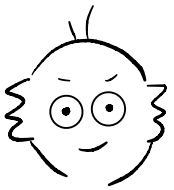
Lipschitz Continuity



Definition of Lipschitz continuity of a function at a point:

A function f from $S \subset \mathbb{R}^n$ into $\mathbb{R}^m$ is Lipschitz continuous at $x \in S$ if there is a constant C such that

$$\|f(y) - f(x)\| \leq C\|y - x\|$$

for all $y \in S$ sufficiently near x.

* Lipschitz continuity at a point depends only on the behavior of the function near that point.

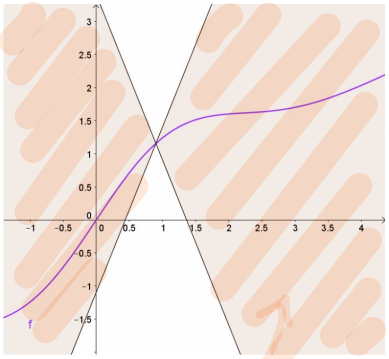
<Properties>

- f is Lipschitz continuous at $x \rightarrow$ continuous at x
- f is a real-valued function defined on $S \subset \mathbb{R}$ and f is differentiable at $x \in S \rightarrow f$ is Lipschitz continuous at x.

$\therefore$ differentiable at $x \Rightarrow$ Lipschitz Continuous at $x \Rightarrow$ Continuous at x.

$$\|f(x) - f(y)\| \leq C\|x - y\|$$

$$\frac{\|f(x) - f(y)\|}{\|x - y\|} \leq C$$



$\longrightarrow$ function f with derivative less than C

$\longrightarrow$ No radical change in derivative

$\longrightarrow$ Stable training is possible for deeplearning

M : area that the derivative of f exists.