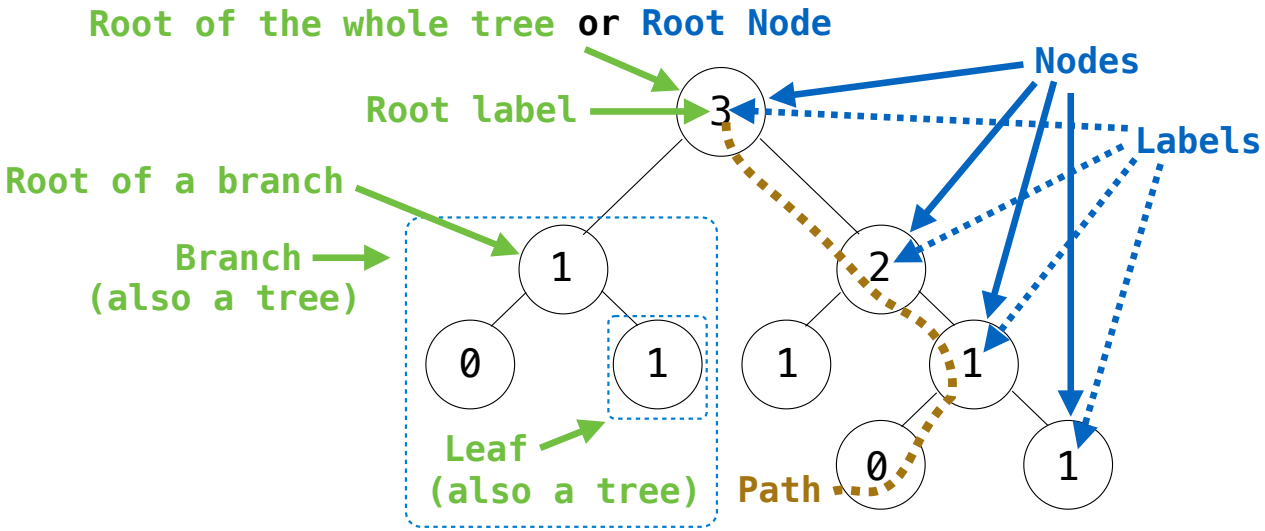
Trees

Announcements

Trees

Tree Abstraction



Recursive description (wooden trees):

A **tree** has a **root label** and a list of **branches**

Each **branch** is a **tree**

A **tree** with zero **branches** is called a **leaf**

A **tree** starts at the **root**

Relative description (family trees):

Each location in a tree is called a **node**

Each **node** has a **label** that can be any value

One node can be the **parent**/**child** of another

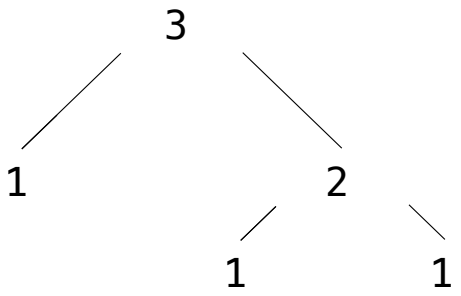
The top node is the **root node**

People often refer to labels by their locations: "*each parent is the sum of its children*"

Implementing the Tree Abstraction

```
def tree(label, branches=[]):
    return [label] + branches

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]
```

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree




```
>>> tree(3, [tree(1),
...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]
```

Implementing the Tree Abstraction

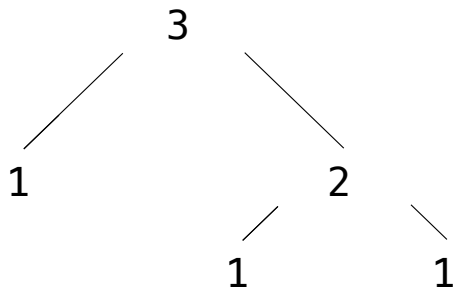
```
def tree(label, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)

def label(tree):
    return tree[0]

def branches(tree):
    return tree[1:]

def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list

- A **tree** has a root **label** and a list of **branches**
- Each branch is a tree




```
>>> tree(3, [tree(1),
...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]
```

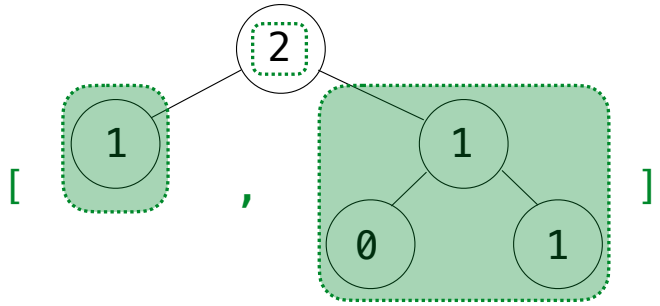
```
def is_leaf(tree):
    return not branches(tree)
```

Using the Tree Abstraction

For a tree t, you can **only**:

- Get the label for the root of the tree: `label(t)`
- Get the list of branches for the tree: `branches(t)`
- Get the branch at index i, which is a tree: `branches(t)[0]`
- Determine whether the tree is a leaf: `is_leaf(t)`
- Treat t as a value: `return t`, `f(t)`, `[t]`, `s = t`, etc.

An example tree t:



(Demo)

Tree Processing

Tree Processing Uses Recursion

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

```
def count_leaves(t):
    """Count the leaves of a tree."""
    if is_leaf(t):
        return 1
    else:
        branch_counts = [count_leaves(b) for b in branches(t)]
        return sum(branch_counts)
```

Writing Recursive Functions

Make sure you can answer the following before you start writing code:

- What recursive calls will you make?
- What type of values do they return?
- What do the possible return values mean?
- How can you use those return values to complete your implementation?

Example: Largest Label

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

```
def largest_label(t):
    """Return the largest label in tree t."""
    if is_leaf(t):
        return label(t)
    else:
        return max([largest_label(b) for b in branches(t)] + [label(t)])
```

Example: Above Root

Processing a leaf is often the base case of a tree processing function

The recursive case typically makes a recursive call on each branch, then aggregates

```
def above_root(t):
    """Print all the labels of t that are larger than the root label."""
    def process(u):
        if label(u) > label(t):
            print(label(u))
        for b in branches(u):
            process(b)
    process(t)
```

Min Practice

Example: Minimum x

Given these two related lists of the same length:

```
xs = list(range(-10, 11))
```

Write an expression that evaluates to the x in xs for which x*x - 2*x + 1 is smallest:

```
>>> xs
[-10,  -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4,  5,  6,  7,  8,  9, 10]
>>> [x*x - 2*x + 1 for x in xs]
[121, 100, 81, 64, 49, 36, 25, 16,  9,  4, 1, 0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
>>> ... some expression involving min ...
1
```