Probset #3

4. (g)

$$K(x, z) = K_3(\phi(x), \phi(z))$$
$$= \phi_3(\phi(x))^T \phi_3(\phi(z))$$

K^3 : a kernel over $\mathbb{R}^d \times \mathbb{R}^d$

$\phi : \mathbb{R}^n \to \mathbb{R}^d$

For any vector z,

$$z^T K z = \sum_i \sum_j z_i K_{ij} z_j$$
$$= \sum_i \sum_j z_i \phi(x^{(i)})^T \phi(z^{(j)}) z_j$$
$$= \sum_i \sum_j z_i \sum_k (\phi(x^{(i)}))_k (\phi(x^{(j)}))_k z_j$$
$$= \sum_k \left(\sum_i z_i \phi(x^{(i)})\right)^2 \geq 0$$

$\longrightarrow$ positive semi-definite

$\longrightarrow$ is necessarily a kernel