2. **[15 points] Poisson regression and the exponential family**

(a) [5 points] Consider the Poisson distribution parameterized by λ:

$$p(y;\lambda) = \frac{e^{-\lambda}\lambda^y}{y!}.$$

Show that the Poisson distribution is in the exponential family, and clearly state what are $b(y)$, η, $T(y)$, and $a(\eta)$.

exponential family → $p(y;\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$

poisson → $p(y;\lambda) = \frac{1}{y!}\exp(-\lambda)\lambda^y$

$\exp(-\lambda)\exp(\log \lambda^y)$

$= \exp(-\lambda + y\log\lambda)$

$= \exp((\log\lambda)y - \lambda)$

$\eta = \log\lambda$

$T(y) = y$

$a(\eta) = \lambda = e^\eta$

$b(y) = \frac{1}{y!}$