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Evaluation of the Statistical Characteristics of Grounding Impulse Impedance of Transmission Line Towers

Kamila Costa, Rafael Alípio, Matheus Duarte, Gabriel Matoso and Rosilene Dias
Department of Electrical Engineering
Federal Center of Technological Education of Minas Gerais
Belo Horizonte, Brazil
kamilacosta1995@gmail.com

***Abstract*—This paper evaluates the representativity of median impulse impedance in lightning response of tower-footing electrodes, for impulses characteristic of first strokes. The analysis is based on the implementation of Monte Carlo method using random front time values to determine the associated values of grounding impulse impedance. Statistical distributions for the impulse impedance values are then obtained, considering counterpoise lengths for different soil resistivities. With these distributions and considering the disruptive effect method, it is evaluated the impact of lightning's randomness on the impulse grounding impedance value and, consequently, on the performance analysis of transmission lines.**

Keywords — grounding impulse impedance, Monte Carlo method, disruptive effect .

I. Introduction

Direct lightning strikes in transmission towers or in shielding wires threaten the continuity of the operation of transmission lines, due to severe overvoltages that can appear across insulators, which may cause their disruption, and consequently lead to short circuits and line outages [1], [2]. When lines are subject to direct lightning strikes, the tower-footing groundings have a major influence whether the overvoltage will result in the insulator flashover or not [3]. Therefore, an accurate and proper tower-footing grounding modelling is important in simulations of the lightning performance of transmission lines (TL).

Lightning impulse currents are usually characterized by a wideband frequency content ranging from dc to several megahertz over which the tower-footing grounding electrodes show a strong frequency-dependent behavior [4]. An accurate representation of grounding systems in lightning studies should incorporate such frequency-dependent behavior, which is done applying accurate electromagnetic models based on the field theory [5]-[8]. However, considering industry applications, this approach may not be feasible. Then, simplified representations of grounding systems are often used in simulations of lightning performance of TL.

Traditionally, the lightning performance of TL is assessed using the widely accepted time-domain electromagnetic transient tools (for instance, ATP-EMPT, EMTP-RV, and PSCAD), due to their capability to deal with complex networks and different system apparatus, along with non-linear devices such as surge arresters. However, since these time-domain tools does not have specific models for representing grounding systems, they are often modeled as a simple lumped resistance which value is equal to the low-frequency grounding resistance (R_{LF}) [9], [10]. This representation is valid only for low-frequency occurrences and does not describe the frequency-dependent behavior of tower-footing electrodes. Therefore, it may lead to non-reliable results if used in simulations of lightning performance of TL.

In this context, concise representations that gather the required simplicity for popular time-domain simulators and the frequency dependent behavior of tower-footing groundings are desired. A recent contribution shows that "*using the tower-footing impulse impedance as a concentrate parameter for a concise representation of tower-footing electrodes in the calculation of the lightning performance of transmission lines yields practically the same results as those obtained under their physical representation*" [3]. The impulse impedance (Z_P) is given by the ratio between the peaks of the ground potential rise (GPR) and the injected current. It depends on the soil resistivity and current front time. However, due to the statistical characteristics of lightning currents, with emphasis on their front time [11]-[13], it is also expected a statistical dispersion of the impulse impedance.

Considering this scenario, the objective of this paper is to assess the statistical dispersion of the impulse impedance, considering the statistical characteristics of lightning current front time. Also, a first attempt is made to investigate to what extent the statistical dispersion of Z_P might affect the simulated lightning overvoltages across the line insulator strings. This paper is organized as follows. Section II presents a methodology based on the Monte Carlo (MC) method to evaluate the statistical dispersion of the tower-footing impulse impedance. Section III shows cumulative distribution functions describing the statistical dispersion of Z_P for typical arrangements of tower-footing electrodes. Section IV address how the statistical dispersion of Z_P might affect lightning overvoltages across line insulators. The main conclusions are presented in Section V.

II. A Methodology to Assess the Statistical Dispersion of the Impulse Impedance

For a given value of soil resistivity, the grounding impulse impedance (Z_P) mainly depends on the lightning current front

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time. Due to the statistical nature of lightning currents, a statistical dispersion of Z_P is expect. Aiming at assessing how strong this dispersion is, a methodology to evaluate how the statistical characteristics of the current front time influence the value of Z_P is developed. This methodology is briefly described below.

First, the Monte Carlo method is used to randomly generate a large number of distinct lightning current waveforms. In particular, the MC method is used to randomly choose the front time of the current wave. Second, each randomly generated lighting current is injected in the grounding system under study and using an accurate electromagnetic model its impulse impedance is determined. Finally, statistical distributions of the impulse impedance are obtained.

A. *Characterization of Lightning Current Waveforms*

As described in [11]-[13], for engineering applications, some parameters are usually adopted to characterize lightning current waveforms. Taking Fig. 1 as reference, the parameters that describe typical curve of first negative downward strokes are:

- I_{p1} and I_{p2}: first and second amplitude peaks;
- T_{10}: front duration (interval between 10% and 90% of I_{p1});
- T_{30}: front duration (interval between 30% and 90% of I_{p1});
- T_{50}: total time necessary for the current amplitude to decay to 50% of I_{p2} (not shown in Fig. 1);
- S_{10}: current rate of rise (between 10% and 90% of I_{p1} intercepts);
- S_{30}: current rate of rise (between 30% and 90% of I_{p1} intercepts);
- di/dt_{max}: maximum rate of rise of current on the front.

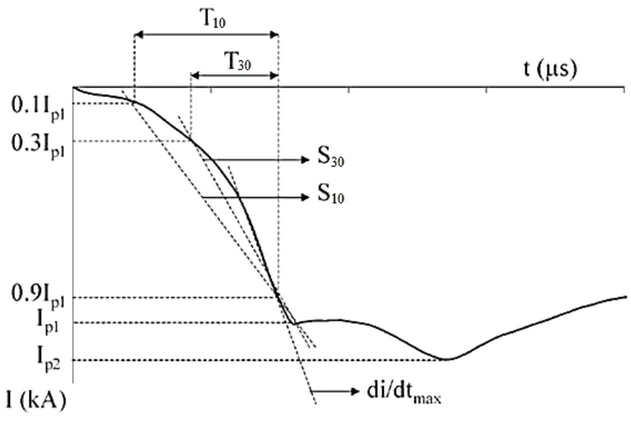

Fig. 1. Parameters of first negative downward strokes waveform. Adapted from [13].

B. *Random Generation of Lightning Current Waveforms*

The Monte Carlo method is implemented to simulate lightning impulses with random front times, in order to evaluate the effect of this variation on the impulse impedance. Due to the higher amplitudes of the negative first strokes compared with subsequent strokes, the former is associated with more severe overvoltages, and then it is the type of current traditionally considered in line performance studies. The database of lightning currents measured at the instrumented tower of Morro do Cachimbo Station (MCS) is used as reference in this paper [13].

According to studies developed from Morro do Cachimbo data, the distribution of first strokes measured parameters can be approximated by a lognormal, assuming a 5% significance [12], [13]. Fig. 2 shows the distribution of 10,000 random values of T_{30} related to first strokes, generated from a lognormal distribution with mean μ = 2.9 μs and logarithmic standard deviation σ_{ln} = 0.44 μs, according to [13]. The values of T_{30} are randomly generated using the MC method.

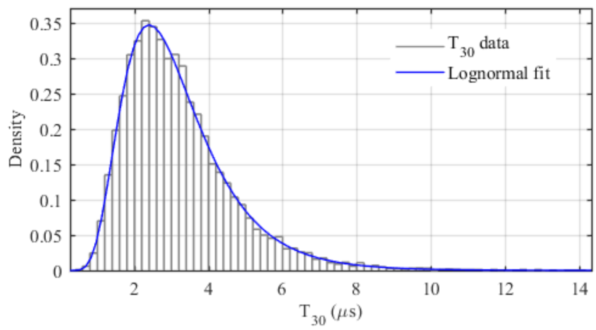

Fig. 2. Lognormal distribution of T_{30} for first strokes.

For each randomly chosen value of T_{30}, a lightning current waveform is synthesized. An accurate assessment of lightning effects on transmission lines depends, among other factors, on the appropriate representation of the lightning current, since the quality of results provided by simulations is conditioned by the representativity of the assumed lightning currents waves. As discussed in [14], according to measurements of instrumented towers, such as those of [13], the first stroke currents are characterized by a pronounced concavity at the front and by the occurrence of multiple peaks, being the second peak usually the highest one, and the maximum steepness occurring near the first peak.

In [15], a sum of seven Heidler functions [16] using (1) and (2) are considered to synthesize typical lightning current waveforms associated with first negative downward strokes:

$$i(t)=\sum_{k=1}^{m}\frac{I_{0k}}{\eta_k}\frac{\left(t/t_{1k}\right)^{n_k}}{1+\left(t/t_{1k}\right)^{n_k}}\exp\left(-t/t_{2k}\right) \tag{1}$$

$$\eta_k=\exp\left[-\left(\tau_{1k}/\tau_{2k}\right)\left(n_k\tau_{1k}/\tau_{2k}\right)^{1/n_k}\right] \tag{2}$$

where I_{0k} controls the amplitude, τ_{1k} and τ_{2k} are constants related to front and decay time, respectively, n_k is the exponent which controls the inclination of each component k added to i(t) and η_k is the factor of amplitude correction.

Table I presents the parameters allowing the synthesis of a typical first stroke current waveform, taking the median characteristics of the first stroke currents measured at MCS as reference [13]. The black dotted line of Fig. 3 illustrates the obtained double-peaked current waveform, which closely reproduce the main median parameters of first strokes measured at MCS.

More recently, a skillful approach is proposed to obtain double-peaked current waveforms, maintaining the main signatures of first stroke currents, but considering not only median parameters [17]. This approach for synthetizing current waveforms is especially suitable for lightning studies considering the MC method.

TABLE I. PARAMETERS OF FITTED HEIDLER FUNCTIONS FROM MEDIAN CHARACTERISTICS OF FIRST STROKES MEASURED AT MCS [15]

k	Heidler Functions Parameters			
	I_{0k} (kA)	τ_{1k} (μs)	τ_{2k} (μs)	n_k
1	6	3	76	2
2	5	3.5	10	3
3	5	4.8	30	5
4	8	6	26	9
5	16.5	7	23.2	30
6	17	70	200	2
7	12	12	26	14

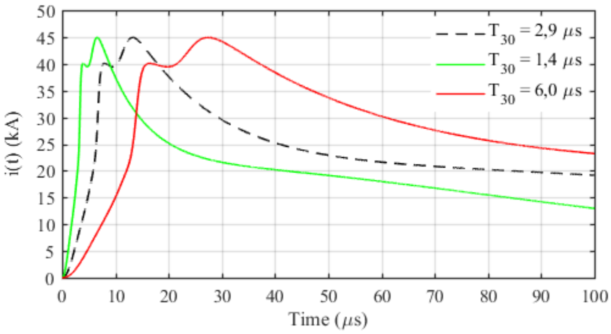

Fig. 3. Current wavefroms of first negative downward strokes (plotted with inverted polarity) adjusted to fit the median lightning current parameters and obtained from the approach proposed on [17].

From this methodology, there are applied adequate multipliers to the base values shown on Table I, varying between certain limits, to modify the lightning current median waveform. These multipliers are implemented as [17]:

$$I_0 = \alpha\left[6 \quad 5 \quad 5 \quad 8 \quad 16.5 \quad 17 \quad 12\times\delta\right] \text{ kA} \tag{3}$$

$$\tau_1 = \beta\left[3 \quad 3.5 \quad 4.8 \quad 6 \quad 7 \quad 70 \quad 12\right] \text{ μs} \tag{4}$$

$$\tau_2 = \beta\left[76 \quad 10 \quad 30 \quad 26 \quad 23.2 \quad 200 \quad 26\right] \text{ μs} \tag{5}$$

$$n = \alpha\left[2 \quad 3 \quad 5 \quad 9 \quad 30 \quad 2\times\gamma \quad 14\right] \tag{6}$$

To reproduce consistent waveforms, according to MCS database, the limits of variation for the multipliers are:

$$0.219775 < \alpha < 4.395508 \tag{7}$$

$$0.47 < \beta < 4.22 \tag{8}$$

$$0.7 < \delta < 1.3 \tag{9}$$

$$0.7 < \gamma < 1.5 \tag{10}$$

The multipliers are obtained from the following equations:

$$\alpha = 0.02475 I_{p1}\text{, with } I_{p1} \text{ in kA} \tag{11}$$

$$\delta = 3.6537\left({I_{p2}}/{I_{p1}} - 0.8568 \right) \tag{12}$$

$$\beta = 0.3328 T_{30}\text{, with } T_{30} \text{ in μs} \tag{13}$$

In this work, it is adopted the variation of the multiplier β to act on the wavefront, by T_{30} random values. However, its variation also affects the decay time, as the multiplier is used for both τ_{1k} and τ_{2k}. According to [17], the multiplier γ provides an indirect adjustment of T_{50}, by n_k, with low influence on I_{p1} and I_{p2} values. However, as the impulse impedance is associated with the current wavefront, the tail parameters are not relevant in this study, then it is adopted $\gamma = 1$.

In addition, it is worth mentioning that the impulse impedance does not depend on the amplitude of lightning current. This assumption is valid if soil ionization effects are neglected, as done conservatively in relevant studies of lightning performance of transmission lines. Then, considering the MC approach to synthesize the random waveforms, the current amplitude remains constant and equal to the median value of MCS database. The green and red curves in Fig. 3 correspond to current waveforms obtained using the described procedure, respectively with front time lower and higher than the median one.

C. *Modelling of Tower-Footing Electrodes*

Fig. 4 shows the typical grounding arrangement of transmission lines' tower footings, with counterpoises of length "L", usually installed on self-supported towers of transmission lines. The indicated dimensions "d" and "D" related to tower base and the distance between counterpoises when arranged parallel to the line axis, respectively.

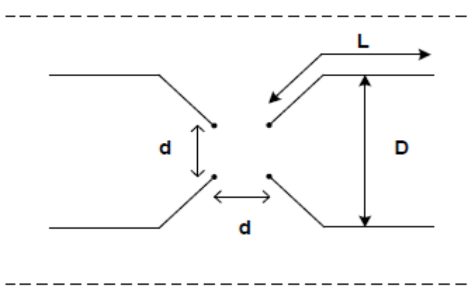

Fig. 4. Typical grounding arrangement of self-supported towers of transmission lines.

The wideband response of the tower-footing grounding system is determined using the accurate Hybrid Electromagnetic Model (HEM) [7]. The Hybrid Electromagnetic Model divides the grounding system into a set of small segments and solves Maxwell's equations numerically via the vector and scalar potentials using the thin wire approximation. The calculations are performed in frequency domain, allowing the direct inclusion of frequency-dependent soil parameters, which is done in this work according to the causal model proposed by Alipio and Visacro in [18]. Time-domain results are obtained by means of a numerical inverse Fourier or Laplace transform. For each synthesized lightning current, the GPR is calculated from HEM model and, finally, the impulse impedance is computed by the ratio between the peaks of the GPR and the injected current.

The described methodology is applied for different grounding arrangements, with variation of length "L" according to the local soil resistivity ρ_0. The definition of the simulated grounding arrangements is based on practical criteria, which contemplate the concept of effective length:

- ρ_0 = 300 Ωm: L = 20 m;
- ρ_0 = 1000 Ωm: L = 40 m;
- ρ_0 = 3000 Ωm: L = 60 m.

In simulations, counterpoises are at 0.5 m depth, d = 6 m and D = 20 m.

III. Z_P Dispersion

From the proposed methodology, Z_P distributions related to each grounding arrangement are generated. Fig. 5 shows the probability density of 10,000 Z_P random values, better fitted as a normal distribution. The statistical distributions of Z_P are obtained considering the statistical dispersion of T_{30}, described by a lognormal distribution, as depicted in Fig. 2.

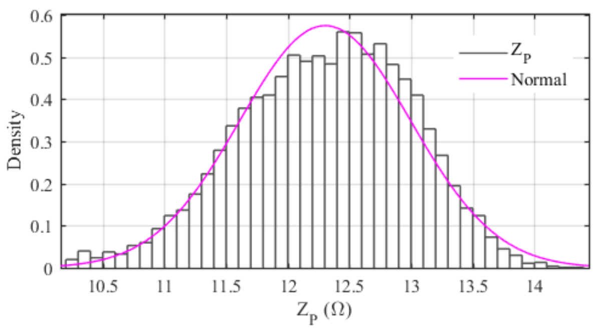

Fig. 5. Normal distribution PDF of Z_P, considering ρ_0 = 1000 Ωm, L = 40 m.

The impulse impedance cumulative probabilities, also obtained from 10,000 Z_P random values and fitted as a normal distribution, are presented in Figs. 6, 7 and 8, respectively for soil resistivity values of 300, 1000 and 3000 Ωm.

Table II shows median and fitted impulse impedance values, obtained from the normal distribution with mean μ and standard deviation σ and a confidence level of 95%.

According to the results, the dispersion of the Z_P values around the mean value raises with the increase of the soil resistivity. This indicates that estimates of the maximum GPR at the tower base might be affected by the statistical dispersion of Z_P, notably in case of soils of higher resistivity. On the other hand, in the case of lightning performance of transmission lines, the voltage across the insulator strings is the most important parameter. In this context, next section address how the statistical dispersion of the impulse impedance might affect the overvoltages across line insulators, when tower-footing grounding is represented by its impulse impedance.

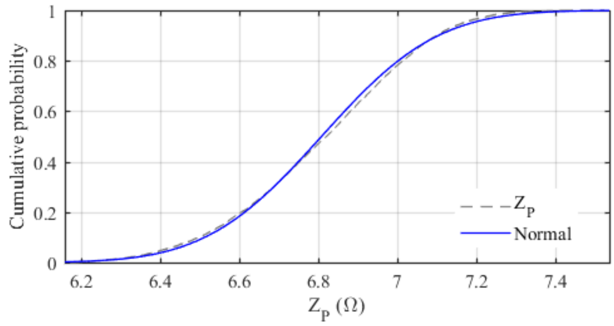

Fig. 6. Normal distribution CDF of Z_P, considering ρ_0 = 300 Ωm, L = 20 m.

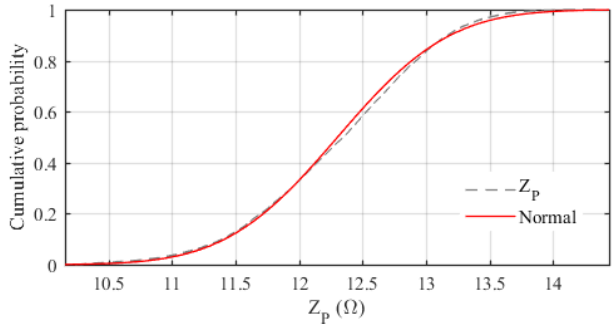

Fig. 7. Normal distribution CDF of Z_P, considering ρ_0 = 1000 Ωm, L = 40 m.

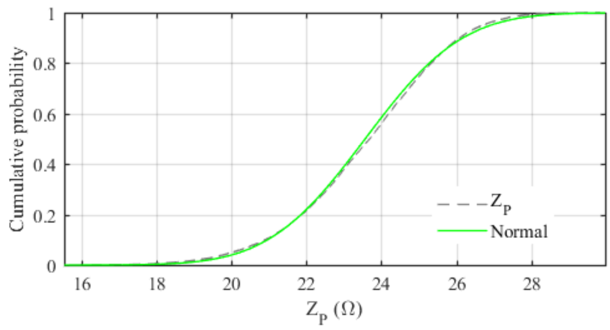

Fig. 8. Normal distribution CDF of Z_P, considering ρ_0 = 3000 Ωm, L = 60 m.

TABLE II. MEDIAN AND FITTED GROUNDING IMPULSE IMPEDANCE

ρ_0 (Ωm)	Median Z_P (Ω)	Fitted Z_P (Ω)			
		Min (5%)	μ	σ	Max (95%)
300	6.82	6.42	6.81	0.23	7.19
1000	12.35	11.15	12.30	0.69	13.45
3000	23.73	20.21	23.57	2.03	26.91

IV. Z_P Dispersion Effects on Lightning Overvoltages Developed at Line Insulators

Results of previous section show that the statistical nature of the lighting current, with emphasis on T_{30}, affects the estimated values of the tower footing impulse impedance. In this section, a first attempt is made to investigate to what extent the statistical dispersion of Z_P might affect the simulated lightning overvoltages across line insulator strings.

A. Tested System and Methodology of Simulation

To assess how the statistical dispersion of the impulse impedance affects the calculated voltage across line insulator strings, a typical overhead 138-kV Brazilian transmission line is considered. Fig. 9 shows the tower geometry, which has one ACSR conductor per phase (LINNET) and one 3/8" EHS shield wire. The coordinates of the line cables (in meters) are indicated in the same figure (values within parenthesis are midspan heights). The shield wires and phase cables are represented using the LCC tool of Alternative Transients Program [19], where their positions are entered according to the tower geometry, and the frequency-dependent line model of JMarti is used [20].

Line parameters are calculated in a frequency range from 1 Hz to 10 MHz, which cover the frequency content of the injected lightning current waveform. The tower is modeled as a lossless single-phase transmission line, for which the surge impedance is calculated using the revised Jordan's formula, which is extended in [21] to take into account vertical multiconductor systems.

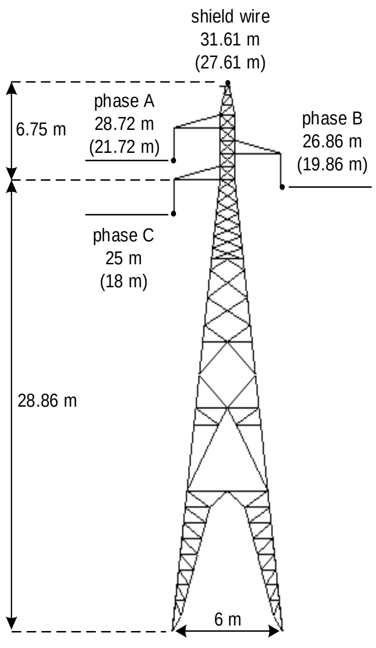

Fig. 9. Geometry of the simulated 138-kV line tower.

The lightning current is injected at the top of the tower. A pair of adjacent towers (identical to the stricken one) is included in simulations to consider the effect of overvoltage propagation along the line conductors and also the reflections from the adjacent towers. Each span length is 400 m, and all aerial conductors are impedance matched 400 m away from the adjacent towers.

According to the first stroke cumulative distribution of MCS, T_{30} varies from 1.42 to 5.95 µs (values exceeded in 95% and 5% of cases, respectively). Equation (14) below express the correlation between I_{p1} and T_{30} for currents measured at MCS [13]. Considering (14), the range of variation of T_{30} from 1.42 to 5.95 µs corresponds to a variation of I_{p1} from about 20 to 100 kA.

$$T_{30} = 0.12\left(I_{p1}\right)^{0.87} \tag{14}$$

Then, a set of current waveforms is synthesized, according to the approach described in Section II.B, varying the peak value from 20 to 100 kA, in steps of 1 kA. All synthesized waveforms consider the correlation between first stroke peak current and its front time, described by (14).

Each current waveform is injected at the tower top and the corresponding voltage across the lower line insulator string is calculated. From the overvoltage, the disruptive effect (DE) associated is determined and plots of "DE *versus* Current Amplitude" are obtained. The concept of the DE method is based on the idea that there exists a critical destructive effect DE_C. If a nonstandard voltage surge contains a DE that exceeds DE_C, flashover occurs, and alternately if the surge contains a DE that is less than DE_C, no flashover occurs [22]. The general equation for the destructive effect is

$$DE = \int_{t_0}^{t} \left[e(t) - V_0 \right]^k dt \tag{15}$$

where e(t) is the voltage across insulator and t_0 is the instant of time when the value of e(t) exceeds V_0. For a typical 138-kV line of CFO = 650 kV, the constants of the DE method are assumed as [22]: $DE_B = 1.1506\left(CFO\right)^{k_d}$; k_d = 1.36; V_0/CFO = 0.770.

Four representations of the tower-footing groundings are assumed: i) a lumped resistance of value equal to the low-frequency grounding resistance (R_{LF}); ii) a lumped resistance of value equal to the impulse impedance calculated for a median first stroke current; iii) a lumped resistance of value equal to the impulse impedance, but considering its statistical dispersion according to the injected lightning current; iv) a wideband model, which accurately consider the frequency-dependent behavior of the grounding system.

In case of modelling iv), which is assumed as reference since it is based on the least neglects in comparison to more simplified approaches, the frequency response of the tower footing grounding system is first obtained using the HEM model (refer to Section II.C). Then a pole-residue model is obtained using the vector fitting method and, finally, from the pole-residue model, an electrical network is synthetized, which can be promptly included in ATP simulations [23].

B. Results

Fig. 10 and Fig. 11 illustrate the obtained results, respectively for a soil of 1000 Ωm and 3000 Ωm. The horizontal line included in the graphs corresponds to the so-called critical DE, which, if exceeded, implies the occurrence of flashover. Results for the soil of 300 Ωm are not included, since there is no occurrence of flashover for the considered range of amplitudes of lightning current.

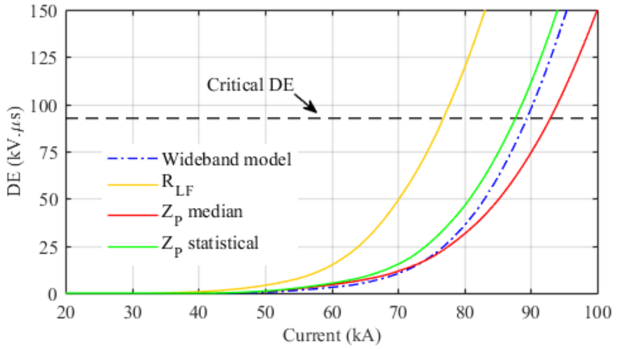

Fig. 10. Disruptive effect (DE) *versus* Current Amplitude for a soil of 1000 Ωm.

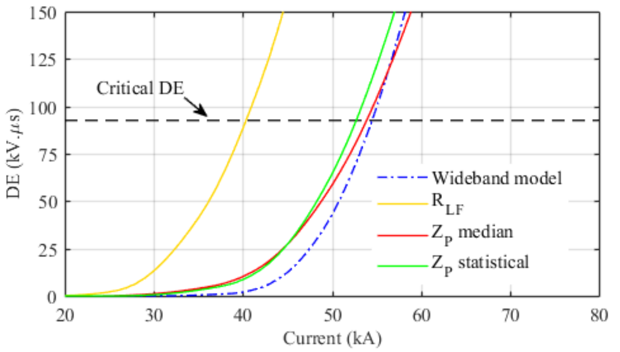

Fig. 11. Disruptive effect (DE) *versus* Current Amplitude for a soil of 3000 Ωm.

The results raised in this paper are based on the concept of critical current (I_{PC}), which corresponds to a threshold for the transmission line under analysis: it is expected an insulator string flashover for lightning which peak value surpasses the I_{PC}. It can be seen that representing the grounding system by its low-frequency grounding resistance leads overestimated results, i.e., the estimated critical current for this representation is much lower than the one estimated considering the wideband representation of the grounding (assumed as reference). This conclusion is in accordance with [3], which first presents this result.

Besides, it is observed that the curves associated with the grounding representation by the impulse impedance, assuming its statistical dispersion or not, show a very similar behavior to the curve obtained with the complete wideband model. This result allows outlining two important conclusions. The first one is originally reported in [3] and corresponds to the fact that using the tower footing impulse impedance as a lumped parameter for a concise representation of the tower-footing electrodes yields to similar estimates of I_{PC} as those obtained using the wideband model.

Second, it can be seen that representing the tower footing grounding by its impulse impedance considering its statistical dispersion leads to similar results than the ones raised for impulse impedance determined for the median first stroke current. This second conclusion complements the first one and conveys generality to it. According to the results, as well as the statistical dispersion of Z_P, its median value leads to results in very good agreement with those obtained using a rigorous tower-footing electrode representation, considering the conditions simulated in this paper.

V. CONCLUSIONS

The accurate methodology presented in this paper indicates that the statistical nature of the lighting current, with emphasis on T_{30}, affects the estimated values of the tower footing impulse impedance, especially considering soils of higher resistivity. However, to assume statistical dispersion of Z_P or not, leads to similar results, both in very good agreement with those obtained from the rigorous tower-footing electrodes representation by the complete wideband model. Therefore, under the conditions implemented in this study, it seems that statistical dispersion of Z_P, compared to the median value representation, does not imply significant differences in terms of analysis of transmission line lightning performance.

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