2019 International Symposium on Lightning Protection (XV SIPDA), São Paulo, Brazil, 30th September – 4th October 2019.

Performance Analysis of the Moving Computational Domain Technique for the Calculation of Sferics

Wenhao Hou[1,2], Farhad Rachidi[2], Mohammad Azadifar[2], Marcos Rubinstein[3], and Qilin Zhang[1]
[1]Nanjing University of Information Science and Technology, Nanjing 210044, China
[2]Electromagnetic Compatibility Laboratory, Swiss Federal Institute of Technology, Lausanne 1015, Switzerland
[3] University of Applied Sciences Western Switzerland, Yverdon-les-Bains 1401, Switzerland

***Abstract*— In this paper, we analyzed the performance of the moving computational domain technique (MCDT) for the calculation of lightning-generated electromagnetic fields in the Earth-ionosphere waveguide (EIWG) using the finite-difference time-domain (FDTD) method. In the FDTD method with MCDT, a spatial window moving along with the wavefront is defined and, at each time step, the field components are updated only in that window. The FDTD efficiency can be improved by at least a factor of two with respect to the conventional FDTD, while the accuracy remains unchanged. The improvement in computational efficiency is dependent on the size of the simulation domain, the cell size and the time window of interest.**

Keywords—Lightning electromagnetic field; MCDT; Earth-ionosphere waveguide; FDTD efficiency

I. Introduction

The finite-difference time-domain (FDTD) technique has become a common method for the evaluation of lightning fields over long distances, including studies on the propagation effects along irregular terrain and the interaction between lightning electromagnetic fields and the lower ionosphere (e.g., [1]-[3]). This is mainly because of the relative ease with which a complex terrain, an inhomogeneous soil, and ionospheric parameters can be handled by the FDTD method.

The FDTD computational burden and simulation time increase quickly with the increase of the simulation domain size. In order to improve the efficiency of FDTD for the calculation of VLF-LF wave propagation over thousands of kilometers in the Earth-ionosphere waveguide, Bérenger proposed a moving computational domain technique that only updates the field components within a reduced domain at each time step [4], [5].

Recently, we have applied the moving computational domain technique (MCDT) in the FDTD method for the calculation of lightning-generated groundwave propagation over irregular terrain [6]. A minimum moving window that can satisfy the computational requirement was defined. With respect to the method in [5], the time delay of the wavefront arrival caused by the propagation effect and the irregular terrain was considered when determining the width of the moving window. The accuracy of the improved method for the application over mountainous terrain was verified. The gain in the CPU time can be as high as a factor of 4 to 5, while the accuracy remains unchanged with respect to the conventional FDTD. In this paper, we will analyze the performance of MCDT for the calculation of lightning-generated electromagnetic fields in the Earth-ionosphere waveguide (EIWG), considering different cell sizes and time windows of interest.

II. Description of the Method

A. Conventional FDTD method

In this study, the lightning-generated electromagnetic pulse (LEMP) propagation in the Earth-ionosphere waveguide (EIWG) is simulated using the two-dimensional FDTD (2-D FDTD) model [5], [6]. Fig. 1 shows the overall configuration of this method, which is established in spherical coordinates with the center of the Earth as the origin. The lightning channel is placed on the left edge of the simulation domain and the upper and right boundaries are surrounded by the convolutional perfectly matched layer (CPML) absorbing boundary [7]. The soil is assumed to be homogeneous and characterized by a finite conductivity σ and a relative permittivity ε_r. The upper domain

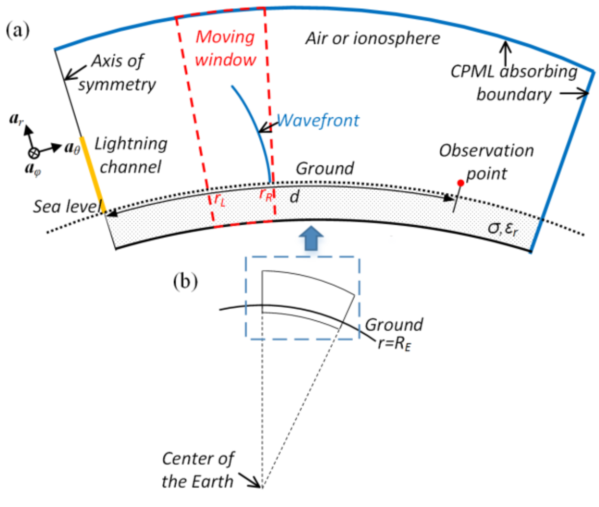

Fig. 1. Configuration of the 2-D FDTD model with the moving computational domain technique. The model is established in spherical coordinates with the center of the Earth as the origin. R_E is the radius of the Earth. The lightning channel is placed on the left edge of the simulation domain and the upper and right boundaries are surrounded by the CPML absorbing boundary. The red dashed region defines the window which moves along with the wavefront at the speed of light c. The soil is characterized by a finite conductivity σ and a relative permittivity ε_r. d is the great circle distance from the lightning channel to the observation point, shown by the red dot.

above the ground surface is the air or ionosphere. d is the great circle distance from the lightning channel to the observation point. Note that all the distances described in this paper are the great circle distances away from the lightning channel base along the ground surface. More details of the field updating equations and the mesh scheme can be found in [4].

In this study, the lower ionosphere was modeled as a vertically inhomogeneous medium with exponential conductivity profile expressed as follows [8]:

$$\sigma(h) = \sigma_0 \exp\left[\beta(h-h')\right] \tag{1}$$

where σ_0=2.22 μS/m, h is the height above the ground surface in km, and h' and β are the reference height and the steepness of the electron density profile, respectively.

B. Application of the MCDT

Usually, we are only interested in the field waveform within a time window (t_w) after the wave propagates to the observation point. According to the propagation properties of the electromagnetic wave, the electromagnetic energy within a considered time window distributes within a corresponding window in the spatial domain. As a result, only the field components in that window need to be updated at every time step. In addition, the electromagnetic energy propagates in the simulation space; therefore, the spatial window moves with the wavefront. The red dashed region in Fig. 1 shows the position of the spatial window.

According to the MCDT presented in [6], the right border of the window can be defined as:

$$r_R(t) = \min\left[ct, L\right] \tag{2}$$

where t is the present time in FDTD and L is the width of the whole FDTD domain. The left border of the spatial window can be defined as:

$$r_L(t) = \max\left[0, c(t - t_w - t_{delay})\right] \tag{3}$$

where the time window of interest t_w is defined by the users according to their specific requirements, t_{delay} is the arrival-time delay of the wavefront compared to d/c, accounting for the propagation effect of the finitely-conducting ground and the Earth's curvature. Note that both r_R and r_L are the great circle distances along the ground surface between the lightning channel base and the borders.

C. Validation of our 2-D FDTD codes

In order to validate our conventional 2-D FDTD method codes for the LEMP propagation in the EIWG, we compare our result with that presented by Tran *et al.* [9]. Fig. 2 shows the comparison of the sferic waveform calculated at 200 km under nighttime ionosphere conditions. Since the Earth's curvature was not considered in [9], we set a very large value of the Earth's radius in our code to represent the flat ground case. It can be seen that the waveform calculated by our conventional FDTD method overlaps with that adapted from [9], thus validating our FDTD codes.

III. Application and Performance Analysis

In this section, we will analyze the performance of the MCDT for the calculation of LEMP in the EIWG considering different cell sizes and time windows. The lightning channel was simulated by a vertical array of current sources [10] and the return stroke velocity was set to be v = 150 m/μs. The typical subsequent return stroke channel-base current was adopted [11]. The modified transmission line model with exponential decay (MTLE) [12] assuming a current decay constant λ = 2 km was

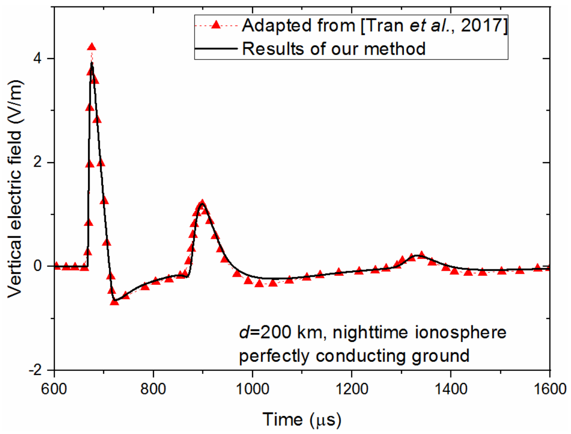

Fig. 2. The sferic waveforms calculated at 200 km under nighttime ionosphere condictions. The red line with triangle markers is the result adapted from [9]. The black line is the calculated result using our method. The overlapping of the waveforms validates our code.

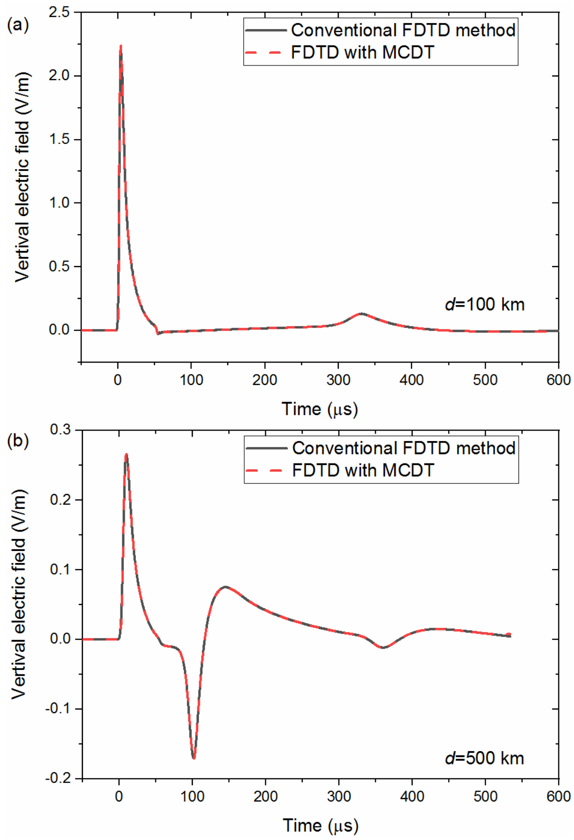

Fig. 3. The vertical electric field calculated by the FDTD method with MCDT (red dash lines) and that calculated by the conventional FDTD method (black solid lines), at distances of (a) 100 km and (b) 500 km. The simulation domain was 510 km × 100 km. The cell size was set to 50 m, and the time window was set to 500 μs.

TABLE I. GAIN IN CPU TIME AFTER ADOPTING THE MCDT FOR DIFFERENT CELL SIZES AND TIME WINDOWS

Simulation domain size	**Cell size**	**Time window**[a]	**Observing distance**	**Total iteration steps**	**CPU time**		**Gain in CPU time**
					without MCDT	***MCDT***	
510 km ×100 km	50 m	0.5 ms	500 km	26380	18.9 h	6.0 h	3.2
510 km ×100 km	100 m	0.5 ms	500 km	13190	2.9 h	0.9 h	3.3
510 km ×100 km	100 m	1.0 ms	500 km	16190	3.6 h	1.8 h	2.0
1010 km ×100 km	100 m	1.0 ms	1000 km	25780	9.6 h	3.0 h	3.2

[a.] In general, the 0.5-ms time window is large enough to simulate the one-hop sky waves for observation distances larger than 100 km. In addition, the 1-ms time window can be used to simulate the two-hop or the three-hop sky waves.

used [13]. The ground conductivity was set to 0.001 S/m and the relative permittivity was set to 10. The typical nighttime ionosphere condition with parameters h'= 85 km and β=0.5 [8] was adopted.

Fig. 3 shows the vertical electric field calculated by the FDTD method with MCDT and the result calculated using the conventional FDTD method at two distances, 100 km and 500 km. The simulation domain was 510 km long and 100 km high. The cell size was set to 50 m. The time window was set to 500 μs, and the spatial window width was 151 km, which was extended by 1 km considering the arrival-time delay caused by the propagation effect [6]. It can be seen from the figure that the waveforms calculated by adopting the MCDT were in perfect agreement with those calculated using the classical FDTD, without adopting the MCDT, thus confirming the accuracy of the MCDT. The total CPU time adopting the MCDT was reduced by a factor of 3, from 18.9 h down to 6.0 h.

Table I further summarizes the total CPU time and the improvement in computational efficiency when adopting different cell sizes and time windows. In theory, when adopting the 50-m and 100-m cell sizes, the upper frequency limits that can be analyzed reasonably accurately are 600 kHz and 300 kHz, respectively [9]. It can be seen from the table that the use of MCDT can improve the computational efficiency at least by two times. In addition, adopting the MCDT can largely reduce the CPU time, especially when simulations with fine cell sizes are required or when the width of the whole simulation domain is much larger than that of the spatial window.

IV. CONCLUSION

In this paper, we analyzed the performance of the moving computational domain technique (MCDT) for the calculation of lightning-generated electromagnetic fields in the Earth-ionosphere waveguide (EIWG). With respect to the conventional FDTD, adopting the MCDT can at least improve the FDTD efficiency by a factor of two without changing the simulation accuracy. The improvement in computational efficiency is dependent on the size of the simulation domain, the cell size and the time window of interest. However, considering the very easy and straightforward implementation of the MCDT, it can be a good choice to improve the FDTD simulation efficiency. This method can be used to efficiently calculate the lightning-generated field over large distances and/or mountainous terrain, and thus to evaluate the performance of lightning locating systems [1]. In addition, with the measured far field data, the proposed FDTD-based approach can also be used for remote sensing of the lower ionosphere [2], [14].

REFERENCES

[1] D. Li, M. Rubinstein, F. Rachidi, G. Diendorfer, W. Schulz, and G. Lu, “Location accuracy evaluation of ToA-based lightning location systems over mountainous terrain,” *J. Geophys. Res.*, vol. 122, pp. 11760–11775, 2017.

[2] M. Azadifar et al., “Analysis of lightning-ionosphere interaction using simultaneous records of source current and 380 km distant electric field,” *Journal of Atmospheric and Solar-Terrestrial Physics*, vol. 159, pp. 48–56, Jun. 2017.

[3] W. Hu and S. A. Cummer, “An FDTD model for low and high altitude lightning-generated EM fields,” *IEEE Trans. Antennas Propag.*, vol. 54, no. 5, pp. 1513–1522, 2007.

[4] J. P. Berenger, “FDTD computation of VLF-LF propagation in the earth-ionosphere waveguide,” *Ann. Telecommun*., vol. 57, no. 11–12, pp. 1059–1090, 2002.

[5] J. P. Berenger, “Long-range propagation of lightning pulses using the FDTD method,” *IEEE Trans. Electromagn. Compat*., vol. 47, no. 4, pp. 1008–1012, 2005.

[6] W. Hou, M. Azadifar, M. Rubinstein, Q. Zhang, F. Rachidi, “An efficient FDTD method to calculate lightning electromagnetic fields over irregular terrain adopting the moving computational domain technique”, *IEEE Trans. Electromagn. Compat*., early access, doi: 10.1109/TEMC.2019.2917282.

[7] J. A. Roden and S. D. Gedney, “Convolution PML (CPML): An efficient FDTD implementation of the CFS-PML for arbitrary media,” *Microw. Opt. Technol. Lett.*, vol. 27, no. 5, pp. 334–339, 2000.

[8] J. R. Wait, and K. P. Spies, “Characteristics of the Earth-ionosphere waveguide for VLF radio waves,” National Bureau of Standards, Boulder, Colorado, NBS Tech. Note, 300, Dec. 1964.

[9] T. H. Tran, Y. Baba, V. B. Somu, and V. A. Rakov, “FDTD Modeling of LEMP Propagation in the Earth‐Ionosphere Waveguide With Emphasis on Realistic Representation of Lightning Source,” *J. Geophys. Res.: Atmos.*, *122*(23), 12918-12937, 2017.

[10] Y. Baba and V. A. Rakov, “On the use of lumped sources in lightning return stroke models,” *J. Geophys. Res.*, vol. 110, 2005, Art. no. D03101.

[11] F. Rachidi, W. Janischewskyj, A. M. Hussein, C. A. Nucci, S. Guerrieri, B. Kordi, and J. S. Chang, “Current and electromagnetic field associated with lightning return strokes to tall towers,” *IEEE Trans. Electromagn. Compat.*, vol. 43, no. 3, pp. 356–367, Aug. 2001.

[12] C. A. Nucci and F. Rachidi, “Experimental validation of a modification to the transmission line model for LEMP calculations,” in *Proc. 8th Int. Symp. Electromagnetic Compatibilty*, Zurich, Switzerland, March 7–9, 1989, pp. 389–394.

[13] F. Rachidi and C. A. Nucci, “On the Master, Lin, Uman, Standler and the modified transmission line lightning return stroke current models,” *J. Geophys. Res.*, vol. 95, pp. 20 389–20 394, Nov. 1990.

[14] F. Han and S. A. Cummer, “Midlatitude nighttime D region ionosphere variability on hourly to monthly time scales,” *J. Geophys. Res.*, vol. 115, no. A09323, 2010.