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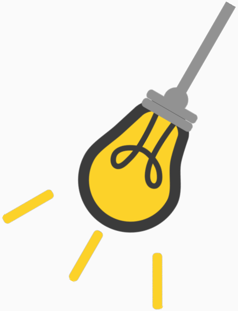
At the end of this lecture, you should be able to:

- Describe variants of the t-test
    - 1-tailed vs 2-tailed
    - 1- vs 2-sample
    - Paired vs unpaired
- Choose an appropriate type of t-test for a given problem
- Describe and use non-parametric alternatives to the t-test



Our earlier example

Is the factory filling each bottle with enough Guinness?

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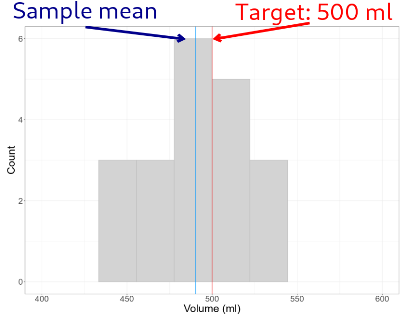

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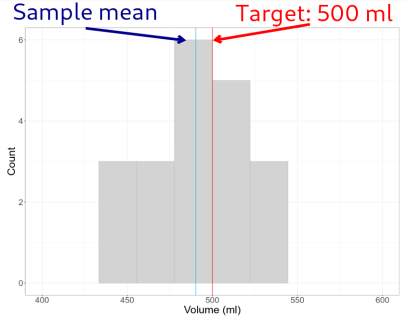

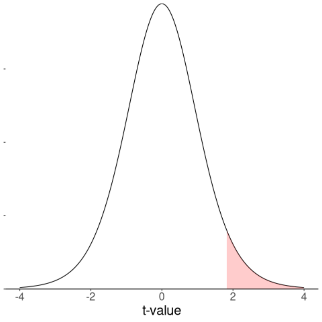

Variants of the *t*-test

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1-tailed vs 2-tailed

One-tailed vs two-tailed t-test

Compare the following two questions:

- Is the factory filling each bottle with **enough** Guinness?
- Is the factory filling each bottle with a volume **different** from 500ml?

One-tailed vs two-tailed t-test

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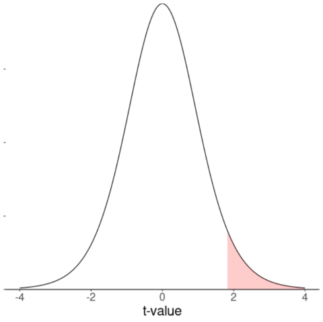
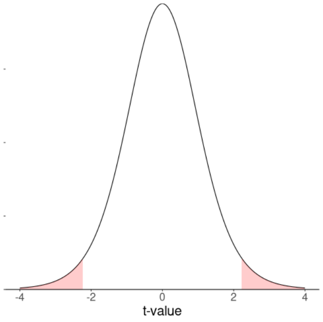
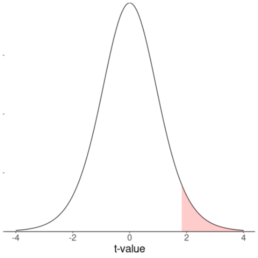


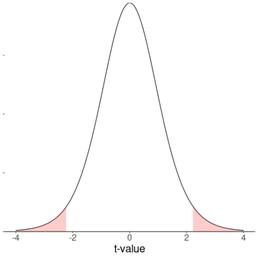
1-tailed - Area of interest is only on one side of the distribution (ex. 5%)



2-tailed - Area of interest is on both sides of the distribution (ex. 2.5% on each side)

Which one should I use?





One-tailed

- If we look for outcomes in one direction only, e.g. A > B (or A < B)
- H_0: A is not greater (or smaller) than B
- H_1: A is greater (or smaller) than B

Two-tailed

- If we look for outcomes in both directions, e.g. A $\neq$ B
- H_0: A is equal to B
- H_1: A is not equal to B

Critical values for 1-tailed and 2-tailed tests

For the same significance level, the critical values for 1-tailed and 2-tailed tests are different!

One tailed Two tailed

d.f.	$t_{.100}$	$t_{.050}$*	$t_{.025}$**	$t_{.010}$	$t_{.005}$	d.f.
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10

Variants of the *t*-test

1-sample vs 2-sample

1-sample vs 2-sample t-test

Compare the following two questions

Is the factory filling each bottle with a volume different from 500ml?

Is the factory in Dublin filling each bottle with a volume different from the factory in Glasgow?

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difference from the ref. value

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

sample standard deviation (estim, of the population σ)

standard error of the mean

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Is the factory in Dublin filling each bottle with a volume different from the factory in Glasgow?

diff. of means

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

combined error

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Equal variances
Student's t-test

Unequal variances
Welch's t-test

Example:

Sample 1: 4, 6, 8, 10

Sample 2: 1, 5, 3, 4

H_0: $\mu_1 = \mu_2$

H_A: $\mu_1 \neq \mu_2$

What should we do to test this hypothesis?

Variants of the *t*-test

Paired vs unpaired

Paired vs unpaired t-test

Paired *t*-test: Same subjects are measured at different times or under different conditions.

Unpaired *t*-test: Different subjects are measured at different times or under different conditions.

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Paired *t*-test example You want to study the effect of a drug on the blood pressure of 10 patients. You measure the blood pressure of each patient before and after the administration of the drug.

Paired t-test - calculating t statistics

The paired t-test is effectively a 1-sample t-test on the differences between the two measurements (with reference value = 0).

avg. difference between paired measurements

$$t = \frac{\bar{d}}{\left(\frac{s_d}{\sqrt{n}}\right)}$$

standard deviation of the differences

t-test variants in R

Use the `t.test()` function in R to perform a *t*-test.

```
t.test(x, y = NULL,
alternative = c("two.sided", "less", "greater"),
mu = 0, paired = FALSE, var.equal = FALSE,
conf.level = 0.95, ...)
```

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1 sample

`mu` = reference value

2 sample

Must provide two vectors of values as `x` and `y`

For Welch's t-test`var.equal` = `FALSE` ()

1-tailed

`alternative` = `"greater"` or `"less"`

2-tailed

`alternative` = `"two.sided"` (default)

Paired

`paired` = `TRUE`

Unpaired

`paired` = `FALSE` (default)

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State the hypothesis. What is H_0 and what is H_A?

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2. What direction are you looking at? 1-tailed or 2-tailed?
3. Are you comparing 1-sample or 2-samples?
4. Are the observations paired or unpaired?

What to do when assumptions are not met?

- **Normality of the sample(s)**
  - If the deviation from normality is small, the t-test is robust to it, and we can still use it.
  - If the deviation from normality is large, we can try transforming the data and see if the data become more normal. Examples of transformations include log, square root, inverse, etc.
  - If the data cannot be transformed, we can use a **non-parametric** test instead (see next slides).

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  - If the data cannot be transformed, we can use a **non-parametric** test instead (see next slides).
- **Homogeneity of variances**
  - We can use Welch's t-test instead of Student's t-test.

A **non-parametric** (or distribution-free) test is a statistical test that does not assume that the data follow a particular distribution.
As alternatives to the t-test, we can use the following non-parametric tests:

- **Wilcoxon signed-rank test** - for paired data or one-sample data
- **Mann-Whitney U test** - for unpaired data

Non-parametric tests

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The idea of these two tests is very similar, and in R you can use the same function `wilcox.test()` to perform both tests.

Wilcoxon: `wilcox.test(x, y, paired = TRUE, ...)`
Wilcoxon, one-sample: `wilcox.test(x, mu = 0, ...)`
Mann-Whitney: `wilcox.test(x, y, paired = FALSE, ...)`

The idea behind non-parametric tests

Wilcoxon and Mann-Whitney tests are based on the **rank** of the observations.
They do not assume a particular distribution of the data, but they do assume that the observations are **independent** and that the **distribution of the observations is the same** (or similar) in the two groups.
They compare medians rather than means.

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They compare medians rather than means.

Note: they have **less statistical power** than the *t*-test, so they are less likely to detect a difference when there is one. However, they are more robust to deviations from normality and to outliers.

How non-parametric tests work

Mann-Whitney test:

- Combine all observations and rank them from smallest to largest
- Sum the ranks in each group
- Calculate the test statistic U and compare it to the critical value (we're not going to do this by hand, but R will do it for you!)

How non-parametric tests work

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Wilcoxon test:

- Calculate the difference between the two observations in each pair
- Rank the differences from smallest to largest
- Sum the ranks of the positive differences and the ranks of the negative differences
- Calculate the test statistic W and compare it to the critical value

Learning objectives

You should now be able to:

- Describe variants of the t-test
  - 1-tailed vs 2-tailed
  - 1- vs 2-sample
  - Paired vs unpaired
- Choose an appropriate type of t-test for a given problem
- Describe and use non-parametric alternatives to the t-test