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Three Direction Finding Methods of Thunder Source Using Microphone Array

Yao YAO
State Grid Hubei Electric Power Research Institute
State Grid Hubei Electric Power Co., LTD.
Wuhan, China
jefferson_yao@163.com

Jingwei FU, Chao CAI, Fuzhou AI
State Grid Hubei Electric Power Co., LTD.
State Grid Corporation of China
Wuhan, China
fjw@hb.sgcc.com.cn

Han ZHANG, Zejun LU
Wuhan NARI Limited Liability Company
State Grid Electric Power Research Institute
Wuhan, China

***Abstract*—In this paper, we have investigated three methods, namely Generalized Cross-Correlation (GCC), Frequency Domain Beam-forming and Approximated Maximum Likelihood (AML), for estimating the direction of thunders measured with a microphone array. The performances of these methods are compared by simulations in terms of the error of angle estimation and also by signal experiments in terms of bi-dimensional representation, coherence measure and error point number for lightning channels measured simultaneously by a high-speed video camera. Comparison results show that both Beam-forming and AML obtain more accurate and complete direction finding than GCC, which is extremely consistent with the reconstructed lightning channel from the camera data. Moreover, Beam-forming and AML obtain additional information regarding thunder sources on top of the lightning channel, which cannot be observed in the camera data due to obstruction. However, the computation of AML is in general much more complicated than the other two methods. Therefore, we conclude that the Frequency Domain Beam-forming based technique suits future practical applications.**

Keywords—lightning; thunder; channel geometry; direction finding; GCC; Beam-forming; AML

I. INTRODUCTION

Audible thunder begins with a shockwave in air due to sudden thermal expansion of the plasma in lightning channel [1]. It is suggested that not only return strokes but also other events, such as a leader step and a K-process, could generate acoustic signals with amplitudes sufficiently large to be captured by microphone array [2]. The geometry of lightning channels can be reconstructed from thunder acoustic data obtained at three or more measuring stations [3]. Moreover, thunder sources can be tracked with acoustic signals, using two popular techniques known as thunder ranging and ray tracing [4]. The latter in general provides more accurate estimation of the thunder source and is capable of tracking more source locations [5].

To use the ray tracing technique, the thunder source direction of lightning channel is estimated using microphone arrays and then the directional ray is traced back to the acoustic source by the time difference between the arrivals of the lightning electromagnetic signal and the acoustic signal [6]. Therefore, a key component of accurate and quick ray tracing is how to efficiently obtain the direction of thunder sources. The direction of acoustic sources on the lightning channel is determined from the time differences of thunder pulses received by an array of microphones. Few [6] proposed an algorithm involving Cross-Correlation Function to estimate the time differences, which was applied and further developed by Nakano [7], [8], Teer [9], Nakamura[10], MacGorman [11], Zhang [12] and Bodhikaj [13].

It is shown in [14] that the channel reconstruction is difficult in the case where the acoustic signals with comparable amplitudes from different channel sections arrive at the microphone array approximately at the same time. Moreover, another disadvantage with the cross-correlation function algorithm is that the estimated time lags corresponding to peaks in the correlation function are very difficult to study from a statistical point of view. To address these issues, a new algorithm called Cross-Power Spectrum (CPS) is developed to calculate the time lags, which could distinguish different channels with different frequencies of acoustic waves [15].

In this paper, we investigate two novel algorithms, namely Frequency Domain Beam-forming and Approximated Maximum Likelihood (AML), to obtain the directions of acoustic sources over the lightning channels. We compare these two novel algorithms with the frequently-used method using both analog signals and real thunder signals. The error of angle estimation is analyzed in the simulation experiment. Moreover, all of these algorithms are applied to the same lightning acoustic signals to reconstruct the lightning channels and we compare these results with the channels reconstructed stereo-graphically from photographs taken by high-speed camera. Results in terms of the accuracy, location number and timeliness show a definite superiority of the Frequency Domain Beam-forming based technique.

II. METHODOLOGY

We measure the acoustic thunder signals with a microphone array consisting of four microphones A, B, C and O which are located at A(0,0,1m), B(0,1m,0), C(1m,0,0) and

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O(0,0,0) as shown in Fig. 1. Then, the aforementioned three algorithms are applied to estimate the directions of the acoustic sources. The details of these algorithms are described as follows.

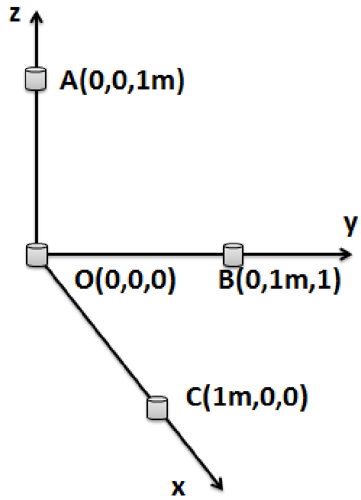


Fig. 1. The geometry of positions of microphones

A. *Generalized Cross-Correlation Algorithm*

The estimation algorithm of time delay is one category of well-studied DOA (Direction of Arrival) estimation method, which has many important applications in different areas such as radar/sonar signal processing, speech signal processing, etc. Generalized Cross-Correlation (GCC) algorithm is the most classical among this category, which is widely used in practice because of its low complexity and robustness. The time difference of arrivals to different microphones can be estimated with GCC algorithm and then the directions of the sources are obtained based on the relative position of the microphone array adopting the least squares method.

In general, the signals at different microphones can be written as:

$$\begin{aligned} x_1(t) &= s(t) + n(t) \\ x_2(t) &= s(t - \tau_{12}) + n(t) \end{aligned} \tag{1}$$

Where x_i(t) represents the signal measured by the ith microphone, $s(t)$ represents the target source signal and $n(t)$ represents the additive noise. Then the cross correlation function of the two microphone measurements is:

$$R_{12}(\tau) = \int_{-\infty}^{+\infty} x_1(t) x_2(t+\tau) dt \tag{2}$$

Applying Fourier transform and using the properties of cross-correlation function (2) can be rewritten as:

$$R_{12}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1(\omega) X_2(\omega)^* e^{j\omega\tau} d\omega \tag{3}$$

Where $X_i(\omega)$ represents the Fourier transform of $x_i(t)$.

Suppose the cross correlation function of different microphone measurements can be obtained, then, the time delay of the signals can be estimated as follows:

$$\tau_{12}' = \arg\max_{\tau \in D} R_{12}(\tau) \tag{4}$$

By iterating over all the microphone measurements, we can derive the time delay of different microphones in the array: $\{\tau_{ij}' | i \neq j,\ i,j = 1,2,\ldots,P\}$.

Suppose that the position in a predefined coordinate of the ith microphone is $\boldsymbol{r}_i = (x_i, y_i, z_i)$, then the time delay of the signals between the ith microphone and the jth microphone is:

$$\tau_{ij} = \frac{1}{c}\left[\left(x_i - x_j\right)\cos\varphi\cos\theta + \left(y_i - y_j\right)\sin\varphi\cos\theta + \left(z_i - z_j\right)\sin\theta\right] \tag{5}$$

Where c represents the acoustic speed, θ and φ represent the elevation and azimuth angles of the target source respectively. According to the previous analysis, θ and φ can be obtained by optimizing the cost function:

$$\left(\hat{\theta}, \hat{\varphi}\right) = \arg\max_{\theta \in [-90,90], \varphi \in [0,360]} \sum_{\substack{i,j=1 \\ i \neq j}}^{P} \left\| \hat{\tau}_{ij} - \tau_{ij}(\theta, \varphi) \right\| \tag{6}$$

B. *Approximated Maximum Likelihood Algorithm*

The broadband DOA estimation method based on the maximum likelihood algorithm is optimal, which usually assumes Gaussian white noise. One representative algorithm is Approximated Maximum Likelihood (AML) proposed by Kung [16]. In this algorithm, with the assumption that the broadband signal noises follow Gaussian distributions in each frequency band, one can obtain a cost function using maximum likelihood rule to estimate the azimuth and elevation of the sources. However, this technique becomes computationally intractable when the number of the target sources is more than two. This is due to the fact that the cost function can only be searched by nonlinear multidimensional optimization method. Meanwhile, the cost function should be nonlinear multidimensional searched for optimal results which increases the computational complexity. Therefore, this algorithm must be the most time-consuming among these three methods. Some recent work by Zhang [17] has suggested using greedy strategy to approximate the optimal solution.

The broadband signal model can be expressed as:

$$x_p(n) = \sum_{m=1}^{M} s_m(n - \tau_{pm}) + w_p(n) \tag{7}$$

Where n=0,…L-1, p=1,…P, m=1,…M, P represents the number of the microphones, M represents the number of the sources, s_m represents the mth acoustic signal, τ_{pm} represents the time in which the mth signal arrived at the pth microphone, $w_p(n)$ represents the Gaussian noise t with zero mean and variance σ_2.

The FFT transform of (7) can be expressed in matrix form:

$$\mathbf{X}(k) = \mathbf{A}(k)\mathbf{S}(k) + \boldsymbol{\eta}(k) \tag{8}$$

Where X(k)=[$X_1(k)$, …, $X_p(k)$]^T represents the value of the broadband signal in the frequency band of k, A(k)=[a$_1$(k), …,

$a_M(k)$] represents the orientation matrix, am(k)=[$e^{-2\pi f_s k/N\tau_{1m}}$, $e^{-2\pi f_s k/N\tau_{2m}}$,…, $e^{-2\pi f_s k/N\tau_{Pm}}$]^T represents the steering vector, $S(k)=[S_1(k), \ldots, S_M(k)]^T$ represents the spectrum of the acoustic signal.

According to the Central Limit Theorem, if the noise signals $\boldsymbol{w}(n)$ are independent and identically distributed, then regardless matter what distribution the noise signal $\boldsymbol{w}(n)$ obeys, $\boldsymbol{\eta}(k)$ should obey Gaussian distribution that with zero mean and variance $L\sigma^2$.

We define $\mathbf{X}(k)=\mathbf{A}(k)\mathbf{S}(k)$. By stacking up the $N/2$ positive frequency bins of the signal model into a single column, we can rewrite the sensor data into a space-temporal frequency vector as $\mathbf{X}=\mathbf{G}(\boldsymbol{\theta})+\xi$, where $\mathbf{G}(\boldsymbol{\theta})=[\mathbf{S}(1)^T, \ldots, \mathbf{S}(N/2)^T]^T$, and $\mathbf{R}_\xi=E[\xi\xi^H]= L\sigma^2 I_{NR/2}$. We assume, initially, that the unknown parameter space is $\boldsymbol{\theta}=[\theta,\varphi,\mathbf{S}_1^T,\ldots,\mathbf{S}_M^T]^T$. Given that $\boldsymbol{\eta}(k)$ follows complex Gaussian distribution, the likelihood function can be written as:

$$f(\Theta)=\frac{1}{(2\pi)^{NR/2/2}\left|\mathbf{R}_\xi\right|^{1/2}}\exp\left\{-\frac{1}{2}\left[\mathbf{X}-\mathbf{G}(\boldsymbol{\theta})\right]^H\mathbf{R}_\xi^{-1}\left[\mathbf{X}-\mathbf{G}(\boldsymbol{\theta})\right]\right\} \quad (9)$$

We calculate the logarithm of the likelihood function and ignore the constant term, then:

$$L(\Theta)=-\frac{\left\|\mathbf{X}-\mathbf{G}(\boldsymbol{\theta})\right\|^2}{\mathbf{R}_\xi} \quad (10)$$

Then the optimization criterion can be obtained:

$$\max_{\boldsymbol{\theta}} L(\boldsymbol{\theta})=\min_{\boldsymbol{\theta}}\sum_{k=0}^{N/2}\frac{\left\|\mathbf{X}(k)-\mathbf{A}(k)\mathbf{S}(k)\right\|^2}{\mathbf{R}_\xi(k)} \quad (11)$$

Where $R_\xi(k)=L\sigma^2$ is a constant, then we have:

$$\max_{\boldsymbol{\theta}} L(\boldsymbol{\theta})=\min_{\boldsymbol{\theta}}\sum_{k=0}^{N/2}\left\|\mathbf{X}(k)-\mathbf{A}(k)\mathbf{S}(k)\right\|^2 \quad (12)$$

After further simplification the cost function of the angle of the target sources can be obtained:

$$\max_{(\theta,\varphi)} J(\theta,\varphi)=\max_{(\theta,\varphi)}\sum_{k=0}^{N/2}\left\|\mathbf{P}\left[k,(\theta,\varphi)\right]\mathbf{X}(k)\right\|^2 \quad (13)$$

Where $\mathbf{P}[k,(\theta,\varphi)]=\mathbf{A}(k)\mathbf{A}(k)'$, $\mathbf{A}(k)'=[\mathbf{A}(k)^H\mathbf{A}(k)]^{-1}\mathbf{A}(k)^H$.

C. Frequency Domain Beam-forming Algorithm

Frequency Domain Beam-forming algorithm is a typical broadband DOA estimation method, which is particularly suitable for single target element DOA estimation because of its accuracy and robustness. This broadband signal beam-forming algorithm is essentially an extended method based on the narrowband beam-forming algorithm. The basic idea of Frequency Domain Beam-forming algorithm is to divide the broadband signal into a plurality of narrow-band signals by Fourier transform, and then adopt the narrowband beam-forming algorithm for each narrowband signal to generate DOA estimations. These estimations are averaged accordingly to obtain the DOA estimation of the broadband signal.

The frequency domain models of the broadband signal have been given in (8). Then the covariance matrix of the broadband signal in the frequency band of k can be calculated:

$$\mathbf{R}(k)=\mathrm{E}\left[\mathbf{X}(k)\mathbf{X}(k)^H\right] \quad (14)$$

The covariance matrix is estimated by the following formula when the number of snapshots is limited:

$$\hat{\mathbf{R}}(k)=\frac{1}{L}\sum_{l=1}^{L}\mathbf{X}_l(k)\mathbf{X}_l(k)^H \quad (15)$$

According to the covariance matrix estimation $\hat{\mathbf{R}}(k)$ in the kth frequency band and the orientation matrix $\mathbf{a}(k)$=[$e^{-2\pi f_s k/N\tau_1}$, $e^{-2\pi f_s k/N\tau_2}$,…, $e^{-2\pi f_s k/N\tau_P}$]^T, the cost function is obtained:

$$\max_{(\theta,\varphi)} J(\theta,\varphi)=\sum_{k=1}^{N/2}\mathbf{a}(k,\theta,\varphi)^H\,\hat{\mathbf{R}}(k)\mathbf{a}(k,\theta,\varphi) \quad (16)$$

III. EXPERIMENTS AND RESULTS

In this section, we will compare performances of the three aforementioned algorithms from the perspectives of both computer simulation and experiments.

A. Simulation Expreiments

In this section, the performances of the proposed algorithms are tested through computer simulation in which the array is consisted of four microphones with relative position as shown in Fig. 1. The wideband sources are Gaussian processes with zero means. The frequency range of the signal is 0-5000Hz corresponding to the real thunder signal. The noises are Gaussian signals temporally and spatially. In the following simulations, the Signal to Noise Ratio (SNR) varies from -20 to 5dB.

Suppose a wideband source impinges on the array with direction $\theta=30^\circ$, $\varphi=50^\circ$, where θ and φ represent the elevation and azimuth angles of the source.

The root mean square error (RMSE) of angle estimation is compared among the three methods. RMSE is defined as:

$$\theta_{RMSE}=\sqrt{\frac{1}{N}\mathrm{E}\left\{\left|\theta-\hat{\theta}\right|^2\right\}} \quad (17)$$

In the following simulation, N=100 Monte Carlo runs are performed to calculate the RMSE for each SNR.

Fig. 2 and Fig. 3 show the RMSE of θ and φ varying with SNR for the three methods.

As shown in Fig. 2 and Fig. 3, the errors of the two angles by the three methods are almost parallel in the regime that SNR≥-5dB. However, the error by the Generalized Cross-Correlation (GCC) ranges from 5° to 10° which is much larger than the errors by the other two methods when the SNR varies from -5 to -20dB. In practical, the SNR of the real signals is mostly less than -5dB. Therefore, the angle estimation results by the AML and Beamforming are more accurate and practical than the GCC.

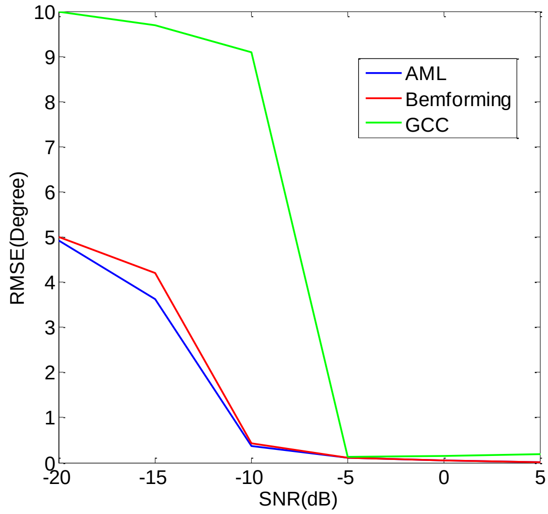


Fig. 2. REMS of θ by the three methods versus SNR

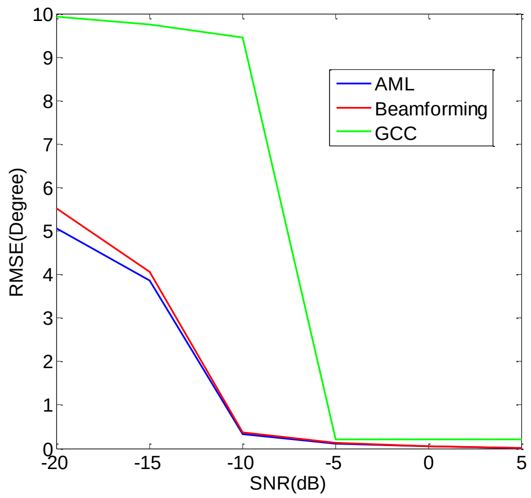


Fig. 3. REMS of φ by the three methods versus SNR

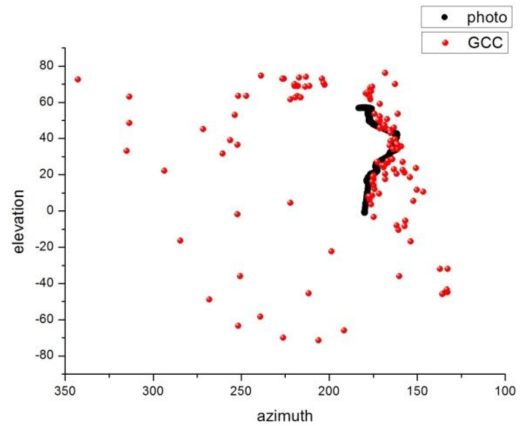
B. *Real Signal Expreiments*

We perform acoustic measurements using a microphone array consisted of four condenser microphones as shown in Fig. 4. Suppose that Mic2 locates at the origin of a specific spatial coordinates. Then, the locations of these four microphones are: $Mic_1(0,0,1m)$, $Mic_2(0,0,0)$, $Mic_3(1m,0,0)$ and $Mic_4(0,1m,0)$.

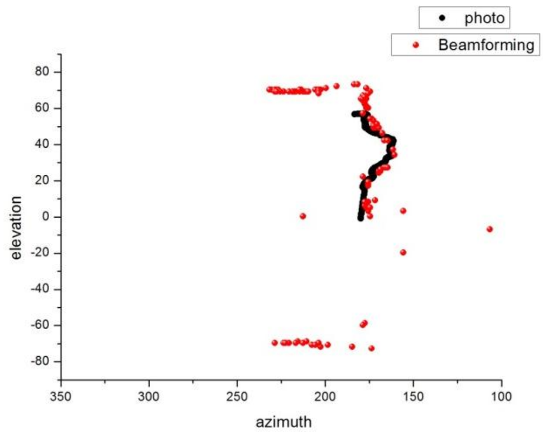
The microphone array was installed in Guangzhou to detect the rocket-triggered lightning since 2008. From 2011 to 2013, the setup was used to conduct research on the natural lightning in Wuhan. Abundant of lightning data has been collected since initial installation. Meanwhile, the high-speed camera was set up to capture photographs of lightning channels. The microphone array and high-speed camera are used to provide simultaneous observations of lightning behavior.

Fig. 4. The microphone array

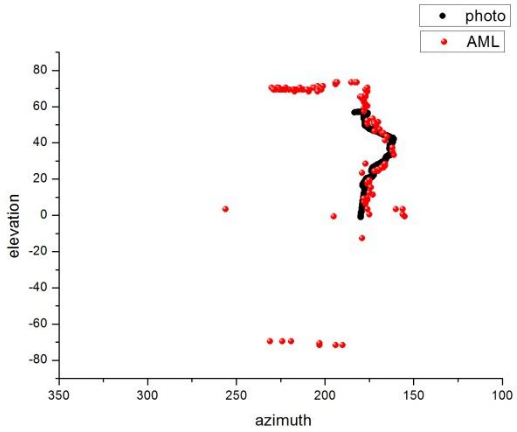
In this paper, we show one specific rocket-triggered lightning occurred on August 12, 2008, which included eight return strokes. The whole thunder signal lasted approximately two seconds.



(a) The direction finding of thunder sources using GCC



(b) The direction finding of thunder sources using Frequency Domain Beam-forming



(c) The direction finding of thunder sources using AML

Fig. 5. Channel geometry reconstruction comparison between acoustic direction finding method and photographic method

We apply the three methods to the thunder signals for estimating the angle of the lightning channel. The estimation results are shown in Fig. 5. In these figures, we use red dots to represent the direction finding of thunder sources by three algorithms, for comparison, we also plot the lightning channel geometry captured by camera with black dots. We can see that all these algorithms could obtain more location results than the optical channel. However, the results by GCC technique are discontinuous and contain more error points than the other two algorithms.

As shown in Fig. 5(b) and Fig. 5(c), most of the estimated thunder sources using Beam-forming and AML algorithms located along the optical channel. Moreover, there were several continuous locations at the elevation of 70 degrees, which were not observed in the photograph. These thunder sources formed a horizontal line which might be produced by the initial process in the cloud. Therefore, the initial process would be reconstructed by these two algorithms even though it cannot be captured by photographical methods due to possible cloud obstruction.

However, there are several error points at the elevation of -70 degrees, which are symmetrical with the aforementioned points at the elevation of 70 degrees. These could be caused by two reasons: 1. the acoustic signals are reflected by the ground, which results strong interference signals; 2. the frequency range chosen for calculation is from 50 Hz to 10 kHz because the aperture of the array is one meter. Therefore, when the frequency of the received signal is even higher, the periodicity of the phase appears the phenomenon of angle faintness resulting in error points at the angle symmetric to the correct one.

IV. CONCLUSIONS

This work provides a comparison among three techniques of acoustic source direction finding. We use computer simulation and experiments with real data to evaluate their performances. More specifically, we compare the error of angle estimation with compute simulation. We also compare the accuracy and the number of estimation sources with experiments.

In the simulation, the estimation results by the Beam-forming and AML based technique are more accurate than GCC especially when the SNR varies from -5 to -20dB. In the experiments with real data, it is shown that the Beam-forming and AML based technique can provide more accurate and complete direction finding results than GCC according to the comparison between the direction finding results and the photographic information of the same lightning. Both of the comparison results indicate that the Beam-forming and AML based technique outperform the GCC. However, the AML algorithm is computationally intractable and time-consuming than the other two methods. Therefore, based on our study, we conclude that the Beamforming based technique suits future practical applications due to its performance and low computation complexity.

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