Probset #2

2. (c)

Effect of including L_2 regularization in the logistic regression objective has on model calibration.

Cost function of logistic regression with L2 regularization.

$$J(\theta) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right] + \frac{\lambda}{m}\|\theta\|_2$$

<Update Rule>

$$\theta_j := \theta_j - \alpha\left[\left(\frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x_j^{(i)}\right) + \frac{\lambda}{m}\theta_j\right]$$

when $j = 0$

$$\frac{\partial}{\partial\theta_0}J(\theta) = \frac{1}{m}\left\{\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})\right\} - \frac{\lambda}{m}\|\theta\|_2 = 0$$

$$\sum_{i=1}^{m} h_\theta(x^{(i)}) - \sum_{i=1}^{m} y^{(i)} - \lambda\|\theta\|_2 = 0$$

$$\sum_{i=1}^{m} y^{(i)} = \sum_{i=1}^{m} h_\theta(x^{(i)}) - \lambda\|\theta\|_2$$

$$y = h_\theta(x) - \lambda\|\theta\|_2$$

$\therefore$ L2 regularization makes the model not well calibrated.