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# Field-to-Transmission Line Coupling Models: Recent Progress

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***Abstract*—In this lecture, some of the latest developments in the modeling of the field-to-transmission line coupling interaction are presented.**

***Keywords—lightning; induced voltages; field-to-transmission line coupling;***

## I. Introduction

Electromagnetic radiation from lightning couples to overhead and also to buried lines and can produce outages or damages. Out of the several models that have been developed to calculate induced overvoltages on distribution and power lines (see. e.g., [1]), those based on the Transmission Line approximation have been validated and used extensively since they yield accurate results for the estimation of the currents and voltages at the loads in many practical applications using reasonable computer resources.

One of the models based on the Transmission Line approximation, the model of Rusck [2], is only applicable in its original form to lines over a perfectly conducting ground and, in addition, as pointed out by Cooray [3], it is incomplete in the sense that some forcing terms are missing. Several recent improvements to this model will be given in this presentation, including the modification introduced by Cooray et al. [4] to the original model of Rusck to add the missing terms and to extend it to the case of a lossy ground as well as the recent work done to cast the model in terms of scalar and vector potentials.

The models based on the Transmission Line approximation generally consider the line terminations to be straight and vertical. This idealization, however, is not always applicable to real cases. Rather than using full-wave simulations that are extremely costly in terms of computer resources and simulation time for the case of long lines, recent work has tackled the problem of the calculation of induced voltages with arbitrarily-shaped terminations while keeping the simplicity of the TL approximation so that currently used computer codes can continue to be used [5]. The solution, which will be described as part of the presentation, involves the use of the concept of partial inductance.

We will also present the latest developments on the generalization of the TL theory to take into account high frequency effects. This effort has resulted in the elaboration of the so-called 'generalized' or 'full-wave' TL theory, which incorporates high frequency radiation effects, while keeping the relative simplicity of the TL equations [6], [7].

Finally, the talk will also include a summary of recent research activities which are specifically related to lightning-induced voltages.

## II. Field-To-Transmission Line Coupling Models

### A. *Transmission Line Approximation*

Electromagnetic fields from natural sources such as lightning can propagate over long distances and can create disturbances on transmission and distribution lines, frequently leading to outages and or damages to line elements or electronics at consumer premises. An understanding of the mechanisms that govern the coupling of external electromagnetic fields to overhead and underground lines is of great importance as a tool in the development and testing of effective protection methods.

Depending on the required degree of precision, the available resources and the relation between the line dimensions and the frequencies of interest, several approaches can be used for the calculation of the voltages and currents induced by external electromagnetic fields on overhead and underground lines: The Quasi-Static approach, the Transmission Line approach and the Antenna approach, also called antenna theory approach or full-wave approach.

For numerous practical applications, the approach based on the Transmission Line Theory represents an excellent trade-off between the inaccuracies of the simple quasi-static approximation method and the computational cost associated with the antenna approach. Indeed, the Transmission Line approach can yield results with excellent accuracy using reasonable computer resources. The main assumptions on which the transmission line approximation is based can be described for an overhead line composed of horizontal wires parallel to the ground. The assumptions are [8]: 1) that the propagation of the coupled voltages and currents occurs along the line axis, 2) that the net sum of the line currents at any cross section of the line is zero, and 3) that the response of the line to the electromagnetic fields is quasi-transverse electromagnetic (quasi-TEM). The last assumption implies that the electric and magnetic field components from the induced electric charges and currents along the line are approximately in the transverse plane of the line, perpendicular to the line axis (only small components of the electric and magnetic fields can exist in the direction of propagation). The assumption of propagation along the line is a good approximation for lines of electrically small cross-sectional dimensions. The condition of net-zero current at any position along the line is satisfied by lines whose currents can be obtained by the method of images. This is the case for lines with a ground made of a sufficiently high conductivity. Finally, the condition that the response of the line is quasi-TEM is only satisfied up to a threshold frequency, above which higher-order modes can also propagate [8]. For typical transmission lines and for the frequency content of lightning fields, this last condition is generally satisfied.

The Transmission Line approach is the most widely used technique due to its accuracy and flexibility and it is the main subject of that will be dealt with in the presentation.

### B. Field-to-Transmission Line Coupling Models

The first complete model based on the Transmission Line approach for the coupling of external electromagnetic fields to transmission lines was presented by Taylor et al. [9] in 1965. Two other complete coupling models, also based on the Transmission Line approach, the model of Agrawal et al. [10] and the Rachidi Model [11], were introduced, respectively, in 1980 and in 1995. Note that seven years before Taylor et al.'s original paper, Rusck presented a somewhat less general coupling model [12] that, in its original form, was applicable only to the case of straight and vertical sources and lossless lines. The above-mentioned models are widely discussed in the literature (e.g., [13]). In what follows, we will limit our discussion to the Rusck model and its modification proposed recently by Cooray et al. [4].

### C. The Modified Rusck Model

In 1958, Rusck [12] introduced a coupling model applicable to the case of a perfectly conducting ground. Cooray [14] found that Rusck's original model is not complete since some of the forcing terms are missing. Recently, Cooray et al. [15] modified Rusck's model by adding the missing terms and extended the model to make it applicable to the case of a lossy ground. The equations of the model of Rusck and of the modified model are expressed in terms of the scalar and vector potentials.

The field-to-transmission line coupling equations for the modified Rusck's model, written in terms of the scalar and vector potentials in the frequency domain, are summarized here for the case of a lossless conductor above a perfectly-conducting ground:

$$\frac{dV^q(x)}{dx} + j\omega L' I(x) = -j\omega A_x^i(\mathrm{x}, h) \tag{1}$$

$$\frac{dI(x)}{dx} + j\omega C' V^q(x) = -j\omega C' \phi^i(x, h) \tag{2}$$

in which $V^q$ is defined as

$$V^q(x) = -\int_0^h E_z^s(x, z) \cdot dz + \phi^i(x, h) \tag{3}$$

The total voltage can be obtained using

$$V(x) = V^q(x) + \int_0^h j\omega A_z^i dz \tag{4}$$

The boundary conditions at the ends of the transmission line at $x = 0$ and $x = L$ for resistive terminations are given by

$$V^q(0) = -Z_A \cdot I(0) - \int_0^h j\omega A_z^i(0, z) dz \tag{5}$$

$$V^q(L) = Z_B \cdot I(L) - \int_0^h j\omega A_z^i(L, z) dz \tag{6}$$

An equivalent circuit for the modified Rusck model is shown in Figure 1.

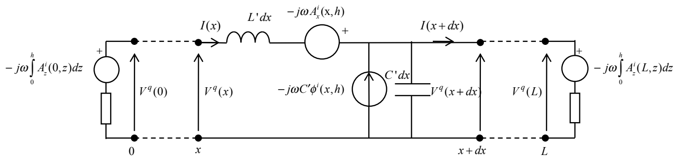



Fig. 1. Equivalent circuit of the modified Rusck coupling model for the case of a lossless conductor above a perfectly-conducting ground. Adapted from [4].

Note that the version of the modified Rusck's model given here is applicable for perfectly conducting lines. Equations taking into account line losses can be found in [15].

An extension of the original Rusck's model to take into account the effect of the finite ground conductivity was also presented by Piantini [16], who demonstrated that lightning-induced voltages on an overhead line over a lossy ground can be obtained by adding to the voltage calculated for the case of perfectly conducting ground a term associated with the horizontal component of the electric field.

The modified Rusck model is completely equivalent to the other models presented already in the literature. The main advantage of the Rusck formulation is that the field-to-transmission line coupling in the case of lightning flashes can be solved using only the vertical component of the vector potential without calculating the total fields.

## III. Modeling of Non-Vertical Risers in the Field-to-Transmission Line Coupling Models

In general, field-to-transmission line coupling models only consider the case of a transmission line terminated by vertical risers at both ends. However, in some cases, the risers at the end of the transmission lines are not vertical and they may have an arbitrary shape. Such a problem can be handled by full-wave methods, which entail high computational cost. A simple and efficient method to take into account non-vertical risers at the ends of the transmission lines was proposed recently by Guo et al. [5].

Consider a transmission line is terminated by a non-vertical riser that has an arbitrary shape at the left end, as shown in Fig. 2.

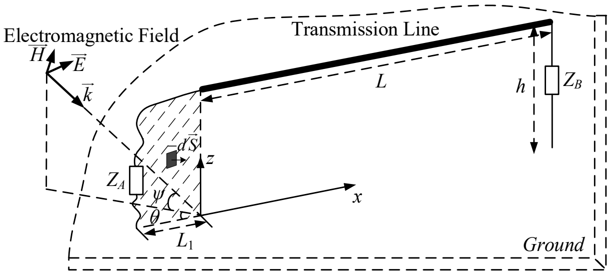



Fig. 2. Transmission line with a non-vertical riser excited by an incident electromagnetic wave.

Since the riser is not vertical, the source at the termination is the line integral of the exciting electric field along the riser's non-vertical geometry. Moreover, the termination impedance, represented by $Z_A$ in Fig. 2, is the series combination of the actual termination and an additional inductive impedance stemming from the more general geometry for the risers. The boundary condition at the left end can be rewritten as

$$V^s(0,\omega) = -(Z_A + j\omega L_A)I(0,\omega) + \int_{Riser} \vec{E}^e \cdot d\vec{l} \qquad (7)$$

in which [5]

$$L_A = \frac{\iint \vec{B}^s \cdot d\vec{S}}{I(0)} \qquad (8)$$

The surface over which the scattered magnetic field is integrated in shown in Fig. 2.

The value of the inductance $L_A$ depends only on the geometry of the riser. Fig. 3 illustrates the equivalent circuit for the left-hand side of the line based on (7).

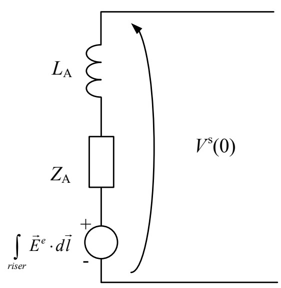


Fig. 3. Equivalent circuit for the left termination when the riser is not vertical. Adapted from [5]

Thus, the integrals of the exciting electric field at the two line ends are evaluated along a path defined by the geometry of the risers.

A validation example is now presented to show the applicability of the proposed method. The configuration of the transmission line is shown in Fig. 2. The terminal riser at the right end is assumed to be vertical, while two different shapes, namely rectangular and triangular, are considered for the riser at the left-end, as illustrated in Figs. 4a and 4b. The equivalent (partial) inductance $L_A$ can be evaluated using the Biot-Savart law. The field-to-transmission-line coupling equations including the treatment of non-vertical risers are solved using the BLT equations [7]. In order to validate the calculation results, the Numerical Electromagnetics Code NEC-4, a full-wave solver based on the Method of Moments, is adopted. In what follows, we will consider a 20-m long wire located at a height of 0.1 m above a perfectly-conducting ground [17]. The conductor radius is 1 mm. The azimuth angle, elevation angle and the polarization angle of the exciting plane wave are 0º, 45º and 0º, respectively.

In the first example, we considered the two cases for the geometry of the left-end riser shown in Fig. 4. In both cases, the value of $L_1$ (see Fig. 4) was set to 0.5 m. The calculated equivalent inductances $L_A$ for the rectangular (Fig. 4a) and for the triangular (Fig. 4b) terminations are, respectively, 0.66 µH and 0.49 µH. The line is terminated at both ends in 100 Ω resistive loads. The frequency range of the wave is 10 kHz-50 MHz, and the amplitude of the $E$-field is 1 V/m across the complete frequency spectrum. The calculated results for the induced currents at the left-end are shown in Fig. 5.

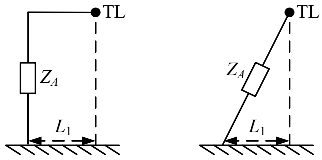



Fig. 4. Cross-section of the two considered geometries for the left-end riser. (a) rectangular, and (b) triangular.

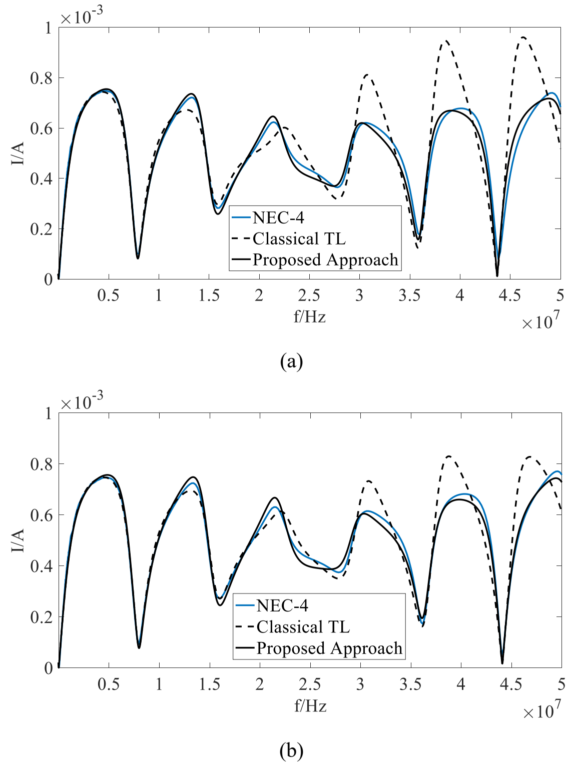



Fig. 5. Left-end induced currents as a function of frequency when the left end geometry is considered to be: (a) rectangular, and (b) triangular. Calculations obtained using the classical transmission line theory, the transmission line theory including the partial inductance, and NEC-4. Adapted from [17]

It can be seen that the results calculated using the classical transmission line theory deviate from the full-wave results obtained using NEC-4. Taking into account the non-vertical riser using the equivalent (partial) inductance leads to significantly more accurate results. In the considered example, the effect of the non-vertical riser appears at frequencies of about 20 MHz, which corresponds to wavelengths which are still much larger than the line cross-section (0.1 m). We can therefore expect that this effect must be included when calculating lightning-induced voltages for non-vertical risers.

## IV. Generalization of the Coupling Models to Take Into Account High Frequency Effects

### *A. The Asymptotic Method*

As discussed earlier, the electromagnetic field coupling to transmission lines is generally evaluated using the transmission line (TL) theory, which applies to uniform transmission lines with electrically small cross-sectional dimensions, and where the dominant mode of propagation is transverse electromagnetic (TEM). Recently, significant efforts have been put into the elaboration of generalizations of the TL theory, to incorporate high frequency effects (e.g., radiation) that are not considered by the classical TL theory, while keeping as much as possible the relative simplicity of the TL equations. Among these studies, Tkachenko et al. [18] derived a TL-like pair of equations for evaluating currents and potentials induced by external electromagnetic fields on a single wire above a perfect conducting ground. Based on perturbation theory, an iterative procedure was proposed to solve the derived coupling equations. Later in 2001, Tkachenko et al. [19] extended their method to take into account the presence of line terminations and discontinuities.

Recently, the Asymptotic method of Tkachenko et al. was generalized to take into account the presence of a multiconductor line and a lossy ground.

### *B. Generalization of the Asymptotic Method to the Case of a Multiconductor Line*

Based on the asymptotic theory of Tkachenko et al., Lugrin et al. [6] presented a theory and an efficient solution approach for the problem of electromagnetic field coupling to a long multiconductor line with arbitrary terminations. The theory is applicable for a high-frequency plane wave electromagnetic field excitation, when the TL approximation is no longer valid.

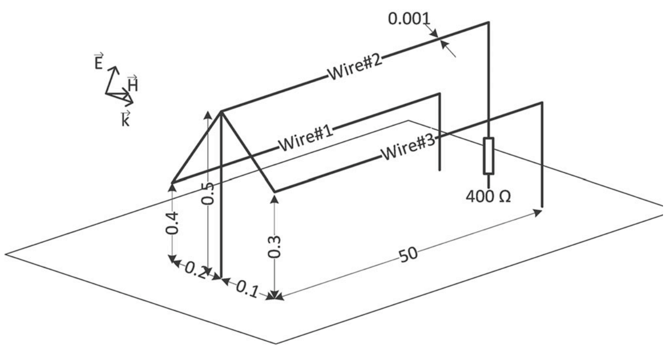



Fig. 6. Three-conductor line system considered in [6].

The proposed method was validated taking as a reference numerical simulations obtained using the Numerical Electromagnetics Code (NEC-4). The authors considered a line made of three wires above a ground plane, with the geometry and terminal conditions shown in Fig. 6. The excitation field was a plane wave with an elevation angle of $\vartheta = 45°$ (the azimuth and polarization angles $\varphi$ and $\eta$ were set to zero).

The current induced along the third wire at 200 MHz is presented in Fig. 7. It can be seen that the proposed method offers results that are in excellent agreement with the "exact" solutions obtained using NEC-4, whereas the classical TL theory does not provide accurate results for the considered frequency.

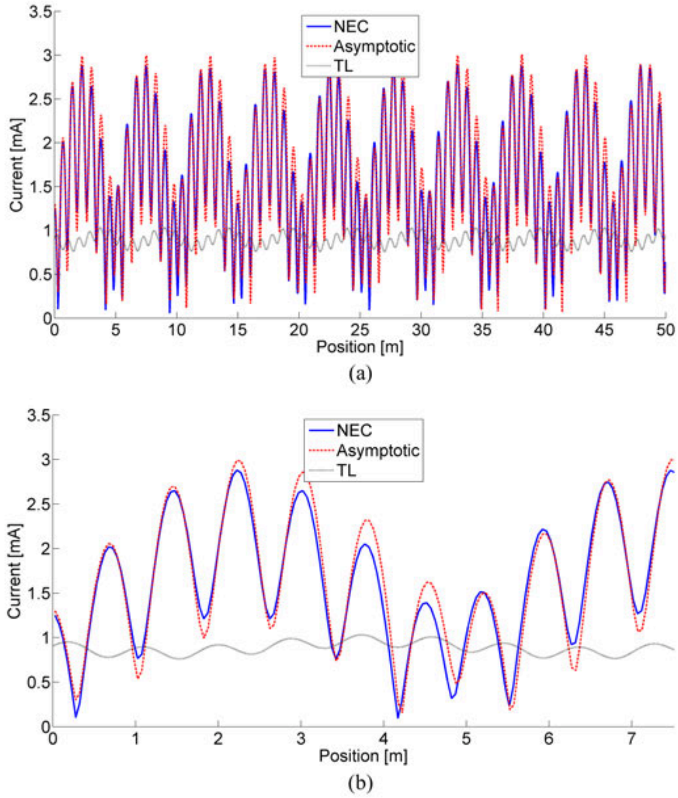
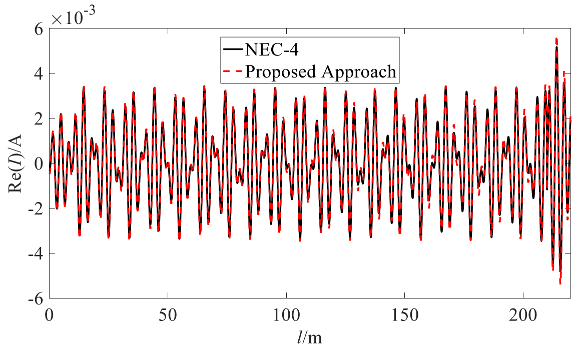


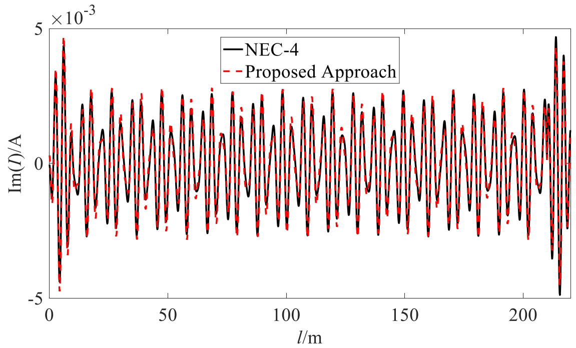

Fig. 7. Amplitude of the current induced along the horizontal part of the line in the third conductor at 200 MHz. Comparison between the solutions provided by NEC, the proposed asymptotic method and the classical TL theory. (a) Whole line, and (b) expanded view of the left part of the line. Adapted from [6].

### *C. Generalization of the Asymptotic Method to the Case of a Lossy Ground*

The generalization of the asymptotic theory of Tkachenko et. al. to handle the case of a lossy ground was presented by Guo et al. [7]. The generalized approach was again validated using a number of simulation cases taking as reference numerical results obtained using NEC-4. Here, we present an example of the validation for the case of a 200-m long, 10-m high, 1-mm diameter conductor above a lossy ground. The ground conductivity and relative permittivity were 0.01 S/m and 10, respectively. The terminal loads at both ends were 50 Ω. The line was excited by a plane wave with an amplitude of 1 V; the polarization angle, the azimuth angle and the elevation angle are 0°, 0° and 45°, respectively. The line response was evaluated using NEC-4 and the proposed method. The current responses along the entire line at a frequency of 100 MHz are shown in Fig. 8. It can be seen that the solution along the entire line calculated using the proposed approach agrees remarkably well with that the results obtained using NEC-4.




(a) Real part of the solution.




(b) Imaginary part of the solution.

Fig. 8. Comparison between the extended asymptotic approach and results obtained using NEC-4. Induced current on a 200-m long, 10-m high, 1-mm diameter conductor above a ground of conductivity $10^{-2}$ S/m and relative permittivity 10. The line is excited by a 1-V/m, 100-MHz plane wave. The polarization angle, the azimuth angle and the elevation angle are 0º, 0º and 45º, respectively. Adapted from [7].

### *D. Mode Transition in High Frequencies*

Xue et al. [20] investigated the effect of mode transition from low-frequency transverse electromagnetic (earth-return wave) to high-frequency Sommerfeld-Goubau (surface wave) propagation on switching surges in gas-insulated buses (overhead cables) by adopting complete formulas for the earth-return impedance and admittance using an extended transmission line (TL) approach. They showed that the taking into account of the mode transition would result in switching surges with frequency components higher than 10 MHz, in agreement with experimental observations. Such a behavior cannot be observed with the classical TL approach.

## V. Recent Progress Specifically Related to Lightning-Induced Voltage Calculations

During the past few years, considerable research has been devoted to improving the methods for the evaluation of lightning-induced voltages on power lines. A special issue of the IEEE Transactions on Electromagnetic Compatibility (Vol. 61, Nr. 3, Guest Editors: R. Procopio, Y. Baba and A. Piantini) with 25 contributions was devoted to the advances in lightning modelling, computation and measurement.

In what follows, a brief summary of some of recent studies which are directly related to lightning-induced voltage computations is given.

Visacro and Silveira [21] presented a study assessing the impact of the frequency-dependence of soil parameters on the lightning performance of transmission lines. They found that the frequency dependence is responsible for a significant decrease of the expected outage rate of the tested lines. Paknahad et al. [22] discussed the effect of the frequency dependence of the soil electrical parameters on lightning currents induced on the shield of buried cables. They showed that, depending on the burial depth of the cable, for poorly conducting soils, the soil dispersion can result either in an increase or in a decrease of the induced current peak.

Tossani et al. [23] proposed a new analytical approach for calculating the transient ground resistance in the time domain that is based on the very accurate Sunde logarithmic expression for the ground impedance matrix of an overhead multiconductor line. Their results show that the proposed formula is in excellent agreement with the numerical inverse Fourier transform of the general Sunde expression and, therefore, it is more accurate than the approximate expressions available in the literature.

The response of a multiconductor line to an indirect lightning was investigated in [24]. It was shown that the cases for which the induced voltage on one conductor is not affected by the presence of other conductors are those of an infinitely long lossless line, of a lossless matched line, and of a lossless open line. For the second case, a reduction of the amplitude of the induced voltages with respect to the case of an infinitely long line is observed, but only near the line terminations, and this is not due to the presence of the other conductors but to the so-called “risers,” which describe the effect of the missing portion of the illuminated line beyond the line terminations. Furthermore, it was shown that the response of a matched lossless multiconductor line differs from that of a single-conductor line at the same height only in the line currents. The effect of the nearby line conductors on the induced voltages is noticeable when the ground losses are taken into account in the surge propagation.

Tossani et al. [25] analyzed the lightning performance of overhead power lines in urban areas, discussing the effect of nearby buildings on the number of direct strikes and on the attenuation of induced voltages. The presence of nearby buildings was also analysed by Thang et al. [26] using a 3-D finite-difference time-domain (FDTD) method. The observed trend was found to be in agreement with reduced-scale experimental data. In another study, Thang et al. [27] used the same 3D-FDTD approach to study the effect of the presence of surge arresters and pole transformers. The computed lightning-induced voltage waveforms were found to agree reasonably well with the corresponding ones measured in reduced-scale experiments.

In [28], Brignone et al. discussed the differences observed between measurements obtained using reduced scale models and calculations. Suitable expressions were derived for the electromagnetic fields generated by a helical antenna that served to emulate a lightning channel, which allowed to compute the corresponding electromagnetic fields in a much more accurate way than in previous studies.

Brignone et al. [29] presented the application of the statistical RSM (response surface method) to assess the effectiveness of the shield wires in the mitigation of lightning-induced overvoltages in MV networks. The analysis was conducted on a realistic configuration and a statistical approach allowed to consider all the possible lightning locations and the incident currents according to their probability density functions.

The influence of the return stroke current waveform on the lightning performance of distribution lines was analysed by Borghetti et al. [30], taking into account both direct and indirect strokes. The authors recommended the use of the simple trapezoidal current waveform which represents a good compromise between computational effort and conservative assessment of the lightning performance.

Andreotti et al. [31] presented a thorough review of exact and approximate analytical solutions for the evaluation of voltages induced on a lossless horizontal conductor, placed over an infinite-conductivity ground, and excited by an external field produced by either a step function or a linearly rising current, moving unattenuated and undistorted along a vertical lightning channel. They also presented an exact solution in the time domain via development in the Laplace-domain. The influence of the return stroke channel tortuosity on lightning-induced voltages was analysed by Andreotti et al. [32]. They showed that the channel tortuosity can have a significant influence on the induced voltages.

## VI. Conclusion

In this lecture, we presented some of the latest developments in the modeling of the field-to-transmission line coupling interaction.

Concerning modeling, we discussed the modification to the original model of Rusck to extend it to the case of a lossy ground as well as the recent work done to cast the model in terms of scalar and vector potentials.

Recent developments to take into account the presence of non-vertical risers were also discussed. The solution involves the use of the concept of partial inductance.

We also presented the latest developments on the generalization of the TL theory to take into account high frequency effects. This effort has resulted in the elaboration of the so-called ‘generalized’ or ‘full-wave’ TL theory, which incorporates high frequency radiation effects, while keeping the relative simplicity of the TL equations.

Finally, a summary of recent research activities which are specifically related to lightning-induced voltages was presented.

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