Mathematics behind adding Gaussian Noise

At each step in the chain we are simply sampling from a Gaussian distribution whose mean is the previous value (i.e. image) in the chain.

$$X_t \sim N(X_{t-1}, 1) \iff X_t = X_{t-1} + N(0, 1)$$

$$N(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2)$$

$p_{X_t}(x_t)$
$= \int p(x_t|x_{t-1}, 1)p(x_{t-1})dx_{t-1}$
$= \int N(x_t; x_{t-1}, 1)p(x_{t-1})dx_{t-1}$
$= \int N(x_t - x_{t-1}; 0, 1)p(x_{t-1})dx_{t-1} \iff convolution$
$= (N(0, 1) * px_{t-1})(x_t)$

$X_t = N(0, 1) + X_{t-1}$

In other words, we have show that asserting the distribution of a timestep conditioned on the previous one via the mean of a Gaussian distribution is equivalent to asserting that the distribution of a given timestep is that of the previous one with the addition of Gaussian noise.