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Unification of engineering return stroke models

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***Abstract*—Engineering models burrow various features from transmission line models and combine them using empirical data to create models that can predict the electromagnetic fields of lightning return stroke in space and time. At present, engineering models are the most successful in predicting the features of lightning electromagnetic fields in close agreement with the experimental data. Today, most scientists who are interested in lightning protection use engineering models in their studies. A large number of engineering return stroke models exist in the literature. Depending on the assumptions made they can be categorized as Current Propagation models (CP-models), Current Generation models (CG-models) and Current Dissipation models (CD-models). The goal of this paper is to describe the different approximations used in constructing the engineering return stroke models and to show that all these models are a special case of a more general model which we call Current model (C-model). We will show how various return stroke model can be precipitated from the C-model by neglecting different features associated with it.**

Keywords*—*Lightning, Return strokes, Engineering Models, Current propagation models, current dissipation models, current generating models

I. INTRODUCTION

A model is a mathematical construct that attempts to predict the outcome of an event using a certain number of input parameters. A model may attempt to explain and predict the outcome of a phenomenon starting with fundamental principles or it may contain as inputs data obtained from experimental investigations. In the case of lightning return strokes, models are used to predict the electromagnetic fields from lightning at different distances and in the remote sensing of lightning currents using the measured electromagnetic fields. More sophisticated models can be used, not only to evaluate the electromagnetic fields but also to predict the temporal variation of the return stroke current along the channel. Depending on their complexity, return stroke models can be divided into five categories. They are, physics based models [1], electromagnetic and antenna models [2, 3], transmission line models [4], waveguide models [5, 6] and engineering models [7-31].

The physics based models attempt to simulate the lightning return stroke using a combination of the laws of conservation, hydrodynamics, thermodynamics and electrodynamics. Unfortunately, at present we do not have successful physical models that predict the temporal variation of the return stroke current and the electromagnetic fields correctly. Electromagnetic models treat the lightning channel as a vertical conductor located above a perfectly conducting ground and the return stroke is simulated using Maxwell's equation as a current pulse propagating along this conductor. The transmission line models treat the return stroke channel as a charged transmission line and simulate the return stroke as a process that discharge the transmission line. These models can incorporate the finite conductivity of the lightning channel into the model description. The waveguide models simulate the return stroke channel as a finitely conducting waveguide and the return stroke is simulated as a current pulse propagating along this waveguide. Engineering models burrow various features from transmission line models and combine them using empirical data to create models that can predict the electromagnetic fields of lightning return stroke in space and time. At present, engineering models are the most successful in predicting the features of lightning electromagnetic fields in close agreement with the experimental data. Today, most scientists who are interested in lightning protection use engineering models in their studies

Engineering return stroke models can be divided into three categories, namely, Current Propagation models (CP-Models), Current Generation Models (CG-Models) and Current Dissipation Models (CD-Models). The goal of this paper is to illustrate the basic principles of the engineering models and to illustrate that all the engineering models can be derived from a one single general model, which we refer to here as Current Model or C-Model.

First, we will give a full description of the C-Model and later show how the other model types could be precipitated from it by additional assumptions.

II. C-MODEL

A. General Principle

The C-model is based on the available knowledge concerning the propagation of a current pulse, while undergoing corona, along a transmission line.

Consider a current pulse traveling along a transmission line. If the current amplitude is less than the critical amplitude necessary for corona emission, the current pulse will propagate with speed of light and, in the case of an ideal transmission line, without attenuation. If the current amplitude is larger than the critical current necessary for corona emission, each element of the transmission line acts as a corona source injecting a corona current into the return stroke channel. Half of the corona current propagates with speed of light into the forward direction (i.e. the direction of propagation of the injected current) and the other half propagates backwards with the same speed. The polarity of the upward moving corona current is opposite to that of the injected current. This corona current interacts with the injected current, which is also moving with the same speed, in such a way that the net current propagates in the forward direction with a reduced speed. The cumulative sum of the downward moving corona current appears as a current pulse at the base of the channel. In summary, this scenario gives rise to three current waveforms in the return stroke channel, namely, upward moving injected current (Current pulse 1), downward moving corona current (Current pulse 2) and upward moving corona current (Current pulse 3). Current pulse 3 has the polarity opposite to that of the Current pulse 1 whereas the Current pulse 2 has the same polarity. In the case of a transmission line in air all the three current pulses propagate with the speed of light. At the base of the transmission line, the total current consists of two parts, namely, injected current and the cumulative current generated by the downward moving corona current. Both these current waveforms have the same polarity.

B. Mathematical representation

In the C-Model, one has the choice of selecting the injected current, $i_i(t)$, the distribution of the linear charge density deposited by the return stroke along the channel (or the corona charge neutralized by the return stroke), $\rho(z)$, the return stroke speed, $v(z)$, and the magnitude and variation of the corona discharge time constant with height, $\tau(z)$ (defined in Equation 1 given below) as input parameters. Only three of these parameters can be selected independently. That is, once three of these parameters are given the fourth parameter can be evaluated analytically from the available information. In the analysis, the speed at which the injected current and the corona currents propagate along the channel is denoted by v_c. In the case of a transmission line in air this speed is equal to the speed of light in free space. Let us represent the average speed of the resultant current front by $v_{av}(z)$. The upward moving corona current generated by a channel element dz located at height z is given by

$$i_{c,up}(z,t) = -\frac{dz\rho(z)}{2\tau(z)}\exp\{-(t - z/v_{av}(z))/\tau(z)\} \quad t > z/v_{av} \qquad (1)$$

The downward moving corona current generated by a channel element at height z is given by

$$i_{c,\mathrm{down}}(z,t) = \frac{dz\rho(z)}{2\tau(z)}\exp\{-(t - z/v_{av}(z))/\tau(z)\} \quad t > z/v_{av} \qquad (2)$$

Now, the Current pulse 1 mentioned earlier is due to the injected current. This current pulse at any height is given by

$$i_1(z,t) = i_i(0, t - z/v_i) \qquad t > z/v_c \qquad (3)$$

The current pulse 2 at any level is created by the cumulative effect of the downward moving corona currents associated with channel elements located above that level. Thus the current pulse 2 at any height in the return stroke channel, $i_2(z,t)$, can be written as

$$i_2(z,t) = \int_z^{h_e} i_{c,down}(t - \xi/v_{av}(\xi) - (\xi - z)/v_c)\,d\xi \quad t > z/v_{av}(z) \qquad (4)$$

The value of h_e in Equation (4) can be obtained from the solution of the following equation:

$$t = \frac{h_e}{v_{av}(h_e)} + \frac{h_e - z}{v_c} \qquad (5)$$

The current pulse 3 at any height z, which is generated by the cumulative effect of the upward moving corona current is given by

$$i_3(z,t) = \int_o^{h_d} i_{c,up}(\xi, t - \xi/v_{av}(\xi) - (z - \xi)/v_c)\,d\xi \quad t > z/v_c \qquad (6)$$

Note that h_d, is the location of the highest point on the channel whose corona current can reach the point z at time t. This can be obtained by solving the equation

$$t = \frac{h_d}{v_{av}(h_d)} + \frac{(z - h_d)}{v_c} \qquad (7)$$

Since the upward moving corona current annihilate the injected current at all points above the return stroke front, the corona current is constrained by the expression

$$i_1(z,t) = i_i(0, t - z/v_c) = -\int_o^{h_d} i_{c,up}(\xi, t - \xi/v_{av}(\xi) - (z - \xi)/v_c)\,d\xi$$

$$tv_{av}(z) \le z < tv_c \qquad (8)$$

Changing the variable we can write

$$i_i(0,t') = \int_o^{h_s} i_{c,up}(\xi, t' - \xi/v_{av}(\xi) + \xi/v_c)\,d\xi \qquad (9)$$

With h_s given by the solution of

$$t' = \frac{h_s}{v_{av}(h_s)} - \frac{h_s}{v_c} \qquad (10)$$

The Equations (9) and (10) can be used to connect the corona current to injected current once the average speed of propagation is assigned in the model.

Equations (1) to (10) completely define the C-model. Once three of the four input parameters are given (i.e. i_i, ρ, v and τ), the fourth parameter can be estimated analytically from the information available in these equations.

Note that the current at the base of the return stroke channel is given by $i_1(0,t) + i_2(0,t)$. That is, the current at the channel base is not the same as the injected current. A return stroke model having all the features of the C-Model was introduced recently by Cooray and Diendorfer [15].

Now, we are in a position to describe the CP, CG and CD models. In the description given we will use the same notation used in the C-Model for the various current pulses (i.e. injected current, upward moving corona current and downward moving corona current) even though the amplitudes and wave shapes of these current pulses may differ in different model types due to different assumptions.

III. CP-MODEL

In CP-Models, the return stroke is represented by a current pulse, injected at the ground end of the leader channel, which propagates along the leader channel with or without attenuation and distortion [7 – 14].

A. Input parameters of the model and mathematical representation

The input parameters of the model are the waveform and the amplitude of the channel base current, the parameters that define the attenuation and dispersion of the current as it propagates along the channel and the variation of the return stroke speed along the channel.

Let us denote the temporal variation of the current pulse injected at the channel base by $i_i(t)$. Let us assume that this current pulse propagates with variable velocity $v(z)$ along the channel and the current ahead of the return stroke front is zero. As the current pulse moves along the channel it may deposit positive charge along the leader channel and this may give rise to a decrease in the amplitude of the current. The propagating current pulse may also suffer dispersion as it propagates along the channel and this may also lead to the change in the shape and the peak amplitude of the current with height. Thus the current at level z is given by

$$i(z,t) = A(z)\,F(z, t - z / v_{az}(z)) \qquad t > z / v_{av}(z) \qquad (11)$$

In the above equation $A(z)$ is a function that represents the attenuation of the current due to the deposition of positive charge along the channel and $F(z,t)$ describes the modified wave shape of the current at height z caused by dispersion, and $v_{av}(z)$ is the average return stroke velocity over the channel section whose upper end is at level z.

One can define the function $F(z)$ that takes care of the variation of the current shape as follows:

$$F(z,t) = \int_o^t i_i(\tau) R(z, t - \tau)\, d\tau \qquad (12)$$

where $i_i(t)$ is the channel base current and $R(z,t)$ is a function that describes how the shape of the current waveform is being modified with height. Actually, this function describes how a delta impulse current function is modified as it propagates along the channel due to dispersion. That is the function $R(z,t)$ describes the current at height z in the presence of dispersion if the channel base current is given by a Dirac delta function. Since no charge is deposited when only dispersion is present along the channel, the function $R(z,t)$ should satisfy the criterion

$$\int_o^\infty R(z,t)\,dt = 1 \qquad (13)$$

Equations (11) to (13) provides the most general description of the current propagation models. In the TLM model both $A(z)$ and $R(z,t)$ is assumed to be unity [11]. In the MTLE model [12] $A(z)$ is represented by an exponential function while in the MTLL model [13] it is represented by a function that decreases linearly with height. Both MTLE and MTLL models assume $R(z,t)$ is equal to unity i.e. no current dispersion. In the model introduced by Cooray and Orville [14] both current attenuation and current dispersion is taken into account.

IV. CG- MODEL

A. General principle

In these models the leader channel is treated as a charged transmission line and the return stroke current is assumed to be generated by a wave of ground potential that travels along it

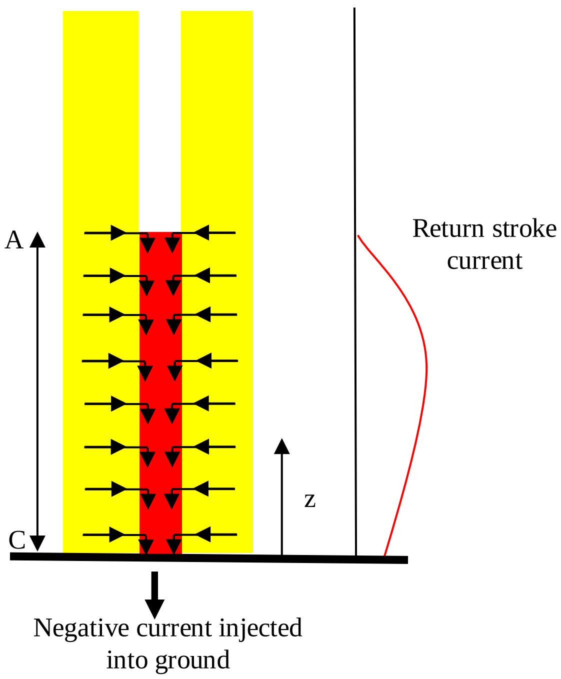

Figure 1: Pictorial description of the current generation concept. According to CG concept the downward moving corona currents generate the return stroke current. Adapted from [15].

from ground to cloud [16 – 29]. The arrival of the wave front (i.e. return stroke front) at a given point on the leader channel changes its potential from cloud potential to ground potential causing the release of bound charge on the central core and the corona sheath giving rise to the current in the channel (this is called the corona current in the literature). These models postulate that as the return stroke front propagates upwards the charge stored on the leader channel collapses into the highly conducting core of the return stroke channel. Accordingly, each point on the leader channel can be treated as a current source which is turned on by the arrival of the return stroke front at that point. The corona current injected by these sources into the highly conducting return stroke channel core travels to ground with a speed denoted by v_c. In most of the CG-Models it is assumed that $v_c = c$ where c is the speed of light in free space. References [16] to [29] provide a description of various types of CG-Models constructed to represent the return strokes.

B. Input parameters of the model and mathematical representation

In CG models one has the choice of selecting the channel base current, $i_2(0,t)$, the distribution of the charge deposited by the return stroke along the channel, $\rho(z)$, the return stroke speed, $v(z)$ or $v_{av}(z)$, and the magnitude and variation of the corona discharge time constant with height, $\tau(z)$ as input parameters.

Since the current at any given level on the channel is the cumulative effect of corona currents associated with channel elements located above that level, the return stroke current at any height in the return stroke channel $i_2(z,t)$ can be written as

$$i_2(z,t) = \int_z^{h_e} i_{c,down}(t - \xi / v_{av}(\xi) - (\xi - z)/v_c) d\xi \qquad t > z / v_{av}(z) \qquad (14)$$

Note that $i_{c,down}(z)$ is the corona current per unit length associated with a channel element at height z. Note that in CG-Models, total corona current injected into the return stroke channel is assumed to travel downwards. Thus, $i_{c,down}(z)$ in CG-Models is connected to the total deposited charge per unit length by the equation

$$i_{c,down}(z) = \frac{\rho(z)}{\tau(z)} \exp\{-(t - z / v_{av}(z)\} \qquad t > z / v_{av}(z) \qquad (15)$$

Note the absence of factor ½ in the equation. In these equations v_c is the speed of propagation of the corona current and $v_{av}(z)$ is the average return stroke speed over the channel section of length z with one end at ground level. The latter is given by

$$v_{av}(z) = z / \int_o^z \frac{1}{v(z')} dz' \qquad (16)$$

The value of h_e in Equation (14) can be obtained from the solution of the following equation:

$$t = \frac{h_e}{v_{av}(h_e)} + \frac{h_e - z}{v_c} \qquad (17)$$

The current at the channel base is given by

$$i_2(0,t) = \int_o^{h_o} i_{c,down}(t - \xi / v_{av}(\xi) - \xi / v_c) d\xi \qquad (18)$$

with h_o given by the solution of

$$t = \frac{h_o}{v_{av}(h_o)} + \frac{h_o}{v_c} \qquad (19)$$

As mentioned earlier, a CG model contains four model parameters and any set of three of these four input parameters will provide a complete description of the temporal and spatial variation of the return stroke current. Most of the CG models use $v(z)$ and either the $\rho(z)$ or $\tau(z)$ in combination with $i_2(0,t)$ as input parameters. Once three of these parameters are specified the fourth can be evaluated either analytically or numerically. Note that in CG-Models, only the current pulse 2 exists in the return stroke channel.

V. Basic Concepts of Current Dissipation Models

A. Basic principle

The CD-Models utilize only the injected current and the upward moving corona current in the C-Model in describing the return stroke [31].

The basic features of the current dissipation models are illustrated in Figure 2. The main assumptions of the current dissipation models are the following: The return stroke is initiated by a current pulse injected into the leader channel from the grounded end. The arrival of the return stroke front at a given channel element will turn on a current source that will inject a corona current into the central core. It is important to stress here that by the statement the arrival of the return stroke front at a given channel element it is meant the onset of the return stroke current in that channel element (i.e. point B in Figure 2). Once in the core this corona current will travel upward along the channel. In the case of negative return strokes the polarity of the corona current is such that it will deposit positive charge on the corona sheath and transport negative charge along the central core. Let us now incorporate mathematics into this physical scenario.

B. Input parameters of the CD-Model and mathematical representation

As in the CG models, in CD models one has the choice of selecting the channel base current, $I_b(t)$, the distribution of the charge deposited by the return stroke along the channel, $\rho(z)$, the return stroke speed, $v(z)$, and the magnitude and variation of the corona discharge time constant with height, $\tau(z)$ as input parameters. Once three of these parameters are specified the fourth parameter can be estimated without ambiguity.

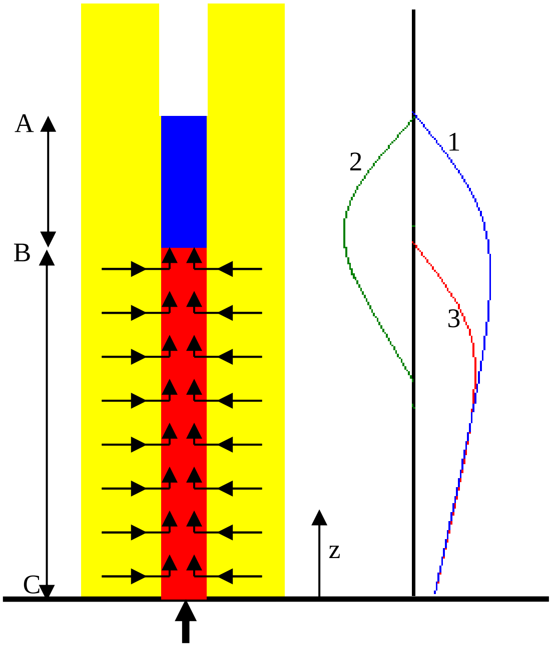

Figure 2: Pictorial description of the current dissipation concept. Waveforms to the right depict different current components along the channel. (1) Injected current. (2) Upward moving corona current. (3) Net current along the channel. (Adapted from [31]).

Consider the diagram to the right in Figure 2. This depicts a situation at any given time t. At this time the tip of the injected current is located at point A and the return stroke front is located at point B. As in the case of a current pulse propagating along a transmission line under corona, at points above the current front or the return stroke front (i.e. points located above B) the current is zero. Now, in the CD models the sum of the corona currents completely neutralize the injected current above the return stroke front. Let us consider any height z located above the return stroke front (i.e. above point B). The net corona current at that height is given by (note that the corona current is defined as negative here because it transports negative charge towards the cloud whereas the injected current transports positive charge upwards)

$$i_3(z,t) = \int_o^{h_d} i_{c,up}(\xi, t-\xi/v-(z-\xi)/v_c)d\xi \qquad (20)$$

Note that h_d, is the location of the highest point on the channel whose corona current can reach the point z at time t. This can be obtained by solving the equation

$$t = \frac{h_d}{v_{av}(h_d)} + \frac{(z-h_d)}{v_c} \qquad (21)$$

Note that in CD-Models, total corona current injected into the return stroke channel is assumed to travel upwards. Thus, $i_{c,up}(z)$ in CD-Models is connected to the total deposited charge per unit length by the equation

$$i_{c,\mathrm{up}}(z) = -\frac{\rho(z)}{\tau(z)}\exp\{-(t-z/v_{av}(z)\} \qquad t > z/v_{av}(z) \quad (22)$$

Note the absence of factor ½ in the equation. The injected current at point z at time t is given by

$$i_1(z,t) = i_i(0,t-z/v_c) \qquad (23)$$

As pointed out in the description of the C-model, the annihilation of the injected current by the upward moving corona current at all points above the return stroke front generates a relationship between the injected current and the upward moving corona current which is given by

$$i_i(0,t) = \int_o^{h_s} i_{c,up}(\xi, t-\xi/v_{av}(\xi)+\xi/v_c)d\xi \qquad (24)$$

With h_s given by the solution of

$$t = \frac{h_s}{v_{av}(h_s)} - \frac{h_s}{v_c} \qquad (25)$$

Note that in the CD-Models, the channel base current is identical to the injected current where as in the CG-Models the channel base current is the same as $i_2(0,t)$. Comparison of Equations, 18 and 24 shows that the relationship between the channel base current and the corona current is given by an identical expression except for the change in the sign of v_c. Moreover, as in the case of current generation models, the input parameters of current dissipation models are the charge deposited on the channel by the return stroke, corona decay time constant, return stroke speed and the channel base current. When three of these parameters are given the fourth one can be obtained in the same manner as it was done in the case of current generation models. But in equations 14 to 19, v_c has to be replaced by $-v_c$ when using these equations in connection with current dissipation models.

VI. Generalization of any Model to Current Generation or Current Dissipation Type

Consider any return stroke model which provides an expression for $i(z,t)$, the return stroke current at any height. Assume also that the speed of propagation of the return stroke front pertinent to that model is v. Cooray [31, 32] showed that this information is enough to describe this model either as a CG model or a CD model with an equivalent corona current which depends on $i(z,t)$ and v. The equivalent corona current per unit length at any given height z that is needed to describe the model as a CG model is given by

$$i_{c,down}(z,t) = -\frac{\partial i(z,t)}{\partial z} + \frac{1}{v_c}\frac{\partial i(z,t)}{\partial t} \qquad (26)$$

In the above equation v_c is the speed of propagation of corona currents along the return stroke channel.

In a similar manner, the equivalent corona current per unit length at any given height z that is needed to describe the model as a CD model is given by

$$i_{c,up}(z,t) = -\frac{\partial i(z,t)}{\partial z} - \frac{1}{v_c}\frac{\partial i(z,t)}{\partial t} \qquad (27)$$

In the above equation v_c is the speed of propagation of corona currents along the return stroke channel. Note that in the description given earlier the upward moving corona current is assumed to have a negative sign; however the equation given above will give the positive value of this corona current. The sign of the corona current given by Equation 27 has to be changed before it is being plugged into the CD-Models.

With this corona current and v as an input the variation of the return stroke current in space and time of the resulting CG or CD model would be identical to that of the original model. However, it is important to point out that the equivalent corona current of the CG model is not the same as the CD model. This shows that even though the special and temporal variation of the return stroke current is the same in all three models (i.e. original and the equivalent CG and CD models) the physics of charge neutralization which is the source of the corona current is different in the three models. That is, the models are not physically equivalent even though they produce the same current distribution along the channel and predict the same electromagnetic fields.

VII. Current Propagation Models as a Special Case of Current Dissipation Models

If the return stroke current associated with a current propagation model is assumed to decrease with height (as in the case of Modified Transmission Line models [12, 13]), the conservation of charge requires deposition of charge along the channel as the return stroke front propagates upward. This leakage of charge from central core to the corona sheath can be represented by a radially flowing corona current. Recently, Maslowski and Rakov [33] showed that this corona current is given by

$$i_{cp}(z,t) = -\frac{\partial i(z,t)}{\partial z} - \frac{1}{v}\frac{\partial i(z,t)}{\partial t} \qquad (28)$$

Where $i_{cp}(z,t)$ is the radially flowing corona current per unit length at height z, $i(z,t)$ is the longitudinal return stroke current at the same height as predicted by the return stroke model and v is the speed of the return stroke front. Note that the direction of flow of the corona current is radial and, in contrast to the current generation or current dissipation models, it does not have a component flowing along the return stroke channel i.e. it is a stationary corona current. Maslowski and Rakov [32] showed that any return stroke model could be reformulated as a current propagation model with an equivalent stationary corona current given by equation 30.

Let us now go back to the current dissipation models. Cooray [31] showed that in general the speed of propagation of the return stroke front in current dissipation models is less than that of the injected current (i.e. v_c). However, he also showed that one can select the parameters of the corona current in such a way that the speed of the return stroke front remains the same as that of the injected current pulse and the corona current. When such a choice is made current dissipation models reduce to MTL models. This can be illustrated mathematically as follows. Let us represent the injected current at the channel base as $i_i(0,t)$. The injected current at height z is given by

$$i_1(z,t) = i_i(0, t - z/v_c) \qquad (29)$$

Assume that the corona current per unit length at level z is given by

$$i_{c,up}(z,t) = -i_1(0, t - z/v_c)\,\mathrm{A(z)} \qquad (30)$$

where A(z) is some function of z. According to this equation the corona current at a given height is proportional to the injected current at that height. Substituting this expression in equation (27) and replacing v by v_c one finds that

$$i_i(0, t - z/v_c)A(\mathrm{z}) = \frac{\partial i(z,t)}{\partial z} + \frac{1}{v_c}\frac{\partial i(z,t)}{\partial t} \qquad (31)$$

One can easily show by substitution that the solution of this equation is given by

$$i(z,t) = A'(z)\,i_i(0, t - z/v_c) \qquad (32)$$

with

$$A'(z) = \int A(z)dz \qquad (33)$$

Note that $i(z,t)$ in the above equation is the total current i.e. sum of the corona current and the injected current. According to equation 32, the total current propagates upward with the same speed as that of the injected current and corona current. Moreover, it propagates upward without any distortion while its amplitude varies with height according to the function $A'(z)$. Indeed, equation 32 describes a MTL model. In this special case Equation 27 reduces to Equation 28 derived by Mazlowski and Rakov [33] because the return stroke speed v becomes equal to v_c. Thus, Equation 28 is a special case of Equation 27 and the latter reduces to the former in the case of MTL models. The above also demonstrates that all the current propagation models available in the literature are special cases of the CD-Model.

VIII. CONCLUSIONS

In this paper we have shown that all the engineering return stroke models can be treated as special cases of a more general model refereed to here as the Current Model or C-Model. It is shown how each engineering model can be precipitated from the C-Model by neglecting different features associated with this model.

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