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An Engineering model to represent positive return strokes – An extension of the MTLL model

Vernon Cooray
Department of Engineering Sciences
Uppsala University, Uppsala, Sweden, vernon.cooray@angstrom.uu.se

***Abstract*—**An engineering model to represent positive return strokes is introduced. The model is an extension of the Modified Transmission Line model with Linear current decay (MTLL). The extension of the model is based on the available experimental data on positive return stroke electric fields and currents. The model is capable of generating electromagnetic fields similar to those observed from positive return strokes. The model could be used easily in studies pertinent to interaction of positive return stroke electromagnetic fields with power lines and the Earth's upper atmosphere.

Keywords—Lightning, return strokes, Engineering models, Positive return strokes, Modified transmission line model

I. Introduction

Engineering return stroke models are utilized in engineering practice to evaluate the electromagnetic fields from lightning return strokes. Such models are utilized in studies related to the interaction of lightning flashes with electrical systems. Lightning can generate large voltages in these electrical systems. In the case of lightning flashes striking in the vicinity of these systems, these voltages are generated by the interaction of lightning electromagnetic fields with these systems. In addition to these engineering studies, physicists are also interested in the lightning electromagnetic fields. The reason being that the recent discoveries show that lightning electromagnetic fields, especially the ones created by positive return strokes, interact with the Earth's middle and upper atmosphere generating luminous events such as sprites and elves [1].

The engineering lightning return stroke models that are frequently used to calculate the electromagnetic fields from negative return strokes are the Transmission Line Model (TLM) [2] and its modifications, namely, MTLE [3] and MTLL [4]. In the MTLE model, the current is allowed to attenuate exponentially with height while in MTLL a linear attenuation is assumed. In contrast, TLM assumes no current attenuation along the channel.

Except for the two models introduced by Cooray [5] and Cooray et al. [6], the engineering return stroke models available in the literature are developed for negative return strokes. In the two models mentioned above, attempts were made to modify the engineering models to fit the positive return strokes by taking into account the differences in the electromagnetic fields and currents of positive return strokes as compared to more common negative return strokes. However, these two models are less than optimal for engineering practice because of the significant amount of numerical calculations necessary to evaluate the current along the channel.

The goal of the present paper is to modify the MTLL model, by utilizing the basic physical concepts used in [5] and [6], to create a positive return stroke model that can be used conveniently and easily in studies related to the interaction of positive return stroke fields with electrical systems and the upper atmosphere.

II. The differences in the radiation fields generated by negative and positive return strokes

While analysing the radiation fields generated by positive and negative first return strokes, Cooray [7] observed that there is a basic difference between the two types of radiation fields. Radiation fields of negative first return strokes rises to a peak value in a few microseconds. After reaching the initial peak, the field decreases monotonically (except for several subsequent peaks), crosses the zero-field line and exhibits a peak in the opposite direction (i.e. overshoot). After reaching the opposite overshoot, the field goes back to the zero-field line. The positive return stroke radiation field has opposite polarity but shows more or less similar behavior until the first zero crossing. However, after the zero crossing, instead of decaying further as in the case of negative return strokes, the field recovers and exhibits a long tail having an initial polarity identical to that of the initial peak of the radiation field. The peak of the tail occurs around 100 – 500 μs and it decays over a time span of about 400 μs – 1000 μs. Cooray [7] suggested that the second increase in the electric field is caused by the enhancement of the current in the return stroke channel by charges transported into it by K-change type discharge activity once the return stroke channel reaches the cloud end of the leader channel. Indeed, many examples of positive return stroke current waveforms measured by Berger [8] exhibit such current enhancements after the initial peak. In this paper, this current-feature is incorporated into the MTLL model to create a positive return stroke model that can be used in engineering practice.

III. The model

In the MTLL model the return stroke is represented by a current pulse injected into the leader channel from its grounded end. The current pulse propagates upwards with constant speed while its amplitude decreases linearly with height. The current at any point of the channel according to this model can be represented mathematically as

$$I(z,t) = 0 \qquad \text{for } t < z/v$$

$$I(z,t) = i_b(t - z/v)(1 - z/H) \qquad \text{for } t \geq z/v \qquad (1)$$

In the above expressions, t is the time, z is the height along the vertical return stroke channel, v is the speed of propagation of the current pulse, H is the height of the return stroke channel and $i_b(t)$ is the current injected into the channel from the grounded end of the leader channel.

The model as presented above is used frequently to study the electromagnetic fields of return strokes once the channel length, return stroke speed and the channel base current are specified.

In order to modify the model to represent positive return strokes, we assume that once the return stroke front reaches the cloud end of the leader channel, it will turn on an in-cloud current source that will inject a current into the return stroke channel. For simplicity we assume that the size of this source is negligible in comparison to the return stroke channel and assume that the source is located at the top of the return stroke channel. This current, when injected into the return stroke channel, is assumed to propagate downwards with a speed u. With this modification, the current distribution along the channel is given by

$$I(z,t) = 0 \qquad \text{for } t < z/v$$

$$I(z,t) = i_b(t - z/v)(1 - z/H) \qquad \text{for } z/v \leq t < t_z$$

$$I(z,t) = i_b(t - z/v)(1 - z/H) + i_c(t - t_z) \qquad \text{for } t \geq t_z \qquad (2)$$

In the above equation $t_z = H/v + (h - z)/u$ and $i_c(t)$ is the current injected from the cloud into the return stroke channel top when it reaches the cloud end. This set of equations describes the positive return stroke model completely.

One could also utilize the MTLE model as the base model for the modification but we have decided to use MTLL model for the following reasons. First, the MTLL model gives rise to a uniform charge distribution along the channel which agrees with the bi-direction leader concept [9]. Second, it generates a close electric field that more or less saturates with increasing time which is in agreement with the experimental observations. Third, in the MTLL model the channel height enters naturally into the model and the current amplitude goes to zero at the cloud end of the channel.

A. *Current waveform along the channel*

Since the initial part of the electromagnetic fields are qualitatively similar in positive and negative return strokes, we represent the channel base current by a current waveform that is used to represent negative first return strokes [10]. This current waveform is given by

$$i_b(t) = i_{01} \frac{(t/\tau_{11})^2}{(t/\tau_{11})^2 + 1} e^{-t/\tau_{12}} \qquad (3)$$

The current waveform injected from the cloud is represented here by

$$i_c(t) = i_{02} \frac{(t/\tau_{21})^2}{(t/\tau_{21})^2 + 1} e^{-t/\tau_{22}} \qquad (4)$$

The parameters corresponding to the channel base current are; i_{01} = 30 kA, τ_{11} =1.8 μs and τ_{12} = 95 μs. These parameters are similar to those used to represent channel base current of negative return strokes. In selecting the parameters of the current injected into the return stroke channel from the cloud we were guided by several observations. First, the total charge transported by an average positive current of about 35 kA within the first 2 ms is about 16 C [8]. Second, the second peak of the current in the experimentally measured current lie in the vicinity of 300 - 500 μs. Moreover, the calculated slow tail of the distant radiation field should be similar to the ones observed experimentally. Based on these considerations, the parameters used to represent the cloud current are; I_{02} = 20 kA, τ_{21} =325 μs, τ_{22} = 455 μs. Based on these parameters the total current that appears at the channel base of the positive return stroke is shown in Figure 1. Observe that similar to the experimental observations, the current rises initially to a peak value and then continue to decay. This represent the time interval in which the return stroke front is traveling upwards along the channel. After a certain time interval the current at the channel base is enhanced again giving rise to a second peak. This current enhancement is created when the current injected from the cloud into the top of the return stroke channel reaches the ground. The initial peak of the current waveform is 35 kA which is identical to the average positive return stroke peak current observed by Berger [8]. The charge transported by this current over the first 2 ms is about 17 C which is also similar to the corresponding average measured charge.

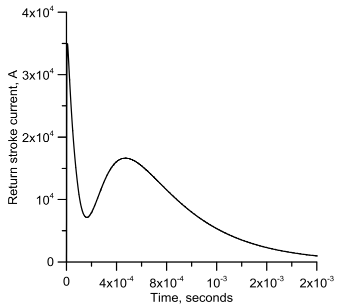

Figure 1: Positive return stroke current at the base of the return stroke channel. In calculating this current, it is assumed that v = 1.0x10^8 m/s, u = c and H = 7 km.

IV. ELECTRIC FIELDS PREDICTED BY THE POSITIVE RETURN STROKE MODEL AT DIFFERENT DISTANCES

The electric fields over perfectly conducting ground at several distances calculated using the model are shown in Figures 2 to 4. On each diagram, the both fields corresponding to $u = c$ and $u = v$ are depicted. In the calculations the height of the return stroke was kept at 7 km.

Observe that the distant radiation field has features which are similar to those observed in measured electric fields from positive return strokes. Initially, the electric field behaves in a manner similar to that of negative return strokes. However, after the zero crossing the field increases again reaches a peak and then continue to decay. In the example given here, the second peak appears around 200 μs and it crosses the zero again around 700 μs.

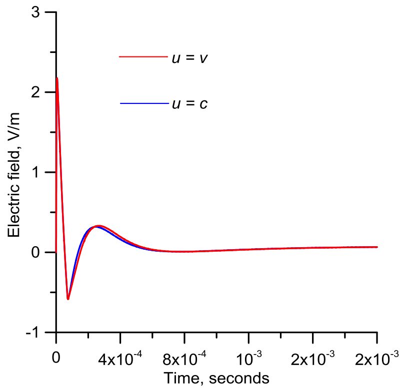

Figure 2: Electric field at 300 km predicted by the model. The electric field is shown for both $u = c$ and $u = v$. In the calculations it was assumed that v = 1.0x10^8 m/s and H = 7 km.

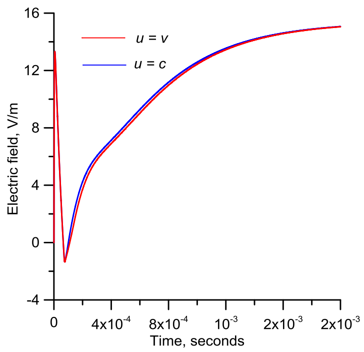

Figure 3: Electric field at 50 km predicted by the model. Electric field is shown for both $u = c$ and $u = v$. In the calculation it was assumed that v = 1.0x10^8 m/s and H = 7 km.

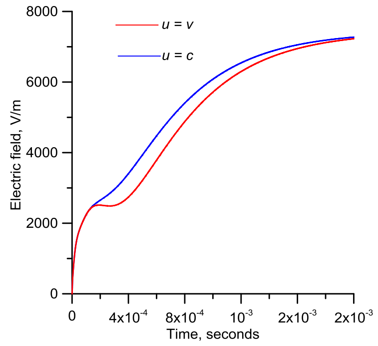

Figure 4: Electric field at 2 km predicted by the model. Electric field is shown for both $u = c$ and $u = v$. In the calculation it was assumed that v = 1.0x10^8 m/s and H = 7 km.

Note that even at 300 km, there is some remnants of the static electric field visible in the tail of the return stroke field. As the distance to the strike point becomes closer the static field starts to increase and there is a significant static field component in the field around 50 km. This observation is also in agreement with the experimental data on positive return stroke fields published by Cooray et al. [11].

Note also that the speed of propagation of the downward moving current waveform does not influence significantly the electric fields. Thus, for all practical purposes, one can assume that $u = c$ which makes it possible to write down an analytical expression for the electric field produced by the downward moving current.

Unfortunately, we do not have much experimental data on positive return stroke fields at closer distances with which we could compare the theoretical predictions. However, this simple model can account for the differences observed between the positive and negative currents and radiation fields. Thus, it could be used in studies pertinent to the interaction of positive lightning return stroke electromagnetic fields with power lines and upper atmosphere.

It is important to point out here that the second current enhancement can also be treated as a large M-component that transfers charge from cloud to ground along the conducting return stroke channel. If this is the case, it is necessary to take into account the reflection of the downward moving current at the base of the channel [12]. This point is currently under investigation.

V. CONCLUSIONS

In this paper we have introduced a simple engineering model, which is a modification of the MTLL model, to describe the positive return strokes. The model is capable of generating fields similar to those observed for positive return strokes.

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