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Transient Analysis of Buried Cables Considering a Nodal Admittance Matrix Approach

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***Abstract*— In this paper, the transient behavior of buried bare and insulated cables is analyzed using transmission line (TL) theory. The proposed approach is based on a passive pole-residue model of the nodal admittance matrix, which can be represented as an electrical network suitable to time-domain simulations in electromagnetic transients programs. The influence of a finitely conducting soil is computed by means of Sunde's ground return impedance equation. An approximate expression is considered to represent the ground admittance. Furthermore, the frequency dependence of the electrical parameters of the soil is considered. The proposed methodology is validated by comparing the results it provides with those obtained by a popular electromagnetic transient program and a field theory approach. The results show that ignoring the ground admittance and the frequency dependence of soil parameters can lead to inaccuracies in transient studies involving buried cables, especially in the case of high-resistivity soils and transients with a wide frequency range.**

Keywords*— *transmission line theory, ground return impedance, ground admittance, frequency dependence of soil electrical parameters, buried cables, transient analysis.

I. Introduction

Bare or insulated cables are commonly found in underground electrical power systems [1], in transmission line grounding (counterpoise wires) [2], and in the interconnection of remote grounding systems in wind farms [3]. All these systems are prone to transients due to lightning, switching or other types of electromagnetic interference [1]. Since lightning currents have high-frequency content, the frequency-dependent behavior of bare or insulated cables buried in the ground needs to be properly characterized for an accurate characterization of ensuing overvoltages.

Transient analysis in underground cable systems is frequently performed with transmission line (TL) theory [4]. This is probably due to the fact that cable models based on TL theory are readily available in popular electromagnetic transient (EMT) programs. This enables the analysis of complex systems involving multiple cables connected to other equipment.

In TL theory, the influence of a finitely conducting soil is described by the ground return impedance and the ground admittance [4]. However, the cable models usually available in EMT-type programs use ground-return impedance expressions that are sufficiently accurate only at low frequencies. Also, such programs ignore the ground admittance [4]. Furthermore, the frequency dependence of the electrical parameters of the soil is neglected [5]. All these approximations can lead to inaccuracies in transient studies that involve a wide frequency range.

Considering this scenario, this paper proposes a model based on the nodal admittance matrix to calculate transients on bare and insulated cables buried in the ground. The proposed model is used to assess the accuracy of the cable models available in EMT-type programs by evaluating the influence of the ground admittance and of the dispersive nature of the soil parameters on the propagation of transient overvoltages. The proposed model can be easily implemented as a user-defined component in EMT-type programs.

This paper is organized as follows. Section II presents the system under study along with the general equations that describe the proposed cable modeling methodology. In Section III, the proposed cable model is validated. In Section IV, the inaccuracies and the applicability of cable models readily available in EMT-type programs are assessed. Section V investigates the validity of the proposed model for the particular case of a bare cable buried in the ground. Finally, conclusions are given in Section VI.

II. Modeling

A. System Geometry

The bare and insulated cables buried in the ground considered in this study are shown in Fig. 1. The soil is characterized by its conductivity (σ_g), permittivity ($\varepsilon_g=\varepsilon_{rg}\varepsilon_0$), and permeability ($\mu_0$). The cable insulation has a permittivity $\varepsilon_{in}=\varepsilon_{rin}\varepsilon_0$. The burial depth is d and the cables have a total length D. The radius of the inner conductor is a and its resistivity is ρ_c; in the case of the insulated cable, the outer radius is b, considering the insulation thickness. In all analysis, the following parameters are assumed: a=0.58 cm, b=0.76 cm, d=1 m, D=100 m, ρ_c=1.7×10^{-8} Ωm and ε_{rin}=2.3, for different values of soil conductivity.

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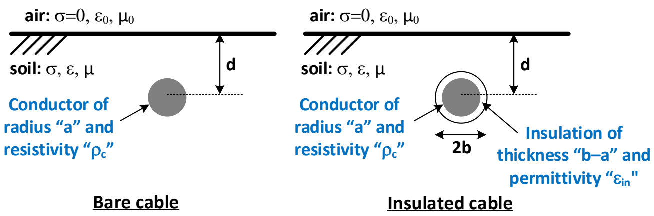

Fig. 1. Bare (a) and insulated (b) cable buried in the ground.

The per-unit-length impedance Z and admittance Y of the insulated cable are given by (1) and (2), respectively.

$$Z = Z_i + Z_e + Z_g \tag{1}$$

$$Y = \frac{j\omega C \times Y_g}{j\omega C + Y_g} \tag{2}$$

For an insulated cable, the impedance given by (1) is the sum of the internal impedance Z_i due to the magnetic field within the conductor, the external impedance $Z_e = j\omega L$ due to the magnetic field within the insulation, and the ground return impedance Z_g due to the magnetic field penetration in the soil [1]. The admittance given by (2) depends on the electric field within the insulation, whose effect is represented by the capacitance C, and on the ground admittance Y_g [1]. For a bare cable, the second term on the right-hand side of (1) is zero, that is, only Z_i and Z_g are relevant to the problem [4]. Also, the admittance given by (2) reduces to the ground admittance [4].

The per-unit-length internal impedance of the conductor is given by [6]

$$Z_i = \frac{\rho_c m}{2\pi a} \frac{I_0(ma)}{I_1(ma)} \tag{3}$$

where $I_0(\cdot)$ e $I_1(\cdot)$ are modified Bessel functions of the first kind and m is calculated as $m = \sqrt{j\omega\mu_0/\rho_c}$.

The per-unit-length inductance and capacitance due to the insulation are calculated using [6]

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \tag{4}$$

$$C = \frac{2\pi\varepsilon_{in}}{\ln\left(\frac{b}{a}\right)} \tag{5}$$

In this paper, the ground return impedance is calculated either with Ametani's approximation of Pollaczek's equation [7] or Sunde's expressions [8], discussed in Sections III and IV. The ground admittance is determined as in [4].

B. Proposed Model

In this paper, it is proposed to calculate the transient response of the bare and insulated cables shown in Fig. 1 using a technique based on the nodal admittance matrix, given by [6]

$$\mathbf{Y_n} = \begin{bmatrix} Y_c\left(1+A^2\right)\left(1-A^2\right)^{-1} & -2Y_c A\left(1-A^2\right)^{-1} \\ -2Y_c A\left(1-A^2\right)^{-1} & Y_c\left(1+A^2\right)\left(1-A^2\right)^{-1} \end{bmatrix} \tag{6}$$

where Y_c is the characteristic admittance and A is the propagation matrix, calculated as follows

$$Y_c = \sqrt{Y/Z} \tag{7}$$

$$A = \exp\left(-D\sqrt{ZY}\right) \tag{8}$$

The nodal admittance matrix is a two-port model of a transmission line or cable. It is obtained from the exact frequency-domain solution of telegrapher's equations, and can be viewed as the mathematical representation of the equivalent-pi circuit that describes the relationship between the voltages and currents at both cable ends at a given frequency. Here, it is calculated in a wide frequency range and then fitted as a pole residue model of the form (9) using the vector fitting technique [9] assuming a common set of poles. The model passivity is enforced by using the technique presented in [10]. From the obtained passive pole-residue model, an electrical circuit that can reproduce the frequency response of the nodal admittance matrix is finally determined using the technique described in [11] and implemented in the Alternative Transients Program (ATP). In (9), $\mathbf{R}_m$ is the matrix of residues, N is the order of approximation, and $\mathbf{D}$ and $\mathbf{E}$ are real matrices. All matrices are of order 2 × 2 because a single cable is considered.

$$\mathbf{Y_n}(s=j\omega) \cong \mathbf{Y_{fit}}(s=j\omega) = \sum_{m=1}^{N} \frac{\mathbf{R}_m}{s-a_m} + \mathbf{D} + s\mathbf{E} \tag{9}$$

III. Comparing the Proposed Approach with Classical ATP Cable Modeling

This section compares results obtained with the approach proposed in Section II-B with those obtained using the cable model available in ATP, simulated with the frequency-dependent model of Marti [12]. In this model, Ametani's approximation to Pollaczek's integral (10) is assumed to represent the ground return impedance [7]. In this equation, $K_0(\cdot)$ e $K_1(\cdot)$ are modified Bessel functions of the second kind, $r_o = b$ is the external radius of the cable, and $\gamma_g = (j\omega\mu_g\sigma_g)^{1/2}$. For consistency in the comparison, this same equation was considered in the calculation of (6) in the proposed model.

$$Z_g = \frac{j\omega\mu_0}{2\pi}\left[K_0\left(\gamma_g r_o\right) - K_0\left(2\gamma_g h\right) + 2\int_0^\infty \frac{\exp\left(-2h|\lambda|\right)}{|\lambda| + \sqrt{\lambda^2 + \gamma_g^{\,2}}} \cos\left(\lambda r_o\right) d\lambda\right], \tag{10}$$

The ground return impedance given by (10) implies that $\sigma_g >> \omega\varepsilon_g$. In other words, it assumes that the conductive current in the soil are much larger than the displacement current. This might be not true for phenomena exciting a wide frequency range or for low-conductivity soils. Also, the underground cable model available in ATP ignores the ground admittance and the frequency dependence of the electrical parameters of the soil. Therefore, the comparisons presented in this section must be viewed only as a means to validate the solution method proposed in Section II-A for the particular conditions implicitly assumed the ATP model.

The nodal admittance matrix (6) was calculated from 1 Hz to 10 MHz considering the ground impedance given by (10), assuming constant soil parameters, and ignoring the ground admittance. A pole residue model with N=100 was obtained, from which an electrical network was synthesized and included in ATP through $INCLUDE statements.

Fig. 2 illustrates the voltages calculated at the receiving end of the 100-m long insulated cable when it is subjected to a unit step voltage at the sending end. An ideal voltage source was connected at the sending end, while the receiving end was left open. Four values were assumed for the soil resistivity, given by $\rho_g = 1/\sigma_g$, namely 100, 300, 1000 and 3000 Ωm.

It is seen in Fig. 2 that the voltages calculated using both models are in excellent agreement. This shows that the proposed approach is able to reproduce the results obtained in ATP with Marti's model. Interestingly, the voltage waveforms are nearly insensitive to the soil resistivity. This may be associated with the simplifications adopted in the ATP cable model, which neglects the displacement current in soil, the soil admittance, and the frequency-dependent nature of the soil electrical parameters.

IV. Underground Cable Simulation with a More Rigorous Soil Representation

To overcome some of the limitations associated with Pollaczek's expression, Sunde's ground return impedance expression can be used, which is given by [8]

$$Z_g = \frac{j\omega\mu_0}{2\pi}\left[K_0\left(\gamma_g r_o\right) - K_0\left(2\gamma_g h\right) + 2\int_0^\infty \frac{\exp\left(-2h\sqrt{\lambda^2+\gamma_g^2}\right)}{|\lambda| + \sqrt{\lambda^2+\gamma_g^2}}\cos\left(\lambda r_o\right) d\lambda\right], \qquad (11)$$

where $\gamma_g = \sqrt{j\omega\mu_0\left(\sigma_g + j\omega\varepsilon_g\right)}$ is the intrinsic ground propagation constant. The ground admittance can be obtained from the ground impedance using the approximate expression [4], [1]

$$Y_g \approx \frac{\gamma_g^2}{Z_g} \quad . \qquad (12)$$

Considering a wide frequency range, the electrical parameters of the soil cannot be assumed constant. In this paper, the frequency-dependent nature of the soil electrical parameters is determined according to the causal model proposed by Alipio and Visacro [5]. The model equations read

$$\sigma_g = \sigma_0 + \sigma_0 \times h\left(\sigma_0\right)\left(\frac{f}{1\ \text{MHz}}\right)^{\zeta} \qquad (13)$$

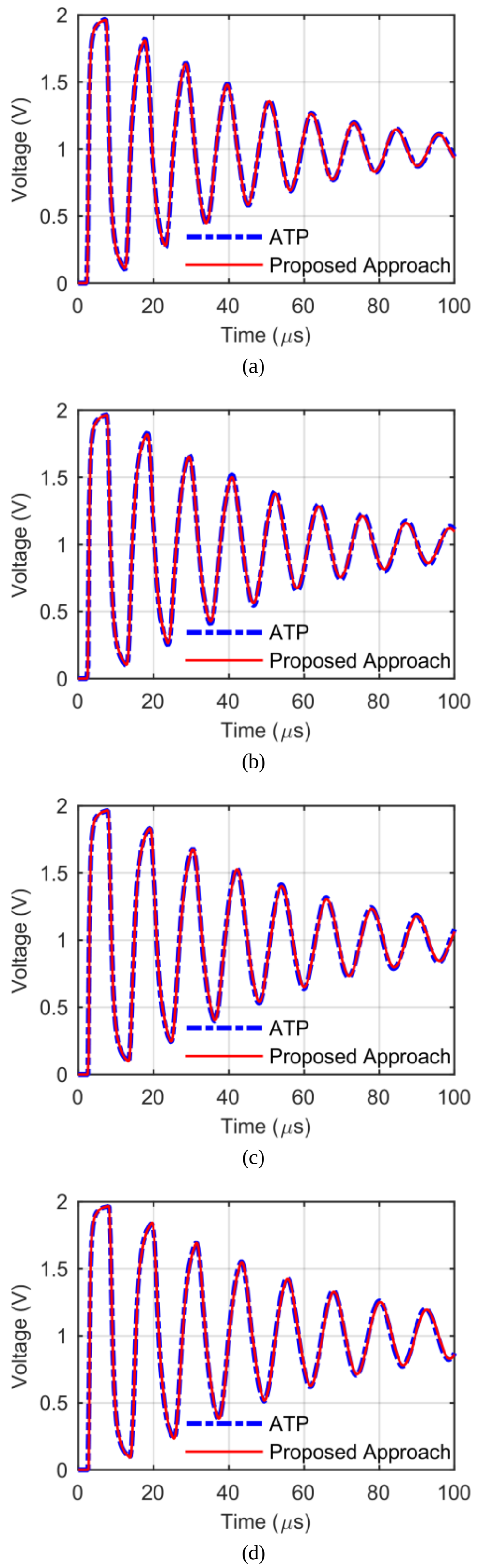

Fig. 2. Voltages at the receiving end of a 100-m long buried insulated cable for the application of a unit step voltage at the sending end with the receiving end left open, considering ATP and the proposed approach, both assuming Pollaczek's equation (10) for the ground return impedance: (a) ρ_g = 100 Ωm; (b) ρ_g = 300 Ωm; (c) ρ_g = 1000 Ωm and (d) ρ_g = 3000 Ωm.

$$\varepsilon_g = \varepsilon'_\infty + \frac{\tan(\pi\zeta/2)\times 10^{-3}}{2\pi(1\text{MHz})^{\zeta}}\sigma_0 \times h(\sigma_0) f^{\zeta-1} \tag{14}$$

In (13) and (14), σ_g is the soil conductivity in mS/m, $\sigma_0=1/\rho_0$ is the DC conductivity in mS/m, ε_g is the soil permittivity, ε'_∞ is the soil permittivity at high frequencies, and f is the frequency in Hz. According to [5], the following parameters are recommended in (13) and (14) to obtain mean results for the frequency variation of σ_g and ε_g: $\zeta=0.54$, $\varepsilon'_\infty=12\varepsilon_0$ and $h(\sigma_0)=1.26\times\sigma_0^{-0.73}$, where ε_0 is the vacuum permittivity.

Once again, the nodal admittance matrix was calculated in a frequency range from 1 Hz to 10 MHz considering the ground impedance and ground admittance as given by (11) and (12), respectively. The calculation also considered frequency-dependent soil parameters as given by (13) and (14). A pole residue model with N=100 was obtained, from which an electrical network was synthesized and included in ATP simulations.

Fig. 3 illustrates the voltages calculated at the receiving end of the insulated cable when it is subjected to a unit step voltage at the sending end, considering the receiving end open. Four values were considered for the DC soil resistivity, namely 100, 300, 1000 and 3000 Ωm. For comparison purposes, voltage waveforms calculated with the cable model available in ATP and discussed in the previous section are also included in the figures.

By analyzing the voltage waveforms illustrated in Fig. 3, it is observed that the results obtained using the proposed methodology considering Sunde's formula and the ground admittance differ significantly from those calculated using ATP. These differences become more expressive with the increase of the soil resistivity. In the proposed model, the voltage waves travel faster and are more attenuated, especially for poorly conducting soils. From these results, it can be inferred that cable models usually available in popular electromagnetic transients programs should be avoided in the analysis of transients with large frequency content or in the case of high-resistivity soils.

V. Bare Cable Buried in The Ground

Sunde's formulation presented in Section IV can also be used to simulate a bare cable buried in the ground. In this case, $r_o=a$ in expression (11). Also, as discussed in Section II, the total impedance is composed only of the internal impedance and the ground return impedance, whereas the total admittance is composed only of the ground admittance. Once again, the nodal admittance was calculated from 1 Hz to 10 MHz considering frequency-dependent soil parameters. Then, a pole residue model was fitted considering 100 poles and an electrical network was synthesized and implemented in ATP.

To assess the accuracy of the results obtained with the proposed methodology, the transient response of the bare cable was also simulated using a MATLAB implementation of the Hybrid Electromagnetic Model (HEM) [13]. The calculations were performed in frequency domain, allowing the direct inclusion of frequency-dependent soil parameters.

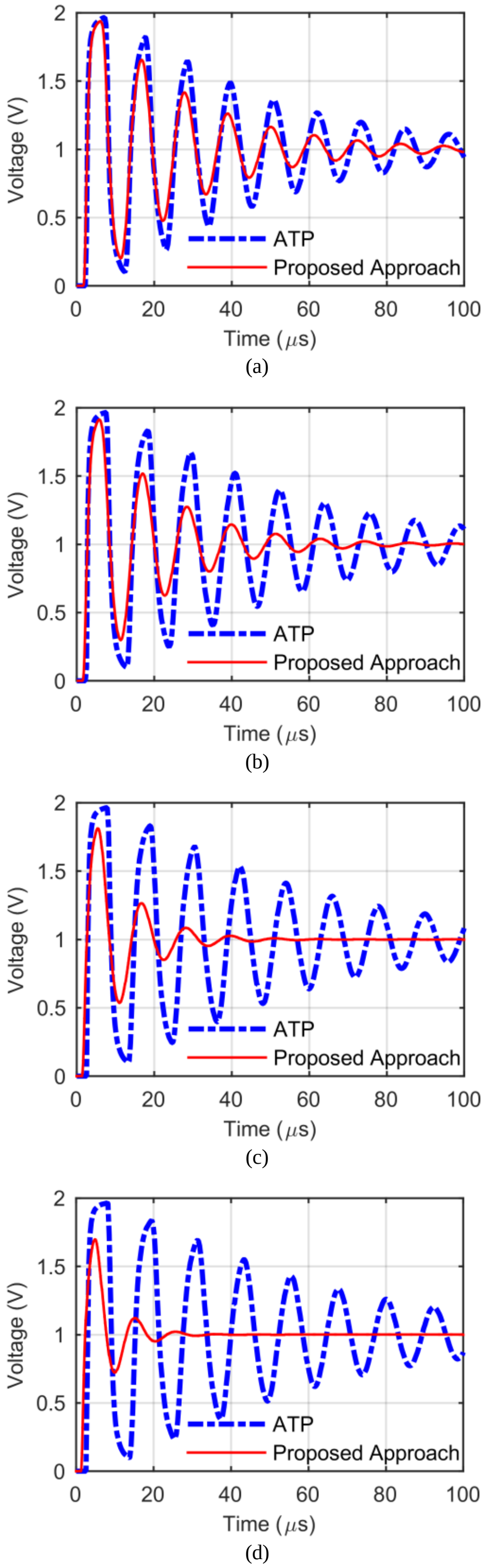

Fig. 3. Voltages at the receiving end of a 100-m long buried insulated cable for the application of a unit step voltage at the sending end with the receiving end left open, considering ATP and the proposed approach, the latter based on the Sunde's theory for the ground impedance (11), including the ground admittance and the frequency dependence of the electrical parameters of soil. (a) ρ_0=100 Ωm; (b) ρ_0=300 Ωm; (c) ρ_0=1000 Ωm and (d) ρ_0=3000 Ωm.

Fig. 4 illustrates the voltages calculated at the receiving end of the considered 100-m long bare cable buried in the ground, subjected to a unit step voltage at the sending end, while the receiving end was left open. Once again, four values were assumed for the DC soil resistivity, namely 100, 300, 1000 and 3000 Ωm.

The results shown in Fig. 4 show a very good agreement between the voltages calculated using the proposed model and the HEM model. This is an interesting result, especially considering that the proposed representation of the bare cable is very simple and can be easily included in EMTP-type programs.

It is also seen in Fig. 4 that, for the bare cable buried in the soil, propagation effects (attenuation and distortion) are much more significant compared to the case of the insulated cable. This is expected since in this case the conductor is in direct contact with soil. Also, due to stronger attenuation, the voltage waveforms do not show the oscillatory behavior observed in the case of the insulated cables.

VI. Conclusions

In this paper, a model based on the nodal admittance matrix was proposed to simulate the propagation of transient voltages along bare and insulated cables buried in the soil. This model was used to test the accuracy of the insulated cable model available in ATP. It was shown that the low-frequency approximation of the ground-return impedance is not appropriate for transient analysis in the case of insulated cables buried in high-resistivity soils. It was also shown that the ground admittance and the frequency dependence of the electrical parameters of the soil cannot be disregarded in transient analysis, as usually done in EMT-type programs. Finally, the proposed representation for bare cables provided good results taking as reference an approach directly based on the field theory. The proposed models can be easily implemented in EMT-type programs and take advantage of such platforms for an efficient time-domain simulation of complex systems.

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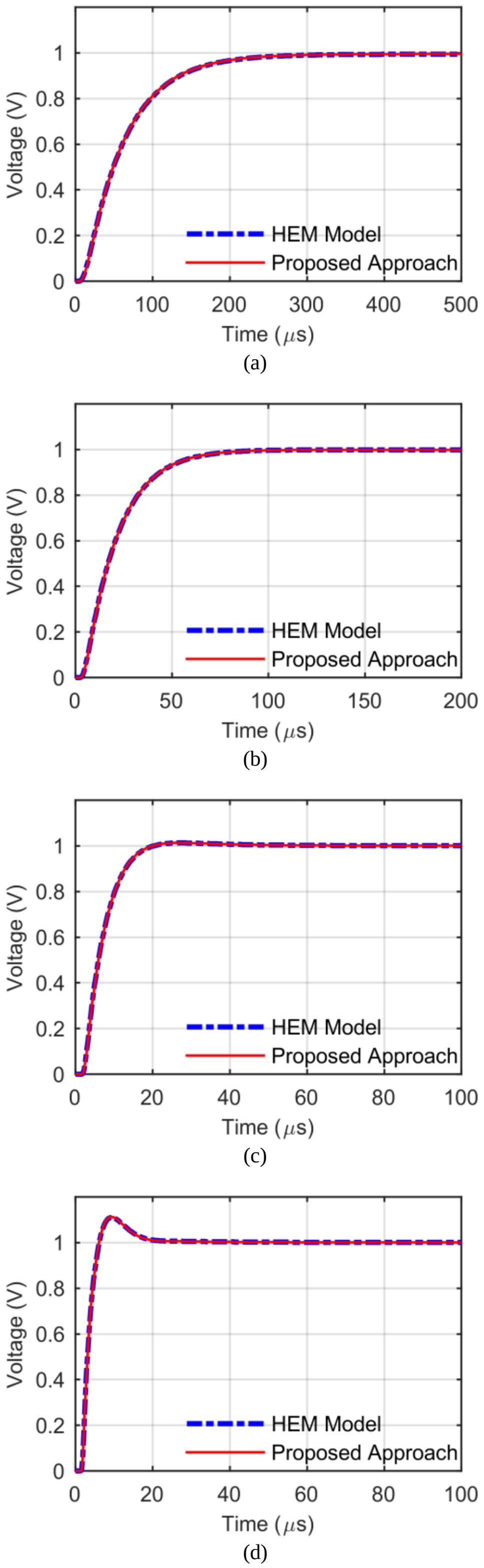

Fig. 4. Voltages at the receiving end of a 100-m long bare cable buried in the soil for the application of a unit step voltage at the sending end with the receiving end left open, considering the HEM model and the proposed approach, the latter based on Sunde’s theory for the ground return impedance (11), including the ground admittance and the frequency dependence of the electrical parameters of soil. (a) ρ_0=100 Ωm; (b) ρ_0=300 Ωm; (c) ρ_0=1000 Ωm and (d) ρ_0=3000 Ωm.

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