2018312824 유채은

➢ 확률변수 X_i 들이 이항분포를 따른다고 가정하자.

즉, $X \sim B(n,p)$ 일 때,

최대우도 추정법을 사용하여 최대우도추정량 $\hat{p}$을 구하시오.

$$X = \{X_1, X_2, \cdots X_m\}$$

$$X \sim B(n, p)$$

$$\text{pdf}: \quad f_X(x_i; p) = \binom{n}{x_i} p^{x_i} (1-p)^{n-x_i}$$

$$\text{Likelihood}: L = \prod_{i=1}^{m} f(x_i; p) = \prod_{i=1}^{m} \frac{n!}{x_i!(n-x_i)!} p^{x_i} (1-p)^{n-x_i}$$

$$\log L = \sum_{i=1}^{m} \log \frac{n!}{x_i!(n-x_i)!} + \sum_{i=1}^{m} x_i \log p + \sum_{i=1}^{m} (n-x_i) \times \log\{1-p\}$$

$$\frac{d}{dp} \log L = \frac{1}{p} \sum_{i=1}^{m} x_i + \frac{1}{1-p} \times (-1) \times \sum_{i=1}^{m} (n-x_i) = 0$$

$$= \frac{\sum_{i=1}^{m} x_i}{p} + \frac{1}{p-1} \left(mn - \sum_{i=1}^{m} x_i\right) = 0$$

$$(p-1) \sum_{i=1}^{m} x_i = -p\left(mn - \sum_{i=1}^{m} x_i\right)$$

$$p \sum_{i=1}^{m} x_i - \sum_{i=1}^{m} x_i = -mnp + p \sum_{i=1}^{m} x_i$$

$$\hat{p} = \frac{\sum_{i=1}^{m} x_i}{mn}$$

$$\hat{p} = \frac{\sum_{i=1}^{m} x_i}{m} \times \frac{1}{n}$$

$$= \bar{x} \times \frac{1}{n}$$

$$\therefore \hat{p} = \frac{\bar{x}}{n}$$

➢ 확률변수 X_i 들이 정규분포를 따른다고 가정하자. 2018312824 유채은

즉, $X \sim N(\mu, \sigma^2)$ 일 때,

최대우도 추정법을 사용하여 최대우도추정량 $\hat{\mu}$ 와 σ^2 을 구하시오.

$X : \{X_1, \dots X_n\}$

$X \sim N(\mu, \sigma^2)$

$$f(x_i; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\text{Likelihood} = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{\sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\log L = n\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$$

$\langle \mu \rangle$

$$\frac{d}{d\mu}\log L = -\frac{d}{d\mu}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

$$= -\frac{1}{2\sigma^2} \times \sum_{i=1}^{n} \frac{d}{d\mu}(x_i^2 - 2\mu x_i + \mu^2) = 0$$

$$= -\frac{1}{2\sigma^2} \times \sum_{i=1}^{n}(-2x_i + 2\mu) = 0$$

$$= 2n\mu = \sum_{i=1}^{n} 2x_i = 0$$

$$\therefore \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\langle \sigma^2 \rangle$ $v = \sigma^2$

$$\log L = n\log\frac{1}{\sigma} + n\log\frac{1}{\sqrt{2\pi}} - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{d}{d\sigma^2}\log L = \frac{d}{d\sigma^2} n\log\frac{1}{\sigma} - \frac{d}{d\sigma^2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2} = 0$$

$$= \frac{d}{dv} n\log\frac{1}{\sqrt{v}} - \frac{d}{dv}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2v} = 0$$

$$= -\frac{1}{2v}n \quad - \frac{d}{dv} \times \frac{1}{v}\sum_{i=1}^{n}(x_i-\mu)^2 \times \frac{1}{2} = 0$$

$$\frac{1}{2v}n = \frac{1}{v^2} \times \frac{1}{2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$nv = \sum_{i=1}^{n}(x_i-\mu)^2$$

$$v = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$$