Forecasting market risk: quantiles, extreme values, and Value at Risk

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1 Introduction

From the 1987 big market crash to 2008 financial crisis, there is a growing interest in risk management. One of the most common risk measures is Value at Risk (VaR). The purpose of this study is to compare different VaR models' risk forecasting ability.

I use the time series of the negative daily stock log returns of Cisco Systems, Inc. Cisco is the largest networking service and products provider in the world. Cisco's stock is a component in DJIA, S&P100, S&P500 index. In the late March 2000, at the peak moment of the doc-com bubble, Cisco became the most valuable company in the world, with a market capital of $555 bn. But just two years later after the bubble burst, its market capital collapsed to $51 bn. This doc-com stock provides an interesting exploratory case to estimate and compare VaRs computed by different approaches.

I choose two sub-period samples of Cisco's daily stock returns in last 20 years: the high-volatility period and the low-volatility period. I use back-testing approach to compare different models' risk forecasting ability in each sub-period. Section 2 briefly discusses the different methods to calculate VaR. Section 3 is the application of the models mentioned in section 2.

2 Methods to calculate VaR

VaR can be defined as the maxima loss of a financial position at a given tail probability p (i.e., the probability of interest), for a given time period (Tsay, 2010). Lots of methods to VaR calculation have been proposed in the literature. These methods can be divided roughly into three categories[1]: Parametric methods such as Riskmetrics and Econometric models; Non-Parametric methods such as quantile estimation, and Semi-Parametric methods based on Extreme Value Theory.

2.1 Parametric methods: Riskmetrics and Econometric approaches

The parametric methods use an explicit distributional assumption to model the entire time series. These methods assume that $r_t|F_{t-1}$ follow some kinds of distribution. If one can find out the $E(r_t|F_{t-1})$ and $Var(r_t|F_{t-1})$, then it is easy to calculate the VaR based on the assumed distribution. VaR is the upper pth quantile (or the $(1-p)$th quantile) of the distribution.

One main problem in these approaches is that the actual distribution of the data is unknown, leading to inaccurate VaR estimation.

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[1] This is a widely used classification, see the review study by (Rocco, 2014).

2.1.1 Riskmetrics method

Riskmetrics method assumes that the conditional return distribution $r_{t+1}|F_t \sim N(0, \sigma_{t+1}^2)$, where $\sigma_{t+1}^2 = (1-\alpha)\alpha_t^2 + \alpha\sigma_t^2$, and $a_t = \sigma_t \varepsilon_t$ is an IGARCH (1,1) process without drift.

$\mathrm{VaR} = Z_{1-p}\sigma_{t+1}$ for the next trading day[2].

2.1.2 Econometric method

Econometric models use ARMA process to model the mean equation, and the GARCH process to model the volatility. If ε_t is Gaussian, then $r_{t+1}|F_t \sim N[\hat{r}_t(1), \hat{\sigma}_t^2(1)]$, and $\mathrm{VaR} = \hat{r}_t(1) + Z_{1-p}\hat{\sigma}_{t+1}$. If ε_t is a Student-t distribution with υ degree of freedom, then

$\mathrm{VaR} = \hat{r}_t(1) + \frac{t_{\upsilon(1-p)}\hat{\sigma}_{t+1}}{\sqrt{\upsilon/(\upsilon-2)}}$. υ can be obtained by the equation $k - 3 = \frac{6}{\upsilon-4}, for\ \upsilon > 4$, where k is the kurtosis of $\{r_t\}$.

2.2 Non-parametric method: Empirical Quantile and Quantile Regression

Quantile estimation is a non-parametric approach to VaR estimation. Different to parametric approach, non-parametric approach does not make specific distributional assumption of the return series and it simply assumes that distribution in the prediction period is the same as in the sample period.

While quantile estimation is easy to use and it avoids the problem of estimating the specific distribution of the time series, it has some shortcomings. One of the biggest problem is that it implicitly assumes that the predicted loss cannot be larger than what historically happened, which is not so in practice. In other words, unless there are some "ancestor" extreme events in the time series, it cannot predict the occurrence of future extreme events.

2.2.1 Empirical quantile method

Given the sorted return series with an increasing order: $r_{(1)} \leq r_{(2)} \leq \cdots \leq r_{(n)}$, where n is the length of time series. If $n(1-p)$ is an integer, the upper p quantile (or the $(1-p)$th quantile) $\mathrm{VaR} = r_{(n(1-p))}$. If $n(1-p)$ is not an integer, VaR can be estimated using weighted neighboring returns (Tsay, 2010).

2.2.2 Quantile regression method

Quantile regression method considers the distribution function $r_{t+1}|F_t$, where F_t includes the explanatory variables. For example, assume $r_{t+1}|F_t$ is affected by predictors $x_{1,t}$ and $x_{2,t}$, then the conditional quantile can be estimated by $\hat{X}_p|F_t = \widehat{\beta_0} + \widehat{\beta_1}x_{1,t} + \widehat{\beta_2}x_{2,t}$.

$\hat{\boldsymbol{\beta}} = \mathrm{argmin}_{\boldsymbol{\beta}} \sum_{t=1}^{n} w_p(r_t - \beta_0 - \beta_1 x_{1,t-1} - \beta_2 x_{2,t-1})$,

where w_p is defined by $w_p(z) = \begin{cases} pz & if\ z \geq 0, \\ (p-1)z & if\ z < 0. \end{cases}$

In this study, I use two predictors to estimate the quantile: lag-1 day (or previous day) stock volatility obtained from fitting a GARCH(1,1) process to the return series; and lag-1 VIX index

[2] For the convenience of later rolling window VaR calculation, in this study, I only discuss the case of one-day horizon VaR.

obtained from Chicago Board Options Exchange (CBOE). For the regression results, see Appendix 1.

2.3 Semi-Parametric method: Extreme value theory

Rather than not making any assumptions of the distribution at all (i.e., Non-Parametric method), or modelling the entire distribution of the data of interest, where common events in the center of the distribution dominate the estimation process given the scarcity of extreme events (i.e., Parametric method), Extreme Value Theory (EVT) based methods focus on analyzing the tail region of distributions (Danielsson, 2011). It is concerned with the behavior of extreme outcomes. In this sense, it can be considered as a semi-parametric method (Rocco, 2014).

The basic idea of EVT is that, regardless of the overall shape of the distribution, EVT assumes the tails of all distributions fall into one of three categories: Gumbel family, Frechet family, and Weibull family.[3] As long as we can estimate three parameters of the distribution (i.e., the shape parameter ξ, the location parameter β, and the scale parameter α), we can use these parameters to calculate VaR. One way to estimate the parameters is to use Maximum-Likelihood Method (MLE). To apply MLE, we have to choose the extreme observations into the estimation process. In literature, there are two widely-used method to pick up: Block-Maxima approach and Peaks-Over-Threshold (POT) approach.

The downside of EVT method is that if we do not have many extreme observations in the sample to estimate the quantiles of interest, then it is very difficult to predict the "black swan" event by using EVT method.

2.3.1 Block-Maxima approach

Estimating VaR using Block-Maxima approach includes the following three steps:

Step one: select the length of the block (i.e., the length of sub-period) n and obtain block maxima $\{r_{n,i}\}$, $i = 1, \ \ldots, \ g$. Where *g=[T/n]*. In this study, I use n =21 (monthly grouping).

Step two: obtain the maximum-likelihood estimates of $(\xi_n, \alpha_n, \beta_n)$.

$$\mathrm{L}(\xi_n, \alpha_n, \beta_n) = \sum_{i=1}^{g} \left\{ -log\alpha_n - \frac{1+\xi_n}{\xi_n} log \left[1 + \frac{\xi_n(\boldsymbol{r_{n,i}} - \beta_n)}{\alpha_n}\right] - \left(1 + \frac{\xi_n(r_{n,i} - \beta_n)}{\alpha_n}\right)^{-1/\xi_n} \right\},$$

where $1 + \frac{\xi_n(\boldsymbol{r_{n,i}} - \beta_n)}{\alpha_n} > 0, -\infty < \xi_n < \infty, \ -\infty < \beta_n < \infty, \alpha_n > 0.$

Step three: calculate VaR: $\mathrm{VaR} = \beta_n - \frac{\alpha_n}{\xi_n}\{1 - [-n \ln{(1-P)}]^{-\xi_n}\}.$

2.3.2 POT approach

Block-Maxima approach has two problems. One problem is that it is somehow wasteful of data as only one observation in each block is chosen. The second highest observation in one block may be higher than the highest one of another block. Another problem is that the choice of block length (i.e., whether using monthly grouping or quarterly grouping) is not clearly defined (Tsay, 2010).

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[3] Some literature use Generalized Extreme Value (GEV) distribution to put all of these three distribution into one formula (see Hosking, Wallis, and Wood, 1985).

The POT method is a method to avoid such problem. It picks up the large observations which are above a given threshold into the MLE estimation. These large observations are called exceedances. POT assumes that these exceedance and the time at which the exceedance occurs $\{(r_{t,i}, t_i)\}$ jointly form a two-dimensional Poisson process. Using the results of the Poisson process, we can write down the MLE function and estimate the three key parameters.

In general, POT method involves the following three steps.

Step one: select the exceedances of the observations over some high threshold η and the times at which the exceedances occur. A number of literature has discussed of how to choose an appropriate threshold (e.g., Scarrott and MacDonald, 2012). One simple procedure of choosing the threshold is to choose the sample quantile as a threshold: the *k*th upper quantile of the order sequence $r_{(1)} \leq r_{(2)} \leq \cdots \leq r_{(n)}$. Frequently used rule k=10% is inappropriate from a theoretical view (Scarrott and MacDonald, 2012). Other simple rules can be: $\mathrm{k} = \sqrt{n}$ or $\mathrm{k} = \frac{n^{2/3}}{\log(\log(n))}$. Using $\mathrm{k} = \sqrt{n}$ may involve too few exceedance. Therefore, I use $\mathrm{k} = \frac{n^{2/3}}{\log(\log(n))}$ to get threshold.

Step two: obtain the maximum-likelihood estimates of (ξ, α, β).

$\mathrm{L}(\xi, \alpha, \beta) = \sum_{i=1}^{N_\eta} \left\{-logD - log\alpha - \frac{1+\xi}{\xi} log\left[1 + \frac{\xi(r_{t,i'} - \beta)}{\alpha}\right]\right\} - \frac{T}{D}\left(1 + \frac{\xi\,(\eta-\beta)}{\alpha}\right)^{-1/\xi}$, where D is the baseline time interval, which is typically the number of trading days in one year. In this study, I assume D=252. In consistent with Tsay (2010), I use the mean corrected log return series at ${r_t}' = r_t - \bar{r}$ to refine the model.

Step three: calculate VaR: $\mathrm{VaR} = \beta - \frac{\alpha}{\xi}\left\{1 - [-D \ln(1-P)]^{-\xi}\right\}$.

3 Application: VaR back-testing using Cisco stock returns

3.1 Data

I choose 5034 observations of daily return of Cisco stock from 1997.01 to 2016.12. The data is obtained from Center for Research in Security Prices (CRSP). I transformed the raw return data into negative daily log returns to perform the VaR analysis. The second column in Table 3.1 reports the basic statistics and preliminary test results of the entire sample.

Table 3.1 Basic statistics and preliminary test results of Cisco stock for different sample period

Statistics and preliminary tests	Entire sample	High-volatility sub-period	Low-volatility sub-period
Sample period	1997.01 to 2016.12	1997.01 to 2002.12	2011.01 to 2016.12
Number of observations	5034	1512	1512
Mean	-0.000319	-0.00046128	-0.00036765
Standard Deviation	0.026	0.0376	0.0159
Skewness	-0.067	-0.1546	0.1158
Kurtosis. If the kurtosis is much larger than 3, then the time series has a fat tail problem.	10.1553	5.733	21.4775
Jarque-Berra test on return series. H0: the return series is normally distributed.	pvalue=0, reject H0	pvalue=0, reject H0	pvalue=0, reject H0
Ljung-Box test on return series. H0: ACFs with 12 lags are all zero, indicating the return series are not correlated.	pvalue=0, reject H0	pvalue=0.0252, reject H0 at 5% significance level.	pvalue=0.7373, accept H0 (the return series are not correlated)
ARMA lags (p, q) determined by BIC.	ARMA(1,1)	ARMA (1,1)	no ARMA process
Ljung-Box test on residual series from ARMA(p, q) process. H0: ACFs with 12 lags are all zero, indicating the residual series are not correlated. Then ARMA model is correctly specified.	pvalue=0, reject H0	pvalue=0.1714, accept H0	no ARMA process
Ljung-Box test on squared residual series from ARMA (p, q). H0: ACFs with 12 lags are all zero, indicating the squared residual series are not correlated. So there is no conditional heterosedasticity.	pvalue=0, reject H0	pvalue=0, reject H0	pvalue=0.8831, reject H0

Table 3.2 shows the one-day-horizon VaR forecast (i.e., forecast for the day 2016.12.30) using the entire sample data with different VaR methods.

In terms of the econometric approach, I choose ARMA (1,1)-GARCH (1,1) with student-t distribution given the fat-tail issue as shown in the preliminary tests. I also include Gaussian ARMA(1,1)-GARCH(1,1) as an alternative check.

We can get some direct observations from the results in Table 3.2.

First, when the upper tail probability p is very small, the VaRs are relatively large, indicating that the change that we observe some large/huge loss is very small (e.g., p=0.001).

Table 3.2 One-day-horizon VaR forecast using the sample data from 1997.01.02 to 2016.12.29.

	p = 0.001	*p = 0.01*	*p = 0.05*
VaR by RiskMetrics	3.70%	2.78%	1.97%
VaR by Student-t ARMA(1,1)-GARCH(1,1)	7.60%	4.26%	2.51%
VaR by Gaussian ARMA(1,1)-GARCH(1,1)	3.85%	2.88%	2.02%
VaR by Empirical Quantile	14.05%	7.71%	3.96%
VaR by Quantile Regression	7.50%	3.06%	1.53%
VaR by Block-Maxima- EVT	10.83%	6.25%	3.01%
VaR by POT- EVT	13.36%	7.79%	3.53%
Actual losses on 2016.12.30	0.79%		

Second, there are substantial VaR forecast differences among different methods. This is because each method has its own assumptions in estimating tail behavior.

Third, VaR forecast difference is getting larger when p is getting smaller, indicating that with very small tail probability, the VaR calculation result is very sensitive to the model choice.

Fourth, Empirical Quantiles method give the highest VaR forecasts. It can be used as a conservative estimate of the true VaR (i.e., lower bounds) in daily risk management.

In addition, VaRs based on student-t distribution is higher than VaRs based on Gaussian distribution. VaR calculated by Gaussian ARMA(1,1)-GARCH(1,1) is close to VaR calculated by Riskmetrics.

3.2 Back-testing using rolling windows

One way to compare the performance of different VaR methods is back-testing. It uses ex-ante VaR forecasts from a particular method and compare them with ex-post actual observations from data history (Danielsson, 2011). In this section, I use rolling window back-testing method to compare different VaR methods.

3.2.1 Identifying two sub-periods for back-testing

Figure 3.1 plot the negative daily log returns. I identify two sub-period samples based the volatility: 1997.01- 2002.12 is a high volatility period; and 2011.01- 2016.12 is low-volatility period. I use these two sub-period samples to do the rolling window VaR estimation.

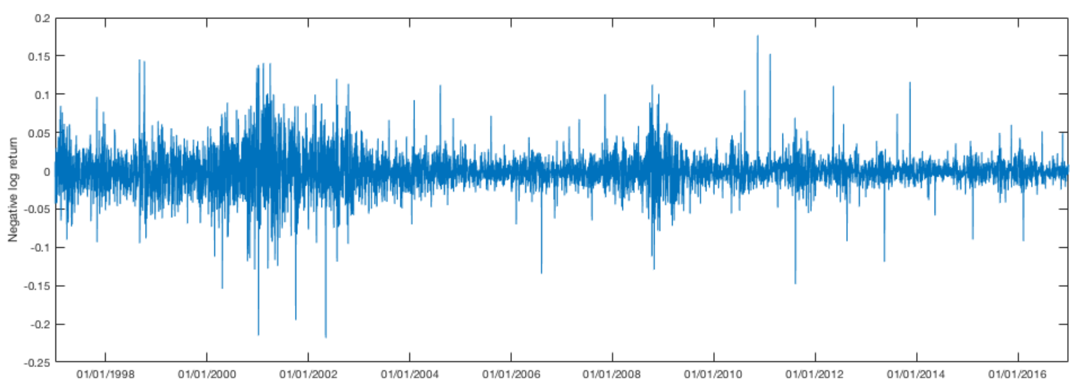

Figure 3.1 The daily negative stock returns for Cisco stock from 02/01/1997 to 30/12/2016.

The purpose of using sub-period sample is to estimate the VaRs for the high-volatility period and VaRs in the low-volatility period using different VaR estimation method. Then I can compare different VaR methods' performance when they are applied to high-volatility period data and to low-volatility period data.

The basic statistics and preliminary test results of the two sub-period samples are reported in the third and the fourth column in Table 3.1. Based on the preliminary test results, for the high-volatility sub-period, I choose six VaR methods: RiskMetrics, Student-t-ARMA (1,1) – GARCH (1,1)[4], Empirical Quantile, Quantile Regression, Block-Maxima-EVT, and POT-EVT.

For the the low-volatility period, similarly, I choose almost the same methods that applied to high-volatility period. One exception is that instead of using Student-t-ARMA(1,1)-GARCH(1,1), I use Student-t-GARCH(1,1) without any ARMA process as the econometric method for the low-volatility period. The serial correlations in return series (for the low-volatility-period) are very weak so that I do not entertain any ARMA model for the mean equation.

3.2.2 Rolling windows

Rolling window analysis of time-series is commonly used in assessing the forecast accuracy of the models.

Take the high-volatility period VaR forecast for example[5].

First, I forecast the VaR for the day 2000-01-01 by using previous three-year data from 1997-01-01 to 1999-12-31. Next, I forecast the VaR for 2000-01-02 by using data from 1997-01-02 to 2000-01-01. So the number of increments between successive rolling window is 1 day.

I repeat this process until finally, I forecast the VaR for the day 2002-12-31. In this way, I got three-year VaR forecasts from 2000-01 to 2002-12 by using data from 1997-01 to 2002-12, and I can compare the VaR forecasts to the realized data (historical data) during 2000-01 to 2002-12. With the same approach, I get the three-year VaR forecast for the low-volatility period. Table 3.3 describes this process.

Table 3.3 Rolling window analysis

High-volatility period			Low-volatility period		
Estimation Window		*VaR forecast*	*Estimation Window*		*VaR forecast*
Start	*End*		*Start*	*End*	
1997-01-01	*1999-12-31*	*VaR(2000-01-01)*	*2011-01-01*	*2013-12-31*	*VaR(2014-01-01)*
1997-01-02	*2000-01-01*	*VaR(2000-01-02)*	*2011-01-02*	*2014-01-01*	*VaR(2014-01-02)*
…	…	…	…	…	…
1999-12-31	*2002-12-30*	*VaR(2002-12-31)*	*2013-12-31*	*2016-12-30*	*VaR(2016-12-31)*

[4] Considering the large kurtosis and the Jarque-Berra test results, I choose Student-t distribution for the econometric methods.

[5] In this rolling window explanation example, I simply assume the first trading day in one year is Jan 01 and the last day is Dec 31, and there is no break in stock market. In practice, the first trading day may be Jan 03/Jan 04 and the last day may be Dec 29/Dec 30 based on holiday schedule.

3.2.3 Graphical analysis and violation ratio analysis

Graphical analysis provides a quick visual inspection of the model performance in VaR forecasting. Figure 3.2 plots the VaRs forecast results for the two sub-periods using the rolling window method (using p = 0.01). The VaRs for the high-volatility period are plotted on the left and VaRs for the low-volatility period are plotted on the right. Y-axis in each plot represents the daily negative log returns and VaR forecasts. It should be noted that the Y-axis scale in each sub-plot is not identical.

Figure 3.2 shows that when extreme event happens, the real losses can exceed VaR. Whenever real losses exceed VaR, a violation is said to have occurred. For VaR back-testing with p=1%, we would expect to observe a VaR violation of 1% of the testing window. Testing window means the period over which VaR is forecast (e.g., in the high-volatility period, the testing window is 2011.01 to 2012.12). If violations are observed more often than 1% (e.g., 5% or higher), the VaR model may underestimate risk at 1% level.

Formal tests of violation include Bernolli Coverage test and Independence of Violations test (see Danielsson, 2011). Another simple way is using *Violation Ratio (VR)* to measure the performance of different VaR models. According to Danielsson (2011),

$$VR = \frac{Observed\ number\ of\ violations}{Expected\ number\ of\ violations},$$

where the $Expected\ number\ of\ violations = p\ \times\ tesing\ window.$

$VR = 1$ is expected, indicating that the model gets very precise estimation of actual risk.

If $VR < 0.5\ or\ VR > 1.5$, the model may overestimate or underestimate the risk.

Table 3.4 shows the results of Violation Ratios for each sub-period.

Table 3.4 Violation Ratios for each sub-period

High-volatility period		*Low-volatility period*	
VaR method	Violation Ratio	VaR method	Violation Ratio
RiskMetrics	0.9259	RiskMetrics	1.0582
Student-t-ARMA(1,1)-GARCH(1,1)	0.3968	Student-t-GARCH(1,1)	0.5291
Empirical Quantile	1.8519	Empirical Quantile	1.0582
Quantile Regression	1.455	Quantile Regression	1.1905
Block Maxima - EVT	3.5714	Block Maxima - EVT	0.7937
POT - EVT	1.1905	POT - EVT	0.7937

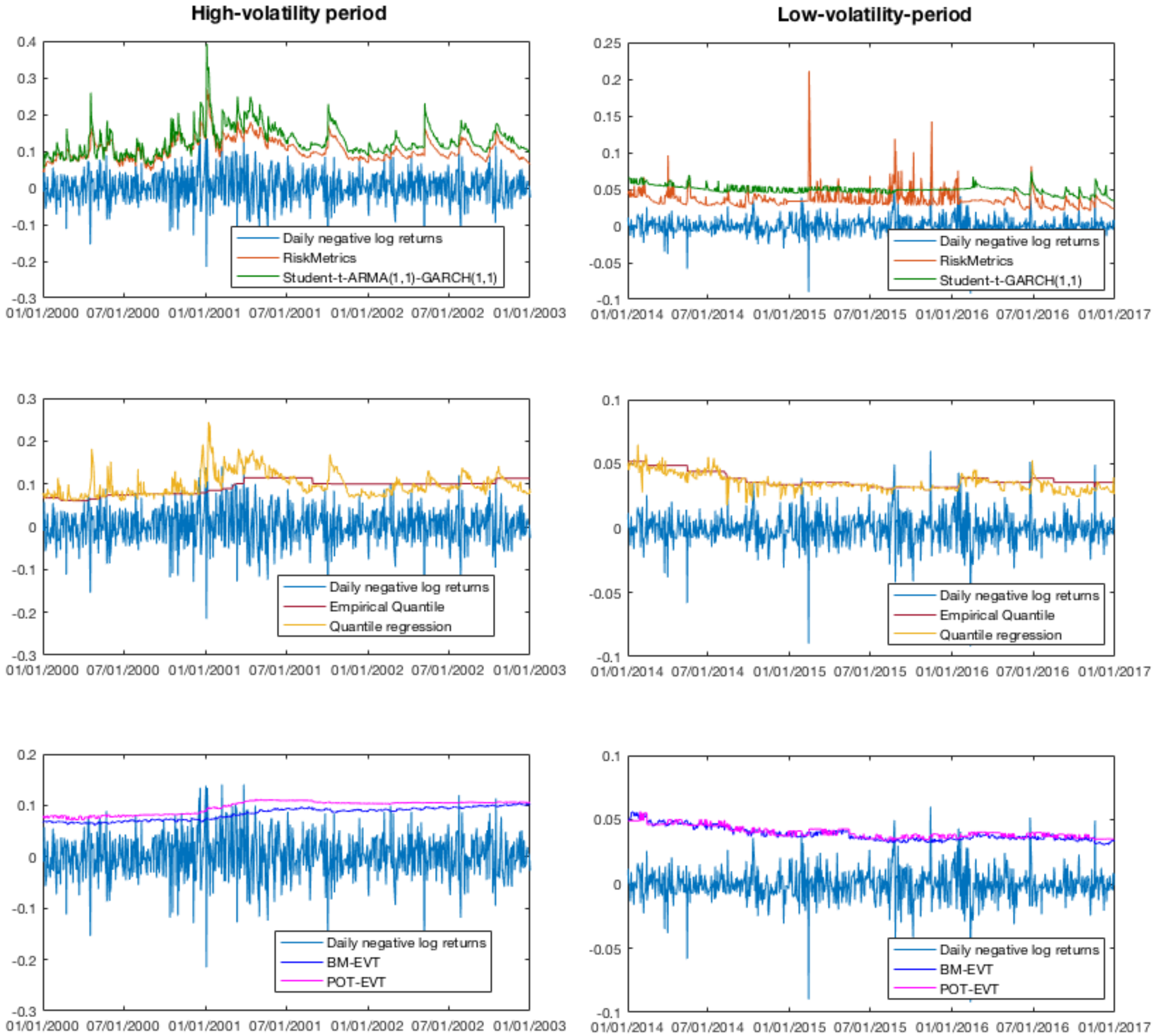

Figure 3.2 VaR back-testing (p = 0.01) for two sub-samples for Cisco stock. Y-axis represents the daily negative log returns and VaR forecasts.

Here are some direct observations from graphical analysis (see Figure 3.2) and violation ratio analysis (see Table 3.4):

First, according to the violation ratios, all methods perform better in the low-volatility period than in the high-volatility period. In addition, the violation ratios' difference among different models is much larger in high-volatility period than in low-volatility period, suggesting that in high-volatility period, we should be more careful in choosing VaR models.

Second, Riskmetric method's violation ratios are close to one for both periods. It turns out Riskmetric method performs better than other methods. Econometrics models, such as

GARCH(1,1) with student-t distribution, the violation ratio is too low for both periods. Indicating these econometrics models overestimate the risks.

Third, Quantile Regression method performs quite well for both periods, while Empirical Quantile is less precise in high-volatility period.

Fourth, the right side plot in the third row in Figure 3.2 shows that Block-Maxima-EVT and POT-EVT methods generate very close VaRs in the low-volatility period. But during the high-volatility period, Block-Maxima-EVT method underestimates risk, given its violation ratio=3.57 in the high-volatility period. This can be explained by the fact that during high-volatility period, the second or third maxima observation in each 'block' is ignored by Block-Maxima-EVT approach. However, these ignored observations are large enough to influence the model's forecast ability (recall the discussion in section 2.3.2).

In addition, the results show that during high-volatility period, the models POT-EVT provides a more reliable VaR forecasts than Block-Maxima-EVT approach. This results might be driven by the fact that POT-EVT method avoid the weakness of Block-Maxima-EVT as discussed above.

4 Implications

In this study, I discuss various methods for calculation VaR and apply these methods to a stock which experience high-volatility and low-volatility periods in its trading history. Using single window analysis (see results from Table 3.2) and rolling window back-testing method, along with graphical and violation ratio analysis (see results from Table 3.4 & Figure 3.2) I find that the choice of tail probability p and the choice of VaR model play important roles in VaR calculation. In addition to the discussions in section 3, here are some takeaways from this study:

First, during high-volatility period, the VaR results is so sensitive to choice of models that we should be careful in choosing VaR models. Since in practice, when we are forecasting risk, we can't get the true loss to compare difference models like we did in back-testing, we may try to apply several different methods to gain insight into the range of VaR.

Second, EVT-POT provides shows a stable and reliable performance in both high-volatility period and low-volatility period. In addition, it makes no particular assumption of the entire data distribution (compared to the strong assumptions made by RiskMetrics method), it is recommended to use this method in daily risk management.

Some proposed future work of this study: 1) Compare model performance in back-testing by using alternative estimating window sizes and testing window sizes to see if the results are different. 2) Use formal tests of violation instead of simply computing comparing violation ratios. 3) As suggested by Tsay (2010), we may include explanatory variables in estimating the three parameters in EVT-POT method to increase the model accuracy. 4) Since VaR is mainly used in financial industry, in particular, it is required and reinforced by Basel II Accord, we can compare these VaR model performance by using data from banks and other financial institutions.

Appendix 1: Quantile regression results

I use the entire sample (negative daily log returns of Cisco stock from 1997.01 to 2016.12) to do quantile regression. Table 1 reports the beta coefficients for each predictor, the corresponding t-stats and p-values, and the VaR forecasts. It shows that the 99^{th} quantile ($p = 0.01$) of Cisco negative daily log returns depends critically on the lag-1 Cisco daily volatility and marginally on the lag-1 VIX index. This conclusion also applys to the 99.9^{th} quantile ($p = 0.001$).

Table 1: Quantile regression using negative daily log returns of Cisco stock from 1997.01 to 2016.12

	p=0.001			p=0.01		
	beta	t-stats	p-value	beta	t-stats	p-value
Constant	0.0087	1.7009	0.089	-0.008	-2.7505	0.006
Sigma t-1	0.6824	3.2174	0.0013	2.1361	17.8413	0
VIX t-1	0.0043	16.7194	0	0.0008	5.685	0
VaR	7.50%			3.06%		

Appendix 2: The coding file list

- Data file: see *"CSCO9716.mat"*.
- Main coding files:

  Table 3.1 results: see file "CSCO_BEG.m" and "CSCO_BIC.m".
  Table 3.2 results: see file "CSCO_VaR.m".
  Figure 3.1 results: see file "CSCO_BEG.m"
  Table 3.4 results: see file "CSCO.m" and "CSCO2.m"
  Figure 3.2 results: see file "CSCO.m" and "CSCO2.m"
  Appendix 1 results: see file "quantileregressionexample.m"
- Function files:
  ARest.m
  ARMA11_obj.m
  ARMA11est.m
  ARMAGARCHt.m
  EVTBM.m
  EVTPOT.m
  GARCH11est.m
  GARCH11obj.m
  GARCHt.m
  JBtest.m
  LBtest.m
  MA_obj.m
  MAest.m
  MLEEVT.m
  MLEGarch11.m
  MLENEWEVT.m
  QuantileReg.m
  RiskMetrics.m

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