Error-reducing codes

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Approach

Error correcting codes:

correct all errors in message.

Error reducing codes:

correct a fraction of the bit errors in a received word, reduce the number of errors.

Algorithms

- LDGM-Staircase Codes (2017)
- Hamming Codes (2016)
- Triangulization codes (2018)

Hamming Codes

- X is message
- G^T * x is encoded message
- Error can be found by multiplying code

  with parity check matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

G matrix

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

H matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

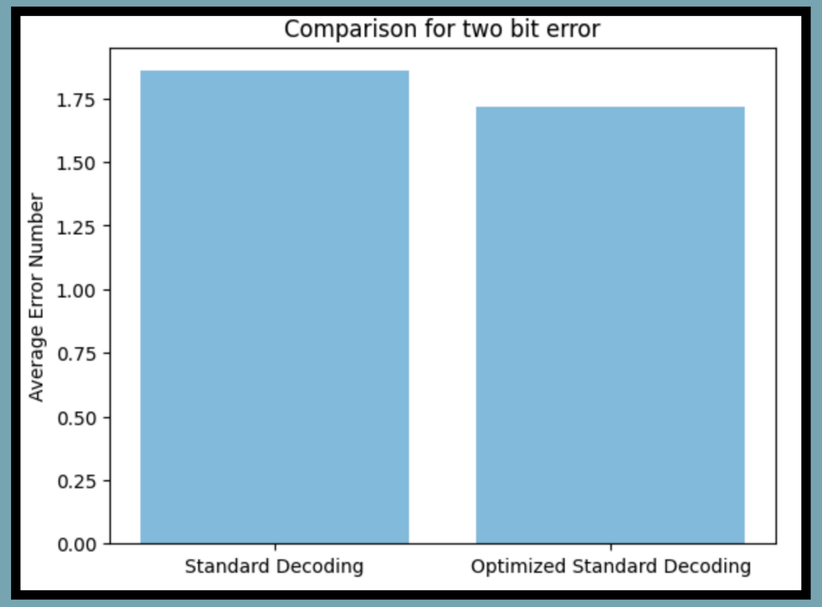
G matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

New G matrix

Modulo-2 sum of the first two rows

Hamming results



Errors with new G matrix

Other approaches

- Minimum of sums decoding
- Minimum of maximums decoding
- Majority bit decoding

Number of errors introduced	Average number of errors in decoded message				
	Standard decoding	Optimized standard decoding	Minimum of sums decoding	Minimum of maximums decoding	Majority bit decoding
2	1.8571	1.7143	1.4286	1.4643	1.2857
3	2.2000	2.1714	1.8571	1.7714	1.7857
4[1]	1.9429	1.8286	2.1429	2.0571	2.1429

Test of different approaches

LDGM codes

- Class of linear block codes
- The main disadvantage: high-complexity encoding
- Structured LDPC: ALDPC, MAC, LDGM

$$H = [I_x \; P^T]$$
$$G = [P \; I_G]$$

Triangulization codes

X = [1, 0, 1, 0]

S = {1, 2, 3}

↓

[1 1 1 0]

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

x*S → 1 0 1 → maj_s (1 0 1) → 1

Encoding procedure

```
decoder(y, G)
  ∀ j:
    If y_j = 1: (x, g_j) > |g_j| / 2

    If y_j = 0: (x, g_j) ≤ |g_j| / 2

  Return: min|x|
```

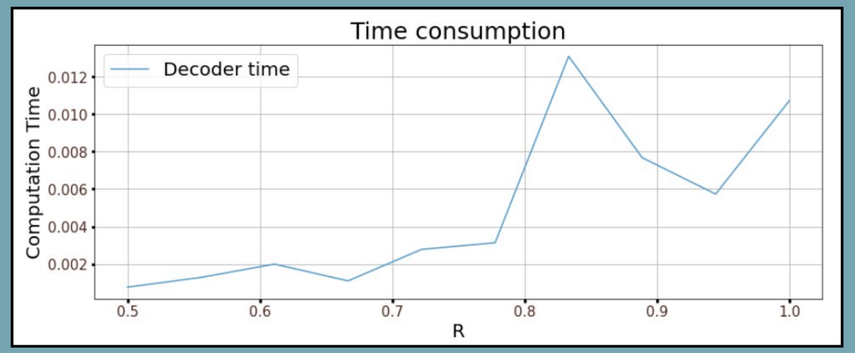
Decoding procedure

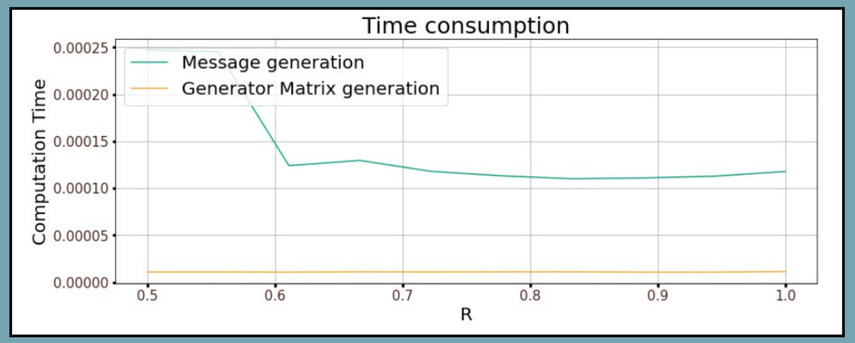
```
Initial Message: [1, 0, 1, 0, 1]

Generator Matrix:
[[0 0 1 0 1 0]
 [1 1 0 0 0 1]
 [0 1 1 1 0 1]
 [1 0 0 1 1 1]
 [1 1 1 1 1 0]]

Encoded Message: [0, 1, 1, 1, 1, 0]
```

Triangularization Results





Results

	Params	Time
LGDM	m=100; P=0.8	1.27 s
Hamming	m=100; P=0.8	1.58 s
Triangular	m=100; P=0.8	1.89 s

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