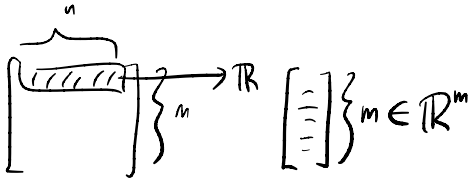
Probset #3

4. (f)

$K(x,z) = f(x)f(z)$

$f: \mathbb{R}^n \to \mathbb{R}$

$X \in \mathbb{R}^m \quad x^{(i)} \in \mathbb{R}^n$



$$K(x,z) = \sum_{i=1}^{m}\sum_{j=1}^{m} f(x^{(i)})f(x^{(j)})$$

$$= f(x^{(1)})f(x^{(1)}) + f(x^{(1)})f(x^{(2)}) + \cdots + f(x^{(m)})f(x^{(m)})$$

$$K_{ij} = K(x^{(i)}, x^{(j)}) = f(x^{(i)})f(x^{(j)}) = f(x^{(j)})f(x^{(i)}) = K(x^{(j)}, x^{(i)}) = K_{ji}$$

→ symmetric

Given any vector z,

$$z^T K z = \sum_i \sum_j z_i K_{ij} z_j$$

$$= \sum_i \sum_j z_i f(x^{(i)}) f(x^{(j)}) z_j$$

$$= \left(\sum_i z_i f(x^{(i)})\right)^2 \geq 0$$

→ positive semi definite

→ is necessarily a kernel.