

Probset #2

3. (a)

Show that $\theta_{\text{MAP}} = \arg\max_{\theta} p(y|x, \theta)p(\theta)$ if we assume that $p(\theta) = p(\theta|x)$

valid for models where the input x are not explicitly modeled by θ . (Note that x and θ are marginally independent, but not conditionally independent when y is given)

$$\theta_{\text{MAP}} = \arg\max_{\theta} p(\theta|x, y)$$

assume: $p(\theta) = p(\theta|x)$

$$p(\theta) = \frac{p(\theta, x)}{p(x)}$$

$$p(\theta, x) = p(\theta) \times p(x) \leftarrow \text{marginally independent}$$

$$= \arg\max_{\theta} \frac{p(\theta, x, y)}{p(x, y)}$$

$$= \arg\max_{\theta} \frac{p(y|\theta, x)p(\theta, x)}{p(x, y)}$$

$$= \arg\max_{\theta} \frac{p(y|\theta, x)p(\theta)p(x)}{p(y|x)p(x)}$$

$$= \arg\max_{\theta} \frac{p(y|\theta, x)p(\theta)}{p(y|x)}$$

$$= \arg\max_{\theta} p(y|\theta, x)p(\theta)$$