$$f_i(x) = \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma_i|^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu_i)^T\Sigma_i^{-1}(x-\mu_i)}$$ $i$: class index

$\Sigma_1 = \Sigma_2 = \Sigma$ $\quad f_1(x) = \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)}$ $\quad f_2(x) = \frac{1}{(2\pi)^{\frac{p}{2}}|\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2)}$

$$\frac{f_1(x)}{f_2(x)} \geq 1 \qquad = \frac{e^{-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)}}{e^{-\frac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2)}} \geq 1$$

$$= \exp\left\{-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1) + \frac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2)\right\} \geq 1$$

$$\log\left\{\frac{f_1(x)}{f_2(x)}\right\} \geq 0$$

$$= -\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1) + \frac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2) \geq 0$$ $\Sigma$: symmetric $\Sigma^{-1} = (\Sigma^{-1})^T$

$$\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1) \leq \frac{1}{2}(x-\mu_2)^T\Sigma^{-1}(x-\mu_2)$$

$$\underset{(1\times p)}{(x-\mu_1)^T}\underset{(p\times p)}{\Sigma^{-1}}\underset{(p\times 1)}{(x-\mu_1)} \leq (x-\mu_2)^T\Sigma^{-1}(x-\mu_2) \in \mathbb{R}$$ Mahalanobis Distance

$$= (x^T-\mu_1^T)\{\Sigma^{-1}x - \Sigma^{-1}\mu_1\} \leq (x^T-\mu_2^T)\{\Sigma^{-1}x - \Sigma^{-1}\mu_2\}$$

$$= \cancel{x^T\Sigma^{-1}}x - x^T\Sigma^{-1}\mu_1 - \mu_1^T\Sigma^{-1}x + \mu_1^T\Sigma^{-1}\mu_1 \leq \cancel{x^T\Sigma^{-1}}x - x^T\Sigma^{-1}\mu_2 - \mu_2^T\Sigma^{-1}x + \mu_2^T\Sigma^{-1}\mu_2$$

$$= -x^T\Sigma^{-1}\mu_1 - \mu_1^T\Sigma^{-1}x + x^T\Sigma^{-1}\mu_2 + \mu_2^T\Sigma^{-1}x \leq \mu_2^T\Sigma^{-1}\mu_2 - \mu_1^T\Sigma^{-1}\mu_1$$

$$\underbrace{-(\mu_1^T\Sigma^{-1}x)^T - \mu_1^T\Sigma^{-1}x}_{=} + (\mu_2^T\Sigma^{-1}x)^T + (\mu_2^T\Sigma^{-1}x) \leq \mu_2^T\Sigma^{-1}\mu_2 - \mu_1^T\Sigma^{-1}\mu_1$$

$$-2\mu_1^T\Sigma^{-1}x + 2\mu_2^T\Sigma^{-1}x \leq \mu_2^T\Sigma^{-1}\mu_2 - \mu_1^T\Sigma^{-1}\mu_1$$

$$-(\mu_1^T\Sigma^{-1} - \mu_2^T\Sigma^{-1})x \leq \frac{1}{2}\{\mu_2^T\Sigma^{-1}\mu_2 - \mu_1^T\Sigma^{-1}\mu_1\}$$

$$(\mu_1-\mu_2)^T\Sigma^{-1}x \geq \frac{1}{2}(\mu_1^T\Sigma^{-1}\mu_1 - \mu_2^T\Sigma^{-1}\mu_2)$$

$$(\mu_1-\mu_2)^T\Sigma^{-1}x \geq \frac{1}{2}\{(\mu_1-\mu_2)^T\Sigma^{-1}\mu_1 + (\mu_1-\mu_2)^T\Sigma^{-1}\mu_2\}$$