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Effect of Rusck's Approximation in the Calculation of Lightning Electric Fields and Induced Voltages

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***Abstract*—This paper investigates the effect of considering the vertical electric field at the ground surface to compute the electric potential and the horizontal electric field above the ground surface, as well as the lightning induced voltages in an overhead line. This procedure, referred here as Rusck's approximation, is currently considered by several authors but, so far, has not been investigated in detail. The paper shows that this approximation leads to significant deviations at close range from the strike but provides accurate results if the distance from the strike is higher than three times the height of the point considered. The paper also shows that the constraints imposed by the electro-geometric model on the close-range indirect strikes yield to lightning induced voltage amplitudes much lower than the typical insulation level of aerial power lines. As a result, Rusck's approximation provides accurate results for the assessment of indirect lightning performance of aerial power lines.**

***Keywords*—lightning; electric field; induced voltages;**

I. Introduction

Lightning induced voltages (LIVs) are a major cause of line flashovers in aerial power distribution lines, which has motivated several researchers to investigate the inducing electromagnetic fields and the resulting induced voltages [1]-[9]. The approach to the problem may be divided into two groups: one that uses numerical methods for the direct solution of Maxwell's equations (e.g., finite-difference time-domain method - FDTD) [10-13], and one that uses analytical expressions, often embedded in a computer code [14-19].

Among those that use analytical equations, it is common to consider an approximation that was introduced by Rusck [1] in the calculation of the inducing electric fields that is also used (explicitly or implicitly) by several other researchers [20-23]. As will be demonstrated in Section II, this approximation is based on the evidence that the vertical electric field (VEF) produced by a lightning flash varies very little with height z above ground, provided that the distance r from the flash is sufficiently greater than the height z considered. As very close flashes lead to direct flashes to the line (instead of nearby flashes), the condition $r >> z$ is normally fulfilled in practice.

An analytical formulation for the VEF without considering Rusck's approximation was developed by Andreotti *et al.* [24] and the comparison with results from Rusck's approximation showed small differences on the LIV. However, Andreotti *et al.* did not investigate the restrictions that apply to Rusck's approximation in practice nor analyzed in detail its effect on the evaluation of the vertical and horizontal electric fields. Therefore, it seems that there is a need for a deep investigation of the limitations and possibilities concerned with Rusck's approximations, both in terms of lightning electric fields and lightning induced voltages (LIV). This is the objective of this paper.

In order to investigate the effect of Rusck's approximation, it is necessary to have a closer look into the development of the electromagnetic field equations due to the lightning return stroke, following the rationale used by Rusck [1]. Therefore, the paper is organized as follows: Sections II analyses the dependency of the close-range VEF field with height, assessing the effect of Rusck's approximation, whereas Section III uses the equations derived in the previous section to investigate the horizontal electric field (HEF) dependency with height. Section IV apply the field equations developed in the previous sections to the calculation of LIVs, highlighting the effect of Rusck's approximation. Finally, Section VI discusses the results and Section VII draws the main conclusions.

II. The Vertical Electric Field at Close Range

A. The Rusck's Approximation

Rusck developed a comprehensive work on LIV [1], which influenced significantly the research on this subject. He assumed a straight lightning channel on perfectly-conducting ground, a step return stroke current, and the transmission line (TL) return stroke model [25]. The vertical electric field (VEF) was calculated in time-domain as the contribution of the actual return stroke and its image below the ground surface:

$$E_Z(z,t) = E_0 + \frac{Z_E\, I_0\, \lambda}{4\,\pi\, v_r}\left\{[(vt-z)^2+\lambda\, r^2]^{-\frac{1}{2}} + [(vt+z)^2+\lambda\, r^2]^{-\frac{1}{2}}\right\}. \quad (1)$$

where Z_E = 377 Ω is the free-space impedance, I_0 is the return stroke current peak-value, v_r is the return stroke relative velocity ($v_r = v / c$, whereas v is the return stroke velocity and c is the light velocity in free-space), r is the radial distance from the flash, z is the vertical distance from the ground surface, and $\lambda = 1 - v_r^2$ is the square of the Lorentz contraction factor. It is worth to mention that this is not the original Rusck's formula, but an adaptation introduced by the authors in order to make it more compact. The constant E_0 is given by

$$E_0(z,t_0) = \frac{-Z_E\, I_0}{4\,\pi\, v_r}\left\{[(vt_0-z)^2+\lambda\, r^2]^{-\frac{1}{2}} + [(vt_0+z)^2+\lambda\, r^2]^{-\frac{1}{2}}\right\}, \quad (2)$$

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where t_0 is the time required for the wave to travel from the channel base to the point of interest, i.e.,

$$t_0 = \frac{\sqrt{r^2 + z^2}}{c} \,. \tag{3}$$

As described by Rusck [1], the VEF becomes insensitive of z as one moves away from the flash and the condition $r >> z$ holds. This condition is often applicable for the LIV calculation, as otherwise the flash would hit the line. Therefore, making $z = 0$ in (1) and (2) leads to a significant simplification with minimum loss of accuracy for the LIV calculation:

$$E_z(r,t) = \frac{Z_E\, I_0}{2\,\pi\, v_r}\left\{\lambda\,[(vt)^2 + \lambda\, r^2]^{-\frac{1}{2}} - r^{-1}\right\}, \tag{4}$$

As the VEF is practically constant up to the line height h, the voltage induced in a straight wire from ground up to h (i.e., the electric potential at height h) is simply given by

$$U_z(r,t) = h\, E_z(r,t)\,. \tag{5}$$

The approximation $z = 0$ that resulted in (4) and (5) is referred in this paper as Rusck's approximation. In essence, it means that the potential at a height h above ground is approximately given by the VEF at ground surface multiplied by the height h. At this point, it is interesting to validate (1) and to assess the errors in Rusck's approximation contained in (4).

Fig. 1 shows the VEF at $z = 10$ m and $r = 50$ m calculated with (1) for a current waveform typical of a subsequent stroke, as proposed by Rachidi *et al*. [26], and $v = 150$ m/μs. The calculation of the field for an arbitrary current waveform based on the expressions for the step waveform presented before is made by using the convolution theorem. This figure also shows the results calculated by Mimouni *et al*. [27] using the finite-difference time-domain (FDTD) method and a good agreement can be seen between the results.

Fig. 2 shows the VEF calculated with (1) for $r = 10$ m and different values of z (0, 5 m, and 10 m). It can be seen that the field intensity decreases with increasing height. Figs. 3 and 4 show the same comparison for $r = 20$ m and 50 m, respectively. As the condition $r >> z$ is approached, Rusck's approximation provides more accurate results.

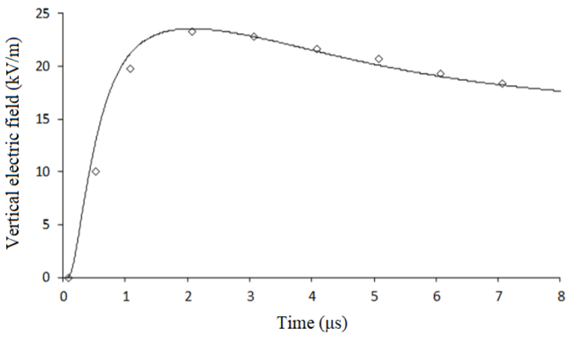

Fig. 1. Vertical electric field at $r = 50$ m and $z = 10$ m for a subsequent stroke current. Continuous line: calculated with (1); Dots: calculated using FDTD by Mimouni *et al*. [27].

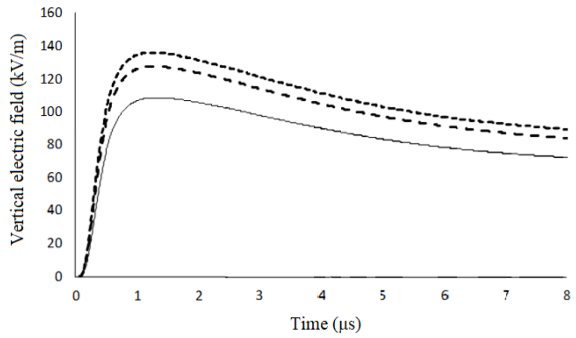

Fig. 2. Vertical electric field at $r = 10$ m for a subsequent stroke current. Continuous line: $z = 10$ m; dashed line: $z = 5$ m; dotted line: $z = 0$ m.

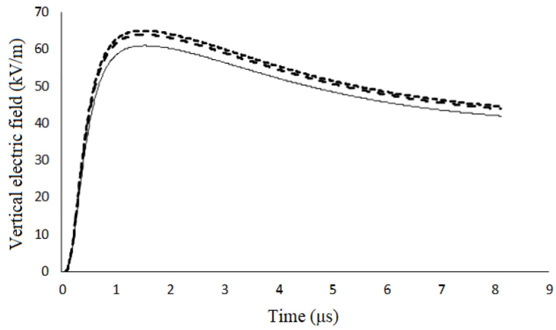

Fig. 3. Vertical electric field at $r = 20$ m for a subsequent stroke current. Continuous line: $z = 10$ m; dashed line: $z = 5$ m; dotted line: $z = 0$ m.

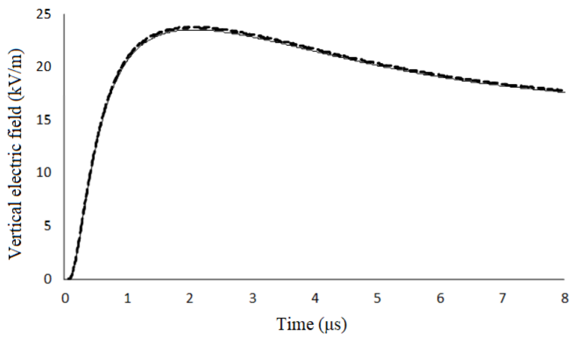

Fig. 4. Vertical electric field at $r = 50$ m for a subsequent stroke current. Continuous line: $z = 10$ m; dashed line: $z = 5$ m; dotted line: $z = 0$ m.

B. *The Height-Dependent Electric Potential*

If the condition $r >> z$ cannot be fulfilled, one must integrate (1) from 0 to z in order to obtain the height-dependent electric potential $U_Z\,(z, t)$. This integration can be carried out analytically from (1) and leads to:

$$\begin{aligned}\int_0^z E_z(z,t)\, dz = U_0(z,r) \\ + \frac{Z_E\, I_0\, \lambda}{4\,\pi\, v_r}\left\{ln\left[(v\,t + z) + [(v\,t + z)^2 + \lambda\, r_0^2]^{\frac{1}{2}}\right]\right. \\ \left. - ln\left[(v\,t - z) + [(v\,t - z)^2 + \lambda\, r_0^2]^{\frac{1}{2}}\right]\right\}\end{aligned} \tag{6}$$

$$\begin{aligned}U_0(z,t_0) = -\frac{Z_E\, I_0}{4\,\pi\, v_r}\left\{ln\left[(v\,t_0 + z) + [(v\,t_0 + z)^2 + \lambda\, r_0^2]^{\frac{1}{2}}\right]\right. \\ \left. - ln\left[(v\,t_0 - z) + [(v\,t_0 - z)^2 + \lambda\, r_0^2]^{\frac{1}{2}}\right]\right\}\end{aligned} \tag{7}$$

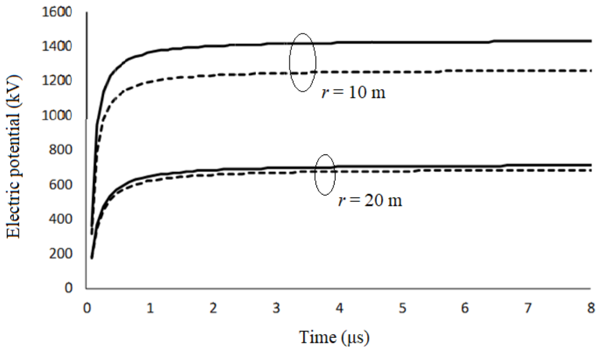

Fig. 5. Electric potential at $z = 10$ m by the VEF for a step-current return stroke, and distances $r = 10$ m and $r = 20$ m. Continuous lines: Rusck's approximation; dashed lines: calculated with (7).

Fig. 5 shows the induced potential calculated with (6) for $z = 10$ m and two values of r: 10 m and 20 m. This figure also shows the respective potentials calculated using Rusck's approximation. Although a significant difference can be seen for $r/z = 1$ (i.e., $r = 10$ m), Rusck's Approximation provides reasonable accurate results for $r/z = 2$ (i.e., $r = 20$ m).

III. The Horizontal Electric Field at Close Range

The horizontal electric field (HEF) produced by the return stroke was not explicitly considered by Rusck, although its effects on the induced voltage were considered in his coupling model. However, for the coupling model described by Agrawal *et al*. [28], the horizontal electric field must be evaluated. It is worth to mention that Agrawal *et al*. coupling model is very suitable for implementing in computer codes and it also bears a physical interpretation that facilitates the analysis of the LIVs.

The HEF was derived from Rusck's expression for the VEF by Barbosa and Paulino [5]. This was done by differentiating the scalar potential at line height with respect to r. In their work, Barbosa and Paulino considered the Rusck's approximation for the scalar potential. In this work, this derivation is repeated considering the height dependence of the VEF. The following expression for the scalar potential is obtained by simply removing the contribution of the vector potential from (6):

$$U_V(z,t) = U_0(z,r) + \frac{Z_E\, I_0}{4\,\pi\, v_r}\left\{ln\left[(v\,t + z)\ + [(v\,t+z)^2 + \lambda\, r_0^2]^{\frac{1}{2}}\right] - ln\left[(v\,t - z) + [(v\,t-z)^2 + \lambda\, r_0^2]^{\frac{1}{2}}\right]\right\}. \qquad (8)$$

Note that, if the height z is made equal to zero in (8), the two terms in logarithmic cancel each other and the voltage is null at ground surface, as expected. Moreover, for the initial instant of time ($t = t_0$) the two terms in (8) cancel each other and the scalar potential starts from zero.

The HEF can be obtained from (8) by differentiating it with respect to r. This is a straightforward calculation that leads to:

$$E_r(z,r,t) = k\left[\frac{\partial F(z,r,t)}{\partial r} - \frac{\partial F(-z,r,t)}{\partial r}\right] \qquad (9)$$

$$\frac{\partial F(z,r,t)}{\partial r} = \left[\frac{\lambda\, r}{\left(v\,t - z + \sqrt{(z - v\,t)^2 + \lambda\, r^2}\right)\left(\sqrt{(z - v\,t)^2 + \lambda\, r^2}\right)} + \frac{r}{\left(\sqrt{z^2 + r^2}\right)\left(z - \sqrt{z^2 + r^2}\right)}\right] \qquad (10)$$

$$k = \frac{Z_E\, I_0}{4\,\pi\, v_r} \qquad (11)$$

As demonstrated in [5], Rusck's approximation can be used to derive the HEF at height z by multiplying the VEF at ground level by the height z, and then deriving the result with respect to r. This leads to:

$$E_r(z,r,t) = \frac{Z_E\, I_0\, z}{2\,\pi\, v_r\, r^2}\left\{1 - \lambda\left[\lambda + \left(\frac{v\,t}{r}\right)^2\right]^{-\frac{3}{2}}\right\}. \qquad (12)$$

Fig. 6 shows the HEF calculated with (9) for a point at $r = 50$ m and $z = 10$ m, considering the return stroke current proposed by Rachidi *et al*. [26], and $v = 150$ m/µs. This figure also shows the results calculated by Mimouni *et al*. [27] using the finite-difference time-domain (FDTD) method and a good agreement can be seen between the results.

Figs. 7, 8, and 9 show the HEF calculated by the two formulations, i.e, (9) and (12), for a step current waveform, $v = 150$ m/µs, point at $z = 10$ m and different distances from the flash: $r = 10$ m, 20 m, and 50 m, respectively. As the relation r / z increases, the two formulations provide closer results. For instance, for $r = 50$ m (Fig. 9) the correspondence between the two formulations is very good and supports the use of Rusck's approximation. On the other hand, for $r = 10$ m (Fig. 7) the differences are very significant and Rusck's approximation should not be used. As observed also for the VEF, in all cases, Rusck's approximation provides higher values of HEF than those computed with the more accurate formulation.

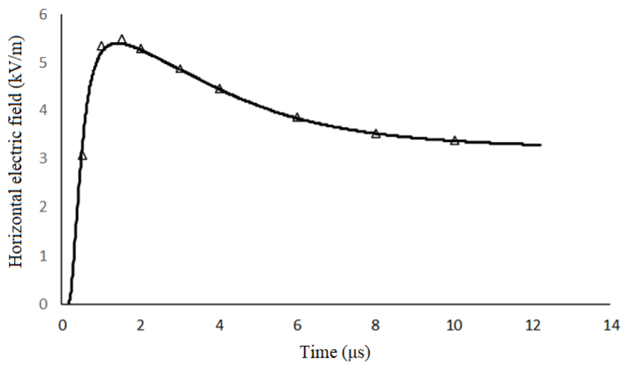

Fig. 6. Horizontal electric field at $r = 50$ m and $z = 10$ m for a subsequent stroke current. Continuous line: calculated with (9); Dots: calculated using FDTD by Mimouni *et al*. [27].

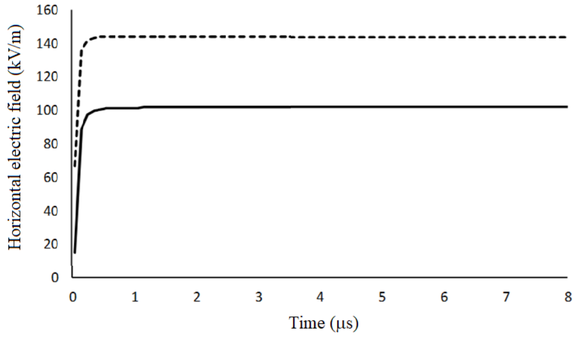

Fig. 7. Horizontal electric field at z = 10 m by a step-current return stroke, for r = 10 m. Continuous line: calculated with (9); dashed line: calculated with Rusck's approximation (12).

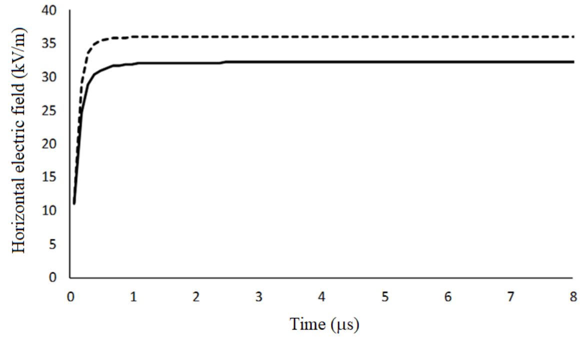

Fig. 8. Horizontal electric field at z = 10 m by a step-current return stroke, for r = 20 m. Continuous line: calculated with (9); dashed line: calculated with Rusck's approximation (12).

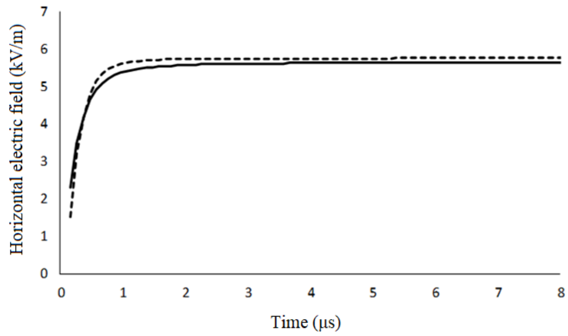

Fig. 9. Horizontal electric field at z = 10 m by a step-current return stroke, for r = 50 m. Continuous line: calculated with (9); dashed line: calculated with Rusck's approximation (12).

IV. Lightning Induced Voltages

An important application of lightning electromagnetic fields is the calculation of LIVs on aerial lines. In this section, the effect of Rusck's approximation on the LIVs is evaluated with the computer code TIDA [22], which calculates the electromagnetic fields according to the time-domain formulation proposed by Barbosa and Paulino [5], [7]. The TIDA code uses the Agrawal coupling model [28] to interact the fields with the line and computes the line response to the scattered voltages through a one-dimensional FDTD routine. In order to assess the effect of Rusck's approximation, the two formulations for the electric field were included in the code: the one considering the accurate expressions presented in the previous sections and the other considering those derived from Rusck's approximation.

Before using the modified code, it is interesting to validate its results by comparing them with results published in the literature. To this aim, Fig. 10 shows the LIV at the center of an aerial line above perfectly conducting ground. The distance between the flash and the line is 50 m, the line height is 30 m, and the return stroke velocity is 120 m/μs. The return stroke current has a step waveform with 10 kA peak-value. In this figure, the continuous line shows the LIV calculated with (4), (5), and (12), i.e., considering Rusck's approximation, whereas the dashed line shows the LIV calculated with (1) and (9), i.e., without considering Rusck's approximation. It is clear that Rusck's approximation enhances slightly the LIV.

Fig. 10 also shows the LIV calculated by Andreotti *et al.* [24] with and without Rusck's approximation, which are represented by dots taken from Fig. 16 of [24]. It is clear that the results obtained from the modified TIDA agrees remarkably well with those provided by Andreotti *et al*.

In order to analyze the effect of Rusck's approximation on the LIVs, one must recognize that there is a physical constraint that limits the minimum value for the ratio r / z, i.e., for sufficiently low r / z value, the lightning flash will strike the line directly, instead of nearby ground. According to the IEEE Guide [29], the minimum distance for an indirect flash is

$$y_{min} = \sqrt{0.19\, r_s^2 + 1.8\, r_s\, h - h^2} \quad (13)$$

where h is the line height and r_s is the striking distance to the conductor, given by:

$$r_s = 10\, I_0^{0.65} \quad (14)$$

where I_0 is the peak-value of the first stroke (in kA). Expressions (13) and (14) allow to plot the minimum ratio r / z as a function of the first stroke peak current, as shown in Fig. 11. As expected, the lower the r / z ratio, the lower the allowed current peak value for an indirect strike. On the other hand, as shown in the previous sections, the higher the r / z ratio, the better the behavior of Rusck's approximation.

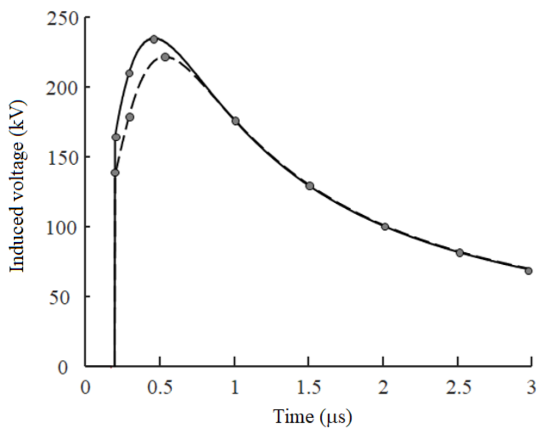

Fig. 10. LIV at the centre of a 30 m high aerial line at 50 m from the lightning flash. Continuous line: using Rusck's approximation with (4), (5), and (12); Dashed line: using height-dependent VEF with (1) and (9); Dots: from Andreotti *et al.* [24] with and without Rusck's approximation.

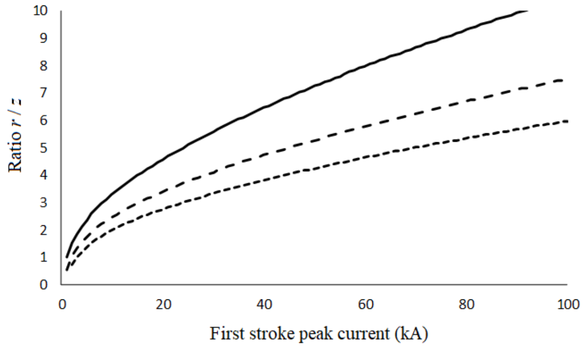

Fig. 11. Minimum ratio r / z as a function of the first stroke peak current, according to the electro-geometric model as per [29]. Continuous line: $z = 10$ m; Dashed line: $z = 15$ m; Dotted line: $z = 20$ m.

An investigation carried out with TIDA showed that Rusck's approximation provides accurate results for $r / z \geq 3$. Based on this and on Fig. 11, Table I provides the corresponding values of line height, minimum distance for the strike, and maximum current peak-values. This table also shows representative values of the critical flashover voltages (CFO) for these lines, assuming that $z = 10$ m, 15 m, and 20 m correspond to 15 kV, 69 kV, and 138 kV rated voltages, respectively.

TABLE I. RESTRICTIONS ON THE MAXIMUM PEAK CURRENT AND MINIMUM DISTANCE IMPOSED BY THE ELECTRO-GEOMETRIC MODEL

Line Height (m)	**Maximum Current peak-value (kA)**	**Minimum Distance from the Line (m)**	**Typical Line CFO (kV)**
10	8	30	100
15	15	45	350
20	24	60	650

Figs. 12 to 14 show the LIV at the center of an aerial line considering the parameters of Table I. The current waveform considered is typical of first strokes [26], the return stroke velocity is 150 m/μs, and the line is matched at both ends in order to avoid reflections. As the effect of the ground resistivity on the LIV partially shadows the effect of the VEF, the ground is considered as perfectly conducting, as this would enhance the differences between the accurate formulation and Rusck's approximation.

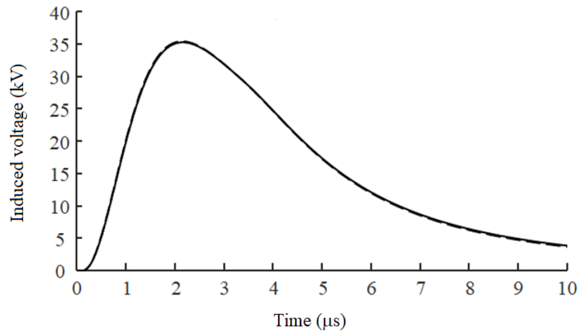

Fig. 12. LIV at the center of a 10 m high overhead line due to a first stroke with 8 kA peak current at 30 m from the line. Solid line: with the formulation presented in this paper; Dashed line: using Rusck's approximation.

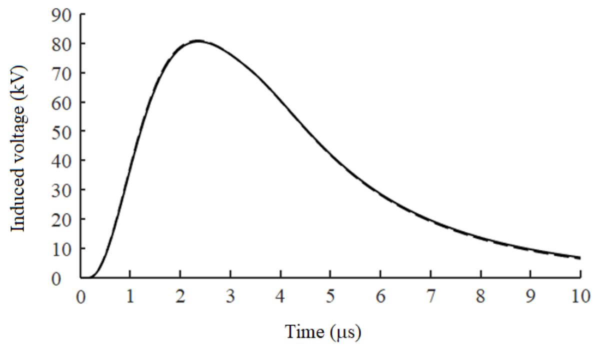

Fig. 13. LIV at the center of a 15 m high overhead line due to a first stroke with 15 kA peak current at 45 m from the line. Solid line: with the formulation presented in this paper; Dashed line: using Rusck's approximation.

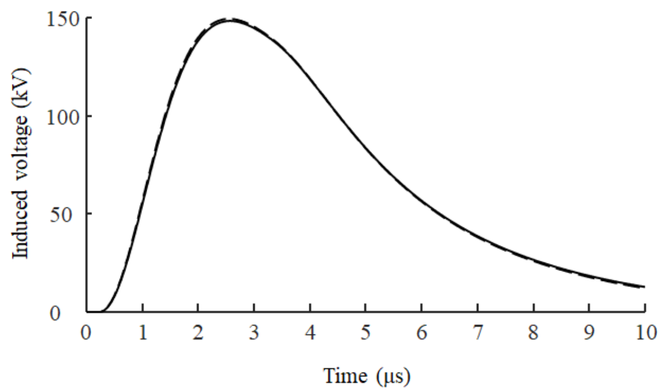

Fig. 14. LIV at the center of a 20 m high overhead line due to a first stroke with 24 kA peak current at 60 m from the line. Solid line: with the formulation presented in this paper; Dashed line: using Rusck's approximation.

The results presented in Figs. 12 to 14 confirm that the ratio $z / r = 3$ is enough to provide accurate results from Rusck's approximation, as the differences observed in the LIV peak-values are less than 1%. Moreover, the resulting LIV peak-values are unlikely to produce flashover in the line insulation, as they are much lower than the typical line CFO (see typical CFO values in Table I). Therefore, Rusck's approximation can be used in the evaluation of the indirect lightning performance of aerial lines without any significant loss of accuracy.

V. DISCUSSION

This paper showed that, although Rusck's approximation might be inaccurate for small r / z ratios, the restrictions imposed by the electro-geometric model assures that this approximation provides very accurate results for the LIVs. However, this conclusion has been drawn assuming an overhead line above flat ground. If the line is run in a corridor through a forest, it may be surrounded by trees at relatively close range. For instance, in Brazil it is common to have a 20 m wide strip for distribution lines, and the trees outside this strip have their heights limited to the distance to the line, so that any falling tree would not hit the line. This means that the condition $r / z \approx 1$ could be achieved for close strikes and significant currents. In this case, Rusck's approximation would overestimate the LIV. However, this a particular situation that is unlikely to have a significant impact on the overall lightning performance of the line, except if a very long line section is surrounded by trees.

As shown in Sections II and III, Rusck's approximation leads to an enhancement of both the VEF and HEF. However, the effects of these field enhancements on the LIV tend to cancel each other. Indeed, for perfect ground, the effect of the HEF is to induce scattered voltages on the line such that they partially cancel the voltage due to the VEF. Therefore, even if the enhancements on the VEF and on the HEF due to Rusck's approximation are significant, their joint effect on the resulting LIV is attenuated. For instance, Fig. 10 shows that Rusck's approximation enhanced the LIV peak-value by 6.7%. Calculating the electric fields for the same conditions show that the VEF and the HEF are enhanced by 9.0% and 17%, respectively.

VI. CONCLUSIONS

This paper showed that Rusck's approximation provides an enhancement on the VEF, HEF, and LIV, but this enhancement becomes negligible when the distance from the strike is greater than 3 times the height considered for the lightning electric field (VEF or HEF) and for the induced voltage on an overhead line (LIV). Considering that the electro-geometric model limits the stroke current for close strikes, it was shown that the LIVs from strikes closer than three times the line height are much lower than the line CFO, so that line flashovers are unlikely. Therefore, Rusck's approximation can be used in the evaluation of the indirect lightning performance of aerial lines without any significant loss of accuracy.

In any case, although Rusck's approximation is sufficiently accurate for the assessment of the indirect lightning performance of distribution lines, it is important to highlight that more accurate evaluation of the VEF, HEF, and LIV may be needed for some applications. One example is the measurement of close-range electric fields and induced voltages, associated with a research project. For these cases, the accurate expressions provided in this paper could be used as reference.

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