Probset #3

4. (d)

$K(x, z) = -a\, k_1(x, z)$ $\quad a \in \mathbb{R}^+$: a positive real number

Given any vector z,

$$z^T K z = \sum_i \sum_j z_i k_{ij} z_j$$

$$= \sum_i \sum_j z_i \{-a\, \phi_1(x^{(i)})^T \phi_1(z^{(j)})\} z_j$$

$$= \sum_i \sum_j z_i \sum_k \{-a\, (\phi_1(x^{(i)}))_k (\phi_1(x^{(j)}))_k\} z_j$$

$$= -a \left(\sum_i z_i \phi_1(x^{(i)}) \right)^2 \leq 0$$

→ Negative semi-definite

→ x Kernel