

# Equivalence between Triangular and Heidler Functions for Calculations of Lightning Overvoltages on Transmission Lines

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**Abstract**—This paper presents a study of the resulting overvoltages on the insulator strings of a 230 kV transmission line, taking into consideration two different models for the lightning currents: Heidler and triangular functions. The analysis was carried out considering only direct strokes to the tower and several grounding conditions. The main objective of the work is to find the front time of a triangular function that results on the same overvoltage peak produced by a Heidler function considering the same current peak. Thus, an equivalence equation between the front times is determined, providing a simplified method for the modelling and simulation of lightning currents. The main advantage of using this equivalency is the reduction of the computational efforts demanded in a series of simulations.

**Keywords**— front time; lightning current; Heidler function; triangular function; lightning overvoltages; and transmisison lines.

## I. INTRODUCTION

The incidence of lightning strokes is responsible for a great number of nonscheduled power system outages, and may result in material, economic and social losses for both consumers and utilities [1, 2]. In order to determine the effects of lightning on electrical systems and to find alternatives to mitigate its effects, many studies have been carried out using full and reduced scale models, analytical calculations and/or simulations [3, 4, 5].

The analysis of lightning overvoltages and the estimation of the performance of transmission and distribution lines demand the utilization of models and equations capable of reproducing the behavior of the system elements. In particular, the modelling of the lightning current is of main importance, as it is the source of the overvoltages. The Heidler function [6,7] is widely used for this purpose, as it can reproduce typical first and subsequent stroke current waveforms. One of its main advantages is the possibility of adjusting almost independently the parameters of the wave front and wave tail, as well as the concavity in the beginning of the wave [8].

However, when a high number of iterative processes and simulations are demanded, the use of a Heidler function may result in long simulation times. In such cases the use of a

simpler current function, such as a triangular waveform, that can reproduce the same overvoltage peaks with a smaller computational demand, could be advantageous.

The influence of the return stroke current waveform on the lightning performance of distribution lines was analyzed in [9]. The stroke current was simulated using the Heidler, the triangular, and the Cigrè functions, and the calculations were performed considering both direct and indirect lightning strokes. The authors concluded that the use of the triangular waveform results in a conservative assessment of the lightning performance of distribution lines and recommended it for insulation coordination studies, in special those based on the Monte Carlo method.

The impact of the use of a triangular and a double-peaked waveform to evaluate the lightning performance of transmission lines directly hit at the overhead ground wire (OHGW) was assessed in [10]. The authors concluded that the use of the triangular waveform results in higher estimates of the outage rates when the insulator strings are modelled using the Disruptive Effect method [11].

The main objective of this paper is to find a relationship between the front times of the currents represented by the triangular and the Heidler functions so that overvoltages with the same amplitudes are produced across the insulator strings of a typical 230 kV transmission line hit by lightning.

## II. DEVELOPMENT

### A. Line Characteristics and Modelling

The line represented in the simulations is a 230 kV real transmission line located between the cities of Arquimedes and Ji-Paraná, both in the northern part of Brazil. The line topology and its main parameters are presented in Fig. 1 and in Table I.

The simulations were run using the EMTP-RV [12], with the line being represented by a frequency dependent model (FD Line). The tower was simulated as a lossless transmission line with constant parameters and surge impedance ( $Z_{tower}$ ) estimated according to [13].

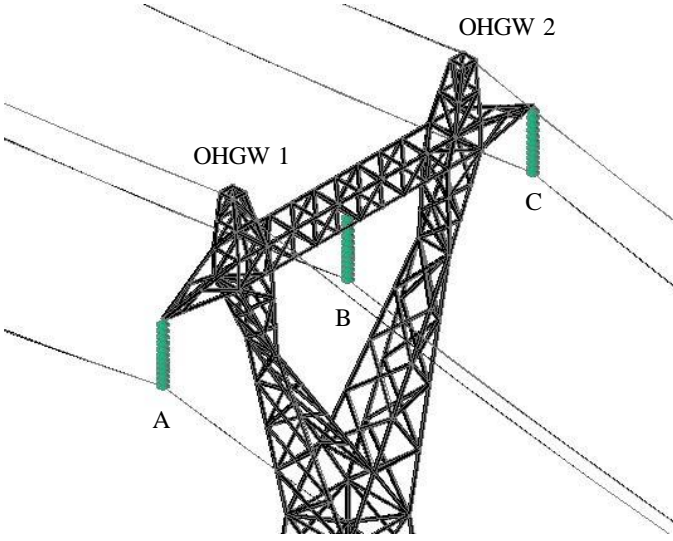


Fig. 1. Topology of the 230 kV transmission line considered in this study.

TABLE I. DATA OF THE 230 kV TRANSMISSION LINE CONSIDERED IN THIS STUDY.

Lina Data					
Conductor	OHW1	OHW2	A	B	C
Height [m]:	30.5	30.5	25.03	25.03	25.03
Distance [m]:	-4	4	-6	0	6
Diameter [mm]:	9.53	9.53	25.15	25.15	25.15
R [ $\Omega$ /km]:	3.44	3.44	0.0897	0.0897	0.0897
Sag [m]:	15.9	15.9	16.79	16.76	16.79
Span [m]:	400				
$\rho$ [ $\Omega$ .m]:	500 and 4000				
$Z_g$ [ $\Omega$ ]:	50 up to 300, step of 50				
$Z_{tower}$ [ $\Omega$ ]:	164.41		$v$ [m/ $\mu$ s]:	240 (0.8c)	

The grounding system was modelled by means of its impulse impedance ( $Z_g$ ), as demonstrated in [14]. Corona and soil ionization effects were not considered.

### B. Current Characteristics and Modelling

For the analysis, 18 Heidler waveforms were used, with the equivalent front time  $t_{d30-90}$  [8, 15] varying from 1.5  $\mu$ s up to 10  $\mu$ s, with a step of 0.5  $\mu$ s. This range of  $t_{d30-90}$  comprises 90% of the first stroke currents [8, 15]. The currents had a fixed peak value of 30 kA and tail time of 90  $\mu$ s. The simulations aimed at finding the front time of a triangular current waveform that produced the same peak voltage across the insulator string of Phase A (Fig. 1) for the case of a lightning strike to the OHGW 1, at the tower. The current peak and tail time of the triangular waveforms were kept constant and equal to those of the Heidler waveforms (30 kA and 90  $\mu$ s, respectively). The lightning current was represented in the simulations by means of an ideal current source, according to [8].

### C. Simulations

The simulated transmission line had 10 line spans, representing 11 towers, plus a 10 km long span on both ends to avoid reflections. This number of spans guarantee that, for the

time of 10  $\mu$ s (maximum simulated time), the reflections produced at the other towers (not considered in the simulations) have a negligible effect on the overvoltages calculated across the insulator string of Phase A in Fig. 1. The lightning current was always injected into the OHGW 1 of the central tower. Fig. 2 shows the representation of the central tower and the adjacent spans, wherein this pattern is replicated.

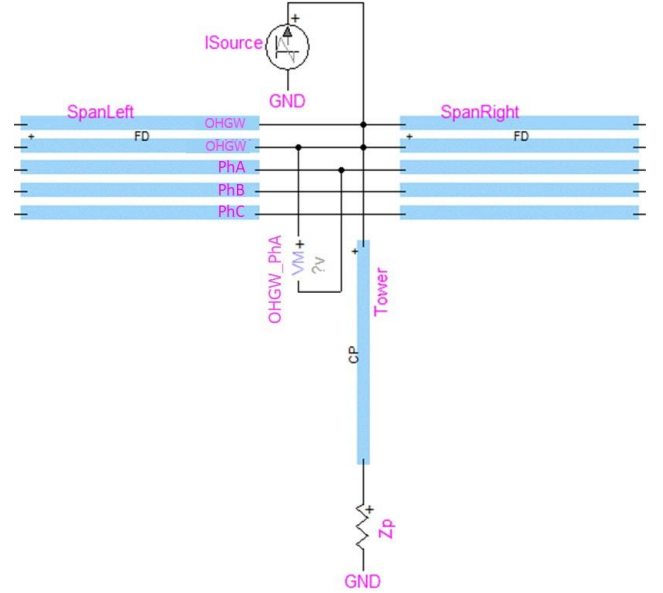


Fig. 2. Model of the 230 kV transmission line.

For all the 18 front times ( $t_H$ ) of the Heidler currents, the front times of the triangular current waveforms ( $t_A$ ) that produce the same overvoltage peaks across the insulator string of Phase A were determined. The simulations were run for the values of soil resistivity ( $\rho$ ) and ground impulse impedance ( $Z_g$ ) shown in Table I, so that 216 situations were considered.

## III. RESULTS AND ANALYSIS

Fig. 3 shows the curves of the triangular front times ( $t_r$ ) as a function of the Heidler front times ( $t_H$ ) for the various values of  $Z_g$  considered. As the simulation results indicated that the soil resistivity has no significant influence on  $t_r$  for the values of ground impulse impedance considered (the maximum observed difference was about 6.3%), each curve in Fig. 3 is the average of the results obtained for the resistivities of 500  $\Omega$ .m and 4,000  $\Omega$ .m. It can be seen that, for  $t_H$  longer than about 4.5  $\mu$ s, the differences between the curves corresponding to the various impulse impedances tend to increase, especially in relation to the curve of  $Z_g = 50 \Omega$ . The average curve, representing the 216 situations considered, is shown in Fig. 4 along with the corresponding trend curve. The determination factor ( $R^2$ ) is 0.9832.

The relationship between the front times of the triangular and Heidler current waveforms, in the range 1.5  $\mu$ s – 10  $\mu$ s, is given by:

$$t_r = 0.0627 t_H^2 + 0.3459 t_H + 1.6481 \quad (1)$$

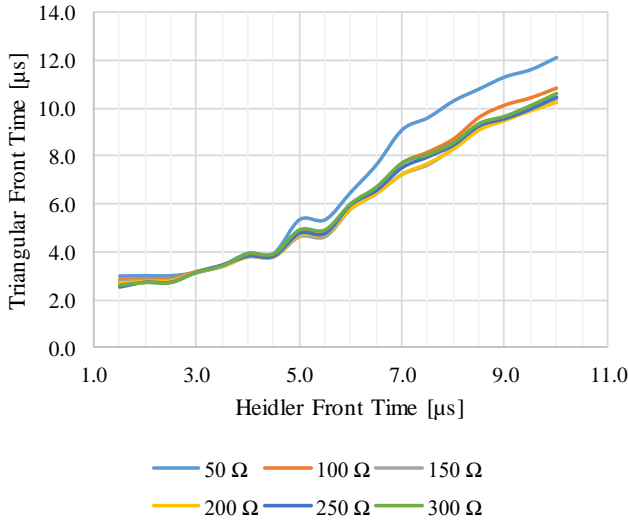


Fig. 3. Equivalence between Heidler and triangular current front times for different values of the ground impulse impedance  $Z_g$ .

Then, in order to check whether (1) could be used in studies related to transmission lines with characteristics similar to the 230 kV line considered in this analysis, the simulations of the 216 situations mentioned in subsection II.C were carried out again with the front times of the triangular current waveforms obtained from (1). The deviations between the peak values of the overvoltages calculated using the triangular and the Heidler waveforms, averaged for the two soil resistivities considered, are presented in Fig. 5.

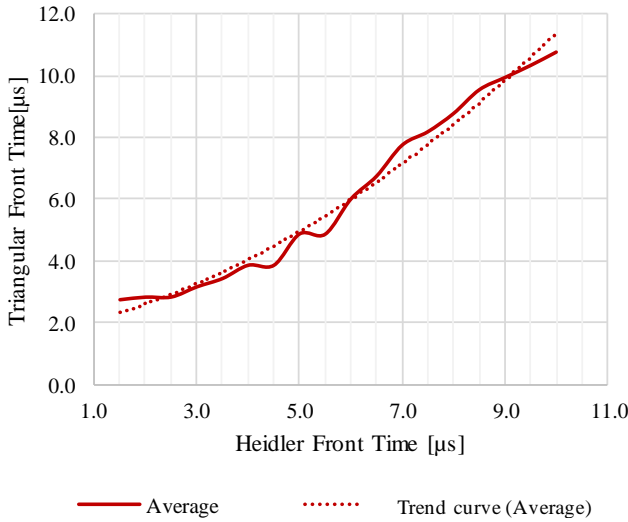


Fig. 4. Average curve of the equivalence between Heidler and triangular front times and the corresponding trend curve given by (1) ( $R^2 = 0.9832$ ).

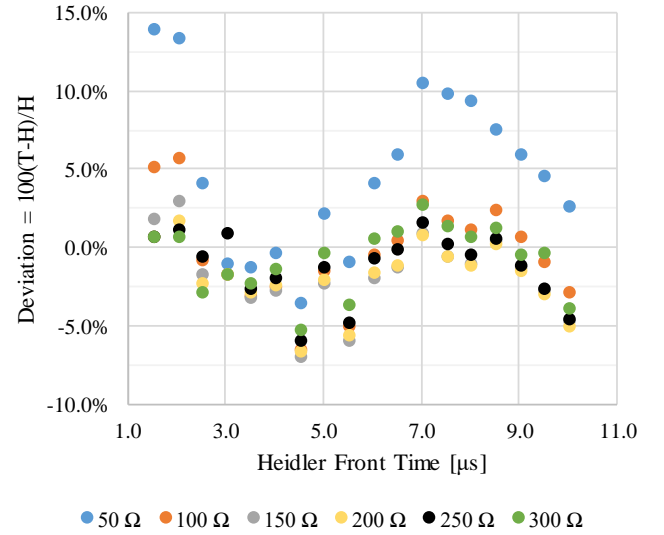


Fig. 5. Deviations (%) between the peak values of the overvoltages calculated using the triangular and the Heidler current waveforms. Front times of the triangular waveforms calculated according to (1).

Fig. 5 shows that most of the deviations are within  $\pm 5\%$ , although higher values are obtained for  $Z_g = 50 \Omega$ . The total range goes from about -15% to +7.5%. The mean deviations for the ground impulse impedances  $Z_g$  are presented in Table II, where it can be seen that for ground impedances equal or higher than  $100 \Omega$  the mean deviations are smaller than 1.9%.

TABLE II. MEAN DEVIATIONS BETWEEN THE PEAK VALUES OF THE OVERVOLTAGES CALCULATED USING THE TRIANGULAR AND THE HEIDLER CURRENT WAVEFORMS. FRONT TIMES OF THE TRIANGULAR WAVEFORMS CALCULATED ACCORDING TO (1).

$Z_g$	50 $\Omega$	100 $\Omega$	150 $\Omega$	200 $\Omega$	250 $\Omega$	300 $\Omega$
Mean	-4.89%	0.22%	1.70%	1.82%	1.12%	0.66%

It is important to emphasize that the analysis considered only the voltage peak, not the waveform. As the front time  $t_f$  is the only adjustable parameter and the current peak is fixed for both the triangular and Heidler functions, in many cases the overvoltages do not present similar waveforms. As known, the occurrence of an insulator flashover depends not only on the voltage magnitude but also on its waveform. Thus, the use of (1) is recommended specially for studies that adopt simplified models to assess the lightning performance of transmission lines, as for instance studies in which insulator strings are simulated by voltage-dependent switches.

The shorter the stroke current front time, the shorter the difference between the times at which the overvoltages calculated with the triangular and Heidler functions reach their peak values. Fig. 6 and Fig. 7 show the voltages corresponding to the Heidler waveform with front times  $t_H$  of 10  $\mu s$  and 6  $\mu s$ , respectively. The overvoltages on phase A and on the overhead ground wire, corresponding to the Heidler waveform with front time  $t_H$  of 2  $\mu s$ , are depicted in Fig. 8.

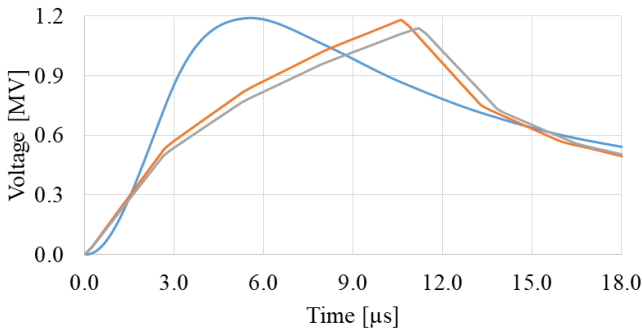


Fig. 6. Overvoltages corresponding to the Heidler waveform with front time of 10  $\mu$ s, soil resistivity of 4,000  $\Omega$ .m, and ground impulse impedance of 300  $\Omega$ . H: Heidler waveform; T: triangular waveform; T1: triangular waveform with the front time obtained from (1).

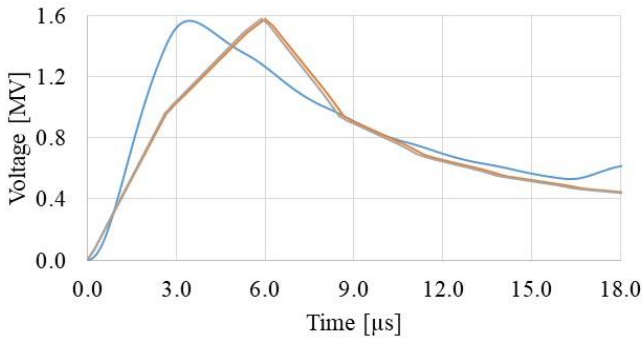


Fig. 7. Overvoltages corresponding to the Heidler waveform with front time of 6  $\mu$ s, soil resistivity of 500  $\Omega$ .m, and ground impulse impedance of 300  $\Omega$ . H: Heidler waveform; T: triangular waveform; T1: triangular waveform with the front time obtained from (1).

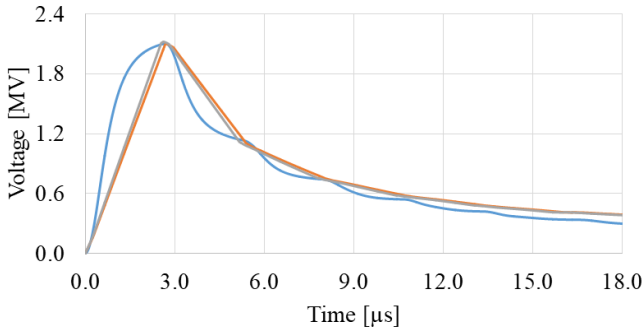


Fig. 8. Overvoltages corresponding to the Heidler waveform with front time of 2  $\mu$ s, soil resistivity of 4,000  $\Omega$ .m, and ground impulse impedance of 300  $\Omega$ . H: Heidler waveform; T: triangular waveform; T1: triangular waveform with the front time obtained from (1).

#### IV. CONCLUSIONS

The main objective of this paper was to find a relationship between the front times of stroke currents represented by the triangular and the Heidler functions. The equivalent front time of the triangular waveform is the one which produces an

overvoltage, across the insulator string of a typical 230 kV transmission line, with the same peak value of the voltage produced by a Heidler function with the same amplitude.

An equation relating the front times of the triangular and Heidler functions was obtained which is valid for current front times in the range 1.5  $\mu$ s – 10  $\mu$ s, soil resistivities between 500  $\Omega$ .m and 4,000  $\Omega$ .m, and ground impulse impedance from 50  $\Omega$  to 300  $\Omega$ . The deviations between the peak values of the overvoltages calculated using the proposed equation for the front time of the triangular waveform and the Heidler current waveform were +15% to -7.5%, with most of them within  $\pm$  5%. The mean deviations were lower than 5% in the whole range of impulse impedance considered and smaller than 1.9% for ground impedances equal or higher than 100  $\Omega$ .

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