2018312824 유채은

➢ 약물반응

- 어느 집단은 남자 60%, 여자 40%로 구성
- 남자의 5%, 여자의 8%는 특정 약물에 거부반응 보임

- 임의로 추출한 한 명이 거부반응을 보일 때 그 사람이 여자일 확률은?

<표로 풀이>

| | 남자 | 여자 | |
|---|---|---|---|
| 거부반응 O | 3% | 3.2% | 6.2% |
| 거부반응 X | 57% | 36.8% | 93.8% |
| | 60% | 40% | 100% |

$$\frac{3.2}{6.2} = \frac{16}{31} = 0.51612\cdots$$

<베이지안 통계로 풀이>

$$P(\text{여자} \mid \text{거부반응}) = \frac{P(\text{거부반응} \mid \text{여자})P(\text{여자})}{P(\text{거부반응})}$$

$$= \frac{P(\text{거부반응} \mid \text{여자})P(\text{여자})}{P(\text{거부반응}, \text{여자}) + P(\text{거부반응}, \text{남자})}$$

$$= \frac{P(\text{거부반응} \mid \text{여자})P(\text{여자})}{P(\text{거부반응} \mid \text{여자})P(\text{여자}) + P(\text{거부반응} \mid \text{남자})P(\text{남자})}$$

$$= \frac{\frac{8}{100} \times \frac{40}{100}}{\frac{8}{100} \times \frac{40}{100} + \frac{5}{100} \times \frac{60}{100}}$$

$$= \frac{32}{32+30}$$

$$= \frac{16}{31}$$

$$= 0.51612\cdots$$

➢ 확률변수 $X_i$ 들이 정규분포를 따른다고 가정하자. 2018312824 유채은

즉, $X \sim N(\mu, \sigma^2)$ 일 때,

베이지안 추정법을 사용하여 베이지안추정량 $\hat{\mu}$ 와 $\widehat{\sigma^2}$을 구하시오.

$X : \{X_1, \dots, X_n\}$

prior: $\mu \sim N(\mu_0, \sigma_0^2)$

$$p(\mu) = \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}\right)$$

Likelihood: $p(x_1, \dots, x_n \mid \mu) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(x_i-\mu)^2}{2\sigma^2}\right)$

posterior: $p(\mu \mid x_1, \dots, x_n) = \dfrac{p(x_1, \dots, x_n \mid \mu)\, p(\mu)}{p(x_1, \dots, x_n)}$

$$p(\mu \mid x_1, \dots, x_n) \propto p(x_1, \dots, x_n \mid \mu)\, p(\mu)$$

$$\propto \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right) \frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}\right)$$

$$\propto \exp\left(\sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2\sigma^2} + -\frac{(\mu-\mu_0)^2}{2\sigma_0^2}\right)$$

$$\propto \exp\left(-\sum_{i=1}^{n}\left\{\frac{x_i^2 - 2x_i\mu + \mu^2}{2\sigma^2}\right\} + -\frac{(\mu-\mu_0)^2}{2\sigma_0^2}\right)$$

$$\propto \exp\left(\frac{1}{2\sigma^2}\left\{-\sum_{i=1}^{n} x_i^2 + 2\mu\sum_{i=1}^{n} x_i - n\mu^2\right\} - \frac{1}{2\sigma_0^2}\left\{\mu^2 - 2\mu\mu_0 + \mu_0^2\right\}\right)$$

$$\propto \exp\left[-\left(\frac{n}{2\sigma^2} + \frac{1}{2\sigma_0^2}\right)\mu^2 + \left(\frac{1}{\sigma^2}\cdot\sum_{i=1}^{n} x_i + \frac{1}{\sigma_0^2}\cdot\mu_0\right)\mu - \left(\frac{1}{2\sigma^2}\sum_{i=1}^{n} x_i^2 + \frac{1}{2\sigma_0^2}\mu_0^2\right)\right]$$

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$$\propto \exp\left[-\left(\frac{n}{2\sigma^2}+\frac{1}{2\sigma_0^2}\right)\mu^2+\left(\frac{1}{\sigma^2}\cdot\sum_{i=1}^{n}X_i+\frac{1}{\sigma_0^2}\cdot\mu_0\right)\mu-\left(\frac{1}{2\sigma^2}\sum_{i=1}^{n}X_i^2+\frac{1}{2\sigma_0^2}\mu_0^2\right)\right]$$

$$\frac{n}{2\sigma^2}+\frac{1}{2\sigma_0^2}=\frac{n\sigma_0^2+\sigma^2}{2\sigma^2\sigma_0^2}$$

$$\underbrace{\propto \exp\left[-\left(\frac{n}{2\sigma^2}+\frac{1}{2\sigma_0^2}\right)\left\{\mu^2-2\times\frac{\sigma_0^2\sum_{i=1}^{n}X_i+\sigma^2\mu_0}{n\sigma_0^2+\sigma^2}\mu+\frac{\sigma_0^2\sum_{i=1}^{n}X_i^2+\mu_0^2\sigma^2}{n\sigma_0^2+\sigma^2}\right\}\right]}$$

$$\shortparallel$$

$$\exp\left(-\frac{(\mu-\hat{\mu})^2}{2\hat{\sigma}^2}\right)$$

$$\hat{\mu}=\frac{\sigma_0^2\sum_{i=1}^{n}X_i+\sigma^2\mu_0}{n\sigma_0^2+\sigma^2}=\frac{1}{n\sigma_0^2+\sigma^2}\mu_0+\frac{\sigma_0^2}{n\sigma_0^2+\sigma^2}\sum_{i=1}^{n}X_i$$

$$\frac{1}{2\hat{\sigma}^2}=\frac{n}{2\sigma^2}+\frac{1}{2\sigma_0^2}$$

$$\frac{1}{\hat{\sigma}^2}=\frac{n}{\sigma^2}+\frac{1}{\sigma_0^2}$$

$$\frac{1}{\hat{\sigma}^2}=\frac{n\sigma_0^2+\sigma^2}{\sigma^2\sigma_0^2}$$

$$\sigma^2\sigma_0^2=\hat{\sigma}^2(n\sigma_0^2+\sigma^2)$$

$$\hat{\sigma}^2=\frac{\sigma^2\sigma_0^2}{n\sigma_0^2+\sigma^2}$$

$$\therefore\ \hat{\mu}=\frac{1}{n\sigma_0^2+\sigma^2}\mu_0+\frac{\sigma_0^2}{n\sigma_0^2+\sigma^2}\sum_{i=1}^{n}X_i$$

$$\hat{\sigma}^2=\frac{\sigma^2\sigma_0^2}{n\sigma_0^2+\sigma^2}$$