

$$p(y; \eta) = b(y) \exp(\eta^T T(y) - a(\eta))$$

$\eta$ : natural parameter (canonical parameter)

$T(y)$ : sufficient statistic (usually  $T(y) = y$ )

$a(\eta)$ : log partition function

$\exp(-a(\eta))$ : usually normalization constant

$$\sum p(y; \eta) = 1 \quad \text{or} \quad \int p(y; \eta) = 1$$

$T, a, b; \eta \rightarrow$  defines the family of distribution

<Gaussian distribution>

$$p(y; \mu) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(y - \mu)^2\right)$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(y^2 - 2\mu y + \mu^2)\right\}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}y^2 + \mu y - \frac{1}{2}\mu^2\right\}$$

$$= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) \times \exp\left(\mu y - \frac{1}{2}\mu^2\right)$$

$$T(y) = y \quad \checkmark$$

$$\rightsquigarrow a(\eta) = \frac{1}{2}\mu^2$$

$$b(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}y^2\right) \quad \checkmark$$

$$\eta = \mu \quad \checkmark$$

$$a(\eta) = \frac{1}{2}\eta^2 \quad \checkmark$$