

# Problem Set #1.

4. (b)

Show that  $\text{Var}(Y|X;\theta)$  can be expressed as the derivative of the mean w.r.t  $\eta$  (i.e., the second derivative of the log-partition function  $a(\eta)$  w.r.t the natural parameter  $\eta$ )

$$p(y|\eta) = b(y) \exp(\eta y - a(\eta))$$

$$\mu_y = \frac{\partial a(\eta)}{\partial \eta}$$

$$\begin{aligned} \frac{\partial}{\partial \eta} \int y p(y|\eta) dy &= \int \frac{\partial}{\partial \eta} y p(y|\eta) dy \\ &= \int (y - \frac{\partial}{\partial \eta} a(\eta)) y p(y|\eta) dy \\ &= \int y^2 p(y|\eta) dy - \frac{\partial}{\partial \eta} a(\eta) \int y p(y|\eta) dy \\ &= E[Y^2] - \frac{\partial}{\partial \eta} a(\eta) \int y p(y|\eta) dy \\ &= E[Y^2] - E[Y] \times E[Y] \end{aligned}$$

$$V(Y|X;\theta) = E[Y^2|X;\theta] - \{E[Y|X;\theta]\}^2$$

$$\begin{aligned} \text{Var}(Y|X;\theta) &= \frac{\partial}{\partial \eta} \int y p(y|\eta) dy \\ &= \frac{\partial}{\partial \eta} \times \frac{\partial a(\eta)}{\partial \eta} \\ &= \frac{\partial^2 a(\eta)}{(\partial \eta)^2} \quad \frac{\partial^2 a(\eta)}{\partial \eta^2} ? \end{aligned}$$