2019 International Symposium on Lightning Protection (XV SIPDA), São Paulo, Brazil, 30th September – 4th October 2019.

Polarity asymmetry in return stroke speed caused by the momentum associated with radiation

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***Abstract*—In lightning positive return strokes, the net momentum transported by the radiation field has the same direction as the momentum associated with electrons whereas the momentum associated with electrons and radiation in negative return strokes has opposite directions. It is shown here that this polarity asymmetry could limit the maximum speed of positive return stroke currents with respect to the negative counterpart.**

Keywords—Lightning, return strokes, radiation field, momentum, polarity asymetry, return stroke speed

I. INTRODUCTION

Basic components of a lightning flash can be separated into streamers, leaders and return strokes. These discharge components exist in two polarities, namely positive and negative. In positive streamers, positive leaders and negative return strokes, electrons travel in the opposite direction to the direction of movement of the discharge while in negative streamers, negative leaders and positive return strokes, they travel in the same direction as the direction of propagation of the discharge [1]. Discharges of negative and positive polarity in general have different features associated with their currents and their speeds of propagation. Moreover, the magnitude of the critical electric fields necessary to support these discharges differ in the negative and positive polarity. Some of these differences are caused by the difference in the direction of propagation of the electrons with respect to the direction of propagation of the discharge [2, 3]. Indeed it is the movement of the electrons that controls the electric discharge because the positive ions remain more or less stationary at the head of the discharge over times which are of interest in generating discharge currents and speed.

The goal of this paper is to illustrate the polarity asymmetry in the speed of electrical discharges caused by the net momentum associated with the radiation emitted by the discharge. Though the theory presented can be applied to any type of electrical discharge, here we will concentrate on the return strokes.

II. THE POLARITY ASYMMETRY ASSOCIATED WITH THE DIRECTION OF MOVEMENT OF CHARGE CARRIERS

In the case of negative return strokes, the return stroke current is maintained by the movement of electrons towards the ground along the leader channel. On the other hand, the return stroke propagates upwards towards the cloud. Thus, the direction of propagation of the electrons is opposite to that of the movement of the return stroke. In the case of positive return strokes, electrons move towards the cloud as well and thus their direction of propagation is the same as the direction of propagation of the return stroke. Due to this polarity asymmetry, the electric and magnetic fields at any point in space have opposite polarity in positive and negative return strokes. However, since the Poynting vector is given by $(\mathbf{E}\times\mathbf{B})/\mu_0$, the direction of propagation of the energy and momentum associated with the radiation fields have the same direction at any given point in space, both in positive and negative return strokes.

In the case of return strokes, the azimuthal symmetry of the electromagnetic field makes the net component of the momentum associated with the electromagnetic radiation field equal to zero except in the direction of propagation of the return stroke front, i.e., in the positive z-direction in the current study. Thus, the momentum dissipated by the radiation field of positive return strokes has the same direction to that of the movement or momentum of the electrons whereas in negative return strokes they have opposite directions. This asymmetry of the direction of the radiation momentum and the direction of momentum associated with the electrons makes significant changes to the speed of propagation of positive strokes in comparison to negative return strokes. Before proceeding further, however, let us consider the momentum transported by the radiation fields of return strokes.

III. MOMENTUM TRANSPORTED BY THE RADIATION FIELD AND ELECTRONS IN THE RETURN STROKE CHANNEL

Electromagnetic fields transport momentum as well as energy. The flux of momentum transported by the electromagnetic field (rate of momentum transport per unit area) is given by $(\mathbf{E}\times\mathbf{B})/c\mu_0$. Due to symmetry, the net momentum transported by the radiation field of a positive or a negative return stroke, assumed to be straight and vertical, is directed along the z-axis. Let us represent the return stroke by a step current of amplitude I_r propagating upward with a constant speed u. If we assume the return stroke channel to be in free space, the rate of change of the net momentum, $P_{rz}(t)$ (directed along the z-axis), transported by the radiation field is given by [4, 5]

$$\frac{dP_{rz}(t)}{dt}=\frac{u^2I_r^2}{8\pi\varepsilon_0c^4}\left[\frac{1}{\beta^2}\ln\frac{(1+\beta)}{(1-\beta)}\left(\frac{3}{\beta^2}-1\right)-\frac{6}{\beta^3}\right] \quad (1)$$

In the above equation, $\beta=u/c$ where u is the speed of propagation of the return stroke and c is the speed of light. If a return stroke channel over a perfectly conducting ground is considered, the result becomes

$$\frac{dP_{rz}(t)}{dt}=\frac{u^2I_r^2}{8\pi\varepsilon_0c^4}\left[\frac{2}{\beta^4}\ln\frac{1}{1-\beta^2}-\frac{2}{\beta^2}\right] \quad (2)$$

Now, let us consider the movement of electrons in the channel. Consider a channel element of unit length of the return stroke channel. The current in this channel element is related to the drift speed v_d by the equation

$$I_r=\pi a^2n_ev_de \quad (3)$$

In the above equation, n_e is the density of electrons in the channel, e is the elementary charge and a is the cross section of the channel. The z-momentum of the electrons in the channel element associated with their drift is then given by

$$p_{ez}=\pi a^2n_ev_dm_e \quad (4)$$

In the above equation, m_e is the mass of the electron. Now, consider the change in momentum of the electrons in the return stroke channel during the time interval $t\to t+dt$. During this time interval, a channel length equal to udt is fed by the source with a current of magnitude I_r. Thus, the increase in the momentum of the electrons during the time interval $t\to t+dt$ is given by

$$dP_{ez}=\pi a^2n_ev_dm_eudt \quad (5)$$

This can be written as

$$dP_{ez}=\frac{m_eu}{e}I_rdt \quad (6)$$

Thus, the rate of increase of the momentum of electrons at time t is given by

$$\frac{dP_{ez}}{dt}=\frac{m_eu}{e}I_r \quad (7)$$

IV. Momentum Balance in Return Strokes

Just before the initiation of the return stroke, the leader travels from cloud to ground bringing charge from cloud to ground. This increases the electric field between the leader and the ground. The electric field created by this charge and the remaining charge in the cloud is the source that drives the return stroke along the channel. Let us consider a positive return stroke where the electrons are moving towards the cloud, i.e. towards the positive z-axis. Let us denote by $F(t)$ the force exerted by this source (i.e. electric field) on the electrons in the channel (note that in the case of positive return strokes it is directed along the negative z-axis), which can be related to the time derivative of the momentum. This force accelerates negative charges or electrons towards the positive z-direction. This force is responsible for the momentum of electrons in the return stroke channel. Let us now consider the momentum balance along the z-axis for the movement of electrons for positive and negative return strokes during the time interval $t\to t+dt$.

A. Positive return strokes

Let us represent the current and the speed of the positive return stroke by I_{rp} and u_p, respectively. We assume that this combination is allowed by the energy conservation. The momentum balance equation for this positive return stroke is given by

$$F(t)=\frac{dP_{loss,z}(t)}{dt}+\frac{dP_{ez}(t)}{dt}+\frac{dP_{rz}(t)}{dt} \quad (8)$$

In the above equation $P_{loss,z}$ is the momentum lost by electrons in collisions and in other interactions during the time when the electrons have increased their speed from zero to v_d. The last two terms represent the rate of change of momentum gained by electrons and the momentum radiated away, respectively. Substituting for the radiation term, we obtain for a return stroke in free space (Equation (1)) and taking into account (7), we obtain

$$F(t)=\left(\frac{dP_{loss,z}(t)}{dt}\right)_{u_p,I_{rp}}+\frac{m_eu_p}{e}I_{rp}$$
$$+\frac{u_p^2I_{rp}^2}{8\pi\varepsilon_0c^4}\left[\frac{1}{\beta_p^2}\ln\frac{(1+\beta_p)}{(1-\beta_p)}\left(\frac{3}{\beta_p^2}-1\right)-\frac{6}{\beta_p^3}\right] \quad (9)$$

In the case of a discharge channel over a perfectly conducting ground, we obtain

$$F(t)=\left(\frac{dP_{loss,z}(t)}{dt}\right)_{u_p,I_{rp}}+\frac{m_eu_p}{e}I_{rp}$$
$$+\frac{u_p^2I_{rp}^2}{8\pi\varepsilon_0c^4}\left[\frac{2}{\beta_p^4}\ln\frac{1}{1-\beta_p^2}-\frac{2}{\beta_p^2}\right] \quad (10)$$

B. *Negative return strokes*

Just to illustrate a point, consider a negative return stroke where the current is transported purely by positrons. In this case the positrons are moving towards the positive z-axis and the momentum balance equations will be identical to the ones given in equations (9) and (10). Now, consider a negative return stroke mediated by electrons. In this case, the electrons are moving towards the ground and the direction of the momentum associated with the electrons is opposite to that of positrons giving rise to an identical return stroke. The momentum balance equation for the negative return stroke with a current I_{rn} and a speed u_n (assuming again this combination is allowed by energy conservation) located in free space is given by

$$-F(t) = -\frac{m_e u_n}{e} I_{rn} - \left[\frac{dP_{loss,z}(t)}{dt}\right]_{u_n, I_{rn}} + \frac{u_n^2 I_{rn}^2}{8\pi\varepsilon_0 c^4}\left[\frac{1}{\beta_n^2}\ln\frac{(1+\beta_n)}{(1-\beta_n)}\left(\frac{3}{\beta_n^2}-1\right)-\frac{6}{\beta_n^3}\right] \quad (11)$$

Observe that the forward momentum is now given by the radiation field alone while the electrons are having a momentum towards the negative z-axis. This can be written as

$$F(t) = \frac{m_e u_n}{e} I_{rn} + \left[\frac{dP_{loss,z}(t)}{dt}\right]_{u_n, I_{rn}} - \frac{u_n^2 I_{rn}^2}{8\pi\varepsilon_0 c^4}\left[\frac{1}{\beta_n^2}\ln\frac{(1+\beta_n)}{(1-\beta_n)}\left(\frac{3}{\beta_n^2}-1\right)-\frac{6}{\beta_n^3}\right] \quad (12)$$

In the case of a return stroke over a perfectly conducting ground, we obtain

$$F(t) = \frac{m_e u_n}{e} I_{rn} + \left[\frac{dP_{loss,z}(t)}{dt}\right]_{u_n, I_{rn}} - \frac{u_n^2 I_{rn}^2}{8\pi\varepsilon_0 c^4}\left[\frac{2}{\beta_n^4}\ln\frac{1}{1-\beta_n^2}-\frac{2}{\beta_n^2}\right] \quad (13)$$

V. RESULTS

Let us consider negative return strokes. Observe that the condition $F(t)-\left[\frac{dP_{loss,z}(t)}{dt}\right]_{u_n, I_{rn}} < 0$ is satisfied for a channel in free space when

$$\frac{u_n^2 I_{rn}^2}{8\pi\varepsilon_0 c^4}\left[\frac{1}{\beta_n^2}\ln\frac{(1+\beta_n)}{(1-\beta_n)}\left(\frac{3}{\beta_n^2}-1\right)-\frac{6}{\beta_n^3}\right] > \frac{m_e u_n}{e} I_{rn} \quad (14)$$

For a discharge channel over a perfectly conducting ground, this condition is satisfied when

$$\frac{u_n^2 I_{rn}^2}{8\pi\varepsilon_0 c^4}\left[\frac{2}{\beta_n^4}\ln\frac{1}{1-\beta_n^2}-\frac{2}{\beta_n^2}\right] > \frac{m_e u_n}{e} I_{rn} \quad (15)$$

These conditions are realized when the momentum associated with the radiation overwhelms the momentum associated with electrons while the momentum loss term becomes larger than the momentum input from the external source. Thanks to the opposite sign of the momentum associated with the radiation field, the source term can still drive a return stroke even when the losses overwhelm the source term. However, the situation is different in the case of positive return strokes. The momentum balance equations show that no positive return stroke can exist when $F(t)-\left[\frac{dP_{loss,z}(t)}{dt}\right]_{u_n, I_{rn}} < 0$ because in this case the momentum balance equation cannot be satisfied. That means, the speed and currents of positive return strokes should be such that $F(t)-\left[\frac{dP_{loss,z}(t)}{dt}\right]_{u_n, I_{rn}} < 0$. In other words, in positive return strokes, the momentum associated with the electrons always has to overwhelm the radiation momentum.

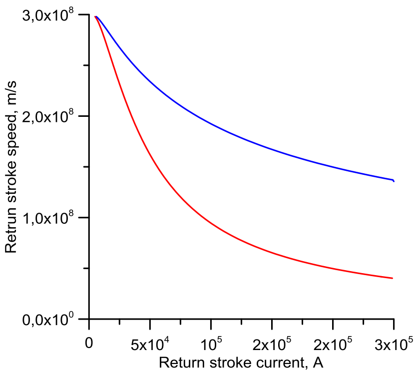

Figure 1: The limiting speed for a given return stroke current where the rate of change of momentum transported by the radiation field overwhelms the rate of change of the momentum of the electrons. The blue (upper curve) corresponds to a return stroke channel in free space and the red curve (lower curve) corresponds to a return stroke channel over perfectly conducting ground.

For a given return stroke current, the return stroke speed at which the radiation term overwhelms the electron momentum term takes place at a certain speed. This speed decreases as the return stroke current increases. Figure 1 depicts the limiting speed above which the rate of change of the radiation momentum overwhelms the rate of change of the electron momentum as a function of current. Note that for a given current, the limiting speed is less in the case of a channel above a perfectly conducting ground. The reason for this is the enhancement of the radiation field for a given current when the channel is located over a perfectly conducting ground. In the case of a channel in free space, the limiting speed for 250 kA, 100 kA, 30 kA, and 6kA are 1.35 x 10^8 m/s, 1.92 x 10^8 m/s, 2.59 x 10^8 m/s, and 2.98 x 10^8 m/s, respectively. In the case of a channel above a perfectly conducting ground, the limiting speeds for 250 kA, 100 kA, 30 kA, and 5kA currents are 4.0 x 10^7 m/s, 9.4 x 10^7 m/s, 2.1 x 10^8 m/s, and 2.98 x 10^8 m/s, respectively. This means, for example, that while a negative return stroke with a 100-kA current can propagate with a speed, say 2.0 x 10^8 m/s, a positive return stroke with an identical current can propagate only with a speed less than 9.4 x 10^7 m/s. Note also that for return stroke currents lower than about 20 kA, the polarity asymmetry does not play any role because the limiting speed is almost equal to the speed of light. The results show that the polarity asymmetry associated with the electrons and the radiation field causes the positive return strokes to propagate with a speed that is lower than the speed possible for a negative return stroke with an ideal current.

VI. Discussion

It is important to point out that what we have presented here is the limiting speeds associated with positive return strokes due to the momentum balance associated with the radiation and electrons. This does not mean that the positive return strokes need to propagate at the limiting speeds. It is possible that, due to other constrains caused by channel properties, the return strokes propagate at speeds less than the limiting values indicated in the results section.

In the analysis, we have considered a step current propagating along the return stroke channel. In reality, the current waveform has a complicated waveshape. However, any complicated waveshape can be divided into step functions with appropriate amplitudes and time shifts, separated by infinitesimal time intervals. The analysis presented is valid for each one of these individual step pulses. However, the momentum balance equation suggests that if a negative return stroke can be represented by a current waveform propagating with constant speed, the same current in a positive return stroke will disperse because different individual steps, depending on their peak current, will be propagating at different speeds. This is true only if the peak current is larger than the value where the momentum associated with the radiation field becomes significant compared to that of the electrons.

VII. Conclusion

In this paper, the effects of net momentum transported by the radiation field on the speed of propagation of the current along the return stroke channel is investigated. It is shown that, given identical initiating conditions, a positive return stroke will travel with a lower speed in comparison to a negative return stroke. The effect can be negligible for small currents but becomes highly significant for large currents.

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