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A Numerical Approach for the Inverse and Forward Kinematic Analysis of 5R Parallel Manipulator

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***Abstract*—In this paper a numerical approach for the position, velocity and acceleration analysis for a 5R parallel manipulator is presented. 5R parallel manipulator is a two degrees of freedom planar mechanism which has been extensively studied by analytical methods. In this work, the use of computational tools such as Matlab and ADAMS is proposed in order to solve the inverse and forward kinematic problem of a particular type of 5R symmetric parallel manipulator. The obtained numerical results are validated by comparing them with analytical methods reported in the literature. This numerical approach allows for the co-simulation with other computational tools for the creation of virtual prototypes and it can save time and resources in the development of 5R parallel manipulator applications.**

I. Introduction

A parallel manipulator is a closed-loop kinematic chain mechanism that is made up of and end-effector (moving platform) and a fixed base, linked together by several independent kinematic chains (limbs). In parallel manipulators, the external load is shared by several limbs of the system which provide them a high structural stiffness and a large load-carrying capacity. Such architecture also make it possible to reduce the mass of the movable links (all the actuators are mainly fixed on the base and many limbs feel only traction or compression, not bending efforts) and as a result, make it possible to use less powerful actuators. Such characteristics promises to create mechanisms with excellent performance in terms of accuracy, rigidity, and ability to manipulate large load [1], [2], [3].

Kinematic analysis refers to the study of the geometry of motion, without considering the forces and torques that cause the motion. In this analysis, the relation between the geometrical parameters of the manipulator with the final motion of the end-effector is derived and analyzed [3]. Moreover, kinematic analysis is the base for the workspace and singularities study, and for the dynamics and control of parallel manipulators.

The five-bar manipulator is a typical parallel manipulator with the minimal degrees of freedom (DoFs), which can be used for positioning a point on a region of a plane. Many researchers have made extensive investigations about 5R parallel manipulator, including inverse and forward kinematics, kinematic performance index (KPI), workspace, assembly modes, singularity synthesis, dynamic analysis and control approaches and so forth [4], [5].

Alici [6] presents an analytical method to obtain joint inputs needed to attain any point in the reachable workspace of a class of five-bar planar parallel manipulators. Sylvesters dialytic elimination method is employed to solve the equations. The proposed method is useful in trajectory planning and control of five-bar planar parallel manipulators in joint space. Cervantes-Sánchez et al. [7], [8] develop a systematic analysis and characterization of the workspace and singularities in the 5R planar parallel manipulator with symmetric architecture. The obtained results are a set of curves which describe the workspace and singularities in cartesian and joint spaces with applications in analysis and design of 5R parallel manipulators. Liu et al. [9] define and present a graphical representation of a set of performance indices for the planar 5R symmetrical parallel mechanism. The obtained atlases can be used to synthesize the link lengths of the mechanism with respect to specified criteria and to achieve the optimum dimensional mechanism with respect to a desired workspace. Macho et al. [10] show how it is possible to obtain for the 5R planar parallel manipulator the complete workspace associated with each solution of the direct kinematic problem or assembly mode. Moreover, an exhaustive analysis of the complete workspace and singular positions of the 5R planar parallel manipulator with two active joints is presented and an application of these principles to path planning is explained.

The kinematic analysis and the study of the workspace and singularities of the 5R parallel manipulator have been used to optimize the design of this kind of parallel robots with respect to different performance criteria and using diverse approaches and methods. [4], [5], [11].

In this work a numerical approach for the kinematic analysis for a 5R parallel manipulator by using computational tools such as Matlab and ADAMS (Automated Dynamic Analysis of Mechanical Systems) is presented. The workspace and the singular positions of the manipulator are obtained too. The obtained numerical results are validated by comparing them with analytical methods reported in the literature. This numerical approach allows for the co-simulation with other computational tools for the creation of virtual prototypes and it can save time and resources in the development of 5R parallel manipulator applications.

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II. Inverse and forward kinematics

A. Analytical solution

The five-bar or 5R manipulator is a planar parallel mechanism with 2 degrees of freedom (DOF) which can be used for positioning a point on a region of a plane. A 5R parallel manipulator consists of five bars that are connected end to end by five revolute joints, two of which are connected to the base and actuated (see Figure 1a)) [4].

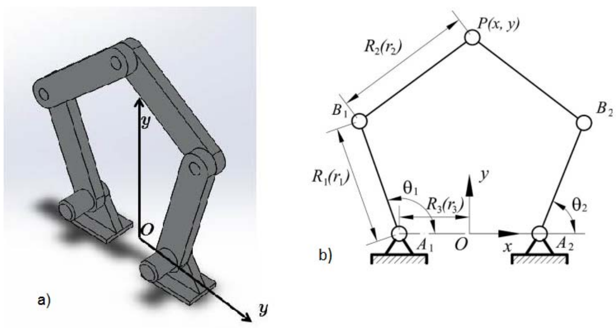

Fig. 1. a) The 5R parallel manipulator.

The kinematic model of the 5R parallel manipulator is developed based on the parameter shown in Figure 1b) and it is described with detail in [2] and [4].

The position of the end effector P in the reference system $O : xy$ can be described by the position vector $\boldsymbol{p}$

$$\boldsymbol{p} = (x \quad y)^T \tag{1}$$

In the same reference system, the position vectors $\boldsymbol{b_i}$ of points B_i are given by

$$\begin{aligned} \boldsymbol{b}_1 &= (r_1 \cos\theta_1 - r_3 \quad r_1 \sin\theta_1)^T \\ \boldsymbol{b}_2 &= (r_1 \cos\theta_2 + r_3 \quad r_1 \sin\theta_2)^T \end{aligned} \tag{2}$$

where θ_1 and θ_2 are the input angles of the actuated joints. The inverse kinematic problem can be solved by writing the constraint equation

$$|\boldsymbol{p}\boldsymbol{b}_i| = r_2, \quad i = 1, 2 \tag{3}$$

Equation (3) can be rewritten as

$$(x - r_1 \cos\theta_1 + r_3)^2 + (y - r_1 \sin\theta_1)^2 = r_2^2 \tag{4}$$

$$(x - r_1 \cos\theta_2 - r_3)^2 + (y - r_1 \sin\theta_2)^2 = r_2^2 \tag{5}$$

If the position of the end effector P is given, the input angles needed to reach this position can be obtained from equations (4) and (5) as

$$\theta_i = 2\arctan(z_i), \quad i = 1, 2 \tag{6}$$

where

$$z_i = \frac{-b_i \pm \sqrt{b_i^2 - 4a_i c_i}}{2a_i} \quad i = 1, 2 \tag{7}$$

and

$$\begin{aligned} a_1 &= r_1^2 + y^2 + (x + r_3)^2 - r_2^2 + 2(x + r_3)r_1 \\ b_1 &= -4yr_1 \\ c_1 &= r_1^2 + y^2 + (x + r_3)^2 - r_2^2 - 2(x + r_3)r_1 \\ a_2 &= r_1^2 + y^2 + (x + r_3)^2 - r_2^2 + 2(x + r_3)r_1 \\ b_2 &= -4yr_1 \\ c_2 &= r_1^2 + y^2 + (x + r_3)^2 - r_2^2 - 2(x + r_3)r_1 \end{aligned}$$

It can be observed that there are four solutions for the inverse kinematic problem of the 5R manipulator which correspond to the four solutions of the equation (7). The configuration shown in Figure 1 can be obtained if the sign "$\pm$" in equation (7) is "$+$" for the case $i = 1$ and is "$-$" para $i = 2$. Such a configuration is denoted as the "$+-$" model. Then there are three others, which are "$-+$","$--$" y "$++$" [4].

The forward kinematic problem can be solved by substitution of the known input angles θ_1 and θ_2 into equations (4) and (5) and by solving for the output position $p = (x \quad y)^T$.

B. Numerical solution

In order to solve and verify the inverse and forward kinematic problem of the 5R parallel manipulator, the algorithm presented in Figure 2 is established. If the proposed desired trajectory $\boldsymbol{X_d}$ used as input for the inverse kinematics follows with an acceptable margin of error to the output trajectory of the forward kinematics, the numerical obtained results will be verified. It must be considered that the solutions for the inverse kinematics are not unique and in order to avoid the convergence to incorrect solutions, it is necessary to choice the angles corresponding to the appropriate configuration. It is important to mention that the same algorithm can be used for differential kinematic problem.

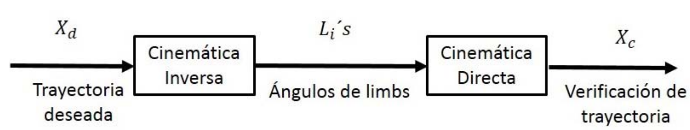

Fig. 2. Algorithm for kinematic problem solution and verification.

In order to solve and verify the complete kinematic problem of the 5R parallel manipulator, two desired trajectories were proposed. The first one is a linear trajectory for each element of final effector position vector $\boldsymbol{p}$. Based on the model "$+-$" the final effector position and velocity are described by

$$x(t) = 10t \quad [cm] \tag{8}$$

$$y(t) = 10t + 73.8644 \quad [cm] \tag{9}$$

The end effector position in its components x and y is shown in Figure 3.

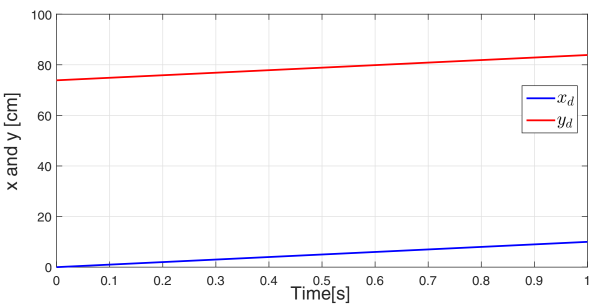

Fig. 3. Desired linear trajectory for the final effector.

The second one is a rest-to-rest trajectory with initial and final velocity and acceleration equal to zero. This desired trajectory in position is defined for a fifth order polynomial and the velocity and acceleration are given by the corresponding fist and second derivative.

$$x(t) = 4.5t^5 - 22.5t^4 - 30t^3 \quad [cm] \tag{10}$$

$$\dot{x}(t) = 22.5t^4 - 90t^3 - 90t^2 \quad [cm/s] \tag{11}$$

$$\ddot{x}(t) = 90t^3 - 270t^2 - 180t \quad [cm/s^2] \tag{12}$$

$$y(t) = -5.412t^5 + 27.0604t^4 - 36.080t^3 + 73.864 \; [cm] \tag{13}$$

$$\dot{y}(t) = -27.0605t^4 + 108.2416t^3 - 108.2415t^2 \quad [cm/s] \tag{14}$$

$$\ddot{y}(t) = -108.242t^3 + 324.7248t^2 - 216.483t \quad [cm/s^2] \tag{15}$$

The components of vector $\boldsymbol{p}$ as a time function for this trajectory are presented in Figure 4.

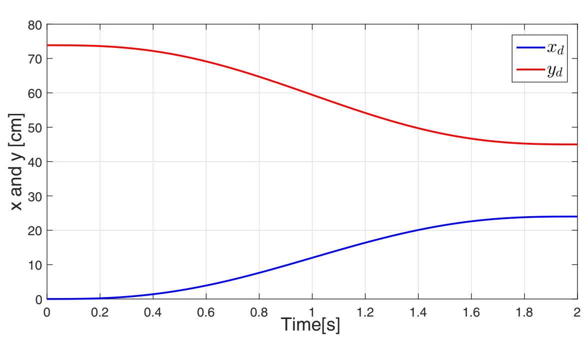

Fig. 4. Desired rest-to-rest trajectory with initial and final velocity and acceleration equal to zero.

The inverse kinematic problem is solved for both proposed trajectories. The numerical parameters for the 5R parallel manipulator are shown in Table I.

TABLE I
GEOMETRIC PARAMETERS OF THE 5R PARALLEL MANIPULATOR

$A_1B_1 = A_2B_2$	Link length $R_1 = 48cm$
$B_1P = B_2P$	Link length $R_2 = 48cm$
$OA_1 = OA_2$	Link length $R_3 = 24cm$

With the desired trajectories given by equations (8) and (9), equation (6) is numerically solved by Matlab.

The obtained joint angles θ_1 and θ_2 are fitted to third order polynomials by Matlab *Curve Fitting Toolbox*.

$$\theta_1(t) = -0.0483t^3 + 0.00275t^2 - 0.313t + 1.885 \quad [rad] \tag{16}$$

$$\theta_2(t) = 0.00679t^3 + 0.03773t^2 - 0.04781t + 1.257 \quad [rad] \tag{17}$$

By following the algorithm presented in Figure 2, the results obtained from the inverse kinematics are uses as inputs to solve the forward kinematic problem given by equations (4) and (5). These equations are numerically solved by the Matlab function *solve()*. The obtained results are the coordinates of the end effector position $p = (x \quad y)^T$ shown in Figure 5.

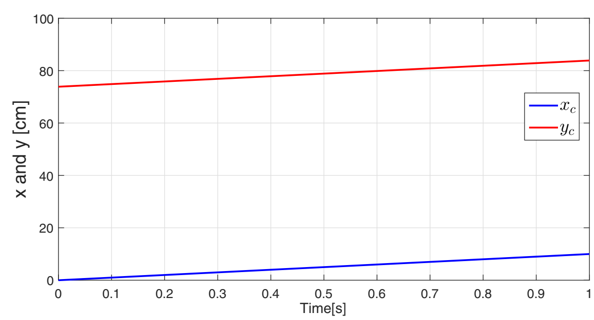

Fig. 5. Obtained end effector trajectory by solving the forward kinematics by Matlab with third order polynomials as inputs.

The same procedure was followed for the second desired trajectory. The obtained joint angles θ_1 and θ_2 are fitted to eight order polynomials given in equations (18) and (19). These functions are used as inputs for the forward kinematics and the end effector position is presented in Figure 6.

$$\theta_1(t) = 0.0389t^8 - 0.2956t^7 + 0.8108t^6 - 0.9338t^5 + 0.4123t^4 \\ -0.1316t^3 + 0.0009381t^2 + 0.0007719t + 1.885 \quad [rad] \tag{18}$$

$$\theta_2(t) = 0.009799t^8 - 0.1293t^7 + 0.5938t^6 - 1.415t^5 + 2.067t^4 \\ -1.633t^3 + 0.1462t^2 + -0.01345t + 1.257 \quad [rad] \tag{19}$$

In order to solve the forward kinematics by ADAMS, a five bar 5R parallel mechanism is built with the parameters of Table I. Functions given by equations (16)-(19) are introduced

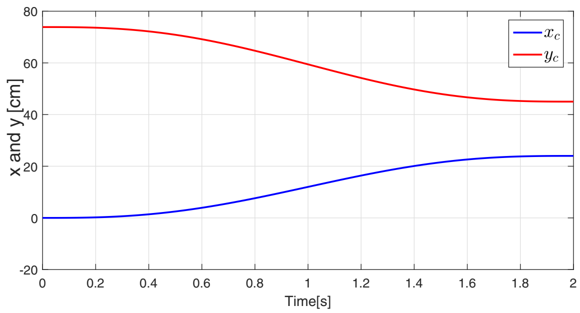

Fig. 6. Obtained end effector trajectory by solving the forward kinematics by Matlab with eight order polynomials as inputs.

as inputs to the actuators placed in the actuated joints. The results for the linear trajectory is shown in Figure 7 and for the rest-to-rest trajectory in Figure 8.

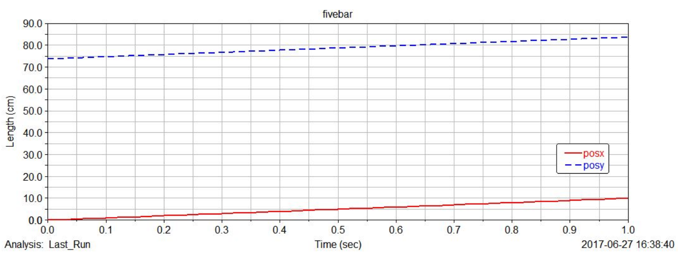

Fig. 7. Obtained end effector trajectory by solving the forward kinematics by ADAMS with third order polynomials as inputs.

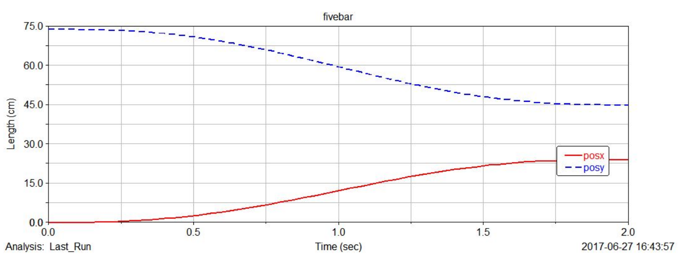

Fig. 8. Obtained end effector trajectory by solving the forward kinematics by ADAMS with eight order polynomials as inputs.

By comparing the obtained numerical results by Matlab and ADAMS with the proposed desired trajectories, the accuracy of the proposed computational tools can be verified. In Table II the error standard deviation of the obtained numerical results is presented.

TABLE II
COMPARATIVE ERROR STANDARD DEVIATION OF THE RESULTS OBTAINED BY ADAMS AND BY MATLAB

Proposed trajectory	Error standard deviation in $x(cm)$	Error standard deviation in $y(cm)$	Cumulative error standard deviation in position (cm)
Linear trajectory	0.0	4.8900e-06	4.8900e-06
Rest-to-rest with zero velocity and acceleration trajectory	2.0506e-5	2.5643e-05	3.2834e-05

III. DIFFERENTIAL KINEMATICS

A. Jacobian matrix

The relationship between the velocities of the actuated joints and the velocity of the end effector can be described by the Jacobian matrix. In order to obtain this matrix, equations (4) and (5) are derived with respect to time to obtain [4]

$$\boldsymbol{A}\dot{\boldsymbol{\theta}} = \boldsymbol{B}\dot{\boldsymbol{p}} \tag{20}$$

where $\dot{\boldsymbol{p}}$ is the vector of velocities of the final effector defined as $\dot{\boldsymbol{p}} = (\dot{x} \quad \dot{y})^T$, $\dot{\theta}$ is the vector of velocities of the actuated joints $\dot{\boldsymbol{\theta}} = (\dot{\theta}_1 \quad \dot{\theta}_2)^T$ and matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are expressed by:

$$A = \begin{bmatrix} y\cos\theta_1 - (x + r_3)\sin\theta_1 & 0 \\ 0 & y\cos\theta_2 + (r_3 - x)\sin\theta_2 \end{bmatrix} r_1 \tag{21}$$

$$B = \begin{bmatrix} x + r_3 - r_1\cos\theta_1 & y - r_1\sin\theta_1 \\ x - r_3 - r_1\cos\theta_2 & y - r_1\sin\theta_2 \end{bmatrix} \tag{22}$$

The Jacobian matrix can be written as

$$\boldsymbol{J} = \boldsymbol{A}^{-1}\boldsymbol{B} \tag{23}$$

B. Singularity analysis

The first kind of singularity is presented when $\boldsymbol{A}$ becomes singular, but $\boldsymbol{B}$ remains invertible. This condition corresponds to a configuration with any limb A_1B_1P y A_2B_2P fully extended or folded. With respect to the workspace of the manipulator, this singularity is produced when the output point P reaches the limit of the workspace [12]. In order to $\boldsymbol{A}$ becomes singular, next expression must be satisfied:

$$\begin{aligned} &xyr_1^2\cos\theta_1\sin\theta_2 - xyr_1^2\cos\theta_2\sin\theta_1 - yr_1^2r_3\cos\theta_1\sin\theta_2 \\ &-yr_1^2r_3\cos\theta_2\sin\theta_1 + y^2r_1^2\cos\theta_1\cos\theta_2 - x^2r_1^2\sin\theta_1\sin\theta_2 \\ &+r_1^2r_3^2\sin\theta_1\sin\theta_2 = 0 \end{aligned} \tag{24}$$

The second kind of singularity is porduced only in closed loop kinematic chains when $\boldsymbol{B}$ is singular, but $\boldsymbol{A}$ remains invertible. There are two cases for this kind of singulariry. The first one is when B_1PB_2 is fully folded, that is, points B_1 y B_2 are coincident. The second case is when B_1PB_2 is fully extended [12]. This condition is expressed by:

$$\begin{aligned} &2yr_3 - yr_1\cos\theta_1 + yr_1\cos\theta_2 + xr_1\sin\theta_1 - xr_1\sin\theta_2 \\ &-r_1r_3\sin\theta_1 - r_1r_3\sin\theta_2 - r_1^2\sin(\theta_1 - \theta_2) = 0 \end{aligned} \tag{25}$$

C. Acceleration analysis

In order to obtain the acceleration Jacobian, equations (4) and (5) are derived two times with respect to time to obtain:

$$\boldsymbol{A}\ddot{\boldsymbol{\theta}} = \boldsymbol{B}\ddot{\boldsymbol{p}} + \boldsymbol{C} \tag{26}$$

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are given by (21) and (22), respectively and

$$\boldsymbol{C} = \begin{bmatrix} \dot{x}f_4 + \dot{y}f_6 - \dot{\theta}_1 f_2 \\ \dot{x}g_4 + \dot{y}g_6 - \dot{\theta}_1 g_2 \end{bmatrix} \tag{27}$$

with

$$\begin{aligned} f_2 &= r_1[\dot{y}\cos\theta_1 - y\sin\theta_1\dot{\theta}_1 - \dot{x}\sin\theta_1 - (x + r_3)\cos\theta_1\dot{\theta}_1] \\ f_4 &= \dot{x} + r_1\sin\theta_1\dot{\theta}_1 \\ f_6 &= \dot{y} - r_1 cos\theta_1\dot{\theta}_1 \\ g_2 &= r_1[\dot{y}\cos\theta_2 - y\sin\theta_2\dot{\theta}_2 - \dot{x}\sin\theta_2 + (r_3 - x)\cos\theta_2\dot{\theta}_2] \\ g_4 &= \dot{x} + r_1\sin\theta_2\dot{\theta}_2 \\ g_6 &= \dot{y} - r_1 cos\theta_2\dot{\theta}_2 \end{aligned} \tag{28}$$

The acceleration of the actuated joints $\ddot{\theta_1}$ y $\ddot{\theta_2}$ can be expressed by

$$\ddot{\boldsymbol{\theta}} = \boldsymbol{A}^{-1}(\boldsymbol{B}\ddot{\boldsymbol{p}} + \boldsymbol{C}) \tag{29}$$

In a similar way, the acceleration of the end effector is given by

$$\ddot{\boldsymbol{p}} = \boldsymbol{B}^{-1}(\boldsymbol{A}\ddot{\boldsymbol{\theta}} - \boldsymbol{C}) \tag{30}$$

D. Numerical solution

In order to determine the singularities or the 5R parallel manipulator it is necessary to found its workspace, this is carried out by evaluating equation (6) for different coordinates of end effector and verifying the existence of a solution for the inverse kinematic problem. If such solution exists, then the proposed coordinate is reachable for the end effector. With the workspace defined, the singular points are searched by evaluating equations (24) and (25). The workspace and the singular points for the 5R parallel manipulator with the parameters given in Table I are presented in Figure 9.

For the velocity analysis, the algorithm of Figure 2 is followed. In this case, only the second trajectory described in Inverse and Forward Kinematics section is considered. The end effector velocities described by equations (11) and (14) are used to find the corresponding velocities of the actuated joints by evaluating equations (20)-(23). The results are shown in Figure 10.

The obtained velocities of the actuated joints are used to solve the forward differential kinematic problem and to find the corresponding end effector velocities by the inverse Jacobian matrix. The cartesian velocities of the end effector obtained by numerically solving the inverse Jacobian matrix by Matlab are presented in Figure 11.

In ADAMS the forward differential kinematic problem is solved by using the data of Figure 10 as velocity inputs.

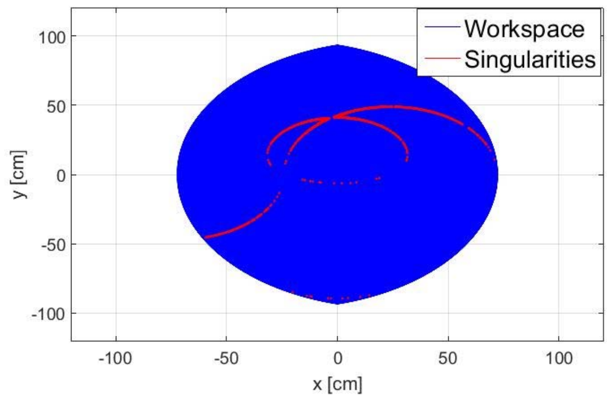

Fig. 9. Workspace and singularities for the 5R parallel manipulator.

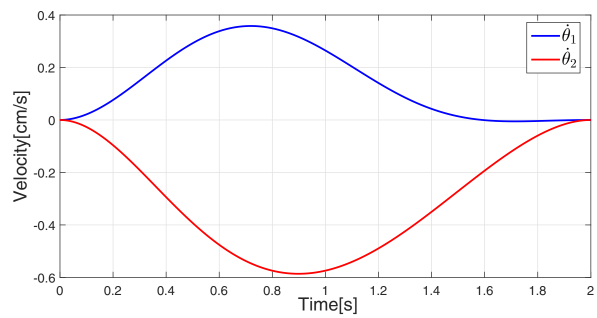

Fig. 10. Actuated joints velocities ($\dot{\theta}_1$ y $\dot{\theta}_2$) for a polynomial trajectory.

The obtained results for the velocities of the end effector are shown in Figura 12. A comparison of the numerical results is presented in Table III.

TABLE III
COMPARATIVE ERROR STANDARD DEVIATION OF THE RESULTS OBTAINED BY ADAMS AND BY MATLAB FOR VELOCITY

Proposed trajectory	Error standard deviation in $\dot{x}(cm/s)$	Error standard deviation in $\dot{y}(cm/s)$	Cumulate error standard deviation in velocity (cm/s)
Polynomial trajectory	0.0011	0.002	0.0023

A similar procedure was applied in order to obtain the solution for the acceleration problem for the 5R parallel manipulator. The end effector accelerations described by equations (12) and (15) are used to find the corresponding accelerations of the actuated joints by evaluating equation (29).

The results for the forward acceleration analysis were obtained numerically. In Figures 13 and 14 the results obtained by Matlab and ADAMS, respectively, are presented. A comparison of the numerical results is presented in Table IV.

IV. CONCLUSIONS

In this paper a numerical approach for the kinematic analysis of the 5R parallel manipulator was presented. The obtained

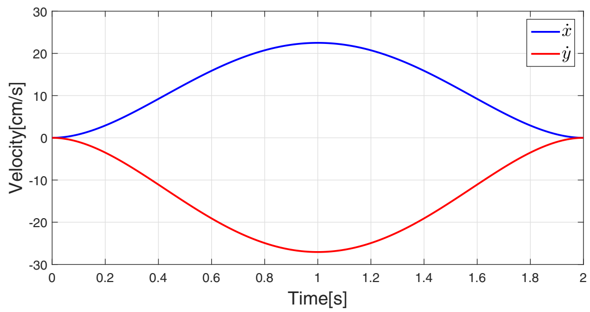

Fig. 11. Velocities of the end effector ($\dot{x}$ y $\dot{y}$) for a polynomial trajectory obtained by Matlab.

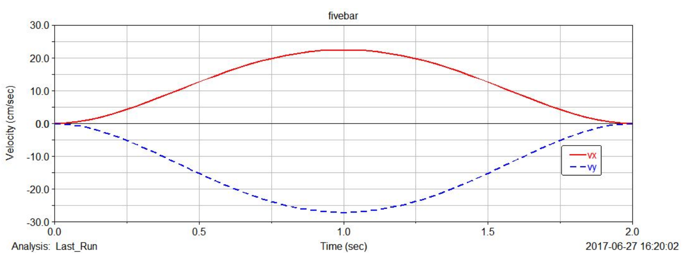

Fig. 12. Velocities of the end effector ($\dot{x}$ and $\dot{y}$) for a polynomial trajectory obtained by ADAMS.

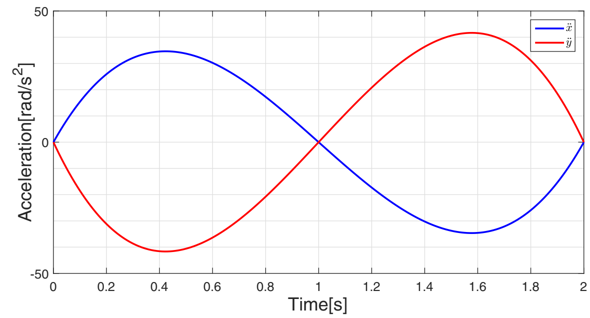

Fig. 13. Accelerations of the end effector ($\ddot{x}$ and $\ddot{y}$) for a polynomial trajectory obtained by Matlab.

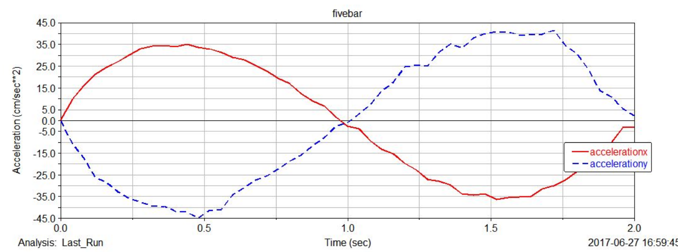

Fig. 14. Accelerations of the end effector ($\ddot{x}$ and $\ddot{y}$) for a polynomial trajectory obtained by ADAMS.

numerical results were validated by comparing them with analytical methods reported in the literature. This numerical approach, in particular the use of ADAMS software, allows for the co-simulation with other computational tools for the creation of virtual prototypes and it can save time and resources in the development of 5R parallel manipulator applications. The proposed approach allows to solve the complete kinematic problem without the need of programming the analytical equations and the obtained results can be used in dynamic analysis, design, optimization and control of this kind of manipulators.

ACKNOWLEDGMENT

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TABLE IV
COMPARATIVE ERROR STANDARD DEVIATION OF THE RESULTS OBTAINED BY ADAMS AND BY MATLAB FOR ACCELERATION

Proposed trajectory	Error standard deviation in $\ddot{x}(cm/s^2)$	Error standard deviation in $\ddot{y}(cm/s^2)$	Cumulative error standard deviation in acceleration (cm/s^2)
Polynomial trajectory	0.5248	0.8778	1.0227

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