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Analytical Approaches for Lightning-Induced Voltages Calculations: Solutions, Tools and Validations

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***Abstract*— In this lecture a survey of all analytical solutions available in the literature for the evaluation of the lightning-induced voltages in case of both step-function and linearly rising channel-base currents is presented. This survey presents also two analytical tools, namely CiLIV and LIV, which adopt some of the mentioned analytical solutions and can be interfaced with commercial software packages, allowing for the evaluation of lightning-induced voltages in case of power system configurations of any complexity. A number of validations of both analytical solutions and tools are finally shown.**

***Keywords*—Lightning; Lightning-induced voltages; Analytical solutions.**

I. INTRODUCTION

Lightning is an important cause of disturbances affecting the quality of power supply. In particular, effects produced by indirect lightning, such as permanent or transient faults, can be very severe. Because of the susceptibility of some loads, even in the case of transient faults, adequate knowledge of the lightning phenomenon and its effects is very important. The effort of the literature in the evaluation of the lightning-induced voltages is significant. Many numerical approaches have been proposed for the evaluation of the voltages induced on overhead lines (e.g., [1-3]). These numerical approaches can accurately model the phenomenon: they are able to model the actual return-stroke waveform, finitely conducting ground effects; they can take into account nonlinearities such as those produced by surge arresters. Nevertheless, analytical solutions (e.g., [4]-[12]) are still useful in the design phase [13], as well as in parametric evaluation and sensitivity analysis (e.g., [14]); they have also been implemented in computer codes for evaluation of lightning-induced effects [15], [16]. Furthermore, analytical solutions provide considerable insight into the phenomenon, which is often obscured in numerical approaches, do not suffer from the lack of convergence or numerical instabilities [17]. However, unfortunately, due to the problem complexity, analytical solutions can be obtained only for a limited number of simple cases.

The most basic case is concerned with the evaluation of the analytical function expressing the waveform of the voltage induced on a lossless horizontal conductor, placed over an infinitely conducting ground, and excited by an external field due to a step-function current moving unattenuated and undistorted at constant speed along a vertical lightning channel (as per the TL model [18]).

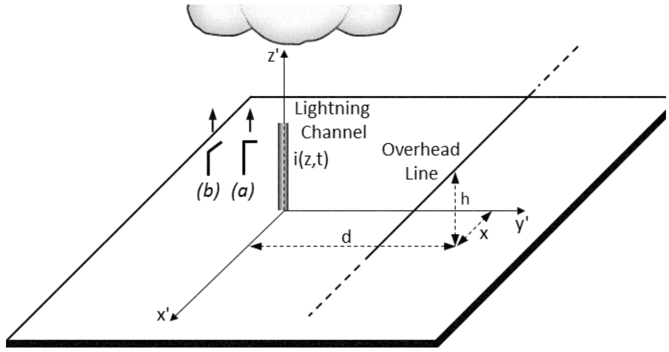


Fig. 1. Lightning channel traversed by a return-stroke current wave near an infinitely long overhead line. (a) Step-function current (b) Linearly-rising current.

This configuration is depicted in Fig.1(a). Approximate analytical solutions for this problem have been derived, in chronological order, by Rusck [4]; Chowdhuri and Gross [5]; Liew and Mar [6]; Haldar and Liew [7] and Høidalen [8]. The exact solution has been obtained by Andreotti *et al.* in [9].

The case of a linearly rising current with constant or drooping tail is more realistic: the configuration is depicted in Fig. 1(b). It has been shown [8], [11] that a linearly rising current with a linearly drooping tail can yield induced voltages which are close to those computed using more sophisticated currents waveforms, such as the Heidler function [19]. Further, by approximating the current waveform by a piecewise linear function, very accurate results can be obtained [11]. Approximate analytical solutions for the linearly rising current case have been derived, in chronological order, by Rusck [4], Chowdhuri and Gross [5], Liew and Mar [6], Haldar and Liew [7] and Sekioka [20]; while the exact solution by Andreotti *et al.* [11].

Some of the mentioned analytical solutions are implemented inside codes which can be interfaced with commercial software packages for the simulation of transients in power networks of virtually any complexity. The most popular codes are the Circuit for Lightning Induced Voltages

(CiLIV) which uses the analytical solutions proposed by Andreotti *et al*. [9], [11] and can be interfaced to Matlab SimPowerSystems and the Lightning Induced Voltage (LIV) [8], which uses the analytical solutions proposed by Høidalen [8], and can be interfaced into ATP-EMTP.

In this paper, all analytical solutions and the mentioned analytical tools will be overviewed, and validation of both solutions and codes will be presented. In particular, in Section II a survey of all analytical solutions for both step-function and linearly rising channel-base currents is presented. In Section III, the analytical tools, CiLIV and LIV are reviewed. Section IV is devoted to some validations of both analytical solutions and tools. In Section V conclusions are drawn.

II. Analytical Solutions

In this section we present all the analytical solutions available in the literature for the step-function current case. Note that, in this survey, we will focus on the solutions which calculate the overall waveforms of the induced voltages. For the solutions which calculated only peak values the reader is asked to refer directly to the relevant bibliography (e.g., Paulino et al. [20]).

A. Step Function Current Case

1) Rusck solution

The analytical solution proposed by Rusck [4] has been developed in the time domain, starting by a coupling model developed by Rusck himself, and known as the “Rusck model” [21]. The validity of the Rusck model has been investigated by Cooray in [21], who compared it with the Agrawal *et al*. coupling model [22], and found that in the Rusck model one of the forcing terms is missing. The Rusck model takes into account the portion of the horizontal electric field generated by the gradient of the scalar potential, but neglects the portion due to the vector potential: this lack makes the model source-dependent: it can give accurate results only when the spatial location of the source that generates the electromagnetic field is such that the contribution of the vector potential to the horizontal field is either zero or can be neglected. When the lightning channel is vertical, this contribution is zero and the solution can be considered correct. The Rusck solution can be found in [4, eqs. (104), (105)].

2) Chowdhuri-Gross solution

The solution proposed by Chowdhuri and Gross [5], which is an approximate solution [9], has been developed in the Laplace domain starting by a coupling model developed by Chowdhuri and Gross themselves known as “Chowdhuri-Gross model” [21]. The validity of the Chowdhuri-Gross model has been investigated by Nucci *et al*. [23], who compared it with the Agrawal *et al*. coupling model [23]. They demonstrated that in the Chowdhuri-Gross model one of the source terms is missing in the description of the electromagnetic coupling between the incident field and the line; furthermore, they showed that this omission could lead to significant errors under certain circumstances. Hence, the Chowdhuri-Gross solution cannot be considered correct. It can be found in [5, eqs. (20)-(25)].

3) Liew-Mar solution

Liew and Mar [6] use the Rusck model in order to obtain their solution. However, the Rusck and Liew-Mar solutions, even if derived using the same coupling model and the same approximations, are completely different in analytical terms as shown by Andreotti *et al*. [9], who also demonstrated that the Liew-Mar solution is incorrect. The solution can be found in [6, eq. (27)].

4) Høidalen solution

The analytical solution proposed by Høidalen [3] has been obtained using the Agrawal *et al*. coupling model [22], and is identical to the Rusck solution (even though a different coupling model was used). It is a correct solution [9], and can be found in [8, eq. (13)].

5) Andreotti et al. solution

The Andreotti *et al*. solution is an exact solution, since it has been derived without approximation [9]; it has been obtained using the Taylor *et al*. coupling model [24]. It was found [9] that the Rusck solution is the first order approximation of the Andreotti *et al*. exact solution. The Andreotti *et al*. solution can be found in [9, eqs. (51)-(55)] and in slightly different form in [12, eqs. (17)-(19)].

It is important to note that all the presented solutions have been obtained by using different models, both for the lightning electromagnetic field evaluation and for the coupling of this field to the line. However, as pointed out in [9], the solutions are *model independent*; hence, they should be identical if derived in exact way and should be also identical if the same approximations are adopted. For example, the solution obtained by Rusck [4] and that obtained by Høidalen [8], based on the same approximations, although using different models for computing lightning electromagnetic fields (monopole [4], [25] and dipole [25], [26] techniques, respectively), and different coupling models (Rusck [4] and Agrawal [22], respectively), are exactly the same. For this reason, the common name for those solutions, Rusck-Høidalen formula, has been used [11]. Table I summarizes the above presented solutions and their features.

TABLE I
SOLUTIONS FOR STEP-FUNCTION CURRENT

Solution (reference)	Electromagnetic field model	Coupling model	Type of solution	Notes
1) Andreotti et al. [9]	Monopole technique [4],[25]	Taylor *et al.* [24]	Exact	
2) Rusck [4]	Monopole technique [4],[25]	Rusck [4]	Approximate (*)	
3) Høidalen [8]	Dipole Technique [25],[26]	Agrawal *et al.* [23]	Approximate (*)	
4) Chowdhury and Gross [5]	Monopole technique [4],[25]	Chowdhury-Gross [5]	Approximate (via *s*-domain)	Incorrect coupling model/ Incorrect procedure
5) Liew and Mar [6]	Monopole technique [4],[25]	Chowdhury-Gross/ Jakubowski correction [6]	Approximate (via *s*-domain)	Incorrect procedure
6) Haldar and Liew [7]	Monopole technique [4],[25]	Chowdhury-Gross/ Jakubowski correction [6]	Approximate (via *s*-domain)	Incorrect procedure

(*) First term of Taylor’s expansion of the Andreotti *et al.*'s exact solution about $h = 0$.

B. Linearly Rising Current Case

1) Rusck solution

In order to obtain the solution in the linearly rising case, Rusck makes a convolution between the solution obtained for the step-function channel-base current [4, eqs. (104) and (105)] and a ramp function. From a system theory perspective, this represents the convolution between the step response of the system and forcing input, which in this case is a ramp. The final solution is presented in [4, eq. (132)].

2) Chowdhuri-Gross solution

Similarly, to the Rusck approach mentioned in the previous point, in order to get the solution for the linearly rising case, Chowdhuri and Gross makes a convolution between the solution obtained for the step-function channel-base current [5, eqs. (20)-(25)] and a ramp function. The final solution can be found in [5, eqs.(32), (33)].

3) Liew-Mar solution

Identical approach to previous ones, namely convolution between solution for the step-function channel-base current and ramp function is carried out. Final solution can be found in [6, eq. (28)].

4) Haldar-Liew solution

Once again, convolution between solution for the step-function channel-base current and ramp function is carried out. Final solution can be found in [7, eq. (18)].

5) Sekioka solution

Sekioka, starting from the Rusck coupling model, obtains the analytical solution in the case of a linearly rising current. The solution can be found in [10, eq. (12)]. We note that Sekioka's solution is the first order approximation of the Andreotti *et al*. exact solution for the linearly rising current case.

6) Andreotti et al. solution

The Andreotti et al exact solution has been obtained by convolution between the solution obtained for the step-function channel-base current [9, eqs. (51)-(55)] and a ramp function. It should be noted that an exact solution for the linearly rising case was found just for the point of the line closest to the lightning channel (x=0); the solution is given in [11, eq. (31)]. We also note that, in the same paper, an approximated solution is presented too.

Table II summarizes the above presented solutions and their features.

TABLE II
SOLUTIONS FOR LINEARLY RISING CURRENT

Solution (reference)	Electromagnetic field model	Coupling model	Type of solution	Notes
1) Andreotti *et al.* [11]	Dipole technique [17],[18]	Taylor *et al.* [24]	Exact	Available only for x=0
2) Andreotti *et al.* [11]	Dipole technique [17],[18]	Taylor *et al.* [24]	Approximate	
3) Rusck [4]	Monopole technique [4],[18]	Rusck [4]	Approximate	
4) Sekioka [10]	Monopole technique [4],[18]	Rusck [4]	Approximate (*)	
5) Chowdhury and Gross [5]	Monole technique [4],[18]	Chowdhury-Gross [5]	Approximate (via *s*-domain)	Incorrect coupling model/ Incorrect procedure
6) Liew and Mar [6]	Monopole technique [4],[18]	Chowdhury-Gross/ Jakubowsky correction [6]	Approximate (via *s*-domain)	Incorrect procedure
7) Haldar and Liew [7]	Monopole technique [4],[18]	Chowdhury-Gross/ Jakubowsky correction [6]	Approximate (via *s*-domain)	Incorrect procedure

(*) First term of Taylor's expansion of the Andreotti *et al.*'s exact solution about $h = 0$.

III. ANALYTICAL TOOLS

A. Circuit for Lightning Induced Voltages (CiLIV) [16]

The CiLIV tool, which is based on the Andreotti *et al*. analytical solutions [9], [11], [12] is a mixed frequency and time domain approach [16]. The idea underlying CiLIV is to get a time domain equivalent circuit, starting from a frequency domain model. This choice is apparent if we consider that approaches based exclusively on a solution in the frequency domain allows one to treat easily conductors and ground losses, but are limited to linear devices, whereas approaches in the time domain can treat non-linear devices such as surge arresters, but involve complex convolution integrations (which can potentially introduce significant errors, e.g., [17]) to account for conductors and ground losses.

This hybrid approach is particularly suitable for the evaluation of the lightning-induced voltages on power lines, where both conductors and ground losses, and the effects produced by non-linear devices, such as surge arresters, have to be taken into account in the modeling.

An important feature of CiLIV is that the solution representing the circuit is divided into two parts. The principal part, which accounts for the signal propagation along the line, and contains irregular terms such as Dirac pulses, is solved entirely analytically [16]; the regular remainder, which accounts for the signal damping and distortion due to losses, described by regular functions [16], is solved either analytically or numerically depending on to the complexity of the model considered [16], [27]. For example, lines that can be treated as lossless, such as lines of lengths less than 2 km [28], have only the principal part, which can be solved entirely analytically. Lossy lines which can be treated as frequency independent have also regular remainders, which can be solved analytically in some cases [27]. The regular remainder of lines with frequency-dependent parameters are generally solved numerically [27].

Some details on the theory underlying the CiLIV tool is briefly reviewed in the following. For further information and details, the reader can refer to [16], [27].

In the general case of a lossy multiconductor line with frequency-dependent parameters, and in presence of lossy ground (both for the lightning electromagnetic field propagation and surge propagation along the line), it is convenient to formulate the generalized telegraphers equations in the frequency (Laplace) domain, according to the Agrawal coupling model [22]:

$$\begin{cases} \dfrac{d\mathbf{V}^s(x;s)}{dx} + \mathbf{Z}(s)\mathbf{I}(x;s) = \mathbf{F}(x;s) \\ \dfrac{d\mathbf{I}(x;s)}{dx} + \mathbf{Y}(s)\mathbf{V}^s(x;s) = \mathbf{0} \end{cases} \tag{1}$$

where the upper case font represent the Laplace transforms and bold represent matrices and vectors. **Z**(s) is the per-unit-length longitudinal impedance matrix and **Y**(s) is the per-unit-length transverse admittance matrix. Since (1) is written in terms of *scattered* voltage $\mathbf{V}^s$, in order to obtain the total voltage $\mathbf{V}$ one must add the *incident* voltage vector $\mathbf{V}^i$:

$$\mathbf{V}(x;s) = \mathbf{V}^s(x;s) + \mathbf{V}^i(x;s) \tag{2}$$

$\mathbf{V}^i$ is due to the vertical component of the lightning electric field, and its value V_i^i for the i^{th} conductor is given by

$$V_i^i(x;s) = -\int_0^{h_i} E_z^i(x,d_i,z;s)\,dz \tag{3}$$

where E_z^i is the vertical component of the incident electric field in the Laplace domain. We also note that the first equation in (1) contains forcing function $\mathbf{F}(x,s)$, which represents the projection along the conductors of the horizontal component of the lightning electric field; for the i^{th} conductor it is given by

$$F_i(x;s) = E_x^i(x,d_i,h_i;s) \tag{4}$$

where E_x^i is the projection along the conductor axis of the horizontal component of the incident electric field.

The longitudinal per-unit-length matrix $\mathbf{Z}$, appearing in (1), can be written as

$$\mathbf{Z}(s) = s\mathbf{L} + \mathbf{Z}_w + \mathbf{Z}_g \tag{5}$$

where $\mathbf{L}$ represents the usual external inductance matrix [11]; the rigorous expression for $\mathbf{Z}_w$, in case of cylindrical wire, is given by Schelkunoff [29]. In CiLIV, the approximation proposed by Nahman and Holt [30] has been used:

$$\mathbf{Z}_w(s) = \mathbf{R} + \mathbf{K}\sqrt{s} \tag{6}$$

where $\mathbf{K}$ is a matrix which depends on the wires conductivity and the line geometry [30]. The general expression for the ground impedance matrix $\mathbf{Z}_g$, under the transmission line approximation, has been derived by Sunde [31]. It is known [32] that for typical overhead conductors (few meters above ground, $\sigma_g = 10^{-3} \div 10^{-2}$ S/m, $\varepsilon_r = 1 \div 10$), the ground admittance matrix $\mathbf{Y}_g$ could be considered as infinite so that

$$\mathbf{Y}(s) = \mathbf{G} + s\mathbf{C} \tag{7}$$

This is the expression used in CiLIV, although the code is not limited to this specific choice. The above equations show how a lossy line in the presence of a lossy ground can be easily described in the frequency (Laplace) domain. When loads are linear and time-invariant, the solution can be obtained directly in this domain. Unfortunately, many components in power networks are non-linear. For example, surge arresters are highly non-linear, and the problem has to be necessarily solved in the time domain. As shown in [16], it is possible, starting from a frequency-domain description, to obtain the time-domain *2n*-port representation of a multiconductor excited line, that in the lossless case reads [16]:

$$\begin{cases} \mathbf{i}_0(t) - \mathbf{Y}_0(\mathbf{v}_0(t) - \mathbf{v}_0^i(t)) = \mathbf{j}_0(t) + \mathbf{j}_0^*(t) \\ \mathbf{i}_L(t) - \mathbf{Y}_0(\mathbf{v}_L(t) - \mathbf{v}_L^i(t)) = \mathbf{j}_L(t) + \mathbf{j}_L^*(t) \end{cases} \tag{8}$$

$$\begin{cases} \mathbf{j}_0(t) = -2\mathbf{i}_L(t-T) + \mathbf{j}_L(t-T) + \mathbf{j}_L^*(t-T) \\ \mathbf{j}_L(t) = -2\mathbf{i}_0(t-T) + \mathbf{j}_0(t-T) + \mathbf{j}_0^*(t-T) \end{cases} \tag{9}$$

The corresponding circuit in the time domain is shown in Fig. 2.

The current sources due to the line axial components of the lightning horizontal electric field are given by

$$\begin{cases} \mathbf{j}_0^*(t) = \mathbf{Y}_0 \displaystyle\int_0^L \mathbf{f}(\eta, t-\eta/c)\,d\eta = \\ \qquad \mathbf{Y}_0 \begin{bmatrix} \vdots \\ \displaystyle\int_0^L e_x^i(\eta,d_i,h_i,t-\eta/c)\,d\eta \\ \vdots \end{bmatrix} \\ \mathbf{j}_L^*(t) = -\mathbf{Y}_0 \displaystyle\int_0^L \mathbf{f}(\eta, t-T+\eta/c)\,d\eta = \\ \qquad -\mathbf{Y}_0 \begin{bmatrix} \vdots \\ \displaystyle\int_0^L e_x^i(\eta,d_i,h_i,t-T+\eta/c)\,d\eta \\ \vdots \end{bmatrix} \end{cases} \tag{10}$$

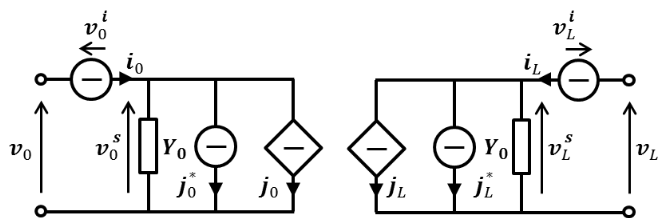


Fig. 2. 2n-ports equivalent representation of an excited lossless multiconductor line (time domain).

where e_x^i is the projection along the conductor axis of the horizontal component of the incident electric field in the time domain. The voltage generators $\mathbf{v}_0^i(t)$ and $\mathbf{v}_L^i(t)$ due to the lightning vertical electric field are given by

$$\begin{cases} \mathbf{v}_0^i(t) = \begin{bmatrix} \vdots \\ -\int_0^{h_i} e_z^i(0, d_i, z, t)dz \\ \vdots \end{bmatrix} \\ \mathbf{v}_L^i(t) = \begin{bmatrix} \vdots \\ -\int_0^{h_i} e_z^i(L, d_i, z, t)dz \\ \vdots \end{bmatrix} \end{cases} \quad (11)$$

where e_z^i is the vertical component of the incident electric field in the time domain. In the case of lossy lines the model is slightly different and the reader can refer to [16] for details.

B. *Lightning Induced Voltages (LIV) [8]*

The LIV code has been obtained using the Agrawal *et al*. coupling model [22]. A fundamental assumption in the LIV code is that the overhead line is lossless (although finitely conducting ground is considered in the propagation of the lightning electromagnetic field) . This is a reasonable approximation for short lines (less than 2 km) [28]. In the LIV code longer lines can be modeled with line segments interconnected by concentrated resistances. The lossless line assumption is important in the LIV code since it allows analytical handling of the wave propagation and reflections. Indeed, the overhead line can be modeled electrically by the classical Bergeron model as shown in Fig. 3, where the voltage sources are given by

$$U_{rA}(t) = U_{ind}^{\sigma}\left(x_A, t\right) + U_B(t-\tau) + Z' \cdot i_B(t-\tau) \quad (12)$$

$$U_{rB}(t) = U_{ind}^{\sigma}\left(-x_B, t\right) + U_A(t-\tau) + Z' \cdot i_A(t-\tau) \quad (13)$$

where symbols meaning can be found in [8].

C. *Comparison between CiLIV and LIV codes*

Table III show a summary, in comparative terms, of the salient features of both CiLIV and LIV code.

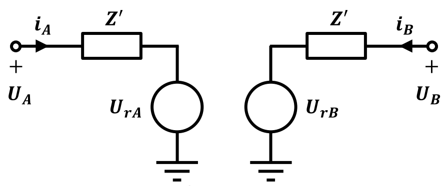


Fig. 3. Equivalent electric model of an overhead line in the LIV code.

TABLE III
SUMMARY OF THE SALIENT FEATURES OF CILIV, LIV TOOLS

	CiLIV (analytical)	LIV (analytical)
Channel-base current representation	Step function. Linearly-rising function. Any other function by piecewise linear approximation	Step function. Any other function by convolution integral
Return-Stroke Model and leader induction effects	Transmission Line (TL) model for return stroke	Transmission Line (TL) model for return stroke
Vertical Electric Field Computation	Exact analytical solution assuming a perfectly conducting ground	Approximate analytical solution assuming a perfectly conducting ground
Horizontal Electric Field Computation	Exact analytical solution for a perfectly conducting ground. Cooray-Rubinstein formula in case of lossy ground split into two terms, with one solved exactly and the other solved by numerical algorithm (trapezoidal rule)	Analytical solution for a perfectly conducting ground. Cooray-Rubinstein formula in case of lossy ground solved by approximate functions used in numerical algorithm (trapezoidal rule)
Field-to-Transmission Line Coupling Model	Agrawal *et al.* model [22]	Agrawal *et al.* model [22]
Ground Losses in the surge propagation	Rachidi *et al.* [33]	Neglected
Implementation	MATLAB SimPowerSystems environment	ATP-MODELS environment

IV. VALIDATION OF THE ANALYTICAL SOLUTIONS AND TOOLS

A. *Barker et al.'s experiment [34]*

Barker *et al.* [34] measured induced voltages produced by rocket-triggered lightning using configuration shown in Fig. 4.

In particular, the lightning channel was at a distance of about 145 m from the two wire test distribution line and the line was about 740 m. The two conductors were vertically placed above ground, with the upper conductor simulating the phase conductor placed at a height of 7.5 m, and the lower conductor simulating the neutral at a height of 5.7 m. Both conductors had a radius of about 0.5 cm. The line was supported by 15 wooden poles spaced about 49 m apart: the lower conductor was grounded at both ends (pole 1 and 15) and at pole 9 which was in close proximity of the center of the line. The values of dc resistances of these three groundings were estimated to be between 30 and 75 Ω [34]. The two conductors were connected at each end to a 455 Ω resistor. We analyze event 93–05 [34]. The measured channel base current for this event, shown in Fig. 5, has been approximated by a linearly rising current with a drooping tail: the front time is 0.7 μs,

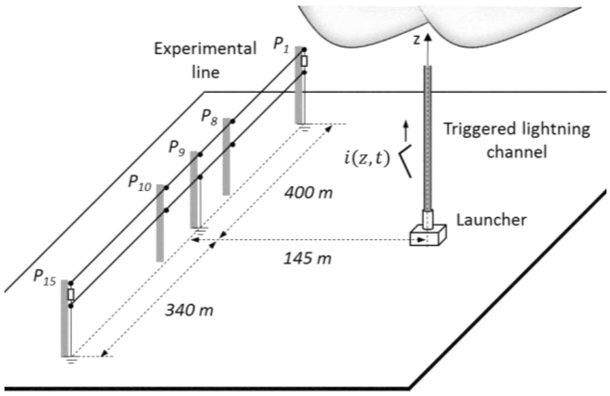


Fig. 4. Configuration of the Barker *et al.* experiment (not to scale).

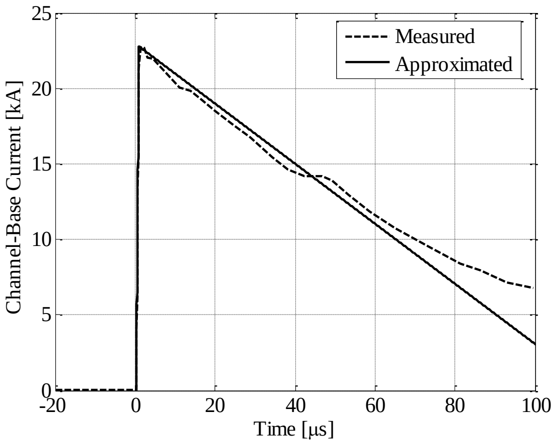


Fig. 5. Approximation of channel-base current measured by Barker *et al.* for event 93-05 [34] by a linearly-rising current with drooping tail.

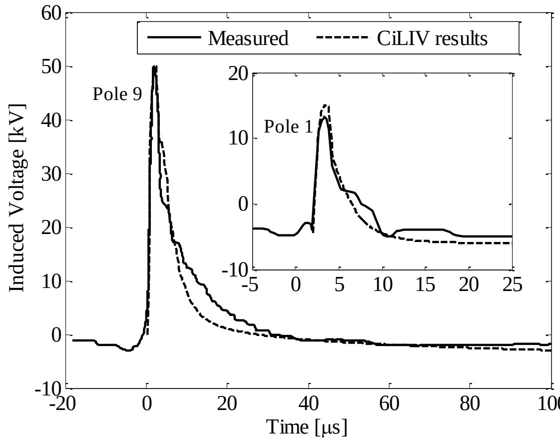


Fig. 6. Experiment 93-05 of [34]: induced voltage at poles 9 and 1. Comparison between measured voltage and CiLIV.

the tail time is 58 μs, and the peak value is 22.8 kA.

In Fig. 6, the induced voltage measured at poles 9 and 1 is compared to that evaluated by using CiLIV. The comparison has been made assuming that β = 0.45, σ_g = 0.003 S/m, ε_{rg} = 10, the terminal resistors equal to 455 Ω, and the grounding resistance at poles 1, 9, and 15 equal to 75 Ω. These values are consistent with those assumed by other authors (e.g., [35]). A good agreement is observed for Pole 9, both for the peak voltage and the overall waveshape. As for Pole 1, a slight discrepancy for the peak voltage is seen, however, the overall waveform is reasonably reproduced. This comparison with the experiment show the accuracy of CiLIV.

B. Yokoyama experiment [36]

Fig. 7 shows the small-scale model of a distribution line and the lightning channel of the experiment presented in [36]. The scale was 1:200; the length of the line was 5 m to simulate a 1-km distribution line. The configuration with no ground wire is first analyzed. In Fig. 8, we show the phase-to-ground induced voltage at the line center (x=0), including the experimental data, the Andreotti *et al*. exact solution for the linearly rising current [11], the results of numerical simulation presented by Yokoyama, and those obtained using CiLIV. One can observe a good agreement between the predictions of all the tools considered and the measurement. In Fig. 9, we analyze the same configuration, but this time with ground wire installed. Same comparison is made as before (apart from the exact solution, which is not available for this configuration). Again, a good agreement is found.

C. A comparison between CiLIV and a numerical code

Real-world configurations of distribution lines are very complex. For example, wide use of surge arresters is common in order to mitigate the induced overvoltages. As already mentioned surge arresters are complex to treat, since they introduce non-linearities in the models. In the following simulation, we will use the results presented in [37] and [38], in which a numerical code, namely the LIOV code, has been tested for the configuration depicted in Fig. 10. Note that this single-conductor configuration is also representative of a

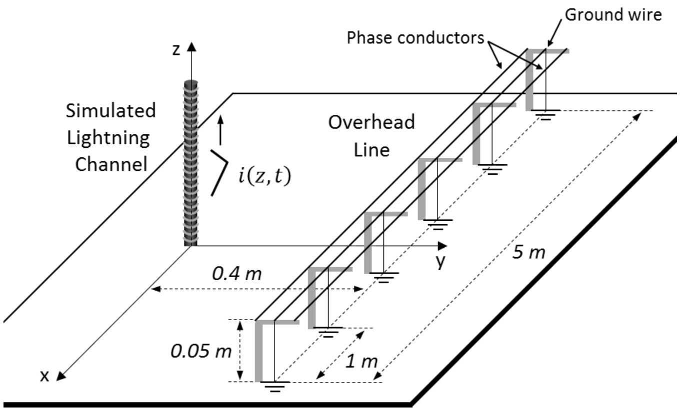


Fig. 7. Yokoyama small-scale experiment (not to scale). The line was matched at both ends.

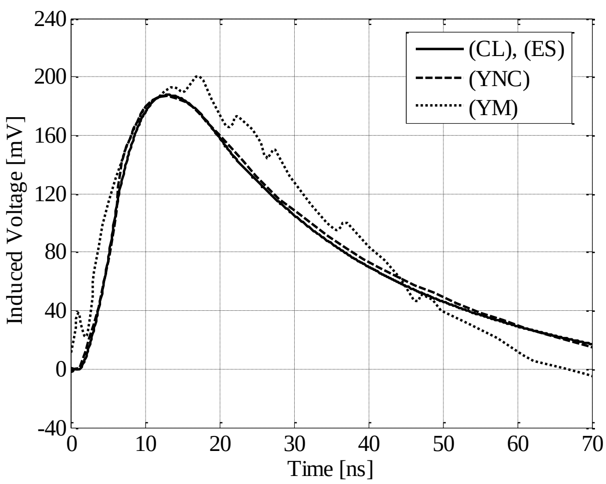


Fig. 8. Comparison of predictions of CiLIV (CL), Andreotti *et al.'s* exact solution (ES), Yokoyama numerical model (YNM), and Yokoyama measurement (YM). The induced phase-to-ground voltage is evaluated at x = 0 for the experiment with no ground wire. Parameters used are the same as in [36].

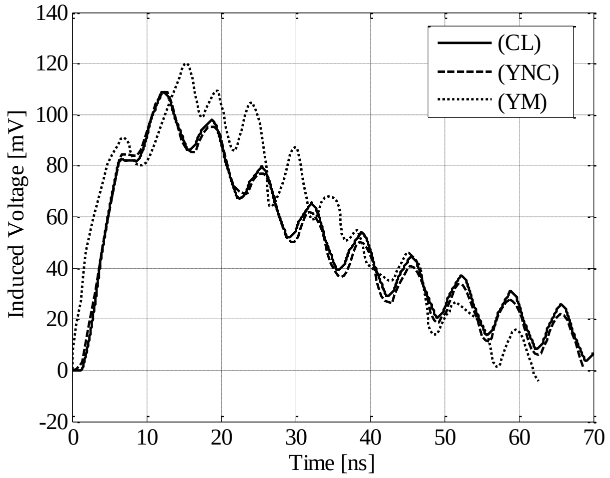


Fig. 9. Comparison of predictions of CiLIV (CL), Yokoyama numerical model (YNM), and Yokoyama measurement (YM). The induced phase-to-ground voltage is evaluated at x = 0 (ground wire is installed).

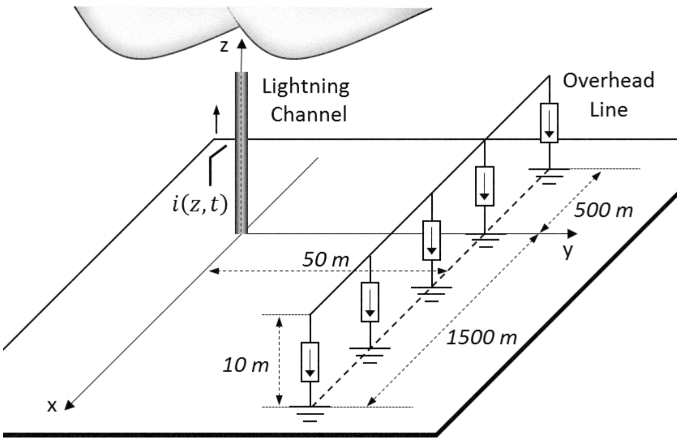


Fig. 10. Single-conductor configuration with surge arresters (not to scale).

three-phase line, since the external lightning excitation would be practically the same on all three conductors (because the differences in distances between the conductors and lightning channel are small) and hence would produce essentially identical effects.

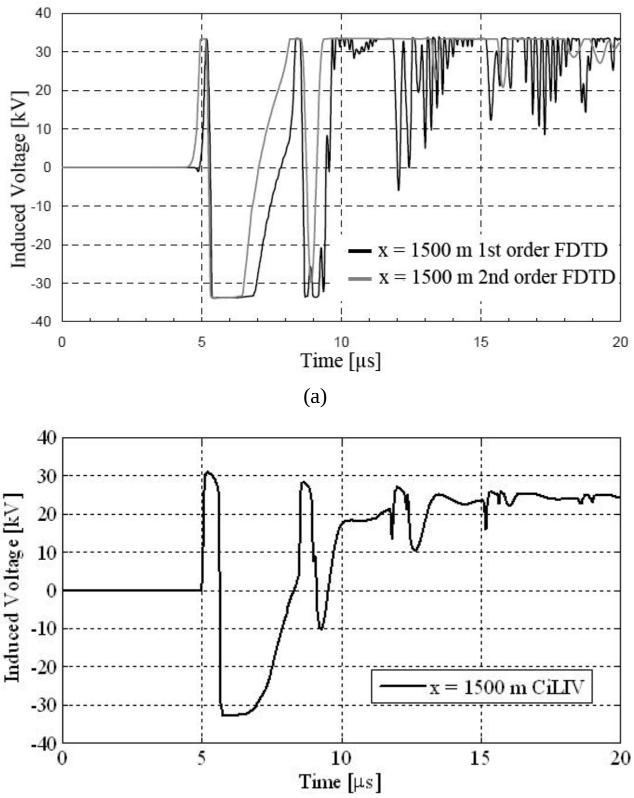
The line is 2-km-long, 10-m high, single conductor of radius 0.5 cm above perfectly conducting ground, equipped with 5 surge arresters equally spaced. The lightning channel is in an "off-the-center position" (500 m from the line center), and at a distance of 50 m from the line. All parameters used are the same as those in [37] an [38]; in particular, characteristic data of surge arresters can be found in [38, Table 3.1].

In Fig. 11(a), we show the induced voltage produced at the right termination of the line (viewing from the lightning channel) by using two versions of the LIOV code, one implementing a 1st order FDTD, and the other a 2nd order FDTD. There is an appreciable difference between the two approaches, visualized by the black and grey curve, respectively, especially after 10 μs. In particular, the 1st order LIOV appears unstable with its numerous spikes, compared to the 2nd order LIOV, which appears more regular. Indeed, it was recognized [37], [38] that *the 2nd order scheme improved the computed results in terms of numerical stability.*

In Fig. 11(b), for the same conditions and parameters, we show the results obtained using CiLIV. There is a good coincidence between CiLIV and both 1st and 2nd order LIOV at the beginning of the induced voltage process, but then the solutions tend to diverge. In particular, by comparison to the CiLIV, the numerical instability of the 1st order scheme is confirmed. Better stability of an analytical code such as CiLIV compared to a numerical code such as LIOV can be seen by comparing the oscillations observed. In the case of CiLIV the effects produced by the surge arresters are apparent. Surge arresters produce oscillations at regular intervals, as they should do. This effect is seen in the case of the numerical code just for the for the first two oscillations, than it seems that numerical problems of convergence take over.

V. CONCLUSIONS

This lecture has presented on overview of all analytical solutions available in the literature for the evaluation of the lightning-induced voltages in case of both step-function and linearly rising channel-base currents. Two analytical tools based on these analytical solutions, namely CiLIV and LIV has been analyzed. A number of validations of both analytical solutions and tools have been carried out.



(b)

Fig. 11. Comparison of (a) LIOV (adapted from [37]), and (b) CiLIV. The induced voltage is evaluated at right termination (x = 1500 m) of the line of Fig. 10 (h = 10m, d = 50m, I_0 = 60 kA, t_f = 0.5 μs, $t_t = \infty$, β = 0.4, r_w = 0.5 cm).

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