Probset #3

4. (b)

$$K(x,z) = K_1(x,z) - K_2(x,z)$$
$$= \phi_1(x)^T\phi_1(z) - \phi_2(x)^T\phi_2(z)$$

Given any vector z,

$$z^T K z = \sum_i \sum_j z_i K_{ij} z_j$$
$$= \sum_i \sum_j z_i \{\phi_1(x^{(i)})^T\phi_1(z^{(j)}) - \phi_2(x^{(i)})^T\phi_2(z^{(j)})\} z_j$$
$$= \sum_i \sum_j z_i \{\sum_k (\phi_1(x^{(i)}))_k(\phi_1(z^{(j)}))_k - \sum_k (\phi_2(x^{(i)}))_k(\phi_2(z^{(j)}))_k\} z_j$$
$$= \sum_k \left(\sum_i z_i \phi_1(x^{(i)})_k\right)^2 - \sum_k \left(\sum_i z_i \phi_2(x^{(i)})_k\right)^2$$

$\rightarrow$ Not positive semi definite.

4. (c) $K(x,z) = aK_1(x,z)$ $\quad a \in \mathbb{R}^+$: positive real number

$$z^T K z = \sum_i \sum_j z_i K_{ij} z_j$$
$$= \sum_i \sum_j z_i \{a\phi_1(x^{(i)})^T\phi_1(x^{(j)})\} z_j$$
$$= \sum_i \sum_j \sum_k z_i \{a(\phi_1(x^{(i)}))_k(\phi_1(x^{(j)}))_k\}(z^{(j)})$$
$$= a \times \left(\sum_i z_i (\phi_1(x^{(i)}))_k\right)^2 \geq 0$$

$\rightarrow$ Positive semi definite.