Written Homework #2.

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1. eye pos (-5, 3, -2). lookat point (-5, 0, -2), up vector (1, 0, 0)

Let lookat be l and eye pos be e

$l - e = (-5, 0, -2) - (-5, 3, -2) = (0, -3, 0)$

$\vec{w}$ Norm $\frac{(0, -3, 0)}{\sqrt{9}} = (0, \frac{3}{3}, 0) = (0, 1, 0)$

$t \times w = \begin{bmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = (0 \cdot 0 - 0 \cdot 1)i + (0 \cdot 0 - 1 \cdot 0)j + (1 \cdot 1 - 0 \cdot 0)k$

$0i + 0j + 1k = (0, 0, 1)$

$\vec{u}$ Norm $\frac{(0, 0, 1)}{\sqrt{1^2}} = (0, 0, 1)$

$w \times u = \begin{bmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (1 \cdot 1 - 0 \cdot 0)i + (0 \cdot 0 - 0 \cdot 1)j + (0 \cdot 0 - 1 \cdot 0)k$

$1i + 0j + 0k = (1, 0, 0)$

$\vec{v}$ Norm $\frac{1, 0, 0}{\sqrt{1^2}} = (1, 0, 0)$.

$$R^{-1} \begin{bmatrix} u_1 & u_2 & u_3 & 0 \\ v_1 & v_2 & v_3 & 0 \\ w_1 & w_2 & w_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T^{-1} \begin{bmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$M_{w2v} = (M_{v2w})^{-1} = R^{-1}T^{-1}$

$$= \begin{bmatrix} 0 & 0 & 1 & -2 \\ 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. near $z = -2$ far $z = -16$ left $x = -2$ right $x = 2$ top $y = 2$ bot $y = -2$

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\ 0 & \frac{2n}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{-(f+n)}{f-n} & \frac{-2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} \frac{2(-2)}{2+2} & 0 & \frac{2-2}{2+2} & 0 \\ 0 & \frac{2(-2)}{2+2} & \frac{2-2}{2+2} & 0 \\ 0 & 0 & \frac{-(-16-2)}{-16+2} & \frac{-2(-16)(-2)}{-16+2} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{9}{7} & \frac{32}{7} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

3. NDC to display. transformation matrix for a viewport 800 pixels wide (W) and 600 pixels high (H). (upper left org.)

$$\begin{bmatrix} 1 & 0 & 0 & \frac{799}{2} \\ 0 & 1 & 0 & \frac{599}{2} \\ & & & \end{bmatrix} \begin{bmatrix} 400 & 0 & 0 & 0 \\ 0 & 300 & 0 & 0 \\ & & & \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ & & & \end{bmatrix} = \begin{bmatrix} 400 & 0 & 0 & \frac{799}{2} \\ 0 & -300 & 0 & \frac{599}{2} \\ & & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & \frac{599}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 400 & 0 & 0 & 0 \\ 0 & 300 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 400 & 0 & 0 & -\frac{1}{2} \\ 0 & -300 & 0 & \frac{599}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 400 & 0 & 0 & 399.5 \\ 0 & -300 & 0 & 299.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. given $(-3, 5, 1)$

$$\begin{bmatrix} 0 & 0 & 1 & -2 \\ 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1-2 \\ -3-5 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ 5 \\ 1 \end{bmatrix}$$

5.
$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{9}{7} & \frac{32}{7} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ -8 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -\frac{9}{7}(5) + \frac{32}{7} \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -\frac{45}{7} + \frac{32}{7} \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ \frac{-13}{7} \\ -5 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -1.857 \\ -5 \end{bmatrix}$$

6.
$$\begin{bmatrix} 1 \\ 8 \\ \frac{-13}{7} \\ -5 \end{bmatrix} \cdot \frac{1}{-5} = \begin{bmatrix} -\frac{1}{5} \\ -\frac{8}{5} \\ \frac{13}{35} \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ -1.6 \\ 0.371 \\ 1 \end{bmatrix}$$

7.
$$\begin{bmatrix} 400 & 0 & 0 & \frac{799}{2} \\ 0 & -300 & 0 & \frac{599}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{5} \\ -\frac{8}{5} \\ \frac{13}{35} \\ 1 \end{bmatrix} = \begin{bmatrix} 400(-\frac{1}{5}) + \frac{799}{2} \\ (-300)(-\frac{8}{5}) + \frac{599}{2} \\ (\frac{1}{2})(\frac{13}{35}) + \frac{1}{2} \\ 1 \end{bmatrix} = \begin{bmatrix} -80 + 399.5 \\ 480 + 299.5 \\ 0.1857 + 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 319.5 \\ 779.5 \\ 0.6857 \\ 1 \end{bmatrix}$$

8.