

Problem set 1 - #3. Poisson Regression

$$p(y; \eta) = b(y) \exp(\eta^T T(y) - a(\eta))$$

η : natural parameter (canonical parameter)

$T(y)$: sufficient statistic (usually $T(y) = y$)

$a(\eta)$: log partition function

$\exp(-a(\eta))$: usually normalization constant

$$\sum p(y; \eta) = 1 \quad \text{or} \quad \int p(y; \eta) = 1$$

$T, a, b; \eta \rightarrow$ defines the family of distribution

(a)

<Poisson distribution>

$$p(y; \lambda) = \frac{e^{-\lambda} \lambda^y}{y!}$$

$$= \frac{1}{y!} \{ \exp(-\lambda) \cdot \lambda^y \} \quad T(y) = y$$

$$= \frac{1}{y!} \{ \exp(-\lambda) \cdot \exp(y \log \lambda) \}$$

$$= \frac{1}{y!} \{ \exp(-\lambda) \cdot \exp(y \log \lambda) \}$$

$$= \frac{1}{y!} \{ \exp(-\lambda + y \log \lambda) \}$$

$$= \frac{1}{y!} \{ \exp(y \log \lambda - \lambda) \}$$

$$\begin{aligned} T(y) &= y \quad \checkmark \\ \rightarrow a(\eta) &= \lambda \\ \eta &= \log \lambda \quad \checkmark \\ e^\eta &= \lambda \\ a(\eta) &= e^\eta \quad \checkmark \\ b(y) &= \frac{1}{y!} \quad \checkmark \end{aligned}$$