Problem set 1 - #3. Poisson Regression

$p(y;\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$

η : natural parameter (canonical parameter)

$T(y)$: sufficient statistic (usually $T(y)=y$)

$a(\eta)$: log partition function

$\exp(-a(\eta))$: usually normalization constant

$\sum p(y;\eta) = 1$ or $\int p(y;\eta) = 1$

$T, a, b; \eta$ → defines the family of distribution

(a)

⟨Poisson distribution⟩

$$p(y;\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}$$

$$= \frac{1}{y!}\{\exp(-\lambda)\cdot\lambda^y\} \qquad T(y)=y$$

$$= \frac{1}{y!}\{\exp(-\lambda)\cdot\exp(\log\lambda^y)$$

$$= \frac{1}{y!}\{\exp(-\lambda)\cdot\exp(y\log\lambda)\}$$

$$= \frac{1}{y!}\{\exp(-\lambda + y\log\lambda)\}$$

$$= \frac{1}{y!}\{\exp(y\log\lambda - \lambda)\}$$

→ $T(y) = y$ ✓

$a(\eta) = \lambda$

$\eta = \log\lambda$ ✓

$e^\eta = \lambda$

$a(\eta) = e^\eta$ ✓

$b(y) = \frac{1}{y!}$ ✓